Vortices in normal part of proximity system

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The order parameter induced in the normal part of superconductor-normal-superconductor proximity system can form vortices in the magnetic field, which are similar but not the same as vortices in bulk superconductors, or it can be modulated *laminarly* with the field dependent decay length.

The question of superconductivity induced in the normal part (N) of superconductor-normal-superconductor (SNS) proximity system has recently been revived by observations of vortices in N.[1] The order parameter Δ induced in N is not uniform even in zero field and is strongly suppressed nearly everywhere in N except the immediate vicinity of interfaces. Hence, the formal problem of the order parameter distribution within vortices in N is qualitatively different from that of bulk superconductors and so are the physical properties of "N-vortices". These properties are of interest both for the basic physics and for applications, enough to mention wires in superconducting magnets which are in fact SNS systems.

Describing proximity effects, one encounters the question of the length scale on which the induced order parameter varies. This problem is reviewed in the first part of this paper for any field and temperature. In the following part, a linear combination of the eigenfunctions of the equation for Δ is constructed to represent vortices in N. In fact, the seminal work of Abrikosov on type-II superconductors suggests the form of this combination.[2] However, Abrikosov combined eigenfunctions of the 1st Landau level, whereas in the problem of interest here these functions are different.

As mentioned, the induced Δ is strongly suppressed everywhere in N except the vicinity of interfaces. Out of this vicinity, equations of superconductivity in N can be linearized. Formally, the situation is similar to that at the upper critical field H_{c2}, where the magnetic field is uniform and the small Δ satisfies a linear equation

$$-\xi^2\Pi^2\Delta = \Delta\,, \quad \text{or} \quad \Pi^2\Delta = k^2\Delta\,. \tag{1}$$

at any temperature T.[3] Here, $\boldsymbol{\Pi} = \boldsymbol{\nabla} + 2\pi i\boldsymbol{A}/\phi_0$, $\boldsymbol{A}$ is the vector potential, ϕ_0 is the flux quantum, and $k^2 = -1/\xi^2$. Notwithstanding the form, this equation differs from the linearized Ginzburg-Landau equation (GL); in the latter the coherence length ξ diverges as $T \to T_c$. At H_{c2} and $T \neq T_c$, $\xi(T)$ is finite and is found by solving the self-consistency equation of the theory

$$\frac{\hbar}{2\pi T}\ln\frac{T_c}{T} = \sum_{\omega>0}\left(\frac{1}{\omega} - \frac{2\tau S}{\beta - S}\right), \quad \beta = 1 + 2\omega\tau\,. \tag{2}$$

Here, $\hbar\omega = \pi T(2n+1)$ are Matsubara energies and τ is the scattering time for non-magnetic impurities. According to Helfand and Werthamer,[3]

$$S(\xi) = \frac{2\beta}{\ell q}\int_0^\infty e^{-s^2}\tan^{-1}\frac{s\ell q}{\beta}ds, \quad q^2 = \frac{2\pi H_{c2}}{\phi_0} = \frac{1}{\xi^2}, \tag{3}$$

where $\ell = v\tau$ is the mean-free path.

Eq. (1) is equivalent the Schrödinger equation for a charge in uniform magnetic field; $H_{c2} = \phi_0/2\pi\xi^2$ corresponds to the minimum eigenvalue. The corresponding eigenfunctions belong to the first Landau level. A linear combination of these functions, constructed by Abrikosov, represents the lattice of vortices.[2]

The normal metal within the proximity system may have its own $T_{c,N} < T$ and $H_{c2,N}(T)$. We are interested here in the part of the phase diagram where the material is in the normal phase outside of the region under $H_{c2,N}(T)$. In this domain, the superconductivity induced by proximity with S is still described by Eqs. (1) and (2), however with a more general $S(H,T,\tau)$:[4,5]

$$S(H,T,\tau) = \sqrt{\pi}\,\text{Re}\int_0^\infty ds\,\frac{(1+us^2)^\sigma}{(1-us^2)^{\sigma+1}}\text{erfc}\,s\,, \tag{4}$$

$$\sigma = \frac{1}{2}\left(\frac{k^2}{q^2} - 1\right), \quad u = \frac{\ell^2q^2}{\beta^2}. \tag{5}$$

Here $\text{erfc}\,s = 2\int_s^\infty e^{-t^2}dt/\sqrt{\pi}$. Solving the self-consistency Eq. (2) with the new S, one can evaluate $\xi(H,T,\tau)$ for any H, T, and τ.

