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A proposal to probe quantum non-locality of Majorana fermions in tunneling experiments

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Topological Majorana fermion (MF) quasiparticles have been recently suggested to exist in semiconductor quantum wires with proximity induced superconductivity and a Zeeman field. Although the experimentally observed zero bias tunneling peak and a fractional ac-Josephson effect can be taken as necessary signatures of MFs, neither of them constitutes a sufficient "smoking gun" experiment. Since one pair of Majorana fermions share a single conventional fermionic degree of freedom, MFs are in a sense fractionalized excitations. Based on this fractionalization we propose a tunneling experiment that furnishes a nearly unique signature of end state MFs in semiconductor quantum wires. In particular, we show that a "teleportation"-like experiment is not enough to distinguish MFs from pairs of MFs, which are equivalent to conventional zero energy states, but our proposed tunneling experiment, in principle, can make this distinction.

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Introduction: Majorana fermions [1] (MF) are localized particle-like neutral zero energy states that occur at topological defects and boundaries in superconductors. A MF creation operator is a hermitian second quantized operator $\gamma^\dagger = \gamma$ which anti-commutes with other fermion operators. The hermiticity of MF operators implies that they can be construed as particles which are their own anti-particles [1–4]. The key issues at this time in the condensed matter context are two fold, first, we must predict and characterize materials supporting MFs and second, we must detect them experimentally. In this paper we address the second issue of experimental detection by proposing a nearly sufficent experimental signature for MFs.

MFs have recently been proposed to exist in the topologically superconducting (TS) phase of a spin-orbit (SO) coupled cold atomic gases [5], semiconductor 2D thin film [6, 7] or 1D nanowire [7–9] with proximity induced s-wave superconductivity and Zeeman splitting from a sufficiently large magnetic field. In principle, the MFs in such systems may be detected either by measuring the zero-bias conductance peak (ZBCP) from tunneling electrons into the end MFs [7, 10–13], by detecting the predicted fractional ac Josephson effect [8, 9, 14–20]. The semiconductor Majorana wire structure, which will be the system of our focus, is of particular present interest since there is experimental evidence for both the ZBCP [21–24] and the fractional ac Josephson effect in the form of doubled Shapiro steps [21].

Despite their conceptual simplicity, neither the ZBCP nor the fractional ac-Josephson effect experiments constitute a sufficient proof of MFs at the ends of topological superconducting wires. A non-quantized $(2e^2/h)$ zero bias peak, such as that observed in the recent experiments [22–24], can in principle arise even without end state MFs [25–27]. Similarly, a fractional ac-Josephson effect can exist even in Josephson junctions made of ordinary quasi-1D p-wave superconductors such as organic superconductors [15] or the non-topological phase of the semiconductor nanowire [28]. Given these caveats as well as the considerable complexity of existing experiments, there have been several alternative proposals to detect the presence of MFs [29–32]. Based on the inherent quantum non-locality of MFs, in this work we propose an alternative tunneling experiment on semiconductor Majorana wires that furnishes a nearly sufficient signature of end-state MFs. We discuss in detail why only topological systems would show such quantum non-locality, which would even be absent for systems with conventional Andreev states at each end.

Non-local electron transfer Non-locality arises in MFs because they differ from conventional complex (Dirac) fermions in that they have no occupation number associated with them. To define a quantum state of a system with MFs we must consider a pair of MFs. The pair of MFs γ_a and γ_b at the ends a and b of a nanowire (NW) shown in Fig. 1 can be combined into a zero-energy complex fermion operator $d^\dagger = \frac{1}{2}[\gamma_a + i\gamma_b]$ associated with the pair of MFs γ_a and γ_b [14]. The quantum state of the system is then determined by the eigenvalue of $n_d = d^\dagger d = 0, 1$. Since the fermion parity $F_P = (-1)^{n_d}$ associated with the operator $d^\dagger$ is related to the MFs by

$$F_P = (-1)^{n_d} = i\gamma_a\gamma_b, \tag{1}$$

we see that the fermion parity of the whole system is determined by non-local correlations between the fractionalized MFs γ_a and γ_b. In fact, the fractionalization of the F_P into a pair of spatially separated operators $\gamma_{a,b}$ in one-dimensional systems with localized fermion excitations, is a unique characterization of the topological

