Odd-parity topological superconductor with nematic order: Application to Cu_{x}Bi_{2}Se_{3}

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Odd-parity topological superconductor with nematic order: application to $Cu_xBi_2Se_3$

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$Cu_xBi_2Se_3$ was recently proposed as a promising candidate for time-reversal-invariant topological superconductors[1]. In this work, we argue that the unusual anisotropy of the Knight shift observed by Zheng *et al*[2], taken together with specific heat measurements, provides strong support for an unconventional odd-parity pairing in the two-dimensional E_u representation of the D_{3d} crystal point group[1], which spontaneously breaks the three-fold rotational symmetry of the crystal, leading to a subsidiary nematic order. We predict that the spin-orbit interaction associated with hexagonal warping plays a crucial role in pinning the two-component order parameter and makes the superconducting state fully-gapped, leading to a topological superconductor. Experimental signatures of the E_u pairing related to the nematic order are discussed.

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Time-reversal-invariant (T-invariant) topological superconductors in two and three dimensions are a new class of unconventional superconductors which exhibit a full superconducting gap in the bulk and gapless helical quasi-particles on the boundary[3–5]. Because these quasi-particles do not possess conserved charge or spin quantum numbers, they cannot be distinguished from their anti-particles and hence are regarded as itinerant Majorana fermions.

There is currently intensive effort in finding T-invariant topological superconductors in real materials[6–10]. Recent theoretical works[1, 11] have established that the key requirement for topological superconductivity in inversion-symmetric systems is odd-parity pairing symmetry. Only a few odd-parity superconductors are known to date. Two prime examples are Sr_2RuO_4 and UPt_3. However, both materials seem to have nodes and/or spontaneously break time-reversal symmetry, hence do not qualify as T-invariant topological superconductors.

Recently, the doped topological insulator $Cu_xBi_2Se_3$, which is superconducting with a maximum T_c of 3.8K[12], was proposed as a candidate topological superconductor with odd-parity pairing[1]. Since then this material has been intensively studied. Specific heat measurements down to 0.3K found a full superconducting gap[13]. The upper critical field appears to exceed the Pauli limit, which is interpreted as consistent with triplet pairing[14]. Much interest is sparked by the observation of a zero-bias conductance peak in a point-contact spectroscopy experiment on $Cu_{0.3}Bi_2Se_3$[15], which is attributed to the putative Majorana fermion surface states from topological superconductivity. However, a later scanning tunneling spectroscopy measurement on $Cu_{0.2}Bi_2Se_3$ found a full gap in the tunneling spectrum at very lower temperature, without any sign of in-gap states[17]. The discrepancy between these two *surface sensitive* experiments has led to considerable debate and controversy about the nature of superconductivity in $Cu_xBi_2Se_3$[18–22]. In view of the current status, direct probes of the pairing symmetry in the *bulk* are much needed.

In a very recent nuclear magnetic resonance (NMR) study of $Cu_{0.3}Bi_2Se_3$, Zheng's group discovered an unusual anisotropy in the Knight shift as a small applied field is rotated within the *ab*-plane[2]. The Knight shift is isotropic above T_c, and decreases in the superconducting state. Remarkably, the change in the Knight shift is largest when the field is parallel to a particular crystal axis. This uniaxial anisotropy is incompatible with the three-fold rotational symmetry of the crystal, and thus provides a direct evidence of spontaneous crystal symmetry breaking associated with unconventional superconductivity in $Cu_xBi_2Se_3$.

In this work, we identify the pairing symmetry of $Cu_xBi_2Se_3$ from the existing NMR and specific heat measurements, theoretically establish a novel fully-gapped topological superconductor phase, and predict experimental signatures for further study. Our main finding is that among all possible pairing symmetries, the odd-parity pairing in the two-dimensional (2D) E_u representation, first introduced in Ref.[1], is the only one compatible with the rotational symmetry breaking observed in NMR measurement[2] *and* the full superconducting gap found in specific heat measurement[13]. Since this E_u pairing generates a subsidiary nematic order, we call the resulting state a "nematic superconductor".