At $H_{c2,N}(T)$, $\xi^2 = \phi_0/2\pi H$ i.e. $k^2/q^2 = -1$, and the parameter $\sigma = -1$.[6] Eq. (2) in dimensionless form,

$$-\frac{1}{2}\ln t = \sum_{n=0}^\infty\left(\frac{1}{2n+1} - \frac{tS}{\lambda + t(2n+1) - \lambda S}\right), \tag{6}$$

($\lambda = \hbar/2\pi T_{cN}\tau$ is the scattering parameter, $t = T/T_{cN}$) should give $H_{c2,N}(T)$ if one sets $\sigma = -1$ in S of Eq. (4). Solving this numerically (see Appendix A) for the clean limit one obtains the lower curve of Fig. 1.

If $H \to 0$, σ diverges, whereas $u \to 0$. It is readily shown that S of this case has a closed form:[5]

$$S(0,T,\tau) = \frac{\beta}{k\ell}\tanh^{-1}\frac{k\ell}{\beta}\,. \tag{7}$$

Solving Eq. (2) with $S(0,T,\tau)$ one obtains the decay length $\xi = 1/k$ of the order parameter in the N part of proximity systems in zero field at any T and τ.[7,8]

Thus, $k^2 = -\xi^2 < 0$ at the curve $H_{c2,N}$ whereas it must be positive in zero field at $t > 1$, where it describes Δ attenuation in the N phase. This suggests that a curve exists on the plane (H,T) where $k^2 = 0$.[4] To check this we set $\sigma = -1/2$ in S of Eq. (4) and solve it numerically for $q^2(t)$. The result for the clean limit is the upper curve in Fig. 1, see remark 9.

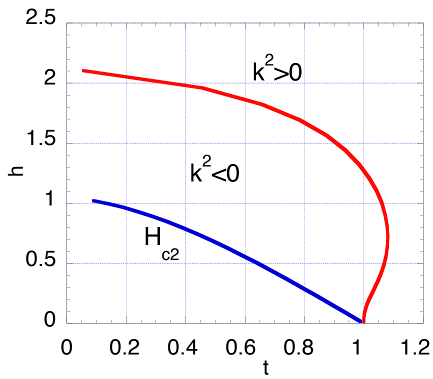

FIG. 1. (Color online) The lower curve is $H_{c2,N}$ of the clean limit in units $2\pi T_{cN}^2\phi_0/\hbar^2v^2$; at this curve $k^2 = -q^2$ and $\sigma = -1$. At the upper curve $k^2 = 0$ and $\sigma = -1/2$. Between the upper and lower curves, k^2 is negative. Above and to the right of the upper curve, k^2 is positive.

Given $k(H,T)$, one can study behavior of the induced order parameter in the N phase, i.e. solutions $\Delta(x,y)$ of the equation $\Pi^2\Delta = k^2\Delta$. Choosing $\boldsymbol{A} = -Hy\,\hat{\boldsymbol{x}}$ we have

$$\left(\frac{\partial}{\partial x} - i\frac{2\pi H}{\phi_0}y\right)^2\Delta + \frac{\partial^2\Delta}{\partial y^2} = k^2\Delta\,. \tag{8}$$

The equation does not contain x explicitly, so that

$$\Delta = \Delta_0\, e^{ipx}\,\chi(y) \tag{9}$$

with $\chi(y)$ satisfying

$$d^2\chi/dy^2 - q^4(y - p/q^2)^2\chi = k^2\chi\,. \tag{10}$$

In terms of $\tilde{y} = y - p/q^2$ the general solution is:

