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Renormalization-group methods provide a viable approach for investigating the emergent collective behavior of classical and quantum statistical systems in both equilibrium and nonequilibrium conditions. Within this approach we investigate here the dynamics of an isolated quantum system represented by a scalar ϕ^4 theory after a global quench of the potential close to a dynamical critical point. We demonstrate that, within a pre-thermal regime, the time dependence of the relevant correlations is characterized by a short-time universal exponent, which we calculate at the lowest order in a dimensional expansion.

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The nonequilibrium dynamics of isolated, strongly interacting quantum many-body systems is currently under intensive experimental and theoretical investigation (see, e.g., Refs. [1–3]), primarily motivated by recent advances in the physics of cold atomic gases [4]. A natural question which arises in this context concerns the eventual thermalization of these systems after a sudden change (*quench*) of a control parameter. In fact, although isolated systems evolve with unitary dynamics [5, 6], their local properties can be described, after some time, by suitable statistical ensembles [7–9]. Interestingly enough, the eventual approach to a thermal state might involve intermediate *pre-thermal* quasi-stationary states, proposed theoretically [10] and experimentally observed [11–13]. These states appear to be related to the integrable part of the post-quench Hamiltonian [14–21], which alone [22] would drive the system towards a state, sometimes well described by the so-called generalized Gibbs ensemble (GGE) [23–31]. Inspired by the analogy with renormalization-group (RG) flows, prethermalization has been ascribed to a non-thermal unstable fixed point [32–34] towards which the evolution of the system is attracted before crossing over to the eventual, stable, thermal fixed point.

While most of the properties of an isolated many-body system after a quench depend on its microscopic features, some acquire a certain degree of universality if the post-quench Hamiltonian is close to a critical point. Examples include the density of defects [1], dynamics of correlation functions [19, 35], statistics of the work [36–38], rephasing dynamics [39], dynamical phase transitions [40–47], or the dynamics of solitons [48]. Despite this progress, an important open issue is the possible emergence of a *universal* collective behavior at *macroscopic short-times* controlled by the memory of the initial state, i.e., a kind of *quantum aging*. This is known to occur for quenches in classical systems in the presence of a thermal bath [49–52] and, more recently, for quantum impurities [53, 54] or open quantum systems [55, 56]. A quench introduces a "temporal boundary" by breaking the time-translational invariance (TTI) that characterizes equilibrium dynamics, causing the emergence of short-time universal scaling, analogous to universal short-distance scaling in the presence of spatial boundaries in equilibrium [57–59]. To our knowledge, non-equilibrium dynamical scaling and aging have never been investigated in the absence of a thermal bath. In this work, we fill this gap by showing the emergence of these features after a quench of an *isolated* quantum many-body system.

At the lowest order in a dimensional expansion, we construct the RG equations for a wide class of isolated quantum systems after a quench, discussing the resulting flow and comparing it with the equilibrium one at a certain effective temperature T_{eff}. Remarkably, these RG equations are characterised by a stable non-Gaussian fixed point which is associated with the occurrence of a *dynamical phase transition* (DPT). Similarly to the case of classical and quantum systems in contact with thermal baths mentioned above, we show the appearance of universal algebraic laws associated with such non-thermal fixed point, which determines the temporal scaling of the relevant quantities, and which is later on destabilized by the thermalizing dynamics.

The model.— In d spatial dimensions consider a system belonging to the equilibrium universality class described by the effective $O(N)$-symmetric Hamiltonian

$$H(r,u) = \int \mathrm{d}^d x \left[\frac{1}{2}\boldsymbol{\Pi}^2 + \frac{1}{2}(\nabla\boldsymbol{\phi})^2 + \frac{r}{2}\boldsymbol{\phi}^2 + \frac{u}{4!N}\boldsymbol{\phi}^4\right], \tag{1}$$

where $\boldsymbol{\phi} = (\phi_1, \ldots, \phi_N)$ is a bosonic field with N components, $\boldsymbol{\Pi}$ its conjugate momentum, $u > 0$, and r the parameter which controls the distance from the critical point. The system is prepared at $t < 0$ in the ground state of the non-interacting Hamiltonian $H_0 \equiv H(\Omega_0^2, 0)$, in a highly disordered phase ($\Omega_0^2 > 0$), and at time $t = 0$ the parameters are suddenly changed, resulting in the post-quench Hamiltonian $H \equiv H(r,u)$. The quench is performed towards a disordered or critical phase such that, in the absence of symmetry-breaking fields, the order parameter $\bar{\boldsymbol{\phi}}(t) \equiv \langle\boldsymbol{\phi}\rangle$ vanishes during the dynamics.

