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Probing Majorana Physics in Quantum Dot Shot Noise Experiments

Dong E. Liu, Meng Cheng, and Roman M. Lutchyn
Microsoft Research, Station Q, University of California, Santa Barbara, CA, 93106, USA
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We consider a quantum dot coupled to a topological superconductor and two normal leads and study transport properties of the system. Using Keldysh path-integral approach, we study current fluctuations (shot noise) within the low-energy effective theory. We argue that the combination of the tunneling conductance and the shot noise through a quantum dot allows one to distinguish between the topological (Majorana) and non-topological (e.g., Kondo) origin of the zero-bias conduction peak. Specifically, we show that, while the tunneling conductance might exhibit zero-bias anomaly due to Majorana or Kondo physics, the shot noise is qualitatively different in the presence of Majorana zero modes.

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Introduction. The search for topological superconductors hosting non-Abelian quasiparticles (defects binding Majorana zero modes) has become an active and exciting pursuit in condensed matter physics[1–3]. There has been enormous theoretical and experimental activity in this direction recently [4] fueled, in part, by the potential application of topological superconductors for the fault-tolerant topological quantum computation schemes[5]. A large number of theoretical proposals for engineering topological superconductors in the laboratory has been put forward [6–15], and there has been a significant amount of experimental activity in this area recently [16–25]. One of the simplest ways to detect the presence of Majorana zero modes (MZMs) in topological superconductors (TSC) is tunneling spectroscopy. Indeed, the presence of MZMs leads to a quantized zero-bias conductance $G = 2e^2/h$ [26–33]. The pioneering Majorana experiment based on a semiconductor/superconductor heterostructure proposal [10, 11] was performed in Delft [16] where the observation of zero-bias peak in a finite magnetic field was reported, consistent with the theoretical predictions[31]. However, other effects might also lead to the zero-bias anomaly which spurred the debate[34–41] as to the precise origin of the (un-quantized) zero-bias conduction peak observed in recent tunneling experiments [16–21]. Therefore, additional experiments testing other properties on MZMs[17, 36, 37, 42–54] are necessary in order to reach a consensus.

In this Letter, we propose a new scheme, which combines the tunneling conductance and current fluctuation measurements, to distinguish between the topological (Majorana) and non-topological origin of the zero-bias peak. We consider a quantum dot (QD) coupled to a MZM and two normal leads, see Fig. 1. One can extract information about MZMs by measuring the shot noise between two normal leads. This approach allows one to eliminate a number of false-positive features by simply changing either Majorana coupling or the QD couplings. Indeed, while the Kondo effect as well as resonant-tunneling physics exhibit zero-bias peaks

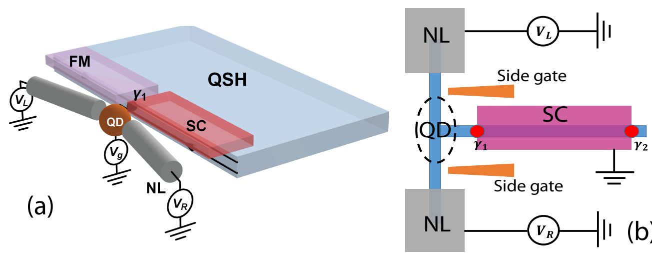

FIG. 1. Proposed experimental setup for the shot noise measurement. (a) A MZM is formed at the domain wall between a ferromagnetic insulator and a s-wave superconductor at the QSH edge. (b) A MZM is formed at the ends of topological superconducting wire, and QD is formed near the wire T-junction.

in the tunneling conductance, their current fluctuations are *qualitatively* different. Thus, it is suggestive to use shot noise as a diagnostic tool for MZMs. The physics of the QD coupled to the MZM has been discussed in Refs. [42, 44, 47, 55]. It has been shown that Majorana coupling significantly modifies the low-energy properties of the QD and drives the system to a new (different from Kondo) fixed point [47]. Building on top of the slave boson formalism developed in Ref. [47], we compute here the shot noise in the system shown in Fig. 1. It is well-known that noise measurements usually provide additional information for correlated systems [56–60] and often allow one to identify the nature of the charge carriers. The shot noise for the non-interacting systems such as the normal lead-TSC and non-interacting QD-TSC have been considered in Refs.[51, 61–64]. In this paper, we address this important and non-trivial question and obtain analytically the power spectrum of shot noise in the presence of the Coulomb interactions in QD by taking into account the interplay between Kondo and Majorana physics.

