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Spontaneous currents in a superconductor with s+is symmetry

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We analyze $s + is$ state proposed as a candidate superconducting state for strongly hole-doped $Ba_{1-x}K_xFe_2As_2$. Such a state breaks time-reversal symmetry (TRS) but does not break any other discrete symmetry. We address the issue whether TRS breaking alone can generate spontaneous currents near impurity sites, which could be detected in, e.g., μSR experiments. We argue that there are no spontaneous currents if only TRS is broken. However, supercurrents do emerge if the system is put under external strain and C_4 lattice rotation symmetry is externally broken.

Introduction The search for a truly unconventional superconductivity which breaks time-reversal symmetry (TRS) in addition to $U(1)$ overall phase symmetry of a superconducting order parameter continues to attract a lot of attention in the physics community.[1–3] Systems which break TRS exhibit a wealth of fascinating properties and are highly sought after for applications[4–14]. The TRS breaking pairing states have been proposed in the two-dimensional ^{3}He (Ref. 2) and for the fractional quantum Hall effect at 5/2 filling.[15,16] In solid-state realizations, TRS-breaking (TRSB) $p_x + ip_y$ superconductivity has been proposed for Sr_2RuO_4 (Ref. 17) and found to be in agreement with the measurements of the polar Kerr effect, specifically designed to measure TRS breaking.[18] There are no experimental realizations yet of TRSB spin-singlet superconductivity, although $d_{x^2-y^2} + id_{xy}$ state has been proposed theoretically for hexagonal systems near van-Hove doping, including doped graphene,[19–21] SrPtAs [22] and, possibly, cobaltates.[23]

The search for TRS breaking superconductivity intensified with the discovery of Fe-based superconductors (FeSCs). These systems have multiple Fermi surfaces, and intra-pocket and inter-pocket interactions vary along each Fermi surface as the consequence of different orbital compositions of low-energy excitations.[24] This variation opens a possibility that the pairing interaction is attractive in more than one channel. In FeSCs, the two leading candidates are s^{+-} and $d_{x^2-y^2}$ channels. The majority of researchers believe that in moderately doped FeSCs s^{+-} superconductivity wins, but some experiments on heavily hole-doped system $Ba_{1-x}K_xFe_2As_2$ with $x = 1$ were interpreted in favor of $d_{x^2-y^2}$ superconductivity[25]. This interpretation is not universally accepted[26], but if it is correct, then one can expect that there will be a mixed state at $x \leq 1$, in which both s and d components are present. According to calculations[27] relative phase between s and d-components is $\pm\pi/2$ (an $s + id$ state). In such a state time-reversal(TR) and C_4 lattice rotational symmetry are simultaneously broken, together with $U(1)$ phase symmetry, and this gives rise to a number of non-trivial properties, including circulating supercurrents near a nonmagnetic impurity.[28]

In a separate line of research, several groups argued that the gap symmetry in $Ba_{1-x}K_xFe_2As_2$ remains s^{+-} for all x. At optimal doping s^{+-} gaps on all hole pockets have the same sign, opposite to that on electron pockets. At $x = 1$ (i.e., in KFe_2As_2) only hole pockets are present[29] and, if the gap remains s^{+-}, it must change sign between the two inner hole pockets[30]-a qualitatively different s-wave state. It has been argued[31] that the transformation of one s^{+-} structure into the other is not continuous at low enough T and involves an intermediate state in which the phases of the gaps on the two inner hole pockets differ by $0 < \alpha < \pi$ (see Fig. 2). Such a state breaks TRS, despite that it has pure s-wave symmetry, and was termed $s + is$.