H for $u=0$ as well as H_0 can be diagonalized in momentum space in terms of two sets of creation/annihilation operators with dispersion relation $\omega_k(r)=\sqrt{k^2+r}\equiv\omega_k$ and $\omega_k(\Omega_0^2)\equiv\omega_k^0$, respectively, where k is the modulus of the momentum. By requiring the continuity of $\boldsymbol{\phi}$ and $\mathbf{\Pi}$ during the quench $\Omega_0^2\to r$, these two sets of operators are related by a Bogoliubov transformation [60]. The relevant two-time correlation functions which characterize the ensuing dynamics are the retarded and the Keldysh nonequilibrium Green's functions [61], defined respectively as $iG_{\alpha\beta,R}(1,2)=\vartheta(t_1-t_2)\langle[\phi_\alpha(1),\phi_\beta(2)]\rangle$ [where $\vartheta(t>0)=1$ and $\vartheta(t<0)=0$] and $iG_{\alpha\beta,K}(1,2)=\langle\{\phi_\alpha(1),\phi_\beta(2)\}\rangle$, with $n\equiv(\mathbf{x}_n,t_n)$ and α, β specifying the components of the field. These functions are non-zero only for $\alpha=\beta$ and they do not depend on α in the symmetric phase, i.e., $G_{\alpha\beta,K/R}=\delta_{\alpha\beta}G_{K/R}$. Their Fourier transforms read:

$$G_R(k,t_1,t_2)=-\vartheta(t_-)\frac{\sin(\omega_k t_-)}{\omega_k}, \tag{2}$$

$$iG_K(k,t_1,t_2)=\frac{K_+\cos(\omega_k t_-)+K_-\cos(\omega_k t_+)}{\omega_k}, \tag{3}$$

for $u=0$, where $t_\pm=t_1\pm t_2$ and $K_\pm(k)=(\omega_k/\omega_k^0\pm\omega_k^0/\omega_k)/2$. Note that G_K (but not G_R) depends on the pre-quench state and is not TTI. Hereafter we primarily focus on the case $\Omega_0\gg\Lambda$, where Λ is the momentum cutoff introduced further below; on a lattice, this implies that the spatial correlation length in the initial state is smaller than the lattice spacing. As the RG fixed-point value of r turns out to be of order Λ^2 (see further below), this case actually corresponds to $\Omega_0^2\gg r$ and therefore to a *deep quench* of the coefficient of $\boldsymbol{\phi}^2$ in Eq. (1). The stationary part $\Omega_0\cos(\omega_k t_-)/(2\omega_k^2)$ of iG_K turns out to have the same form as in equilibrium [60] at a high temperature $T=\Omega_0/4\gg\Lambda$ (see also Refs. [62, 63]). A similar conclusion holds for the (non-thermal) occupation number n_k of the post-quench momenta, which is approximately thermal for $k\ll\Omega_0$. Accordingly, the behavior of the system after the quench is expected to bear some similarities to the equilibrium one at temperature T. Depending on d and N, the latter encompasses an order-disorder transition at $r=r^*_{\rm eq}(T)$ [64–66] which displays the critical properties of a classical system in $d+1$ spatial dimensions for $T=0$, while those of a classical system in d dimensions for $T>0$ because, in this case, the additional dimension has a finite extent T^{-1}. On this basis, after the quench, one heuristically expects a collective behavior to emerge at some value $r^*(\Omega_0)$ of r, as in a $d+1$-dimensional film of thickness $\sim\Omega_0^{-1}$. In addition, the non-stationary part $-\Omega_0\cos(\omega_k t_+)/(2\omega_k^2)$ of iG_K (absent in equilibrium) turns out to be responsible for the short-time universal scaling behavior discussed below.

