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Effects of colored noise on Landau-Zener Transitions: Two and Three-Level Systems

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We investigate the Landau-Zener transition in two- and three- level systems subject to a classical Gaussian noise. Two complementary limits of the noise being fast and slow compared to characteristic Landau-Zener tunnel times are discussed. The analytical solution of a density matrix (Bloch) equation is given for a long time asymptotic of transition probability. It is demonstrated that the transition probability induced/assisted by the fast noise can be obtained through a procedure of *Bloch's equation averaging* with further reducing it to a master equation. In contrast to the case of fast noise, the transition probability for LZ transition induced/assisted by the slow classical noise can be obtained by averaging the *solution* of Bloch's equation over the noise realization. As a result, the transition probability is described by the activation Arrhenius law. The approximate solution of the Bloch's equation at finite times is written in terms of Fresnel's integrals and interpreted in terms of interference pattern. We discuss consequences of a local isomorphism between $SU(2)$ and $SO(3)$ groups and connections between Schrödinger and Bloch descriptions of spin dynamics. Based on this isomorphism we establish the relations between $S = 1/2$ and $S = 1$ transition probabilities influenced by the noise. A possibility to use the slow noise as a probe for tunnel time is discussed.

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I. INTRODUCTION

The interest to Landau-Zener (LZ) model[1–4] is constantly increasing over last decades as it establishes one of the most important fundamental notions in nonstationary quantum mechanics. Original LZ model describes the probability of transition between two quantum isolated states coupled by a linearly sweeping external transverse field of a constant amplitude and a time-dependent longitudinal field which passes through resonance with the transition frequency. Even though the resulting LZ formula seems to be quite simple, it has proved to be applicable in analysing the experimental data on charge transfer particle collisions[5]. The model has been employed in various studies related to charge transport in nanostructures[6–9], Bose-Einstein condensates[10–12], spin tunnelling of nanomagnets[13,14], quantum quenches[15–17]. Especially, LZ becomes a corner stone of theories discussing adiabatic quantum computing[18–20] due to a possibility to enhance a read out of qubits via the Zener flip tunneling[21]. Such a mechanism has previously been implemented for flux qubits[22] and may serve also for inverting spin population by sweeping the system through the resonance (rapid passage) in ultracold molecules[23].

In realistic systems however, spins states remain constantly coupled to their environment. Among various mechanisms of dephasing and decoherence of LZ transitions between Zeeman-split spin states, the coupling of two-level system both with phonon bath and nuclear subsystem should be mentioned. If the spin-nuclear coupling strength is weak enough and the relaxation of the nuclear bath is fast, then the nuclear dynamics effects can be reduced to a fast random field[24–28]. In the opposite situation, if the nuclear subsystem is slow enough compared to the characteristic tunnel time, its influence on LZ transition can be accounted by an effective model of a slow classical noise[29,30]. For the simplest LZ scenario, the nuclear dynamics can be considered in the Born-Oppenheimer approximation, thus that the spins are presumed isolated and transitions activated by an external magnetic field. Besides, the noise associated with both a hyperfine and dipole fields plays an important role in description of dynamical response of nanomagnets[30].

Experiments with molecular magnets[31] revealed the presence of hysteresis phenomena in nanoscale molecular magnets among which are Mn_{12} and Fe_8. LZ transitions at the avoided crossing between the Zeeman-split spin levels produced by hyperfine interactions have been pointed out as responsible of plateaus on hysteresis loops[31,32]. Hence, a number of proposals have been suggested, clarifying the effects of nuclear bath, noises and decoherence effects on the transition probability in linearly driven systems including two and multi-states systems[33–39]. Several compact analytic results have been derived to describe these effects, namely; the Kayanuma's formula[24–26] for a strong diagonal noise and Pokrovsky-Sinitsyn formula[27,28] for the coupling to a fast colored noise with off-diagonal components.