Our main results are summarized in Table I. We find that in the case of symmetric couplings to the leads $\Gamma_L = \Gamma_R$ the shot noise power spectrum $P(\omega \to 0)$ in the presence of MZM coupling exhibits a universal

TABLE I. Shot noise power spectrum $P(\omega \to 0)$ and conductance G [43, 45, 46] for $\Gamma_L = \Gamma_R$.

	spinless system (non-interacting $U = 0$)	spinful system: Kondo regime ($\|\epsilon_d\| \gg \lambda, \Gamma$)
with MZM	$P(0) = \frac{e^2}{2h}$ and $G = \frac{e^2}{2h}$, (independent of ϵ_d)	$P(0) = \frac{e^2}{2h}$ and $G = \frac{3e^2}{2h}$
without MZM	$P(0) = 0$ and $G = \frac{e^2}{h}$ for $\epsilon_d = 0$	$P(0) = 0$ and $G = \frac{2e^2}{h}$

value $e^2/2h$ which is independent of the QD energy level ϵ_d, and corresponds to the transmission probability $T(0) = \frac{1}{2}$. This is to be contrasted with the resonant level model which exhibits a strong dependence on ϵ_d. In the Kondo limit, tunneling conductance exhibits the zero-bias anomaly but the shot noise power spectrum is zero $P(\omega \to 0) = 0$. Thus, the combination of both shot noise and conductance through a QD allows one to distinguish between the Majorana and other physics. We believe that our results are relevant for the ongoing Majorana experiments since conductance and current fluctuations can be readily accessed.

Theoretical Model. We consider a setup shown in Fig. 1 in which a QD is coupled to a MZM γ_1 localized at the domain wall between a magnetic insulator and an s-wave superconductor at the edge of a Quantum Spin Hall (QSH) insulator [7] or localized at the ends of the topological superconducting wire [10, 11, 16]. We assume here that the superconducting gap Δ is large, and develop an effective low-energy theory for the system valid at $E \ll \Delta$:

$$H = H_{\text{Leads}} + H_{\text{Dot}} + H_{\text{L-D}} + i\lambda(d_\uparrow + d_\uparrow^\dagger)\gamma_1 + i\delta\gamma_1\gamma_2. \quad (1)$$

Here $H_{\text{Leads}} = \sum_{\alpha=L,R}\sum_{k,\sigma}\epsilon_k c^\dagger_{k\sigma,\alpha}c_{k\sigma,\alpha}$, $H_{\text{QD}} = \sum_\sigma \epsilon_d d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow$, and $H_{\text{L-D}} = \sum_{\alpha=L,R}\sum_{k,\sigma}(t_{\alpha k}c^\dagger_{k\sigma,\alpha}d_\sigma + h.c.)$ describe the leads, QD, and the Lead-QD coupling, respectively. The operators $c^\dagger_{k\sigma,\alpha}$ ($d^\dagger_\sigma$) create a spin-σ electron in the α-lead (the dot), $n_\sigma = d^\dagger_\sigma d_\sigma$, ϵ_d is the chemical potential of the QD, U is the QD on-site Coulomb interaction, and $t_{\alpha k}$ (λ) is the tunneling coupling between the leads (TSC) and the QD. The splitting energy δ represents the finite overlap between two MZMs. We note that the time-reversal symmetry is broken by the magnetic insulator which also determines the spin polarization of the MZM. Without any loss of generality we assume γ_1 only couples to the spin up channel of the QD. The lead and QD Hamiltonians remain SU(2)-invariant under spin rotation.