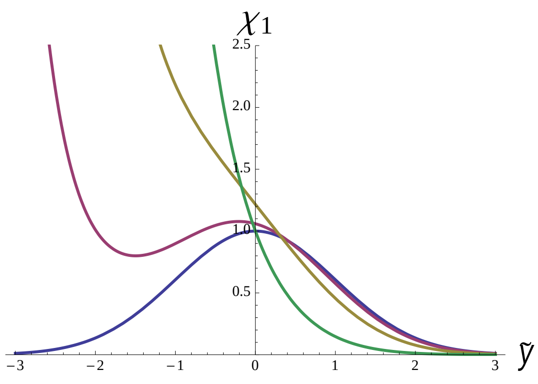

FIG. 2. (Color online) $\chi_1(\sigma,\tilde{y})$ for $q = 1$ in units $2\pi T_c/\hbar v$. Ordering curves by their left edges clockwise: $\sigma = -1$ (H_{c2}), -0.9 (with a minimum at the left), -0.5 ($k = 0$), and 1.

$$\begin{aligned}\chi &= C_1\chi_1 + C_2\chi_2\,,\\ \chi_1 &= e^{-q^2\tilde{y}^2/2}\,\mathcal{H}\left(-\sigma-1, q\tilde{y}\right)\,,\\ \chi_2 &= e^{q^2\tilde{y}^2/2}\,\mathcal{H}\left(\sigma, iq\tilde{y}\right)\,,\end{aligned} \tag{11}$$

with arbitrary constants $C_{1,2}$ and σ of Eq. (5). The Hermite functions $\mathcal{H}(\mu,\ w)$ can be expressed in terms of the parabolic cylinder functions and reduce to Hermite polynomials for $\mu = 0, 1, 2, \ldots$, see Appendix B.

Note that χ_1 with σ being a negative integer are the harmonic-oscillator wave-functions which go to 0 as $\tilde{y} \to \pm\infty$; these are the eigenfunctions of Landau levels. We are interested here in the part of the phase diagram where $\sigma > -1$ and $\chi_1(\tilde{y})$ is real, diverges as $\tilde{y} \to -\infty$, and goes to 0 as $\tilde{y} \to +\infty$, see Fig. 2. For symmetric SNS systems, χ_1 should be discarded.

On the other hand, χ_2 for $\sigma > -1$ has both real and imaginary parts. An example of $\chi_2(-0.7,\ 1,\ \tilde{y})$ is shown in Fig. 3. Both real and imaginary parts diverge at large $\tilde{y}$. χ_2 should be taken into account in finite samples.

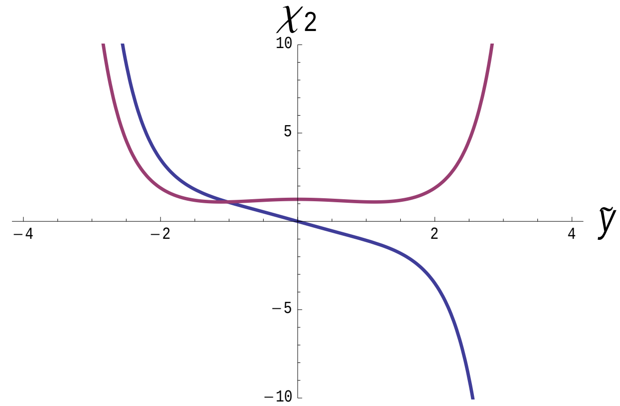

FIG. 3. (Color online) $\chi_2(\tilde{y})$ for $\sigma = -0.7$ and $q = 1$. $\mathrm{Im}\,\chi_2$ is an odd function of $\tilde{y}$, whereas $\mathrm{Re}\,\chi_2$ and $|\chi_2|$ are even.

Consider now a normal metal layer between two thick superconducting banks forming the SNS proximity sandwich. The N slab is infinite in x and z directions whereas $-W/2 < y < W/2$. The temperature of the system $T_{cN} < T < T_{cS}$. In zero field Eq. (8) gives $\Delta \propto \cosh ky$. In a field along z less than $H_{c1,S}$, the field is confined to the N domain, whereas the S banks are in the Meissner state. Since superconductivity is induced in N by proximity with S, one expects vortices to nucleate within the N layer. In small fields, vortices should form a periodic chain in the slab middle.

The N slab is uniform in the x direction, so that the parameter p in the solution (9) can take any value. Consider a linear combination

$$\Delta = e^{ipx}\chi_2(y - p/q^2) + e^{-ipx}\chi_2(y + p/q^2)\,, \tag{12}$$

where the overall constant factor Δ_0 is omitted. Clearly, if $\Delta(x_0, 0) = 0$, the zero should be repeated with the period $\delta x_0 = \pi/p$. If the penetration depth of S banks is small relative to W, the flux quantization gives $\delta x_0 WH = \phi_0$ and

$$p = \frac{\pi WH}{\phi_0} = \frac{q^2W}{2}\,. \tag{13}$$

It is convenient here to normalize lengths to $W/2$, then $p = q^2$. Since the RHS of Eq. (12) is dimensionless, we keep the same notation x, y, p, q as for their dimensional counterparts.