The issue we discuss in this letter is how to detect the $s + is$ state experimentally. One option is to detect low-energy Leggett-type modes in the $s+is$ state[31,32] by, e.g., Raman measurements, or to detect anomalous behavior of the upper critical field[33]. But it would be much more desirable to have experimental probes which would directly detect TRS breaking. In this respect, $s + is$ state presents a challenge. The TRS-broken states studied before break other discrete symmetries in addition TR, e.g., $s + id_{x^2-y^2}$ state breaks C_4 symmetry and $d_{x^2-y^2} + id_{xy}$ breaks mirror symmetries. The $s + is$ state only breaks TRS but keeps lattice symmetries intact. We show, by analyzing the Ginzburg-Landau Free energy functional, that in this situation there are no circulating supercurrents near a non-magnetic impurity, although spontaneous currents are predicted to exist around topological defects.[34]. The absence of the currents can be also traced to the fact that the spontaneous current comes from the intrinsic angular momentum of the Cooper pairs, rather than the broken time-reversal symmetry[35] This intrinsic momentum is zero for $s + is$ state. We show, however, that supercurrents do develop if a system with $s+is$ gap is put under external strain that breaks the C_4 lattice rotational symmetry. Our results are in agreement with recent zero-field muon-spin relaxation study of superconducting $Ba_{1-x}K_xFe_2As_2$ for $0.5 < x < 0.9$ (Ref. [36]). The measurements showed no evidence of spontaneous internal magnetic fields at temperatures down to 0.02 K.

We propose to preform the same μSR measurement under external strain.

$s+id$ superconductor It is instructive to consider first $s+id$ state for which numerical calculations[28] have shown the presence of spontaneous currents around an inhomogeneity. This will help us understand the distinction between $s+id$ and $s+is$ states and set up necessary conditions for the existence of currents in an $s+is$ superconductor.

For a candidate $s+id$ system, the Free energy can be written as the combination of the homogeneous and spatially varying parts: $\mathcal{F} = \mathcal{F}_h + \mathcal{F}_s$, where

$$\begin{aligned}
\mathcal{F}_h &= \alpha_s|\Delta_s|^2 + \alpha_d|\Delta_d|^2 + \beta_1|\Delta_s|^4 + \beta_2|\Delta_d|^4 \\
&\quad + \beta_3|\Delta_s|^2|\Delta_d|^2 + \beta_4\left(\Delta_s^*\Delta_s^*\Delta_d\Delta_d \ + \ \text{c.c.}\right) \\
\mathcal{F}_s &= \gamma_s|\vec{D}\Delta_s|^2 + \gamma_d|\vec{D}\Delta_d|^2 \\
&\quad + \gamma_{sd}\left[(\vec{D}_x\Delta_s)^*\vec{D}_x\Delta_d - (\vec{D}_y\Delta_s)^*\vec{D}_y\Delta_d \ + \ \text{c.c.}\right]
\end{aligned} \quad (1)$$

Here Δ_d is the magnitude of $d_{x^2-y^2}$ gap ($=\Delta_d\cos 2\theta$) and the integration over θ is already carried out. The two order parameters are $U(1)$ fields $\Delta_s = \Delta e^{i\phi_s}$ and $\Delta_d = \Delta e^{i\phi_d}$. In the spatially varying part $\vec{D} \equiv -i\vec{\partial} - \frac{2e}{c}\vec{A}$ and all derivatives act of the center of mass co-ordinate of the Cooper pair.[37] We assume that the parameters α_i and β_i of the homogeneous part $\mathcal{F}_h$ are such that the ground state is a TRSB $s+id$ superconductor with $\phi_d - \phi_s = \pm\pi/2$.

The term with the prefactor γ_{sd} depends on the relative phase $\phi_s - \phi_d$ of the two order parameters, but not on the cumulative phase $\phi_s + \phi_d$. This term is consistent with the symmetry of $s+id$ state as it remains invariant if one changes the relative phase $\phi_d - \phi_s$ by π and simultaneously rotates the reference frame by 90^o. Both symmetry operations change $\Delta_d \to -\Delta_d$, and $\mathcal{F}$ is invariant under the product of these two operations.