We first integrate out Majorana operators γ_1 and γ_2, which leads to the self-energy $\Sigma(\omega)$ (defined below). We assume that the QD is in the single-occupancy regime $U \gg |\epsilon_d| \gg \lambda, \Gamma$ with Γ being the broadening of the QD level due to normal leads $\Gamma = \Gamma_L + \Gamma_R$ with $\Gamma_\alpha = \pi|t_\alpha|^2\rho_F$; here ρ_F is the density state of the leads at the Fermi level. In this limit, one can use a slave boson approximation for an infinite-U Anderson model[65, 66] where the double occupancy of the QD is suppressed. Following standard procedure [65, 66], one can introduce the auxiliary boson b and fermion f_σ in order to $d_\sigma \to f_\sigma b^\dagger$, with the constraint $b^\dagger b + \sum_\sigma f^\dagger_\sigma f_\sigma = 1$. Within the slave boson mean field approximation (SBMF), we replace the bosonic field and the Lagrangian multiplier η by their expectation values. The mean field parameter b and η can be determined self-consistently by minimizing the free energy. The detail of SBMF calculation in the presence of a MZM can be found in Ref. [47] (also see [67]). The SBMF approach decouples the spin-up channel from the spin-down channel and allows one to compute various correlation functions.

Shot noise calculation. We now use the Keldysh formalism [68] to study current fluctuations. Since two spin channels are decoupled within SBMF approximation, we drop the index σ in this derivation. Given that MZM coupling breaks particle number conservation, the QD Green's function now acquires an anomalous contribution (e.g. $i\langle T_c d(t)d(t')\rangle$), and we need to work in the Nambu space $\mathbb{N}$. We introduce a lead-QD basis $\vec{\Psi}^\dagger = (\{c^\dagger_{Lk}, c_{Lk}\}, d^\dagger, d, \{c^\dagger_{Rk}, c_{Rk}\})/\sqrt{2}$, and write the action in this new space $\mathbb{S}$. The effective action can be written in terms of the full Green function $\breve{Q}$

$$S = S_0 + S_{L-D} + S_{\text{source}}, \quad (2)$$

$$S_0 + S_{L-D} = \int_C \int_C dt dt' \vec{\Psi}^\dagger(t)\breve{Q}^{-1}(t,t')\vec{\Psi}(t'), \quad (3)$$

$$\breve{Q}_{kk'} = \begin{pmatrix} Q_{Lk,Lk'} & Q_{Lk,d} & Q_{Lk,Rk'} \\ Q_{d,Lk'} & Q_{d,d} & Q_{d,Rk'} \\ Q_{Rk,Lk'} & Q_{Rk,d} & Q_{Rk,Rk'} \end{pmatrix} \quad (4)$$

All matrix elements above have the same structure, e.g. $Q_{d,d} = \{\{G_{d\bar{d}}, F_{dd}\}, \{F_{\bar{d}\bar{d}}, G_{\bar{d}d}\}\}$, see SI [67] for details of $\breve{Q}_{kk'}$. After restoring the spin index, the retarded Green's functions are given by

$$G^R_{d\bar{d},\sigma}(\omega) = \frac{\omega + \widetilde{\epsilon}_d + i\widetilde{\Gamma} - \Sigma_\sigma(\omega)}{(\omega + i\widetilde{\Gamma} - 2\Sigma_\sigma(\omega))(\omega + i\widetilde{\Gamma}) - \widetilde{\epsilon}_d^2}, \quad (5)$$

$$F^R_{dd,\sigma}(\omega) = \frac{-\Sigma_\sigma(\omega)}{(\omega + i\widetilde{\Gamma} - 2\Sigma_\sigma(\omega))(\omega + i\widetilde{\Gamma}) - \widetilde{\epsilon}_d^2}, \quad (6)$$

where $\Sigma_\sigma(\omega) = \lambda^2_\sigma b^2\omega/(\omega^2 - \delta^2)$ with $\lambda_\uparrow = \lambda b$ and $\lambda_\downarrow = 0$. The effective broadening and energy of the QD level now read $\widetilde{\Gamma} = \Gamma b^2$, $\widetilde{\epsilon}_d = \epsilon_d + \eta$.