The structure of the solution (12) is illustrated in Fig. 4 where the modulus $|\Delta(x, y)|$ is plotted for $W = 2$, $\sigma = 1$, $q^2 = p = 2$. As expected, the distance between singularities (vortices) is $\delta x_0 = \pi/p \approx 1.57$. The solution shown is normalized as to have $\Delta = 1$ at the interfaces $y = \pm 1$. The phase near one of the vortices is shown in Fig. 5.

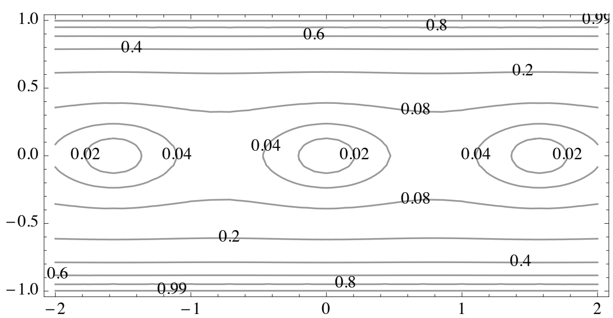

FIG. 4. Contours of constant $|\Delta(x, y)|$ according to Eq. (12) with $\sigma = 1$, $p = q^2 = 2$ for $-2 < x < 2$ and $-1 < y < 1$.

Similar structures for the dirty N layer were obtained by solving Usadel equations.[1]

The linear combination (12) obeys the boundary condition $|\Delta(y = \pm 1)| =$const if $\sigma = 2n+1$ is a positive odd integer. We have for $p = q^2$ at $y = 1$

$$\Delta = e^{2p-ipx}\mathcal{H}(\sigma, 2i\sqrt{p}) + e^{ipx}\mathcal{H}(\sigma, 0) \,. \qquad (14)$$

The integral representation (B2) of $\mathcal{H}$ shows that $\mathcal{H}(\sigma, 0) \propto \cos(\pi\sigma/2) = 0$ for $\sigma = 2n + 1$. Hence, in these cases, the second term in Eq. (14) vanishes and $|\Delta(1)|$ is x independent. For other values of σ this boundary condition can be satisfied only for $p = 0$, i.e. for a *laminar* vortex-free structure with $|\Delta(y)|$ independent of x.

To compare energies of the vortex chain and the laminar structure, we turn to the question of energy. Since Eq. (1) formally coincides with the linearized GL equation, the corresponding energy functional can be written

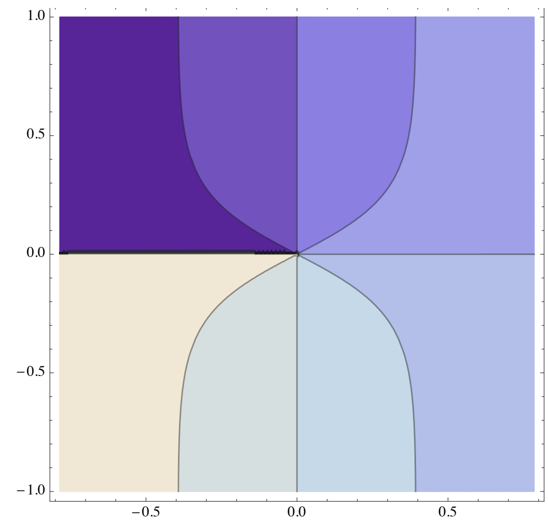

FIG. 5. (Color online) Contours of the constant phase with the step $\pi/4$ in clock-wise order for the vortex at the origin of Fig. 4. The phase jumps by 2π at the straight ray from the vortex center to the left.

as a volume integral

$$\mathcal{F} = AN(0) \int dV \left(k^2|\Delta|^2 + |\mathbf{\Pi}\Delta|^2\right) . \qquad (15)$$