The fully-gapped nature of the E_u superconducting state found here is remarkable, considering that all previous works found nodes in the gap[1, 15, 16]. Moreover, a full gap is required for topological superconductivity. While previous works are based on a rotationally invariant Dirac fermion model for the bulk band structure of $Cu_xBi_2Se_3$, we find that crystalline anisotropy plays an indispensable role in the odd-parity E_u state. We show by general argument and model study that the spin-orbit interaction associated with hexagonal warping[23] pins the direction of the two-component E_u order parameter to a two-fold axis of the crystal, consistent with the Knight shift anisotropy, and makes the superconducting state fully-gapped. Such a nematic superconductor constitutes a new phase of odd-parity pairing.

Pairing Symmetry: It was recognized at the outset that strong spin-orbit coupling must be taken into consideration in discussing the pairing symmetry of $Cu_xBi_2Se_3$. Indeed, the importance of spin-orbit coupling becomes manifest in the Knight shift measurement of electron's spin susceptibility. If spin-orbit coupling were absent, the Knight shift would be fully isotropic for spin singlet as well as triplet pairing, because the triplet d-vector would be free to rotate with the applied magnetic field. In contrast, in the presence of spin-orbit coupling, the notion of spin-singlet or triplet pairing is, strictly speaking, not well-defined. Instead, pairing symmetries are classified according to the representations of the crystalline symmetry group D_{3d}[1], which acts simultaneously on spatial coordinates and electron's spin. The consequence is that the spin structure of the superconducting order parameter is locked to crystal axis, resulting in an anisotropic spin susceptibility.

Among the the six irreducible representations of D_{3d} (A_{1g}, A_{1u}, A_{2u}, A_{2g}, E_u and E_g), only the E_u and E_g representations are multi-dimensional and hence potentially compatible with the spontaneous rotational symmetry breaking observed in the Knight-shift measurement. In order to determine which one of the two is the pairing symmetry of $Cu_xBi_2Se_3$, we first consider Ginzburg-Landau theory for the E_u and E_g superconducting states. The D_{3d} point group symmetry dictates that up to the fourth order, the Landau free energy in both cases must take the form

$$\begin{aligned} F &= r(|\Psi_1|^2 + |\Psi_2|^2) + u_1(|\Psi_1|^2 + |\Psi_2|^2)^2 \\ &+ u_2|\Psi_1^2 + \Psi_2^2|^2 \end{aligned} \tag{1}$$

where $r \propto (T - T_c)$. Here $\Psi = (\Psi_1, \Psi_2)$ is the two-component order parameter, which transforms like a vector under the three-fold rotation. The same form of the free energy also applies to other crystal systems[24, 25]. Importantly, the nature of the superconducting state below T_c depends on the sign of u_2. For $u_2 > 0$, a T-breaking chiral state with a complex order parameter $\Psi \propto (\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}})$ arises, which is *isotropic* within the *ab*-plane. For $u_2 < 0$, a T-invariant state with a real order parameter $\Psi \propto (\cos\theta, \sin\theta)$ arises. This superconducting state spontaneously breaks the rotational symmetry, and possesses a subsidiary nematic order parameter Q:

$$Q = (|\Psi_1|^2 - |\Psi_2|^2,\ \Psi_1^*\Psi_2 + \Psi_2^*\Psi_1). \tag{2}$$

The two components of Q transform as $x^2 - y^2$ and xy respectively. Such a nematic superconductor with uniaxial anisotropy is consistent with with the Knight shift measurement, whereas the isotropic chiral state is not.

We now show that the nematic state with E_g pairing and the one with E_u pairing can be experimentally distinguished by their qualitatively different gap structures, because of the difference in the parity of the order parameter: E_g is even-parity and E_u is odd-parity. To analyze the gap structure, it is convenient to express the pair potential $\Delta(\mathbf{k})$ in the band basis. Since the superconducting gap is much smaller than the Fermi energy in $Cu_xBi_2Se_3$, it suffices to consider only the bands at the Fermi energy. Due to the presence of both time reversal and inversion symmetry, the energy bands are twofold degenerate at every $\mathbf{k}$, which we label by a "pseudospin" index α. Because of spin-orbit coupling, $\alpha = 1, 2$ does not correspond to electron's spin. The pair potential thus reduces to a 2×2 matrix over the Fermi surface: the gap function $\Delta_{\alpha\alpha'}(\mathbf{k})$.