We now consider current fluctuations through the left junction. The current operator is given by

$$I_L = \frac{ie}{\hbar}\sum_k \left(\widetilde{t}_{Lk}c^\dagger_{Lk}d - \widetilde{t}^*_{Lk}d^\dagger c_{Lk}\right) = \vec{\Psi}^\dagger\hat{M}\vec{\Psi}, \quad (7)$$

The 6-by-6 matrix $\hat{M}$ in $\mathbb{N}\otimes\mathbb{S}$ space for lead momentum k is $\hat{\mathrm{M}}_k = (ie/\hbar)\{\{0, M^{12}_k, 0\}, \{M^{21}_k, 0, 0\}, \{0,0,0\}\}$ where

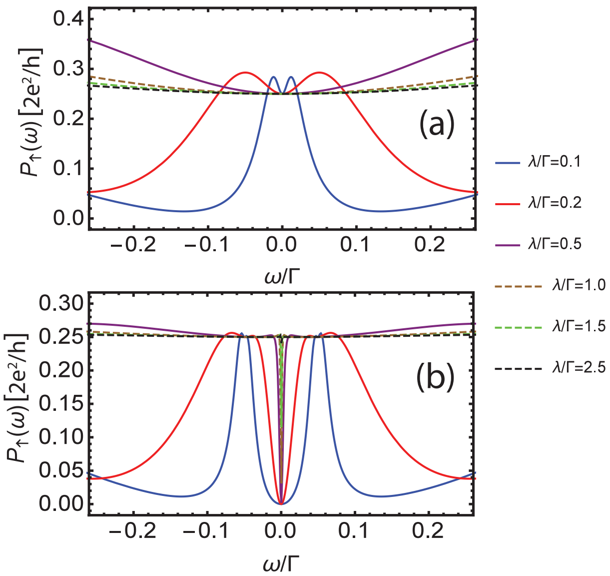

FIG. 2. The power spectrum $P_\uparrow(\omega)$ for $\Gamma_L = \Gamma_R$ and different λ. Panel (a) no splitting of MZMs $\delta/\Gamma = 0$. The non-monotonic dependence of $P_\uparrow(\omega)$ originates from the non-trivial dependence of the P-H contribution $\mathbb{A}_A$ to the shot noise; (b) splitting energy $\delta/\Gamma = 0.05$. Here we used $\epsilon_d = 0$.

$M^{21}_k = \left(\begin{smallmatrix} -\tilde{t}^*_{Lk} & 0 \\ 0 & -\tilde{t}_{Lk}\end{smallmatrix}\right)$ and $M^{12}_k = \left(\begin{smallmatrix} \tilde{t}_{Lk} & 0 \\ 0 & \tilde{t}^*_{Lk}\end{smallmatrix}\right)$. Then, the action for the source term is

$$S_{\rm source} = -\int_C dt A(t) I_L(t) = -\int_{-\infty}^{\infty} dt \vec{\Psi}^\dagger_a \hat{A}_{ab}\hat{M}\vec{\Psi}_b. \quad (8)$$

Here we rewrote the action in terms of the forward and backward components and performed Larkin-Ovchinnikov rotation[68]. As a result, the source $\hat{A} = A^\alpha \hat{\gamma}^\alpha$ is now a matrix in Keldysh $\mathbb{K}$ space, where $\alpha = cl, q$ with $\hat{\gamma}^{cl} = \mathbb{I}$ and $\hat{\gamma}^q = \sigma_1$, see details in SI[67]. The generating function for this problem $Z[A] = \int D[\{c^\dagger_{Lk}c_{Lk}\}d^\dagger d\{c^\dagger_{Rk}c_{Rk}\}]\ e^{iS}$ can be obtained in the following way[68] : $\ln Z[A] = \mathrm{Tr}\ln\left[\breve{\mathrm{I}} - \breve{\mathrm{Q}}\hat{\mathrm{A}}\hat{\mathrm{M}}\right]$, where the unit matrix $\breve{I}$ and the Green function $\breve{Q}$ are defined in $\mathbb{N}\otimes\mathbb{S}\otimes\mathbb{K}$ space, $\hat{A}$ is in $\mathbb{K}$ space, and $\hat{M}$ is in $\mathbb{N}\otimes\mathbb{S}$ space. Finally, the symmetrized current noise for left junction can be written as

$$\begin{aligned} S_I(\omega, eV) &= \int dt e^{i\omega t}\langle \delta I_L(t)\delta I_L(0) + \delta I_L(0)\delta I_L(t)\rangle \\ &= -\frac{1}{4}\frac{\delta^2 \ln Z[A]}{\delta A^q(\omega)\delta A^q(-\omega)}\bigg|_{A=0} \end{aligned} \quad (9)$$