Spin transport processes in magnetic semiconductors designs unavoidably suffer from hyperfine interactions treated as noise source frustrating spins during transmissions[40–42]. The common way to protect information during the propagation consists on adiabatically

applying an external controlling magnetic field. Bethausen et *al*[42] have recently presented an alternative experimental method to protect spin propagation in spin-transistors including diabatic LZ tunable transitions. Indeed, in these experiments a controlling magnetic field is a combination of a spatially rotating magnetic field B_s and a homogeneous field B. Thus, the spin states are subjected to both a constant magnetic field (B) and a fluctuating (Overhauser's) field. A theoretical attempt to attack such problem has been introduced in Refs.[27], [28] for two-level spin systems by means of fast noise associated with random hyperfine interactions. In contrast to it, recent experiments on spin polarization of nuclear subsystem via time-dependent gate voltage in double quantum dots[43–46] have shown that the fluctuations of Overhauser's field are rather slow changing dramatically the properties of LZ transition. The "minimal theoretical model" should however take into account all low energy two-electron states in a dot consisting of three singlet and one triplet ($S = 1$) state.

In this paper, we consider the influence of both fast and slow classical noise on two- and three- level systems. We calculate transition probabilities for the noise-induced and noise-assisted processes by using density matrix (Bloch) equation. The analytical expression for finite time probabilities for two- and three- level systems are interpreted in terms of Fresnel's interference. In addition to two standard definitions of a tunnel time for LZ transition by means of internal or external clock, we discuss a possibility to use a noise as yet another probe for the LZ time.

The paper is organized as follows; The Section II is devoted to discussion of basic equations for LZ transition $S = 1/2$ derived through Schrödinger and Bloch approaches. In Section III, we discuss the noise-induced and noise-assistant LZ transitions in two-level system. The classical noise associated with fluctuations of Overhauser's field is considered as a colored noise with Gaussian realization. Both the cases of one and two- component transverse noise are discussed. Sections IV and V contain the key equations for three-level $S = 1$ system subjected to both fast and slow classical noise. In Section VI, we discuss the LZ transition times defined through internal and external clock in the presence of noise. The details of derivation are sketched in Appendices.

II. BASIC RELATIONS FOR TWO-LEVEL SYSTEMS

A. Schrödinger spin-1/2 picture

The time evolution of $\mathcal{N}$ states of a quantum mechanical system with a coherently driven total spin S can be described by a system of $\mathcal{N}$ coupled differential equations for the amplitudes $C_1^{(S)}(t), C_2^{(S)}(t), ..., C_{\mathcal{N}}^{(S)}(t)$ of the states $\psi_1^{(S)}(t), \psi_2^{(S)}(t), ..., \psi_{\mathcal{N}}^{(S)}(t)$ ($\hbar = 1$):

$$i\frac{d}{dt}\mathbf{C}(t) = \hat{\mathcal{H}}(t)\mathbf{C}(t). \tag{2.1}$$

Here, $\mathbf{C}(t) = [C_1^{(S)}, C_2^{(S)}, ..., C_{\mathcal{N}}^{(S)}]^T$ is a column vector for amplitude probabilities and

$$\hat{\mathcal{H}}(t) = \vec{\Theta}(t) \cdot \vec{S} \tag{2.2}$$

is the total Hamiltonian of the system, $\vec{S}$ the total spin vector involving all the three generators of the group $SU(2)$.

The relevant aspect of Eq.(2.2) with our aim lies in its description of Zeeman splitting of spin states in a linearly sweeping external magnetic field. This aspect intimately refers to the traditional LZ problem and the functions in Eq.(2.2) are explicited as follows:

$$\Theta^x(t) = 2\Delta, \quad \Theta^y(t) = 0 \quad \text{and} \quad \Theta^z(t) = 2\alpha t. \tag{2.3}$$

Here, $\alpha > 0$ is the constant sweep velocity, Δ the tunneling coupling matrix element between states that we assume here as real and lasts from $t = -\infty$ to $t = \infty$.