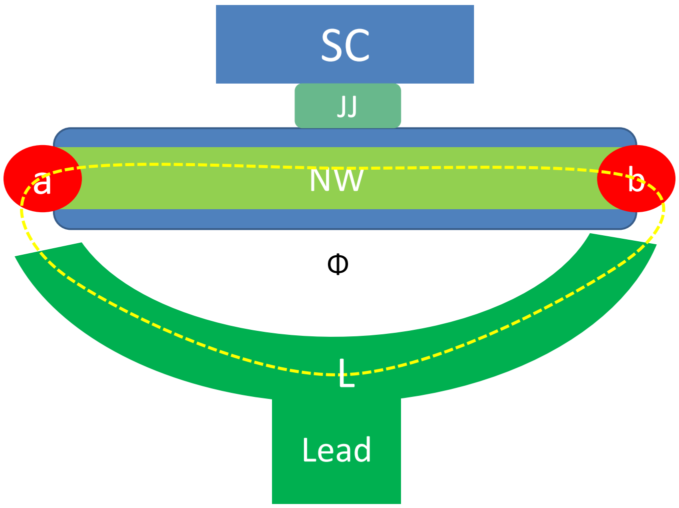

FIG. 1. (Color online) Schematic experiment to detect Majorana-assisted electron tunneling between the MFs a and b along the dashed lines. The electrons in the ring are transported via the Majorana-assisted tunneling process along the dashed line, which enclosed the flux Φ. The dark green metallic loop L is connected to a metallic lead, which together with the superconductor serves as the terminals for the conductance measurement. Similar to the Aharonov-Bohm oscillations in a mesoscopic ring, the interference can be detected as a $2\Phi_0 = hc/e$ periodicity of the conductance as a function of Φ. The nanowire NW is placed on a supercondcuting island (shown in light blue) which has a charging energy E_C.

state of the system [33]. Our central concern is how to probe this non-locality to provide a robust and sufficient criterion of MFs.

An immediate idea involves trying to inject an electron into γ_a and retrieve it from γ_b [34–37]. By connecting leads to the left and the right ends of the TS wire in Fig. 1, one could imagine that an electron injected into the end a flips the occupation number n_d from $n_d = 0$ to $n_d = 1$. The injected electron can then escape from the end b flipping the state back from $n_d = 1$ to $n_d = 0$. Such a process where an electron can enter from one end a and exit at the other lead b, can be interpreted as a transfer of an electron, which we will refer to as Majorana-assisted electron tunneling. However, as has been discussed in previous works [34, 35], such a transfer occurs in a way so as to not violate locality and causality.

The amplitude for the Majorana-assisted electron tunneling [35, 36] can be written in terms of the retarded Green function as

$$G^R_{mn}(\tau) = -i\langle[c^\dagger_m(\tau), c_n(0)]\rangle\Theta(\tau), \qquad (2)$$

where τ is the time-interval between the tunneling events, $\Theta(\tau)$ is the Heaviside step function and $c^\dagger_m$ are the electron operators at the left (i.e. $m = a$) and right (i.e. $m = b$) end of the wire. In the low-energy limit in the topological state, the electron-operators at the ends $c^\dagger_{a,b}$ can be approximated by the end Majorana modes $c^\dagger_{a,b} \sim \gamma_{a,b}$. Thus, the amplitude $G^R_{ab}(\tau)$ represents Majorana-assisted non-local normal electron tunneling between the ends a exits as a hole at the end b and has a non-zero value in the topological phase given by

$$G^R_{ab}(\tau) = -iF_P, \qquad (3)$$

where $F_P \equiv i\langle\gamma_a(0)\gamma_b(0)\rangle$ is the fermion parity of the TS system. Since $G^R_{ab}(\tau)$ is directly related to the fermion parity F_P, the detection of such a non-vanishing amplitude for a non-local Green function $G^R_{ab}(\tau)$ is a signature of the fractionalization associated with MFs.