The case of a quench which does not affect u, i.e., which occurs from the ground state of $H(r_0,u)$ to $H(r,u)$, was studied within the mean-field approximation in Ref. [41] and in the exactly solvable limit $N\to\infty$ in Refs. [43–45]. Quite generically it was shown that, upon crossing a line in the (r_0,r)-plane (at fixed u), the system undergoes a dynamical transition signaled by a qualitative change in the time evolution of the mean order parameter $\bar{\boldsymbol{\phi}}$. In particular, starting from a disordered initial state with $\bar{\boldsymbol{\phi}}=0$ (i.e., $r_0>0$), this transition occurs at a certain $r=r^*<0$, below which the system undergoes coarsening. Although the quench protocol considered here involves a vanishing pre-quench u, a non-vanishing u solely affects the effective value of $r_0=\Omega_0^2$. Accordingly, we expect that the DPT associated with the RG fixed point $Q_{\rm dy}$ discussed further below and emerging after the quench is closely related to the DPT discussed in Refs. [41, 43–45]. Indeed, the critical exponent ν which describes the RG flow around $Q_{\rm dy}$ agrees, up to the first order in the dimensional expansion and for $N\to\infty$, with the exact result found in Ref. [45] at the dynamical transition.

Renormalization-group flow.— In order to highlight the dynamical scaling after the quench and to account for the effects of non-Gaussian fluctuations, we study perturbatively the RG flow of the relevant couplings [61]. In particular, from the Schwinger-Keldysh action associated with H in Eq. (1) we determine the effective action for the "slow" modes by integrating those with a wavevector k within a shell of infinitesimal thickness just below the cutoff Λ. Subsequently, spatial coordinates, time, and fields are rescaled in order to restore the initial cutoff Λ: from the resulting coupling constant one infers the RG equations [67, 68]. An analogous procedure was recently carried out for a quench in $d=1$ [19, 69], for driven quantum systems in $d>1$ (see, e.g., Refs. [70, 71]), and for quantum impurities (see, e.g., Refs. [53, 54]). At one loop and for times larger than the microscopic time $\simeq\Lambda^{-1}$ (before which the dynamics is non-universal), the resulting RG equations read [72]

$$\frac{\mathrm{d}r}{\mathrm{d}\ell}=2r+a_d\frac{N+2}{24N}u\Lambda^d\frac{2\Lambda^2+r+\Omega_0^2}{(\Lambda^2+r)\sqrt{\Lambda^2+\Omega_0^2}}+\mathcal{O}(u^2), \tag{4a}$$

$$\frac{\mathrm{d}u}{\mathrm{d}\ell}=(d_c-d)u-a_d\frac{N+8}{24N}u^2\Lambda^{d-4}\sqrt{\Lambda^2+\Omega_0^2}+\mathcal{O}(u^3), \tag{4b}$$

where $a_d=2/[(4\pi)^{d/2}\Gamma(d/2)]$, d_c is the upper critical dimensionality discussed below, and $\ell>0$ is the flow parameter which rescales coordinates and times as $(x,t)\mapsto(\mathrm{e}^{-\ell}x,\mathrm{e}^{-\ell}t)$. According to this scaling, the RG flow can be parameterized in terms of the time t elapsed from the quench by setting $\ell=\ell_t\equiv\ln(\Lambda t)$. Equations (4) are actually valid up to a typical time t^* discussed later, after which thermalization may take place, according to the dynamical scenario sketched in Fig. 1. For $\Omega_0\ll\Lambda$, inspection of Eq. (4b) shows that the effective coupling constant is $u\Lambda^{d-3}$ and therefore the upper critical dimensionality is $d_c=3$, i.e., the same as in equilibrium at

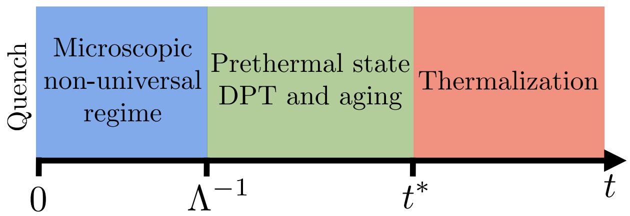

FIG. 1. (Color online) A schematic picture of the various temporal regimes which characterize the evolution of the system after the quench.