where $\delta I_L(t) = I_L(t) - \langle I_L\rangle$, and an extra factor $1/2$ is to remove the doubling of the Hilbert space. The details of the evaluation of Eq.(9) are presented in SI[67]. At zero temperature, the shot noise is given by

$$S_I(eV) = \sum_\sigma \int_{-eV/2}^{eV/2} d\omega P_\sigma(\omega), \quad (10)$$

where $P_\sigma(\omega) = (2e^2/h)\Big(\mathbb{A}^\sigma_{\rm N}(\omega) + \mathbb{A}^\sigma_{\rm A}(\omega)\Big)$. Here $P_\sigma(\omega)$ is the power spectrum of noise for each spin with $\mathbb{A}^\sigma_{\rm N/A}$ being the contributions to the noise from particle-particle (P-P)/particle-hole(P-H) channels, respectively. After tedious calculations (see SI[67] for details), one finds

$$\begin{aligned} \mathbb{A}^\sigma_{\rm N} &= 2\widetilde{\Gamma}_L\widetilde{\Gamma}_R\big(|G^R_{d\bar{d},\sigma}|^2 + |G^R_{\bar{d}d,\sigma}|^2\big) + 4\widetilde{\Gamma}^2_L|F^R_{dd,\sigma}|^2 \\ &\quad -8\widetilde{\Gamma}^2_L\widetilde{\Gamma}^2_R\big(|G^R_{d\bar{d},\sigma}|^4 + |G^R_{\bar{d}d,\sigma}|^4\big) - 16\widetilde{\Gamma}^4_L|F^R_{dd,\sigma}|^4 \\ &\quad -16\widetilde{\Gamma}^3_L\widetilde{\Gamma}_R\big(|G^R_{d\bar{d},\sigma}|^2 + |G^R_{\bar{d}d,\sigma}|^2\big)|F^R_{dd,\sigma}|^2, \end{aligned} \quad (11)$$

$$\begin{aligned} \mathbb{A}^\sigma_{\rm A} &= \widetilde{\Gamma}^2_L\Big[\big(F^R_{dd,\sigma} + F^A_{dd,\sigma}\big)^2 \\ &\quad -8(\widetilde{\Gamma}^2_L - \widetilde{\Gamma}^2_R)\frac{|F^R_{dd,\sigma}|^2}{\Sigma_\sigma}\big(F^R_{dd,\sigma} + F^A_{dd,\sigma}\big) \\ &\quad +16(\widetilde{\Gamma}_L - \widetilde{\Gamma}_R)^2((\widetilde{\Gamma}_L + \widetilde{\Gamma}_R)^2 + \tilde{\epsilon}^2_d)\frac{|F^R_{dd,\sigma}|^4}{\Sigma^2_\sigma}\Big], \end{aligned} \quad (12)$$

where $G^R_{\bar{d}d}$ can be obtained from $G^R_{d\bar{d}}$ by $\epsilon_d \to -\epsilon_d$. The P-H contribution is vanishing at zero frequency $\mathbb{A}^\sigma_{\rm A}(\omega) \sim \omega^2$. Here we assume a symmetric bias $V_L = -V_R$.

Results and Discussions. Before presenting the results for an interacting QD problem, it is instructive to consider first a non-interacting spinless model, for which the results can be easily obtained by setting $\eta = 0$ and $b = 1$ in $P_\uparrow(\omega)$(10). The power spectrum $P(0)$ at $T = 0$ and $\delta = 0$ is

$$P_{\lambda\neq 0}(0) = \frac{2e^2}{h}\frac{\Gamma_L\Gamma_R}{\Gamma^2} = \frac{e^2}{2h}\bigg|_{\Gamma_L=\Gamma_R} \quad (13)$$

$$P_{\lambda=0}(0) = \frac{2e^2}{h}\frac{4\Gamma_L\Gamma_R}{\Gamma^2+\epsilon^2_d}\left(1 - \frac{4\Gamma_L\Gamma_R}{\Gamma^2+\epsilon^2_d}\right) = \frac{2e^2}{h}\frac{\Gamma^2\epsilon^2_d}{(\Gamma^2+\epsilon^2_d)^2}\bigg|_{\Gamma_L=\Gamma_R} \quad (14)$$