For the special case of two levels, the problem Eq.(2.1) leads to a system of two independent equations

$$\frac{d^2}{dz^2}C_1^{(1/2)}(z) + \Big[i\lambda - 1/2 - z^2/4\Big]C_1^{(1/2)}(z) = 0, \tag{2.4}$$

$$\frac{d^2}{dz^2}C_2^{(1/2)}(z) + \Big[i\lambda + 1/2 - z^2/4\Big]C_2^{(1/2)}(z) = 0. \tag{2.5}$$

known as Weber's equations[47], where $z = \sqrt{2\alpha}te^{-i\pi/4}$ and $\lambda = \Delta^2/2\alpha$. Solutions of these equations are computed with respect to the initial conditions. For the choice $C_1^{(1/2)}(-\infty) = 1$ and $C_2^{(1/2)}(-\infty) = 0$ i.e. the particle was initially prepared in the state $\psi_1^{(1/2)}(t)$, one has[1,2]:

$$C_1^{(1/2)}(t) = -\frac{A_+}{\sqrt{\lambda}}e^{-i\pi/4}e^{i\varphi}D_{-i\lambda}(-i\mu t), \tag{2.6}$$

and

$$C_2^{(1/2)}(t) = A_-e^{i\varphi}D_{-i\lambda-1}(-i\mu t). \tag{2.7}$$

Here, $D_n(z)$ is the parabolic cylinder (Weber's[47]) function, φ a phase factor and $\mu = \sqrt{2\alpha}e^{-i\pi/4}$. The parameter λ is introduced hereafter to distinguish between the sudden ($\lambda \ll 1$) and the adiabatic ($\lambda \gg 1$) limits of transitions. The normalization factors A_+ and A_- in Eqs.(2.6) and (2.7) are respectively defined by their modulus, $|A_+| = |A_-| = \sqrt{\lambda}e^{-\pi\lambda/4}$.

The probability $|C_2^{(1/2)}(t)|^2$ that the system will be found in the state $\psi_2^{(1/2)}(t)$ at any given time t is therefore given by

$$P_{\rm LZ}(t) = \lambda e^{-\pi\lambda/2}|D_{-i\lambda-1}(-i\mu t)|^2. \tag{2.8}$$

Symmetries of levels allow for directly finding the probability to remain in the same state. Some asymptotic and exact values of Eq.(2.8) are performed with the aid of asymptotic series expansion of Weber's functions[47]. By putting for instance $t \to \infty$ we recover

$$P_{\rm LZ}(\infty) = 1 - e^{-2\pi\lambda}, \tag{2.9}$$

known as the celebrated LZ formula[1,2].

B. Bloch spin-1/2 picture

The general solution for time-dependent LZ probability (2.8) is written in terms of products of Weber's functions. The asymptotic form of this equation casts nevertheless a very simple exponential form (2.9). In this section, we present an approximate solution for LZ finite time probability (not necessarily in a long time limit) as an exponential of single-parametric real function and discuss accuracy of this solution for sudden and adiabatic limits.

The time evolution of the quantum dynamical system described by the model (2.2) and coherently driven is here governed by the von-Neumann equation for the total density matrix $\hat{\tilde{\rho}}(t)$,

$$i\frac{d\hat{\tilde{\rho}}(t)}{dt} = \Big[\hat{\mathcal{H}}(t), \hat{\tilde{\rho}}(t)\Big]. \tag{2.10}$$

With the help of Eq.(2.10), we find the population difference $\hat{\rho}(t) = \hat{\rho}_{11}(t) - \hat{\rho}_{22}(t)$ as being solution of the differential equation:

$$\frac{d\hat{\rho}(t)}{dt} = -i\Theta_-(t)\hat{\rho}_{21}(t) + i\Theta_+(t)\hat{\rho}_{12}(t), \tag{2.11}$$

where $\hat{\rho}_{21}(t) = \hat{\rho}^*_{12}(t)$ involving $\hat{\rho}^*(t) = \hat{\rho}(t)$ with,

$$\hat{\rho}_{12}(t) = \frac{i\int_{-\infty}^{t}\exp\Big(i\int_{-\infty}^{t_1}\Theta^z(\tau')d\tau'\Big)\Theta_-(t_1)\hat{\rho}(t_1)dt_1}{2\exp\Big(i\int_{-\infty}^{t}\Theta^z(\tau')d\tau'\Big)}, \tag{2.12}$$