Depending on the parity of the order parameter, the gap function of a T-invariant superconductor takes two different forms:

$$\Delta^e(\mathbf{k}) \;=\; \Delta(\mathbf{k}) \cdot I, \text{ where } \Delta(\mathbf{k}) = \Delta(-\mathbf{k}), \tag{3}$$

$$\Delta^o(\mathbf{k}) \;=\; \vec{d}(\mathbf{k}) \cdot \vec{\sigma}, \text{ where } \vec{d}(\mathbf{k}) = -\vec{d}(-\mathbf{k}). \tag{4}$$

The even-parity gap function $\Delta^e(\mathbf{k})$ is a real *scalar*, while the odd-parity gap function $\Delta^o(\mathbf{k})$ is parameterized by a real *vector* field $\vec{d}(\mathbf{k})$, the d-vector. The superconducting gaps $\delta(\mathbf{k})$ in the two cases are given by $|\Delta(\mathbf{k})|$ and $|\vec{d}(\mathbf{k})|$ respectively.

The scalar nature of even-parity gap function (3) dictates that the T-invariant E_g state of $Cu_xBi_2Se_3$ must have line nodes. To see this, let us recall that for any non-s-wave pairing, the gap function integrated over the Fermi surface must be zero:

$$\int_{\mathbf{k}\in \mathrm{FS}} d\mathbf{k}\ \Delta(\mathbf{k}) = 0. \tag{5}$$

As shown by angle-resolved photoemission spectroscopy experiments[19, 26], $Cu_xBi_2Se_3$ has a connected Fermi surface enclosing $\mathbf{k} = 0$. It then follows from Eq.(5) that $\Delta(\mathbf{k})$ must change sign somewhere on such a Fermi surface, resulting in unavoidable line nodes. As an explicit example, the E_g gap function $\Delta(\mathbf{k}) \propto k_z k_x, k_z k_y$ considered in Ref.[16] has lines of nodes on the $k_z = 0$ and $k_x, k_y = 0$ planes. The existence of line nodes conflicts with the specific heat measurement[13]. This seems sufficient to rule out the E_g pairing in $Cu_xBi_2Se_3$. In contrast, we will show below that the E_u states generically have a full superconducting gap.

Superconducting gap: For the sake of concreteness, we first derive the superconducting gap of the E_u state within a two-orbital model for $Cu_xBi_2Se_3$. Later, we will show that the presence or absence of nodes is a robust property that depends only on symmetry, not microscopic details.

The band structure of $Cu_xBi_2Se_3$ at low energy is described by a $k \cdot p$ Hamiltonian at Γ, which to *first order* in k takes the following form[1]

$$H_0 = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \left[v(k_x s_y - k_y s_x)\sigma_z + v_z k_z \sigma_y + m\sigma_x - \mu \right] c_{\mathbf{k}},$$

where $c^\dagger = (c^\dagger_{1\uparrow}, c^\dagger_{1\downarrow}, c^\dagger_{2\uparrow}, c^\dagger_{2\downarrow})$ consists of two orbitals hereafter denoted as 1 and 2, in addition to electron's spin. Here σ and s are two sets of Pauli matrices associated with orbital and spin respectively. It is worth pointing out that spin-orbit coupling in time-reversal and inversion symmetric systems necessarily involves more than one orbitals, as shown in the two-orbital Hamiltonian here. The physical origin of H_0 is elucidated in Ref.[27]. The chemical potential μ lies in the conduction band due to Cu-doping.