Charging energy: In equilibrium, the degeneracy of the different fermion parity states characteristic of a TS system lead to fluctuations in F_P, that would result in a vanishing average for the tunneling amplitude $G^R(\tau; ab)$. This is remedied [37] by introducing a charging energy E_C on the superconducting island supporting the NW, which makes one of the fermion parities energetically favorable over the other. To compute G^R, we consider the Hamiltonian for the NW in Fig. 1 as

$$H = H_{BCS} + 4E_C(\hat{N} + \hat{n}_W/2 - n_g)^2 - E_J\cos(\phi), \quad (4)$$

where E_C is the charging energy of the wire, H_{BCS} is the BCS Hamiltonian for the proximitized NW, $\hat{n}_W$ is the number of electrons in the NW. Here $\hat{N}$ is the total number of Cooper pairs with the SC island, the NW and the gate and is a variable that is conjugate to the phase ϕ, n_g is the gate charge. To control the charging energy of the NW we couple it to a superconductor with Josephson strength E_J, which can in principle be controlled using a SQUID geometry[44].

While coupling to the superconducting lead in Fig. 1 breaks charge conservation, it preserves fermion parity F_P, which is related to the number of electrons modulo two [45]. In the limit that $E_J \gg E_C$, so that the only effect of charging energy is an energy splitting $\delta = 16\left(E_C E_J^3/2\pi^2\right)^{1/4} e^{-\sqrt{8E_J/E_C}}\cos 2\pi n_g$ between the different fermion parity states $F_P = \pm 1$. Thus the effective Hamiltonian is written as $H_{eff} = H_{BCS} + F_P\delta$. Since F_P and H_{BCS} commute, expanding H in terms of H_{BCS} the Green function $G^R(\tau)$ can be written as

$$G^R_{mn}(\tau) = -i\Theta(\tau)\frac{\langle[c^\dagger_m(\tau), c_n(0)]e^{-(\beta+i\tau)F_P\delta}\rangle_0}{\langle e^{-\beta F_P\delta}\rangle_0}, \qquad (5)$$

where $\langle\ldots\rangle_0$ is the thermal expectation with respect to H_{BCS}.

Coincidence probability: The amplitude $G^R_{ab}(\tau)$ can lead to a so-called coincidence probability $P_c(\tau)$, which maybe measured by using a joint measurement by two point contact detectors at the two ends [35, 41]. Alternatively, the non-local transfer of electron can also be measured by a non-local conductance or transconductance

$\frac{dI_b}{dV_a}$ between the ends a and b in Fig. 1. This measurement does not require closing the loop (L) in Fig. 1 and would require adding a lead to the end a. In such a set-up, a voltage V_a applied to the left-lead a (relative to the SC) results in a current I_b in the right lead b. Using results of Ref. 34 for symmetric $t_a = t_b = t$ we find that

$$\frac{dI_b}{dV_a} = \delta \frac{32V_a}{16\Gamma^2 + (\delta^2 - V_a^2)^2 + 8\Gamma^2(\delta^2 + V_a^2)}, \quad (6)$$

which clearly vanishes for $\delta \to 0$ (i.e. vanishing charging energy $E_C \to 0$). Here $\Gamma \propto t^2$ is the lead-induced broadening of the MFs.