One can see that coupling to MZM dramatically modifies the shot noise. For example, at the symmetric point the shot noise power does not depend on ϵ_d and is given by $e^2/2h$ whereas without MZM $P_{\lambda=0}(0)$ depends on ϵ_d and is zero on resonance $\epsilon_d = 0$. By tuning the coupling asymmetry Γ_L/Γ_R or QD energy level ϵ_d, one should observe a qualitative different behaviour for the cases with and without MZMs, see SI [67] for more details. The shot noise at finite bias eV is given by Eq.(10). In order to understand the $eV \neq 0$ results, we plot the power spectrum $P(\omega)$ in Fig. 2 (a), which shows a two-peak structure. For $\lambda \ll \Gamma$, we find that the width between the two peaks $\sim \lambda^2/\Gamma$, and for $\lambda \gg \Gamma$, this width becomes $\sim \Gamma$.

We now discuss results at finite splitting $\delta \neq 0$. As shown in Fig. 2 (b), the spectral function for a finite δ exhibits two peaks at small ω. When $\lambda \ll \Gamma$, the position of the peak is at $\pm\delta$. Thus, in order to observe the predicted value $P(0) = e^2/2h$, one should adjust the voltage to be $\lambda^2/\Gamma \gg eV \gg \delta$. When $\lambda \gg \Gamma$, the width of the splitting is $\Gamma\delta^2/\lambda^2$. Thus, the condition to observe $P(0) = e^2/2h$ value is $\Gamma \gg eV \gg \Gamma\delta^2/\lambda^2$. We plot the shot noise as both a function of the λ and δ in Fig. 3.

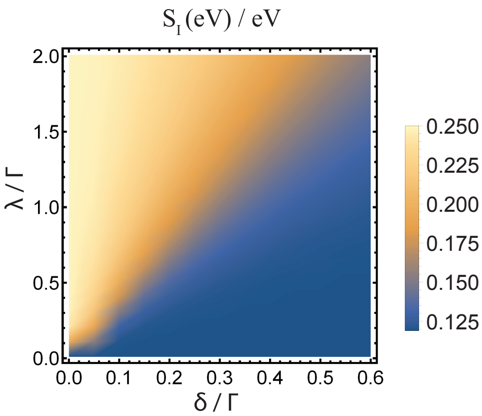

FIG. 3. Spinless non-interacting QD: the dependence of the shot noise $S_I(eV)/eV$ (measured in units of $2e^2/h$) at finite bias $eV/\Gamma = 0.1$ on λ and δ. Here $\Gamma_L = \Gamma_R$, $\epsilon_d/\Gamma = -0.4$.

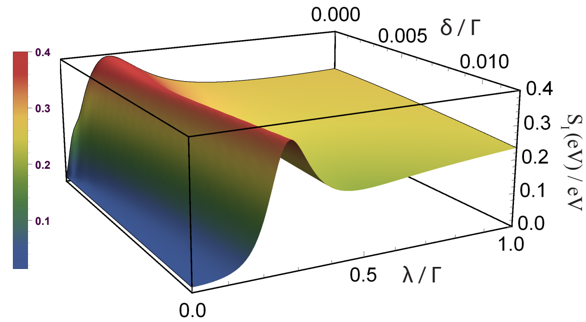

FIG. 4. Spinful QD in the single-occupancy regime: The dependence of the shot noise $S_I(eV)/eV$ (measured in units of $2e^2/h$) as a function of λ and δ. Here $\epsilon_d/\Gamma = -10.0$, $\Gamma_L = \Gamma_R$, $eV/\Gamma = 0.001$, and $\Lambda/\Gamma = 30.0$. The non-monotonic behavior as a function of λ originates from the P-H contribution $\mathbb{A}_A$. The contribution to $S_I(eV)$ from the spin-down channel is negligibly small in this parameter regime.

One can see that the larger the splitting energy δ, the larger Majorana coupling λ is needed to observe the predicted value for the shot noise $P(0) = e^2/2h$. The effect of varying ϵ_d is discussed in [67].