and $\Theta_\pm(t) = \Theta^x(t) \pm i\Theta^y(t)$. The indices 1 and 2 denote the two- level crossing. Inserting Eq.(2.12) into Eq.(2.11), with reference to the context of LZ theory i.e. $\Theta^z(t) = 2\alpha t$, without loss of generality, we find an equation

$$\frac{d\hat{\rho}(t)}{dt} = -\int_{-\infty}^{t} dt_1 \cos\Big[\alpha(t^2 - t_1^2)\Big]\Theta_+(t)\Theta_-(t_1)\hat{\rho}(t_1), \tag{2.13}$$

which can be included in the family of Volterra's integral-differential equations[48,49]. Eq.(2.10)-(2.13) correspond to Bloch's transformations related to the optical Bloch's[50] equation $\dot{\vec{b}} = -\vec{\Theta}\times\vec{b}$. Where $\vec{b}$ is the Bloch's vector set on unit sphere by the condition ${\rm Tr}\hat{\tilde{\rho}} = {\rm Tr}\hat{\tilde{\rho}}^2 = 1$. The z-component of it being expressed as a linear combination of diagonal matrix elements of $\hat{\tilde{\rho}}(t)$ as $b_z(t) = \hat{\rho}(t) \equiv \hat{\rho}_{11}(t) - \hat{\rho}_{22}(t)$ satisfies Eq.(2.13)

$$\frac{d}{dt}\hat{\rho}^{(0)}(t) = -4\Delta^2\int_{-\infty}^{t}\cos\Big[\alpha(t^2 - t_1^2)\Big]\hat{\rho}^{(0)}(t_1)dt_1, \tag{2.14}$$

for the conventional LZ problem. The superscript (0) refers to LZ problem in the absence of noise.

The integral-differential Eq.(2.14) can be solved iteratively with the condition $\hat{\rho}^{(0)}(-\infty) = 1$ that preserves the total population at any arbitrary time t. A perturbation series expansion investigation with respect to the parameter $\lambda \ll 1$ (see Appendix.A) is achieved as,

$$\hat{\rho}^{(0)}(t) = 1 + 2\Big[-2\pi\lambda F(t) + \frac{1}{2!}\Big(-2\pi\lambda F(t)\Big)^2 + ... \\ -G(t,\lambda)\Big] = -1 + 2\Big[\exp\Big(-2\pi\lambda F(t)\Big) - G(t,\lambda)\Big]. \tag{2.15}$$

The function $F(t)$ is defined as

$$F(t) = \frac{1}{2}\Big[\Big[c\Big(\sqrt{\frac{2\alpha}{\pi}}t\Big) + \frac{1}{2}\Big]^2 + \Big[s\Big(\sqrt{\frac{2\alpha}{\pi}}t\Big) + \frac{1}{2}\Big]^2\Big]. \tag{2.16}$$

$c(\sqrt{2\alpha/\pi}t)$ and $s(\sqrt{2\alpha/\pi}t)$ are respectively the cosine and sine Fresnel's integrals[51]. The function $F(t)$ is more often encountered in the theory of light diffraction, where it relates to the intensity of light passed through a semi-infinite plane bounded by a sharp straight edge with t assuming the lateral distance of the edge from the point of observation[52].

The function $G(t,\lambda)$ includes all corrections to the exponential solution and is determined so that as $t \to \infty$ asymptotically one comes back to the conventional LZ formula Eq.(2.9). Then, for convenience, we write our finite time transition probability as follows[53]:

$$P_{\rm LZ}(t) = 1 - \exp\Big\{-2\pi\lambda[F(t) + \ln W(t)]\Big\}, \tag{2.17}$$

with

$$\ln W(t) = -\frac{1}{2\pi\lambda}\ln\Big[1 - G(t,\lambda)\exp\Big\{2\pi\lambda F(t)\Big\}\Big], \tag{2.18}$$

in which

$$G(t,\lambda) = \lambda e^{-\pi\lambda/2}|D_{-i\lambda-1}(-iz)|^2 - \Big(1 - e^{-2\pi\lambda F(t)}\Big). \tag{2.19}$$

One can see that in the limit $\lambda \ll 1$ the correction function $\ln W(t) < F(t)$ for all times. The two forms Eq.(2.17) and Eq.(2.8) are equivalent with the only difference being that Eq.(2.17) is the exponential form of Eq.(2.8). The form (2.17) we obtained will be used for analytic derivations of finite times transition probabilities in the limit of slow noise driven LZ transitions.