Topological versus non-topological systems: However, a coincidence measurement does not directly imply a non-zero $G^R(\tau; ab)$ in more general situations. The amplitude $G^R_{ab}(\tau)$ in Eq. 2, reflects the amplitude for being able to transfer an electron from a to b while leaving the state $|g\rangle$ invariant. On the other hand, the measurement of the coincidence probability, P_c, does not keep track of the internal state of the system. For a general system (i.e. one that may be topological or non-topological), P_c for an electron entering at a and exiting at b can be written more generally as

$$P_c(\tau) = \sum_{g_1, g_2} |\langle g_2 | c_b(0) c_a^\dagger(\tau) | g_1 \rangle|^2, \quad (7)$$

where g_1, g_2 are the internal states of the wire, which are not necessarily identical. While TS systems with MFs have a non-degenerate ground state in a given fermion parity sector, more general systems with zero-energy Dirac end states may have multiple allowed values for g_1, g_2. Therefore, the coincidence probability P_c cannot be considered a unique signature for a topological system.

An important example of the inequivalence of $P_c(\tau)$ and $G^R(\tau; ab)$ is a non-topological superconductor with Andreev zero mode at each end. The quantum state is characterized by the occupancy n_a, n_b of the two conventional zero energy end modes. We can easily have $P_c \neq 0$ in this non-topological setup. Suppose the initial state is $g_1 \equiv (n_a = n_b = 0)$, then the sum for P_c in Eq. 7 would have a non-zero contribution from $g_2 \equiv (n_a = n_b = 1)$. The tunneling of an electron from the lead into the zero-mode at a changes the occupation from $n_a = 0$ to $n_a = 1$. On the other hand, the electron required to change the occupation of the state b from $n_b = 0$ to $n_b = 1$ comes from breaking of a Cooper pair. The other electron from the broken Cooper pair is emitted into the lead in the vicinity near b. Note that the process conserves the number of electrons within the system and cannot be eliminated even by the introduction of a finite charging energy [37]. Therefore in order to clearly distinguish this case from the process of Majorana-assisted electron tunneling (that also returns a non-zero P_c), we require $G^R(\tau; ab)$ given in Eq. 2 itself to be non-zero. In other words, we require that the system return to the same state g after the tunneling process, so the same electron that enters at a leaves at b. The Green function between the ends of a non-topological systems, $G^R_{ab}(\tau)$, vanishes. This is because introducing a superconducting phase-slip through a non-topological system which transforms $c_b \to -c_b$ and flips the sign of the Green function without affecting the Hamiltonian. In fact, in the Supplementary material [38] we explicitly show how this vanishes even in the case of decoupled pairs of MFs.

Proposed set-up: The Majorana-assisted electron transfer $G^R_{ab}(\tau)$ can be measured by the setup in Fig. 1 consisting of an external semiconducting loop (L) that is connected to the ends a and b of the NW. The Green function $G^R(\tau)$ can be determined by measuring the Andreev conductance from the semiconducting loop L into the superconductor shown in Fig. 1 in the tunneling regime (i.e. small tunneling) with tunneling amplitude $t_{a,b}$ between the ends of the NW and loop. The tunneling Hamiltonian between L and NW is written as

$$H_t = [t_a c_a^\dagger c_{L,a} + t_b c_b^\dagger c_{L,b}] + h.c., \quad (8)$$

where $c_{L,a}, c_{L,b}$ are fermion annihilation operators in the loop L near the ends a, b and the flux affects t_b as $t_b = t_{b,0} e^{i\varphi/2}$ where $\varphi = \frac{2\pi\Phi}{\Phi_0}$.