In this two-orbital model, the E_u pairing arises when electrons in the two orbitals within a unit cell pair up to form a spin triplet, with zero total spin along an in-plane direction $\mathbf{n} = (n_x, n_y)$. The corresponding pair potential, $V_\mathbf{n} = n_x V_x + n_y V_y$, is a superposition of two independent basis functions given in Ref.[1] (therein called "Δ_4 pairing"):

$$\begin{aligned} V_x &= i\Delta_0(c^\dagger_{1\uparrow}c^\dagger_{2\uparrow} - c^\dagger_{1\downarrow}c^\dagger_{2\downarrow}) \\ V_y &= \Delta_0(c^\dagger_{1\uparrow}c^\dagger_{2\uparrow} + c^\dagger_{1\downarrow}c^\dagger_{2\downarrow}). \end{aligned} \tag{6}$$

$V_\mathbf{n}$ is T-invariant and rotational symmetry breaking. $\mathbf{n}$ should be regarded as a nematic director (a headless vector), because the superconducting order parameter $V_\mathbf{n}$ and $V_{-\mathbf{n}}$ only differ by sign and correspond to the same physical state.

We can directly obtain the superconducting gap $\delta_\mathbf{n}(\mathbf{k})$ by diagonalizing the BCS mean-field Hamiltonian $H_{sc} = H_0 + V_\mathbf{n}$. Alternatively, we can derive the gap function $\Delta(\mathbf{k})$ by rewriting $V_\mathbf{n}$, defined by (6) in spin and orbital basis, in terms of band eigenstates of H_0 at the Fermi energy, as done in Ref.[16]. To leading order in Δ_0/μ, the two approaches yield identical results for the superconducting gap on the Fermi surface: $\delta_\mathbf{n}(\mathbf{k}) = \Delta\sqrt{\tilde{k}_z^2 + (\tilde{\mathbf{k}}\cdot\mathbf{n})^2}$, where $\Delta = \Delta_0\sqrt{1 - m^2/\mu^2}$. Here we have introduced a rescaled momentum $\tilde{\mathbf{k}}$ to parameterize the Fermi surface:

$$\tilde{\mathbf{k}} = (vk_x, vk_y, v_z k_z)/\sqrt{\mu^2 - m^2}. \tag{7}$$

$\tilde{\mathbf{k}}$ maps the ellipsoidal Fermi surface of the Hamiltonian H_0 to a unit sphere. The gap $\delta_\mathbf{n}(\mathbf{k})$ vanishes at two points on the equator of the Fermi surface: $\pm\mathbf{k}_0 = \pm k_F \hat{\mathbf{z}} \times \mathbf{n}$. Hence, based on this model, previous works concluded that the E_u states in $Cu_xBi_2Se_3$ have point nodes.

However, we note that H_0 is fully rotationally invariant around the $\hat{\mathbf{z}}$ axis. This is an artifact of the *first-order* $k \cdot p$ theory, which does not include any effect of crystalline anisotropy. In reality, the crystal of $Cu_xBi_2Se_3$ only has a discrete three-fold symmetry, and this crystalline anisotropy is responsible for pinning the direction of the E_u order parameter $\mathbf{n}$. This motivates us to take crystalline anisotropy into account and re-examine the gap structure of E_u pairing.

We find that the gap structure depends on the orientation of the order parameter $\mathbf{n}$ relative to the crystal axes: the point nodes remain present when $\mathbf{n}$ is parallel to a two-fold axes, whereas they become lifted for $\mathbf{n}$ in all other directions, resulting in a full superconducting gap. To illustrate this node-lifting explicitly, we add a "hexagonal warping" term of third order in $\mathbf{k}$ to the Hamiltonian, which is allowed by the D_{3d} point group symmetry of $Cu_xBi_2Se_3$:

$$H = H_0 + \lambda \sum_\mathbf{k} (k_+^3 + k_-^3) c^\dagger_\mathbf{k} \sigma_z s_z c_\mathbf{k}, \; k_\pm \equiv k_x \pm i k_y. \tag{8}$$

Here x is along a two-fold axis, or equivalently, normal to a mirror plane, as shown in Fig.1. This hexagonal warping term arises from the spin-orbit interaction associated with crystalline anisotropy and can be regarded as the bulk counterpart of the warping term for topological insulator surface states[23, 28]. For $\lambda \neq 0$, the Fermi surface becomes hexagonally deformed, and more importantly, the orbital-resolved spin polarization of Bloch states in $\mathbf{k}$ space becomes modified.