The γ_{sd} term in the free energy (1) gives rise to circulating currents around inhomogeneities. For this, we introduce an isotropic impurity at $\mathbf{r} = \mathbf{0}$, obtain coordinate-dependent $\Delta_s(\mathbf{r})$ and $\Delta_d(\mathbf{r})$, and compute the current density $\vec{j} = -\frac{\partial \mathcal{F}_s}{\partial \vec{A}}\,|_{\vec{A}=0}$. We follow Ref. 28 and assume that an impurity introduces coordinate dependencies of the prefactors α_i in (1) via $\alpha_i \to \alpha_i + \alpha_{\rm imp,i}(\vec{r})$ ($i = s, d$), where $\alpha_{\rm imp,i}(\vec{r})$ is a decreasing function of r. For this work we chose $\alpha_{\rm imp,i}(\vec{r}) = \alpha_0 e^{-(r/r_0)^2}$. The current density is then expressed as[37]

$$\vec{j} = \frac{j_0}{2}\frac{\gamma_s - \gamma_d}{\sqrt{\gamma_s\gamma_d}}\left[\vec{\partial}(t_+ - t_-) + \tilde{\gamma}\vec{\partial}_d(t_+ + t_-)\right], \quad (2)$$

where $\vec{\partial}_d \equiv \hat{x}\partial_x - \hat{y}\partial_y$ and

$$t_\pm \equiv t_\pm(r) = \int \frac{d^2k}{(2\pi)^2}\frac{e^{i\vec{k}\cdot\vec{r}}\tilde{\alpha}_{\rm imp}}{k_x^2(1\pm\tilde{\gamma}) + k_y^2(1\mp\tilde{\gamma}) + \delta^2}. \quad (3)$$

In Eq. (3) $\tilde{\alpha}_{\rm imp}$ is the Fourier transform of $\alpha_{\rm imp}$, $\tilde{\gamma} \equiv \gamma_{sd}/\sqrt{\gamma_s\gamma_d}$, and δ is the mass scale set by β_i in (1). One can easily verify that $\vec{j}$ satisfies the continuity equation $\vec{\nabla}\cdot\vec{j} = 0$. The difference $t_+ - t_-$ contains $\tilde{\gamma} \propto \gamma_{sd}$ as the overall factor (see Eq. (3), hence both terms in the square brackets in (2) scale with γ_{sd}, and $|\vec{j}| \propto \gamma_{sd}$. We see that the current is entirely due to the mixed γ_{sd} term in (1), which, we remind, remains invariant under the product of C_4 rotation and TR. The presence of two symmetry transformations, each of which changes Δ_d to $-\Delta_d$, is therefore crucial to obtain a non-zero circulating current around an impurity. The current $\mathbf{j}(\mathbf{r})$ is plotted in Fig. 1 and agrees well with the numerical results.[28]

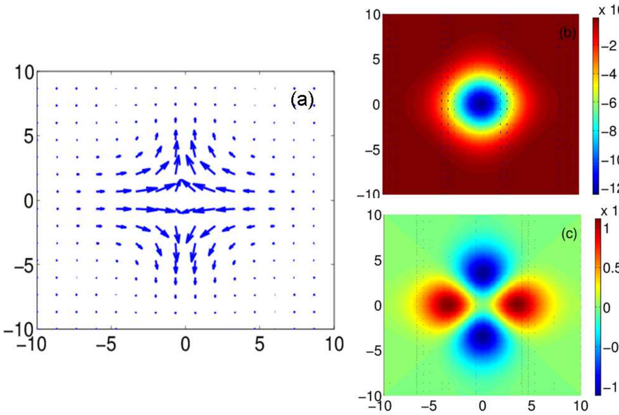

FIG. 1: (a)The profile of supercurrent (blue arrows) in real space in the $s+id$ superconductor around an isotropic impurity located at the origin. The current satisfies the continuity equation and creates a magnetic quadrupole moment centered on the impurity site. We set $\tilde{\gamma} = 1$, sample size $L = 10r_0$, $\delta = 0.5/r_0$. (b) and (c) are the amplitude and phase fluctuations, $\gamma_s m_s = \gamma_d m_d$ and $\sqrt{\gamma_s\gamma_d}\phi_d = -\sqrt{\gamma_s\gamma_d}\phi_d$, respectively. Note the $d-$wave symmetry of phase fluctuations.