$T = 0$. In the opposite case of a deep quench $\Omega_0 \gg \Lambda$, the effective coupling is $\Omega_0 u \Lambda^{d-4}$ and, correspondingly, $d_c = 4$ [72]. This kind of *dimensional crossover* is similar to the one occurring in equilibrium quantum systems upon varying T [64–66] (or in classical statistical systems in spatial confinement, see, e.g., Ref. [73]). Equations (4) with constant Ω_0, $d_c = 4$ (i.e., for a deep quench), and $d < d_c$ admit a non-trivial, stable fixed point $Q_{\rm dy}(\Omega_0) \equiv (r^*_{\rm dy}(\Omega_0), u^*_{\rm dy}(\Omega_0))$ in the (r, u)-plane, which describes a *dynamical phase transition*. In particular, depending on the initial values (r, u) of the parameters, after the non-universal transient of duration $t \simeq \Lambda^{-1}$ depicted in Fig. 1, their post-quench effective values $(r(\ell_t), u(\ell_t))$ determined by solving Eqs. (4) may approach the fixed point $Q_{\rm dy}$ characterized by scaling behavior and aging. When t exceeds t^*, $Q_{\rm dy}$ is generically destabilized as discussed further below. The RG Eqs. (4) are also very similar to those of this same quantum system in equilibrium at temperature T (see, e.g., Ref. [64]) — with Ω_0 playing the role of T — characterized by an *equilibrium* fixed point $Q_{\rm eq}(T) \equiv (r^*_{\rm eq}(T), u^*_{\rm eq}(T))$. Remarkably, up to this order in perturbation theory, the critical exponents ν derived by linearizing these two sets of RG equations around $Q_{\rm dy}$ and $Q_{\rm eq}$ are the same and equal $\nu_{\rm eq} = 1/2 + \epsilon(N+2)/[4(N+8)] + \mathcal{O}(\epsilon^2)$, where $\epsilon \equiv d_c - d$ indicates the deviation from the upper critical dimensionality of the model. One can actually define an *effective temperature* $T = T_{\rm eff}(\Omega_0)$ such that the systems which are critical under equilibrium conditions are also critical after the quench. This implies that the (linearized) critical lines of $Q_{\rm dy}$ and $Q_{\rm eq}$ in the (r, u)-plane are the same, though $Q_{\rm dy}(\Omega_0) \neq Q_{\rm eq}(T_{\rm eff}(\Omega_0))$. Only for $\Omega_0 \gg \Lambda$, these two fixed points coincide, with $T_{\rm eff} = \Omega_0/4$ and $r^*_{\rm dy}(\Omega_0) = r^*_{\rm eq}(T_{\rm eff}) = -\epsilon\Lambda^2(N+2)/[2(N+8)] + \mathcal{O}(\epsilon^2)$. In passing, we mention that the same happens also for $\Omega_0 \ll \Lambda$. In this respect and up to this order in perturbation theory, the dynamical transition (in the notion of Refs. [40–42]) has some of the features of the equilibrium transition occurring at $T_{\rm eff}$, though differences could emerge at higher orders in perturbation theory or in quantities which depend on $Q_{\rm dy}$ or on the post-quench distribution at short length scales, which is definitely not thermal [45] (see further below). It also remains to be seen whether the $T_{\rm eff}$ defined above has any thermodynamic or dynamic role in the system, e.g., entering into fluctuation-dissipation relations [74, 75].

The RG Eqs. (4) have been derived under the assumption that inelastic scattering does not occur, at least in the early stages of the evolution, and that the dynamical exponent keeps its initial value $z = 1$. In fact, up to this order in perturbation theory, the tadpole is the only relevant diagram which is responsible for the occurrence of elastic dephasing during the time evolution and, for a deep quench, it results in the fixed point $Q_{\rm dy}$ discussed above. However, the RG transformations also generate relevant dissipative terms which are expected to drive the system to thermal equilibrium [76, 77]. In the present case, they appear as secular terms growing in time [72], eventually spoiling the perturbative expansion (unless they are properly resummed [32, 78, 79]), and changing the dynamical exponent z towards the diffusive value $z \simeq 2$. Nonetheless, these terms, which are absent immediately after the quench and are therefore generated perturbatively, turn out to be small at short times $\Lambda t \lesssim \Lambda t^* = 1/(\Omega_0 u^*_{\rm dy}) \simeq \epsilon^{-1}$, which include the range of times within which the short-time scaling behavior associated with $Q_{\rm dy}$ sets in [72]. Note that no dissipative terms are actually generated in the cases studied in Refs. [43–45], namely in the $N \to \infty$ limit, because the relevant fluctuations are Gaussian. Accordingly, the prethermal state is stable at all times and no thermalization occurs.