By solving the mean-field Hamiltonian $H_{\rm sc} = H + V_\mathbf{n}$ with the same pair potential as before, we find the superconducting gap in the presence of hexagonal warping:

$$\delta_\mathbf{n}(\mathbf{k}) = \Delta\sqrt{1 - [\tilde{\mathbf{k}}\cdot(\hat{\mathbf{z}} \times \mathbf{n})]^2}, \tag{9}$$

where $\tilde{\mathbf{k}}$ is still defined by Eq.(7), but $\mathbf{k}$ now lives on a new Fermi surface determined by

$$\sqrt{m^2 + v^2(k_x^2 + k_y^2) + \lambda^2(k_+^3 + k_-^3)^2 + v_z^2 k_z^2} = \mu.$$

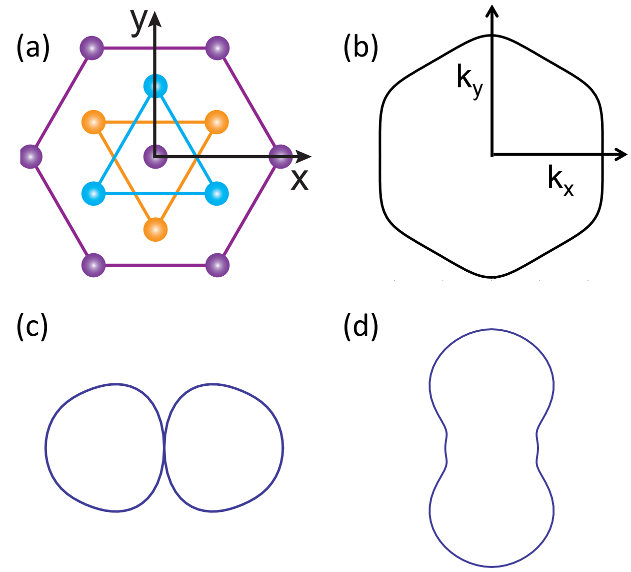

FIG. 1: (a) Crystal structure of $Cu_xBi_2Se_3$ viewed from the c axis. Note that the x axis is normal to a mirror plane. (b) Hexagonal Fermi contour at $k_z = 0$ for the Hamiltonian (8). (c) and (d) show the angle dependence of the anisotropic superconducting gap over the $k_z = 0$ Fermi contour for the E_u order parameter V_x and V_y respectively, defined in (6). The presence of nodes in (c) and the full gap in (d) are robust and model-independent.

It is clear from (9) that the gap $\delta_{\mathbf{n}}(\mathbf{k})$ goes to zero only where $|\mathbf{n}\cdot(\tilde{\mathbf{k}}\times\hat{\mathbf{z}})|=1$. Importantly, we note that for $\lambda\neq 0$, $|\tilde{\mathbf{k}}\times\hat{\mathbf{z}}|$ is less than 1 everywhere on the warped Fermi surface, except at six corners of the hexagon on the $k_z=0$ plane (see Fig.1): $\pm\mathbf{k}=k_F\hat{\mathbf{y}}$ and the star of $\pm\mathbf{k}$ obtained by three-fold rotation, where $|\tilde{\mathbf{k}}\times\hat{\mathbf{z}}|=1$. As a result, the zero-gap condition $|\tilde{\mathbf{k}}\cdot(\hat{\mathbf{z}}\times\mathbf{n})|=1$ is satisfied only when the nematic director $\mathbf{n}$ is parallel to one of the three two-fold axes, such as $\mathbf{n}=\pm\hat{\mathbf{x}}$. In this case, the nodes found previously remain present. In contrast, for $\mathbf{n}=(\cos\theta,\sin\theta)$ in all other directions, i.e., $\theta\neq 0,\pm\pi/3$ or $\pm 2\pi/3$, the nodes are lifted by hexagonal warping, resulting in a full gap.