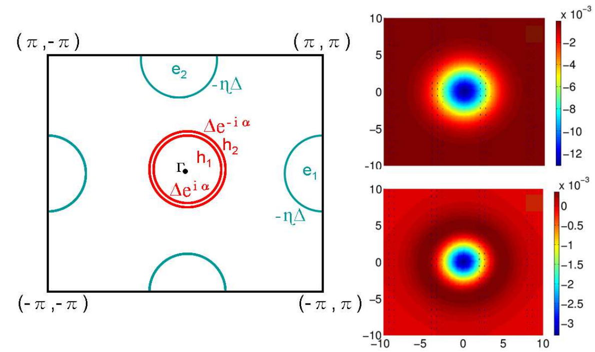

FIG. 2: Left: The Fermi surface of the microscopic model considered in the text. It has two hole pockets around the Γ point and two electronic pockets at $(\pi,0)$ and $(0,\pi)$ points of the Brillioun zone. We tune the parameters of this model to the TRSB state and expand the gaps in spatial fluctuations around the impurity site in TRSB $s+is$ state to obtain the currents. Right: The spatial variation of the amplitude (m_1, top) and the phase (ϕ_1, bottom) of the gap on one of hole pockets without external strain. Observe that there is no phase modulation unlike in $s+id$ state.

$s+is$ superconductor We now apply the same phe-

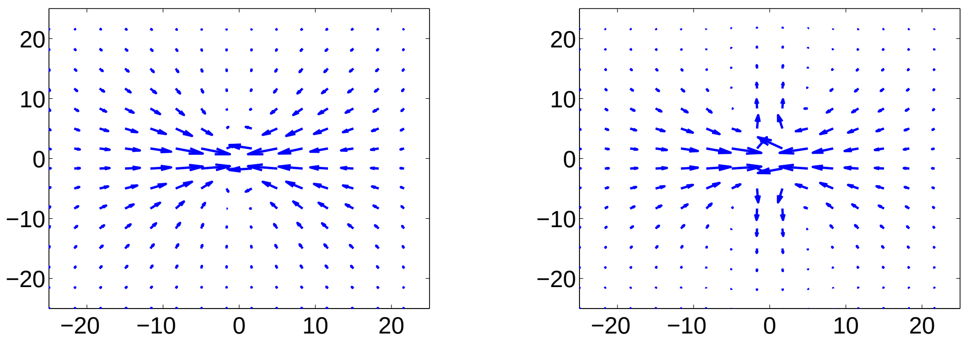

FIG. 3: The profile of a supercurrent for the $s+is$ state. We set the parameter ϵ, related to external breaking of C_4 symmetry, to be $\varepsilon = 0.1\sqrt{a_1 a_2}$, and $0.3\sqrt{a_1 a_2}$ in left and right panels, respectively. We also set $a_h = 0.5\sqrt{a_1 a_2}$, $\delta = 0.1/r_0$, and the sample size $L = 25 r_0$ where r_0 is related to impurity-induced correction to the prefactor α in the quadratic part in the free energy via $\alpha_{\rm imp}(\vec r) \propto e^{-(r/r_0)^2}$. The asymmetry in the current pattern from x to y is due to the applied strain.

nomenological description to $s+is$ superconductor with order parameters Δ_s and $\Delta_{s'}$. The generic form of $\mathcal{F}$ remains the same as in Eq. (1), we only need to replace Δ_d by $\Delta_{s'}$, γ_d by $\gamma_{s'}$, γ_{sd} by $\gamma_{ss'}$, and change the sign between D_x and D_y parts of the mixed term, which for C_4-preserving $s+is$ superconductor becomes

$$\gamma_{ss'}\left[(\vec D_x \Delta_s)^* \vec D_x \Delta_{s'} + (\vec D_y \Delta_s)^* \vec D_y \Delta_{s'} \; + \; \text{c.c.}\right], \quad (4)$$