Short-time scaling of various quantities.— The emergence of a short-time scaling after a deep quench is clearly revealed by a perturbative calculation of iG_K and G_R for $k = 0$, at the critical point $Q_{\rm dy}$. In fact, it turns out that for $t_2 \ll t_1$ and up to $\mathcal{O}(u^{*2}_{\rm dy})$, $G_R(0, t_1 \gg t_2) = -t_1[1 - \theta \ln(t_1/t_2)] \simeq -t_1(t_2/t_1)^\theta$ and, analogously, $iG_K(0, t_1, t_2) \simeq (\Omega_0/\Lambda^2)(\Lambda t_2)^{2-2\theta}(t_2/t_1)^{\theta-1}$, where

$$\theta = \frac{N+2}{(N+8)}\,\frac{\epsilon}{4} + \mathcal{O}(\epsilon^2). \tag{5}$$

These algebraic dependences on time are similar but not identical to the ones observed in classical [50] and quantum [55] systems undergoing *aging* in contact with a thermal bath, with an *initial-slip* exponent θ. As in classical dissipative systems, θ emerges because the fields at $t = 0$ acquire a different scaling dimension compared to those at $t > 0$, due to the breaking of TTI caused by the quench [80]. In the limit $N \to \infty$, Eq. (5) predicts the value $\theta_\infty = \epsilon/4 + \mathcal{O}(\epsilon^2)$ for the exponent θ of the very same model studied in Refs. [43–45], although this universal short-time regime was overlooked by past studies, and constitutes a central result of our paper. The algebraic behavior of $G_{R,K}$ discussed above also appears in the response function $-G_R$ as a function of

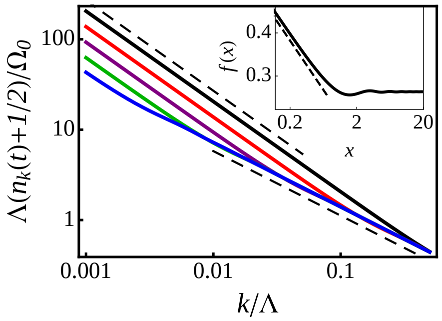

FIG. 2. (Color online) Momentum distribution n_k after the quench, as a function of $k/\Lambda \ll 1$ for $\Lambda t = 2, 8, 32, 128, 512$ (solid lines, from top to bottom). The algebraic short- and long-time behaviors of n_k are highlighted by the upper $\sim k^{-1}$ and lower $\sim k^{-1+2\theta}$ dashed lines, respectively. The inset shows a log-log plot of the scaling function $f(x)$, which approaches $\sim x^{-2\theta}$ for $x \lesssim 1$ (dashed line). With the purpose of highlighting the crossover, we set $\epsilon = 2$ in the perturbative expressions of these curves.

the spatial distance $x = |\mathbf{x}_1 - \mathbf{x}_2|$. For $u = 0$, its expression $G_R^{(0)}(x, t_1 - t_2)$ is TTI and shows typical light-cone dynamics by being enhanced at $x = t_1 - t_2$ where $G_R^{(0)} \propto -\Lambda^3 [\Lambda(t_1 - t_2)]^{-3/2}$ in $d = 4$, while decaying rapidly inside the light-cone for $x \ll t_1 - t_2$, and being vanishingly small outside it for $x \gg t_1 - t_2$. At one loop, G_R is found to acquire an algebraic behavior for $t_2 \ll t_1$, i.e., $G_R(x = t_1 - t_2, t_2 \ll t_1) \simeq (t_2/t_1)^\theta t_1^{-3/2}$ [80]. Analogously, the dynamics of the order parameter $\bar{\boldsymbol{\phi}}(t)$ can be studied by adding a small symmetry-breaking field in the pre-quench Hamiltonian $H(\Omega_0^2, 0) \to H(\Omega_0^2, 0) - \int \mathrm{d}^d x\, h_1 \phi_1(x)$, which gives $\bar{\phi}_1(0^-) \equiv \phi_0 = h_1/\Omega_0^2 \ll 1$. The time evolution of $\bar{\phi}_1$ due to the post-quench Hamiltonian H in Eq. (1) (with no symmetry-breaking field) is determined by $[\partial_t^2 + M^2(t) - u\bar{\phi}_1^2/(3N)]\bar{\phi}_1(t) = 0$ where $M^2(t) \simeq r + u\bar{\phi}_1^2/(2N) + u(N+2)iG_K(x = 0, t, t)/(12N)$. At criticality $r = r^*_{\mathrm{dy}}(\Omega_0)$ and for times such that $\Lambda^{-1} \ll t \ll t_i$ where $\Lambda t_i \sim \mathcal{O}(|\phi_0|^{-1})$ one finds $M^2(t) \simeq \theta/t^2$ and therefore $\bar{\phi}_1 \simeq \phi_0 t^\theta$ [80], i.e., the short-time evolution of $\bar{\phi}_1$ is controlled by θ and corresponds to an initial *increase* of the order with time. If the quench occurs slightly away from criticality, with $r = \delta r + r^*_{\mathrm{dy}}$, the short-time algebraic laws discussed above turn out to be modulated by oscillations of period $\propto |\delta r|^{-\nu z}$ [80].