We plot in Fig.1 the superconducting gaps over the equator of a hexagon-like Fermi surface, for two E_u pairings with $\mathbf{n}=\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ respectively, which are representative of the two contrasting cases. It should be said that the quantitative gap structure are model specific. For example, the gap anisotropy depends on the amount of warping and the microscopic pairing interaction. Nonetheless, the presence of nodes for $\mathbf{n}=\hat{\mathbf{x}}$ and a full gap for $\mathbf{n}=\hat{\mathbf{y}}$, which we have explicitly shown using the model Hamiltonian (8) and the pair potential (6), are robust and model-independent properties of the E_u superconducting state in $\mathrm{Cu}_x\mathrm{Bi}_2\mathrm{Se}_3$, as we will show below.

Stable nodes have a deep origin in the symmetry and topology of the gap function. In a T-invariant odd-parity superconductor, a node in the gap occurs where the d-vector is zero. Importantly, we observe that when strong spin-orbit coupling is present, as in $\mathrm{Cu}_x\mathrm{Bi}_2\mathrm{Se}_3$, the d-vector $\vec{d}(\mathbf{k})$ (whose direction depends on the choice of pseudospin basis at $\mathbf{k}$) is generically a *three-component* vector field in $\mathbf{k}$ space, instead of uniaxial or planar. This is simply because crystalline symmetry group alone is generally insufficient to make any component of the d-vector vanish *everywhere* in $\mathbf{k}$ space. Since $\vec{d}(\mathbf{k})=0$ requires satisfying three equations, it is vanishingly improbably to find a solution on the *two-dimensional* Fermi surface[29]. This implies that stable nodes in T-invariant odd-parity superconductors are unlikely to occur in the presence of spin-orbit coupling, unless there is special crystal symmetry protecting their existence.

An example is when there is a reflection symmetry with respect to a mirror plane, e.g., $\mathbf{x}\to-\mathbf{x}$, *and* the odd-parity order parameter is *invariant* under this reflection. In this case, $\vec{d}(k_x=0,k_y,k_z)$ and $\vec{d}(k_x=\pi/a,k_y,k_z)$ must be parallel to the normal of the mirror plane, due to its *pseudo*-vector nature. Such a two-dimensional uniaxial d-vector field on the $k_x=0,\pi/a$ plane is allowed to have lines of zeros, whose intersection with the Fermi surface will generate stable point nodes in the superconducting gap[29].

The general argument presented above explains the gap structures of different E_u states of $\mathrm{Cu}_x\mathrm{Bi}_2\mathrm{Se}_3$ found in our model studies. The rotationally invariant model H_0 has the artifact of being symmetric with respect to any vertical plane, thus resulting in point nodes regardless of the nematic director $\mathbf{n}$. However, the crystal of $\mathrm{Cu}_x\mathrm{Bi}_2\mathrm{Se}_3$ has only three mirror planes that are 120 degrees apart from each other, which is correctly captured in the refined model (8) with hexagonal warping. For $\mathbf{n}$ normal to a mirror plane such as $\mathbf{n}=\pm\hat{\mathbf{x}}$, the corresponding order parameter V_x is invariant under the reflection $\mathbf{x}\to-\mathbf{x}$; hence the nodes located on the $k_x=0$ plane are protected by this mirror symmetry. For $\mathbf{n}$ in other directions, however, the order parameter is not invariant under any reflection; hence nodes are absent[30].

To capture the important effect of crystalline anisotropy in Ginzburg-Landau theory, we must include higher-order terms in the free energy (1), which start at the sixth order

$$F_6=\kappa\left[(\Psi_+^*\Psi_-)^3+(\Psi_+\Psi_-^*)^3\right],\quad \Psi_\pm\equiv\Psi_1\pm i\Psi_2 \quad (10)$$