Such term is allowed by symmetry as it again depends only on the relative phase of Δ_s and $\Delta_{s'}$ and is invariant under gauge transformation and TR operation. However, in distinction to $s+id$ case, there is no possibility to compensate the change of the phase of Δ_s' by π by a rotation of the coordinate frame by 90^o. As a result, the contributions from this term must vanish in the $s+is$ state, hence it should not give rise to a circulating current. To see this explicitly, we calculate the $\vec j$ using the cross-term with the derivatives in the form of Eq. (4). The current is still given by Eq. (2), but now $t_\pm \sim \tilde\alpha_{\rm imp}/(k^2+\delta^2)(1\pm\tilde\gamma)$. Substituting this into (2) we We find that $\vec j$ indeed vanishes:

$$\begin{aligned}\vec j \;&=\; \frac{j_0}{2}\frac{\gamma_s - \gamma_{s'}}{\sqrt{\gamma_s \gamma_{s'}}}\vec\partial \int \frac{d^2k}{(2\pi)^2}\frac{e^{i\vec k\cdot\vec r}\tilde\alpha_{\rm imp}}{k^2+\delta^2}\left(\frac{-2\tilde\gamma}{1-\tilde\gamma^2}+\frac{2\tilde\gamma}{1-\tilde\gamma^2}\right)\\ &=\; 0. \end{aligned} \quad (5)$$

We now substantiate this reasoning with the analysis of the minimal band-resolved model which displays $s+is$ superconductivity. The model consists of two Γ-centered hole pockets and two electron pockets centered around $(0,\pi)$ and $(\pi,0)$ in the one-Fe Brillouin zone, as shown in Fig. 2. In the physical two-Fe Brillouin zone, the two electron pockets are centered at (π,π) and hybridize into inner and outer pockets. We label the gap functions on hole pockets as $\Delta_{h_1}, \Delta_{h_2}$ and use Δ_{e_1} and Δ_{e_2} for inner and outer electron pockets. As discussed in Ref. 31, an $s+is$ can be realized at large hole doping where $\Delta_{h_1} = \Delta_1 e^{i\alpha_1}$, $\Delta_{h_2} = \Delta_2 e^{i\alpha_2}$, and $\alpha_1 - \alpha_2$ is a fraction of π. The spatially varying part of the free energy for this state is

$$\begin{aligned}\mathcal{F}_s \;=\;& a_1|\vec D\Delta_{h_1}|^2 + a_2|\vec D\Delta_{h_2}|^2 + a_3|\vec D\Delta_{e_1}|^2 + \\ & a_4|\vec D\Delta_{e_2}|^2 + a_h\left[(\vec D\Delta_{h_1})^*\cdot(\vec D\Delta_{h_2}) \;+\; c.c\right] \\ & + a_{h_1e_1}\left[(\vec D\Delta_{e_1})^*\cdot(\vec D\Delta_{h_1}) \;+\; c.c\right] \;+\; [e_1\to e_2] \\ & + a_{h_2e_1}\left[(\vec D\Delta_{e_1})^*\cdot(\vec D\Delta_{h_2}) \;+\; c.c\right] \;+\; [e_1\to e_2] \\ & + a_e\left[(\vec D\Delta_{e_1})^*\cdot(\vec D\Delta_{e_2}) \;+\; c.c\right]\end{aligned} \quad (6)$$

The terms with products $(\vec D\Delta_i)^*\cdot(\vec D\Delta_j)$ with $i\neq j$ depend on the relative phases of Δ_i and Δ_j and their structure reproduces that in Eq. (4).

Differentiating the free energy with respect to vector potential we obtain the current $\vec j = -\frac{\partial\mathcal{F}_s}{\partial\vec A}|_{\vec A=0}$ in the form

$$\begin{aligned} i\frac{j_x}{2e/c} \;=\;& a_1\Delta^*_{h_1}\check\partial_x\Delta_{h_1} + a_2\Delta^*_{h_2}\check\partial_x\Delta_{h_2} + a_3\Delta^*_{e_1}\check\partial_x\Delta_{e_1} \\ & + a_4\Delta^*_{e_2}\check\partial_x\Delta_{e_2} + a_h\left[\Delta^*_{h_1}\check\partial_x\Delta_{h_2} \;-\; \text{c.c.}\right] \\ & + a_{h_1e_1}\left[\Delta^*_{e_1}\check\partial_x\Delta_{h_1} \;-\; \text{c.c.}\right] \;+\; [e_1\to e_2] \\ & + a_{h_2e_1}\left[\Delta^*_{e_1}\check\partial_x\Delta_{h_2} \;-\; \text{c.c.}\right] \;+\; [e_1\to e_2] \\ & + a_e\left[\Delta^*_{e_1}\check\partial_x\Delta_{e_2} \;-\; \text{c.c.}\right], \end{aligned} \quad (7)$$

where $g\check\partial h \equiv g\partial h - h\partial g$. The expression for j_y is obtained by interchanging $\check\partial_x$ by $\check\partial_y$.