Here A is the interface area, $N(0)$ is the density of states at the Fermi level per spin of the N metal, and the integral is extended over the N region. Varying this with respect to Δ^* one obtains Eq. (1). However, there is a substantial difference with GL: integrating by parts the term $\int dV\, \mathbf{\Pi}\Delta \cdot \mathbf{\Pi}^*\delta\Delta^*$, one has to take into account the boundary condition $\Delta = \Delta_S$ at the interface which implies $\delta\Delta^* = 0$. In particular, this boundary condition is used in calculation of the equilibrium energy. Integrating by parts the term $\int dV\, |\mathbf{\Pi}\Delta|^2$ of Eq. (15) one has:

$$\frac{1}{2}\int dV \left[\mathbf{\Pi}\Delta\left(\mathbf{\nabla} - i\frac{2\pi}{\phi_0}\boldsymbol{A}\right)\Delta^* + \mathbf{\Pi}^*\Delta^*\left(\mathbf{\nabla} + i\frac{2\pi}{\phi_0}\boldsymbol{A}\right)\Delta\right]$$
$$= -\int dV\, k^2|\Delta|^2 + \int d\boldsymbol{S}\cdot\mathbf{\nabla}|\Delta|^2/2 \,, \qquad (16)$$

where the surface integral is over the interface and $d\boldsymbol{S}$ is directed to the superconducting side. Thus, we have:

$$\frac{\mathcal{F}}{AN(0)} = \frac{1}{2}\int d\boldsymbol{S}\cdot\mathbf{\nabla}|\Delta|^2 \,. \qquad (17)$$

Note that $\nabla|\Delta|^2$ is always directed toward the S-side of the NS interface, so that $\mathcal{F} > 0$, in other words, induced superconductivity raises the energy of the normal metal.

One then obtains for the N-slab of this paper:

$$\frac{\mathcal{F}}{AN(0)} = L_z \int_{-\infty}^{\infty} dx \frac{\partial |\Delta|^2}{\partial y}\Big|_{y=1} = L_z \left[\frac{\partial}{\partial y}\int_{-\infty}^{\infty} dx |\Delta|^2\right]_{y=1}, \quad (18)$$

where L_z is the sample size in z direction; in the x direction the system is assumed to be large.

One can now compare energies of a laminar structure $p = 0$ with the vortex chain corresponding to p of Eq. (13). To this end, we calculate the dimensionless quantity

$$f = \frac{\mathcal{F}}{AN(0)\Delta_S^2 L_z} = \left[\frac{\partial}{\partial y}\int_{-\infty}^{\infty} dx |\Delta|^2\right]_{y=1}, \quad (19)$$

where $|\Delta|$ is normalized to Δ_S. For the vortex chain (12) with $\sigma = 1$, we have $\chi_2 = 2iq\tilde{y}\, e^{q^2\tilde{y}^2/2}$ and

$$\Delta = \frac{e^{ipx}\chi_2(y-1) + e^{-ipx}\chi_2(y+1)}{\chi_2(2)} \quad (20)$$

(the boundary condition $|\Delta| = 1$ at $y = \pm 1$ is obeyed). We obtain after straightforward algebra:

$$f = \frac{2L_x}{W}\left(1 + H\frac{2\pi W^2}{\phi_0}\right). \quad (21)$$

For the in-field laminar structure $p = 0$, the dimensionless $|\Delta|^2 = y^2 e^{q^2(y^2-1)}$. This yields

$$f_0 = \frac{4L_x}{W}\left(1 + H\frac{\pi W^2}{2\phi_0}\right). \quad (22)$$

Comparing f_0 with f for the vortex chain, we obtain that for $\sigma = 1$ the chain is preferred, if $H < \phi_0/\pi W^2$. In fields larger than $\phi_0/\pi W^2$, the laminar vortex-free structure wins. Physically, this means that vortices within the chain repel each other and since their separation $\delta x_0 = \pi/p = \pi/q^2 \propto 1/H$, the chain energy grows till at $H = \phi_0/\pi W^2$ it reaches the energy of the laminar structure.