The zero-bias conductance can be calculated using the Meir-Wingreen formula and expanding to lowest non-vanishing order in the tunneling amplitude as

$$\sigma(\varphi) = \int d\epsilon \, \text{sech}^2 \frac{\epsilon}{2T} \sum_{m,n} Im[\Gamma_{m,n}(\epsilon) G^R_{n,m}(\epsilon)], \quad (9)$$

where $\Gamma_{m,n}(\epsilon) = t_m \rho_{mn}(\epsilon) t_n^*$ is the imaginary part of the lead-induced self-energy and the retarded Green function in the time-domain is written as $G^R_{n,m}(t) = \Theta(t)\langle\{c_m(t), c_n^\dagger(0)\}\rangle$. Here the indices m, n are summed over the ends a, b and $\rho = Im(G_L(0))$ is the density of states, which can be calculated as the the imaginary part of the retarded Green function in the loop.

Ignoring the energy dependence of the lead density of states $\rho_{mn}(\epsilon)$ and choosing (for simplicity) a symmetric lead and contacts with $|t_a| = |t_b|$, the imaginary part of the lead self-energy Γ can be written as $\Gamma_{aa} = \Gamma_{bb} = \Gamma_0$ and $\Gamma_{ab} = \lambda \Gamma_0 e^{i\varphi/2}$ for appropriately chosen constants Γ_0 and λ. Within this set of simplifying approximations, the conductance is found to be

$$\sigma(\varphi) = \int d\epsilon \, \text{sech}^2 \frac{\epsilon}{2T} \Gamma_0$$
$$Im\left[G^R_{a,a}(\epsilon) + G^R_{b,b}(\epsilon) + \lambda(G^R_{a,b}(\epsilon) e^{i\varphi/2} + e^{-i\varphi/2} G^R_{b,a}(\epsilon))\right]. \quad (10)$$

It is clear from the above formula that $\sigma(\varphi)$ shows 4π-periodic oscillations whenever the non-local tunneling amplitude, $G^R_{ab} \neq 0$ across the NW is finite. This direct measurement of the non-vanishing tunneling amplitude

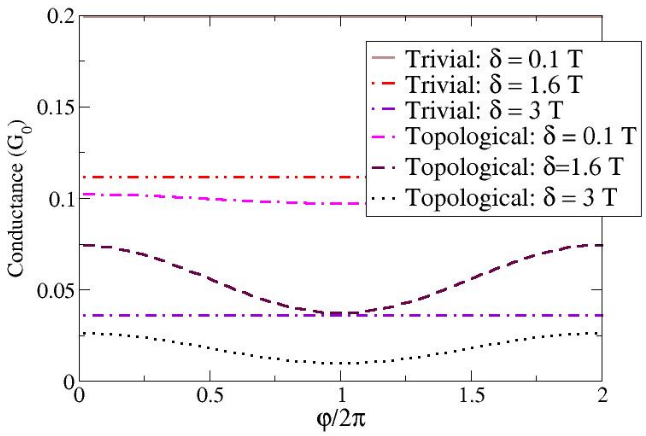

FIG. 2. Tunneling conductance $\sigma(\varphi)$ as a function of flux $\Phi = \varphi\Phi_0/2\pi$ in the loop in Fig. 1 in both the topological phase and the nontopological phase. Here $\Phi_0 = hc/2e$ is the flux quantum and $G_0 = e^2/h$ is the conductance quantum. The conductance shows 4π-periodic oscillation in φ in the topological case that results from non-local transfer of electrons between the ends. In contrast, the spectrum in the nontopological case shows no dependence on phase at the lowest order in tunneling.

G^R_{ab}, which is a measure of the Majorana-assisted non-local tunneling process, would be a direct measurement of the non-local character of Majorana modes.