We perform the same computational steps as for $s+id$ superconductor: introduce an impurity at $\vec r = 0$ and expand the order parameters to linear order around the homogeneous solution for $s+is$ state. We first consider the simplest case when we treat the two hole pockets and the two electron pockets as identical in the spatially varying part of the free energy, i.e., set $a_1 = a_2$, $a_3 = a_4$, $a_{h_ie_j} = a_{he}$ in Eq. (6). The homogeneous solution for identical hole and identical electron pockets is[31] $\Delta_{h_1} = \Delta e^{i\alpha}$, $\Delta_{h_2} = \Delta e^{-i\alpha}$, $\Delta_e = -\eta\Delta$, where η and α are functions of doping. For the critical doping where T_c^{TRSB} is the largest, $\eta = \sqrt{2}$ and $\alpha = \pi/4$. We expand the gaps as $\Delta_{h_1} = \Delta e^{i\alpha}(1 + m_1 + i\phi_1)$, $\Delta_{h_2} = \Delta e^{-i\alpha}(1 + m_2 + i\phi_2)$, $\Delta_{e_1} = \Delta_{e2} = -\eta\Delta(1 + m_e + i\phi_e)$, substitute the expansion into (7) and obtain for the current

$$\begin{aligned} j_x &= j_0 \left[(a_1 + a_h \cos 2\alpha - 2a_{he} r \cos\alpha) \partial_x(\phi_1 + \phi_2) \right. \\ &\quad + (2a_3 r^2 + 2a_e r^2 - 4a_{he} r \cos\alpha)\partial_x \phi_e \\ &\quad \left. + (a_h \sin 2\alpha - 2a_{he} r \sin\alpha)\partial_x(m_1 - m_2) \right], \end{aligned} \tag{8}$$

and j_y is obtained by interchanging x and y. Minimizing $\mathcal{F}_s$ with respect to variations m_i, ϕ_i, m_e and ϕ_e we obtain that $\phi_e = 0$ and $m_1 = m_2$, $\phi_1 = -\phi_2$, where

$$m_1 + i\phi_1 = \int \frac{d^2k}{(2\pi)^2} e^{i\vec{k}\cdot\vec{r}} \frac{a_3 + a_e + 2a_{he}}{G} e^{i\alpha} \frac{-\tilde{\alpha}_{\text{imp}}}{k^2 + \delta^2},$$

and $G \equiv (a_1 + a_h)(a_3 + a_e) - 4a_{he}^2\eta$. Substituting this into Eq. (8) we see that each term in (8) vanishes, i.e., $\vec{j} = 0$. We extended the analysis to non-equivalent hole pockets and to distinct inner and outer electron pockets, and obtained the same result – spontaneous supercurrent around an impurity vanishes[37].

$s + is$ superconductor under external strain The comparative analysis of $s + id$ and $s + is$ cases shows a way how one can generate supercurrents in an $s + is$ superconductor — one has to externally break C_4 rotational symmetry. This can be achieved, for example, by application of a small uniaxial strain. For Eq. (4) breaking of C_4 implies that $\gamma_{s,s'}$ along x and y directions become non-equivalent: $\gamma_{ss}^x = \gamma_{ss} + \gamma_{ss}^*$ and $\gamma_{ss}^y = \gamma_{ss} - \gamma_{ss}^*$ Then the cross-term becomes the sum of (4) and the new term

$$\gamma_{ss'}^* \left[(\vec{D}_x \Delta_s)^* \vec{D}_x \Delta_{s'} - (\vec{D}_y \Delta_s)^* \vec{D}_y \Delta_{s'} + \text{c.c.} \right], \tag{9}$$