The conclusion based on the results of the spinless non-interacting problem is that Majorana coupling qualitatively modifies the shot noise through the QD. Thus, the combination of the conductance and shot noise measurements allow one to clarify the nature of the zero-bias conduction feature, see Table I. Even though the spinful problem is more complicated, we show that this qualitative feature persists in the presence of interactions and allows one to distinguish between the Majorana and Kondo origin of the zero-bias feature in the tunneling conductance. We now consider the QD in the limit of single-occupancy $U \gg |\epsilon_d| \gg \Gamma, \lambda$ and $eV \ll T_K$ with T_K being the Kondo temperature. We first analyze the case of no splitting $\delta = 0$. A recent study based on SBMF approach [47] shows that a crossover from Kondo- and Majorana-dominated regimes can be realized by tuning the coupling λ . For $\lambda \ll \lambda_c \equiv \sqrt{T_k/\Gamma}|\epsilon_d|$, Kondo effect is important [47]: the renormalized coupling corresponds to Kondo temperature $\tilde{\Gamma} \equiv \Gamma b^2 = T_K = \Lambda \exp(-\pi|\epsilon_d|/2\Gamma)$ and the renormalized energy level is $\tilde{\epsilon}_d \equiv |\epsilon_d + \eta| \sim \Gamma b^4$ (Here Λ is the bandwidth and $b \ll 1$ is the variational parameter). When $\lambda \gg \lambda_c$, the parameter $b \sim \lambda/|\epsilon_d|$ is determined by the Majorana coupling rather than the Kondo temperature. One can see that in the perturbative regime $|\epsilon_d| \gg \Gamma, \lambda$ corresponding to $b \ll 1$, the position of the renormalized level is close to the Fermi energy $\tilde{\epsilon}_d \sim \Gamma b^4 \ll \tilde{\Gamma}$ [47]. In both cases the spin-down channel shows perfect transmission (i.e. linear conductance $G = e^2/h$), and, thus, its contribution to the shot noise is zero. On the other hand, the shot noise for the spin-up channel, due to the coupling to MZM, corresponds to the universal value $e^2/2h$ independent of ϵ_d. The conductance and shot noise for spinful QD can be summarized as follows. The linear conductance for $|\epsilon_d| \gg \lambda, \Gamma$ reads

$$G|_{\Gamma_L = \Gamma_R} = \frac{e^2}{h}\left(\frac{1}{2} + 1\right) = \frac{3e^2}{2h}, \tag{15}$$

which is consistent with the numerical renormalization group calculation [45]. The shot noise power is

$$P(0)|_{\Gamma_L = \Gamma_R} = \frac{2e^2}{h}\left(\frac{1}{4} + 0\right) = \frac{e^2}{2h}. \tag{16}$$

The results beyond the $|\epsilon_d| \gg \lambda, \Gamma$ limit can be obtained numerically and are discussed in [67].

We now consider the effect of a finite energy splitting $\delta \neq 0$ and a finite bias $eV \neq 0$ which is important for the experimental detection of the effect we predict here. The shot noise $S_I(eV)/eV$ as a function of λ and δ is shown in the Fig. 4. One can see that in order to resolve the quantized value $P(0) = e^2/2h$, one has to satisfy the following conditions: a) in the regime $b\lambda \ll b^2\Gamma$, the voltage should be $\lambda^2/\Gamma \gg eV \gg \delta$; b) in the $\lambda \gg b\Gamma$ regime, the condition becomes $b^2\Gamma \gg eV \gg \Gamma\delta^2/\lambda^2$. It is thus clear that in the Majorana-dominated regime, i.e. $\lambda \gg b|\epsilon_d| \gg b\Gamma$, the voltage should satisfy condition b), in which case the shot noise power spectrum exhibits a plateau around $S_I(eV)/eV = e^2/2h$, see Fig. 4. One can also notice that the width of the plateau around $S_I(eV)/eV = e^2/2h$ gradually shrinks with increasing δ. In the limit $\lambda \gtrsim |\epsilon_d|$, the renormalized energy level $\tilde{\epsilon}_d$ shifts away from the Fermi level since $b \sim 1$, which, in turn, suppresses the conductance at zero bias and enhances the shot noise, see discussion in SI [67].

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