The form (12) is not the only possibility. Linear combinations with various complex coefficients all satisfy $\Pi^2\Delta = k^2\Delta$. The choice of these coefficients is dictated by boundary conditions. If, e.g., a factor 4 is added to the second term in Eq. (12), one obtains a distribution shown in Fig. 6. In this example $|\Delta|$ is normalized on its value at $y = 1$. The constant value reached at $y = -1$ is clearly larger than 1. Thus, this type of linear combination might be useful in describing the proximity effect in asymmetric SNS$'$ systems with different S-banks for which vortices in N tend to be closer to the bank with smaller order parameter.

The boundary condition $|\Delta| = |\Delta_S|$ at the interface, chosen above for the sake of simplicity, may not hold in real proximity systems, see e.g. Ref. 7. However, for solutions discussed here the relevant feature is that Δ_S is x independent, which is so for a uniform and flat S

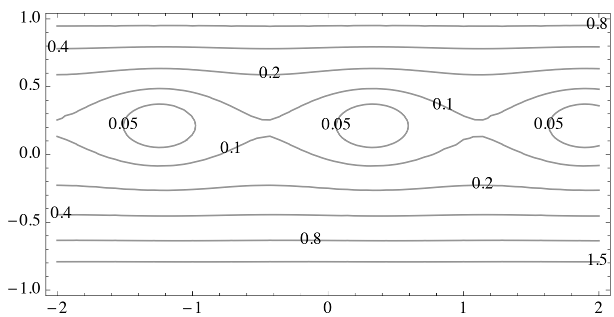

FIG. 6. (Color online) Contours of constant $|\Delta|$ for the parameters of Fig. 4 and a factor 4 in front of e^{-ipx} of Eq. (12).

banks. The numerical factor in the "critical field" ϕ_0/W^2 may differ from $1/\pi$. Moreover, the straight chain may turn unstable with increasing field with respect to e.g. "saw-tooth" perturbations (observed in superconducting strips[13,14]), the question out of the scope of this paper.

Hence, vortices appear at the bottom of the suppressed order parameter valley. They have normal cores in a sense that $\Delta = 0$ at the center of each vortex and the phase changes by 2π if one circles the center. Still, they differ from their Abrikosov "brethren". The order parameter changes differently with the distance from the center along x or y directions. Unlike Abrikosov's case, one cannot define the core size as the distance from the center to a place with depairing current density. The self-energy of these vortices should be quite small because they appear in the region where Δ is suppressed even in zero field. The magnetic field is practically constant in the N layer (exactly so within our model). Hence, methods of observing vortices by detecting the vortex field (decoration or scanning SQIUD microscopy) will probably not work for N-vortices. On the other hand, STM that probes the order parameter value should discern zeros of Δ. In fact, the recent STM data show vortices between superconducting Pb islands separated by the normal wetting layer.[1]

There are many questions remain on properties of vortices within domains of proximity induced superconductivity. Currents through the SNS sandwich in magnetic field should cause vortex motion. If so, what is the drag coefficient? On the other hand, if the N-layer structure of Δ is laminar, the flux-flow dissipation should be absent. Hence, measuring in-field current-voltage characteristics of an SNS sandwich one, in principle, can confirm presence of N-vortices. The above statement that the vortex chain may appear only if $\sigma = 2n+1$ was obtained for a particular SNS geometry (at a given T, vortices appear at a discrete set of fields; alternatively, at a given H - at a discrete set of temperatures). It is not clear whether this discreteness survives for other shapes.

An interesting question concerns superconducting fluctuations in the N phase. According to results of Schmid[10] and Prange[11] based on linearized GL equation, the diamagnetic susceptibility χ_d in the N phase is proportional to ξ (in zero field, χ_d diverges as T approaches T_c from above). Here, a method is offered to evaluate $\xi(H,T)$ at any H and T. It would be of interest to look at possible differences in χ_d within the region where $k^2 = -1/\xi^2$ is negative (between the curves of Fig. 1) and out of it where k^2 is positive.