This last term has the same form as for $s + id$ superconductor and should give rise to a supercurrent. Another way to state the same is to recall that breaking of C_4 mixes $s-$wave and $d-$wave channels, i.e. $s-$wave gaps change from Δ_a to $\Delta_{a,1} + \epsilon\Delta_{a,2}\cos 2\theta_a$, where $a = s, s'$. Selecting the products $\Delta_{s,1}\Delta_{s',2}$ and $\Delta_{s',1}\Delta_{s,2}$, we recover Eq. (9) with $\gamma_{ss'}^* \propto \epsilon$. Note in passing that in KFe_2As_2 ϵ and $\gamma_{ss'}^*$ are additionally enhanced because $s-$wave and $d_{x^2-y^2}$ channels are nearly degenerate[27,30].

We now apply this reasoning to our model for $s + is$ superconductor. To simplify the presentation, we assume that the effect of strain is the strongest on the a_h term in $\mathcal{F}_s$, set $a_h^x = a_h + \varepsilon$ and $a_h^y = a_h - \varepsilon$, and ignore $a_{h_i e_j}$ and a_e terms which involve electron pockets. Within this approximation,

$$\begin{aligned} j_x &= j_0 \left[\partial_x(a_1\phi_1 + a_2\phi_2) + (a_h + \varepsilon)\cos 2\alpha \ \partial_x(\phi_1 + \phi_2) \right. \\ &\quad \left. + (a_h + \varepsilon)\sin 2\alpha \ \partial_x(m_1 - m_2) \right], \end{aligned} \tag{10}$$

and j_y is obtained by interchanging $x \to y$ and $\varepsilon \to -\varepsilon$.

The coordinate-dependent functions m_j and ϕ_j ($j = 1, 2$) are obtained by minimizing $\mathcal{F}_s$ with respect to fluctuations. Performing the same calculations as before we obtain

$$\begin{aligned} j_x &= 2j_0\varepsilon \sin\alpha \frac{a_1 - a_2}{\sqrt{a_1 a_2}} \partial_x \int \frac{d^2k}{(2\pi)^2} e^{i\vec{k}\cdot\vec{r}} \left(\frac{\tilde{\alpha}_{\text{imp}} k_y^2}{Z} \right), \\ j_y &= -2j_0\varepsilon \sin\alpha \frac{a_1 - a_2}{\sqrt{a_1 a_2}} \partial_y \int \frac{d^2k}{(2\pi)^2} e^{i\vec{k}\cdot\vec{r}} \left(\frac{\tilde{\alpha}_{\text{imp}} k_x^2}{Z} \right), \end{aligned} \tag{11}$$

where $Z = a_1 a_2 k^4 - [a_h k^2 + \varepsilon(k_x^2 - k_y^2)]^2$. We see that the current is non-zero only when $\alpha \neq 0, \pm\pi$, $\varepsilon \neq 0$ (i.e. when TRS *and* C_4 symmetry are simultaneously broken) and $a_1 \neq a_2$ (i.e. when the two hole pockets have inequivalent fluctuations). The latter is analogous to the condition $\gamma_s \neq \gamma_d$ for $s + id$ superconductor; see Eq. (2). We also emphasize that, although electron pockets do not contribute to $\vec{j}$ in Eq. (11), they are essential to have $\alpha \neq 0, \pm\pi$. We show the current profile for $\vec{j}$ given by Eq. (11) in Fig 3. It is quite similar to the one for $s + id$ superconductor. One can explicitly verify that $\vec{\nabla} \cdot \vec{j} = 0$ is satisfied.

Conclusions In this paper we analyzed $s + is$ state proposed as a candidate superconducting state for strongly hole-doped $Ba_{1-x}K_xFe_2As_2$. This state breaks TRS but does not break any other discrete symmetry. We addressed the issue whether TRS breaking alone can generate spontaneous supercurrents, which can potentially be detected in μSR experiments. Our conclusion is that there are no currents if only TRS is broken. We argue, however, that spontaneous supercurrents do emerge in an $s + is$ superconductor if the system is put under external strain and C_4 lattice rotation symmetry is externally broken. We call for μSR measurements in KFe_2As_2 under external strain.

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