Depending on $\kappa>0$ or $\kappa<0$, $\mathbf{n}$ is pinned either parallel or perpendicular to one of the three mirror planes, e.g., along the $\hat{\mathbf{y}}$ or $\hat{\mathbf{x}}$ axis. It is natural to expect that the fully gapped state with $\mathbf{n}=\hat{\mathbf{y}}$ has a lower free energy below T_c than the nodal state with $\mathbf{n}=\hat{\mathbf{x}}$. The nematic state with $\mathbf{n}=\hat{\mathbf{y}}$ has two degenerate gap minima at $\pm k_F\hat{\mathbf{x}}$, and spontaneously lowers the point group symmetry from D_{3d} (rhombohedral) to C_{2h} (orthorhombic). This crystal symmetry breaking naturally leads to an anisotropic spin susceptbility. Importantly, the C_{2h} point group in the symmetry breaking phase has only one principal axis—the two-fold axis $\hat{\mathbf{x}}$ that lies within the ab-plane. It is exactly along this axis that the change in Knight shift was found to be largest in the NMR experiment[2]. This agreement lends additional support to the E_u pairing symmetry we have identified. A quantitative calculation of spin susceptibility in the anisotropic E_u state depends on microscopic details, which we leave to future study.

The anisotropic E_u state found here is a novel example of odd-parity pairing with a full gap. Among the various phases of superfluid He-3, the T-invariant B phase is isotropic, while the anistropic A phase is T-breaking. Perhaps the closest analog to $\mathrm{Cu}_x\mathrm{Bi}_2\mathrm{Se}_3$ is the A phase of UPt_3[31], whose order parameter is real and breaks the sixfold crystal rotational symmetry[32]; however, this phase is known to have nodes.

Topological superconductivity: With an odd-parity pairing symmetry and a full gap, the E_u superconducting state in $\mathrm{Cu}_x\mathrm{Bi}_2\mathrm{Se}_3$ satisfies all the requirements for T-invariant topological superconductivity stated in Ref.[1]. The exact topology depends further on the nature of the Fermi surface. At low doping, the normal state has an ellipsoidal Fermi pocket centered at Γ, which under E_u pairing will become a three-dimensional (3D) topological superconductor, with Majorana fermion surface states on all crystal faces. At high doping, the Fermi surface is most likely open and cylinder like, as indicated by recent photoemission[19] and de Haas-van

Alphen measurements[33, 34]. If this is the case, the E_u pairing will give rise to a quasi-two-dimensional topological superconductor, which is equivalent to stacked layers of 2D topological superconductors along the c axis, correspond to $v_z = 0$ in our model (8). Side surfaces of this state host an even number of 2D massless Majorana fermions. The top and bottom surfaces are fully-gapped, but a step edge on these surfaces hosts 1D helical Majorana fermions. It has been noted[19] that the scenario of quasi-2D topological superconductivity may explain both the point contact and scanning tunneling spectroscopy measurements. In either 3D or quasi-2D case, more direct evidence of Majorana fermions would be desirable.

Experimental signatures: The ab-plane gap anisotropy of the E_u pairing can be directly probed by directional dependent thermal conductivity[35] or tunneling spectra. Here we focus on testing the E_u pairing symmetry in $\mathrm{Cu}_x\mathrm{Bi}_2\mathrm{Se}_3$ via the subsidiary nematic order. Symmetry dictates a linear coupling between a uniaxial strain ϵ_{ij} in the ab-plane and the superconducting order parameter:

$$F_{\mathrm{s}} = g\left[\frac{\epsilon_{xx} - \epsilon_{yy}}{2}(|\Psi_1|^2 - |\Psi_2|^2) + \epsilon_{xy}(\Psi_1^*\Psi_2 + \Psi_2^*\Psi_1)\right].$$

As a result of this coupling, an uniaxial strain in the ab-plane acts as a symmetry breaking field for the nematic order, which should be able to align the nematic director of the superconducting order parameter near T_c, thereby changing the pattern of the anisotropic Knight-shift. In addition, the superconducting transition temperature should increase linearly under a small uniaxial strain, independent of its direction. The investigation of such strain-related effects on superconductivity seems within experimental reach[36] and may shed light on the pairing symmetry of $\mathrm{Cu}_x\mathrm{Bi}_2\mathrm{Se}_3$. Furthermore, the nematic order parameter allows for half-integer disclination, around which the superconducting order parameter changes sign. Hence these disclinations may trap a half-integer flux quantum ($h/4e$). Finally, it would be interesting to consider whether the nematic order or other orders related to the E_u pairing can emerge prior to the onset of superconductivity, similar to such phenomena in other systems[37–40].

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