Remarkably, the momentum distribution n_k of the quasi-particles also shows signatures of the exponent θ in the dependence on k at criticality. Immediately after the quench, n_k takes the expected form of a GGE with a momentum-dependent effective temperature T^k_{eff} [62, 81] which becomes independent of k and equal to $T_{\mathrm{eff}}(\Omega_0)$ for deep quenches. Interactions eventually modify this behavior. In particular, for a deep quench at the critical point Q_{dy}, a perturbative calculation yields $n_k(t) + 1/2 = (\Omega_0/\Lambda)(\Lambda/k)^{1-2\theta} f(kt)$, where the scaling function f can be consistently estimated up to $\mathcal{O}(\epsilon)$ as the exponential of the one-loop correction and is such that $f(x \ll 1) \simeq x^{-2\theta}$, with a finite value for $x \gg 1$. Accordingly, for fixed t, $n_k(t)+1/2$ as a function of k crosses over from an algebraic behavior $\sim k^{-1}t^{-2\theta}$ for $k \lesssim t^{-1}$ to $\sim k^{-1+2\theta}$ for $k \gtrsim t^{-1}$. This crossover is shown in Fig. 2 along with a plot of $f(x)$. It is interesting to note that the dynamics of $n_k(t)$ in Fig. 2 closely resemble the one observed at non-thermal fixed points (see, e.g., Ref. [82]).

The scaling properties of $G_{R,K}$ discussed above bear remarkable differences compared to those in the classical case: for example, G_R decreases $\propto t_2^\theta$ upon decreasing the smaller time t_2, whereas the opposite happens in the corresponding classical response function [50]. Nonetheless, the algebraic time dependence of $\bar{\boldsymbol{\phi}}$ is the same as in the classical case and, in addition, the corresponding exponent θ has the same value up to one-loop in spite of the fact that the dynamics are significantly different. Indeed here the dynamical exponent is $z = 1$ and energy is conserved, whereas $z > 1$ in the classical case with a thermal bath.

The universal short-time behavior described here could be investigated, for the $O(N = 2)$ universality class, in experimental realizations of the Bose-Hubbard model via ultra-cold atoms in optical lattices [5, 83, 84]. Alternatively, the relative phase of tunnel-coupled condensates is known to be effectively described by Eq. (1) with $N = 1$ and in $d = 1$ its dynamics has already been successfully studied in experiments [85, 86]. Similar protocols can also be adapted for fluids of light in non-linear optical systems [87]. Finally, recent experimental realizations of systems with $SU(N)$ symmetry [88, 89] could be used in order to investigate the emergence of a short-time universal collective behavior in systems governed by an effective theory different from Eq. (1).

Conclusions.—The RG analysis presented here demonstrates in a simple setting the emergence of a novel scaling behavior after a deep quench of an isolated quantum system. This phenomenon, due entirely to elastic dephasing, is an example of a macroscopic short-time non-thermal fixed point; the corresponding behavior of various physical observables is controlled by a universal exponent θ, which we calculated at the first order in a dimensional expansion [see Eq. (5)]. The non-thermal fixed point is eventually destabilized towards a thermal regime, driven by dissipative terms generated in the effective action.

As the scaling regime unveiled here occurs at macroscopic short times, its numerical investigation should not be hampered by the computational limitations which typically prevent the investigation of the post-quench dynamics at long times.

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