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Appendix A:

To account for the branch point at $s = 1/\sqrt{u}$, the integral (4) is rewritten as:

$$S = \sqrt{\frac{\pi}{u}} \int_0^1 \frac{d\eta(1+\eta^2)^\sigma}{(1-\eta^2)^{\sigma+1}} \left[\mathrm{erfc}\frac{\eta}{\sqrt{u}} - \cos(\pi\sigma)\,\mathrm{erfc}\frac{1}{\eta\sqrt{u}} \right]. \tag{A1}$$

For the calculation of $H_{c2,N}(T)$, it is convenient to measure length in units of $\hbar v/2\pi T_{cN}$. Then, we have:

$$\sqrt{u} = \frac{q}{\lambda + t(2n+1)}\,, \quad q^2 = \frac{\hbar^2 v^2 H}{2\pi T_{cN}^2 \phi_0} \approx \frac{H}{H_{c2,N}(0)}, \tag{A2}$$

where $\lambda = \hbar/2\pi\tau T_{cN}$, $H_{c2,N}(0)$ is the zero-T clean limit upper critical field, and $t = T/T_{cN}$.

Appendix B:

The general solution of

$$f''(w) - 2\mu\, w^2 f'(w) + 2\mu\, f(w) = 0 \tag{B1}$$

is $f = C_1\mathcal{H}(\mu, w) + C_2 e^{z^2}\mathcal{H}(-\mu-1, i\,w)$. For a non-negative integer $\mu = n$, the Hermite functions $\mathcal{H}(\mu, w)$ reduce to Hermite polynomials. An integral representation useful for our purpose is:

$$\mathcal{H}(\sigma, w) = \frac{2^{\sigma+1}}{\sqrt{\pi}} e^{w^2} \int_0^\infty e^{-t^2} t^\sigma \cos\left(2wt - \frac{\pi\sigma}{2}\right) dt. \tag{B2}$$

Appendix C:

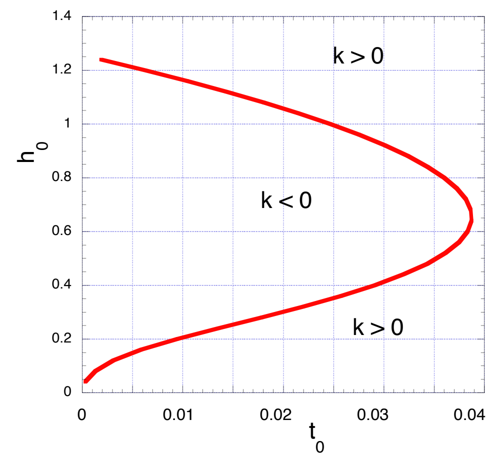

FIG. 7. (Color online) The curve $k(t_0, h_0) = 0$ for $T_{cN} = 0$. Note that this curve holds for any mean-free path; the actual temperature and field are $T = \hbar t_0/2\pi\tau$ and $H = \phi_0 h_0/2\pi\ell^2$. Hence, on approaching the clean limit, this curve shrinks to the origin so that $k > 0$ everywhere. On the other hand, the domain of $k < 0$ expands with increasing scattering.

The assumption of a finite T_{cN} in the main text is in fact not necessary. However, the formal treatment of the case $T_{cN} = 0$ should take into account that $\Delta = 0$ when the effective coupling is zero. Nevertheless, proximity with S results in non-zero Green's functions $F(\omega)$ in the normal metal. One can show that this leads to $\beta - S = 0$ and to different exponential decay lengths of F for different $\omega = \pi T(2n+1)/\hbar$. The longest length corresponds to $n = 0$, so that calculating the depth of pairs penetration one can disregard all $n \neq 0$.[4,12]

Since in this situation there is no standard energy scale related to Δ or T_c (and no length scale $\hbar v/T_c$), one can use the following reduced temperature and field:

$$t_0 = \frac{2\pi\tau}{\hbar} T, \quad h_0 = \frac{2\pi\ell^2}{\phi_0} H. \tag{C1}$$

In these variables, $\beta = 1 + t_0$ and $u = h_0/(1+t_0)^2$. To find $k(t_0, h_0)$ one has to solve $\beta - S = 0$ with S taken at $n = 0$. Consider, as an example, the curve $k(t_0, h_0) = 0$ at which $\sigma = -1/2$. Using the form (A1), we have

$$S_0 = \sqrt{\frac{\pi}{u}} \int_0^1 \frac{d\eta}{\sqrt{1-\eta^4}}\, \mathrm{erfc}\frac{\eta}{\sqrt{u}}\,. \tag{C2}$$

This integral is expressed in terms of generalized hypergeometric functions, which are easily treated with the help of Mathematica. Solving numerically $1 + t_0 = S_0(u, \sigma)$ one obtains the curve of Fig. 7.

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