Results: To illustrate the periodic oscillations of $\sigma(\varphi)$ generated by the presence of non-degenerate Majorana modes, we calculate the conductance of an InSb nanowire [8, 9] with effective mass $m^* = 0.015m_e$, Rashba spin-orbit coupling $\alpha = 0.5\,eV - \mathring{A}$, pair potential in the NW $\Delta = 3\,$K. For simplicity, the loop is taken to be a semiconductor with effective mass m^*, but without spin-orbit coupling or Zeeman so that the spin-dependence of Γ can be ignored. The chemical potential in the NW is taken to be $\mu_{NW} = 2$ K, while the loop is at a chemical potential $\mu_{loop} = 6$ K. Further details of the model are provided in the Supplementary material [39]. The magnitude of the tunneling matrix elements $|t_{a,b}|$ are chosen to produce an experimentally reasonable zero-bias conductance of order $\sigma_0 \sim 0.1G_0$ at a temperature $T = 50$mK where G_0 is the conductance quantum. The conductance $\sigma(\varphi)$ in the set-up shown in Fig. 1 is plotted as a function of φ in Fig. 2 for both the cases where NW is in the topological and non-topological phase. Details of the numerical evaluation of Eq. 10 are provided in the Supplementary material [40]. The result in Fig. 2, shows that the conductance $\sigma(\varphi)$ including the charging energy shows a 4π-periodic oscillation only in the topological case, as expected from non-local Majorana-induced quasiparticle transfer across the wire.

The set-up in Fig. 1 can be also used to separate out Majorana-assisted electron transfer from direct transfer by tunneling of quasiparticles through NW. In the topological case, the presence of a finite tunneling amplitude $G^{(R)}_{ab}$ depends on the charging energy parameter δ, which can be controlled by a SQUID configuration [44]. As seen in Fig. 2, the 4π-periodic oscillations that are characteristic of the TS phase are completely suppressed for small δ/T. In contrast, oscillations generated by direct tunneling of quasiparticles between the ends of the wire is not expected to be affected by δ.

Comparison with the fractional Josephson effect: The signature of a TS phase in Fig. 2 appears as a 4π-periodic oscillations in conductance σ. Formally, this resembles the 4π-periodic current-phase relationship predicted for the fractional Josephson effect in TS systems[11, 14]. However, the 4π-periodicity of the current in the Josephson junction in a TS system relies on fermion parity protection, which is typically accomplished by using a non-equilibrium AC Josephson measurement [11]. In principle, protecting the fermion parity by a charging energy would allow the observation of the fractional Josephson effect in equilibrium. Observation of the fractional Josephson effect protected by E_C y cannot occur in previously proposed linear Josephson junctions[11, 14, 17–20], which always have an additional pair of uncoupled MFs contributing to the fermion parity. The loop geometry in Fig. 1 would in principle allow the 4π- periodic current phase relationship to be measured. However, such a current phase relation, would be relatively difficult to measure since the Josephson current would have to be measured in a closed loop circuit. Finally, we note that the 4π-periodicity in both the non-local transport and the Josephson case does not violate the Byers-Yang theorem [47] because of the long ranged Coulomb charging energy E_C, which is not accounted for in the BCS mean-field theory.

Summary and Conclusion: In this paper we have proposed a scheme for uniquely identifying the Majorana assisted non-local electron tunneling between two MFs at the ends of a wire in the TS phase. In principle, such a non-local transfer of electrons may be observable by a coincidence measurement [35, 36, 41]. However, as we have shown here that the Majorana assisted electron tunneling process using either a coincidence detection [35, 41] or by measuring the transconductance with a charging energy [37], while interesting, cannot be taken as a definitive signature of MF modes because even conventional near-zero energy states trapped near the spatially separated leads can also produce such non-local signature. Instead we have proposed an interferometry experiment [37] appropriately generalized to geometries without edge modes. We have shown that such a measurement can distinguish conventional and Majorana zero modes. Our proposed non-local correlation experiment in terms of tunneling, which requires the inclusion of charging energy to fix the fermion parity, provides a direct verification of the non-locality of MFs in TS wires. We emphasize that the non-locality of the end state MFs arises from the non-locality of the fermion parity, which is unique to topological systems and cannot be emulated by conventional systems [33].

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[39] Sec. II of the Supplementary material gives details of the nanowire and loop model, which are standard.
[40] Sec. III of the Supplementary material gives details of the computation of the Green function and conductance.
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