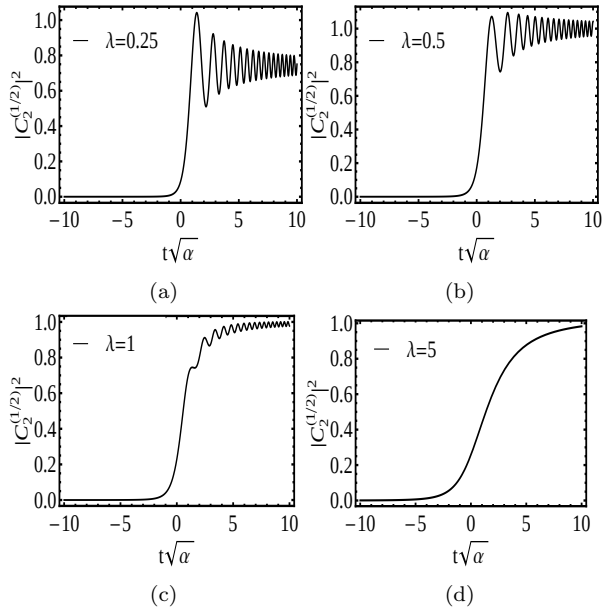

FIG. 1. Time evolution of the LZ transition probability in the diabatic basis of a two-level system. The small values of the Landau-Zener parameter $\lambda = \Delta^2/2\alpha$ correspond to sudden limit (rapid drive), panel (a). The large values of λ describe adiabatic limit (slow drive), panel (d). The intermediate LZ regimes are presented on panels (b) and (c). The time is in units of $1/\sqrt{\alpha}$.

The Fresnel's integrals give rise to Fresnel's oscillations (see Fig.1 below) and suggest interferences between states around the anticrossing region. The Fresnel-type oscillatory factors $e^{\pm i\alpha t^2}$ involved in Fresnel's integrals originate from the phase

$$\int_0^t \Theta^z(t')dt' = \alpha t^2, \tag{2.20}$$

accumulated by the two components of the wave function during a linear sweep.

The function $P_{\rm LZ}(t)$ could have also been found by numerically solving the time-dependent Schrödinger's equation (2.1). A typical result is depicted by Fig.1 above, where the time evolution of the transition probability $P_{\rm LZ}(t)$ discriminates two regions. In the domain $t \leq 0$, all probabilities are smooth monotonic functions of time. First, around the crossing time $t = 0$, one sees a sharp transition of $P_{\rm LZ}(t)$ which rises from zero to its maximum value and later behaves asymptotically with decaying oscillations around the saturation value $P_{\rm LZ}(\infty)$. This last performance characterizes the system for small values of λ (sudden limit), while for largest λ (adiabatic limit), oscillations are strongly mitigated. The first region identified characterizes the jump time $\tau_{\rm jump}$ while the second determines the relaxation time $\tau_{\rm relax}$[54]. The jump time $\tau_{\rm jump}$ called hereafter LZ transition time is denoted as $\tau_{\rm LZ}$. In adiabatic limit (slow passage), this time should be a function of the velocity α and the splitting energy Δ (see discussion in Section VI):

$$\tau_{\rm LZ} = \mathcal{L}(\alpha, \Delta). \tag{2.21}$$

The rapid drive of a two-level system produces repeated LZ transitions after passing the resonance, drives the system into a coherent superposition of states where they may interfere and generates Stückelberg's fringes[55–60]. As a consequence, appearing fringes encode information about the system's evolution and energy spectrum[3,55–60]. The system will not then feel the gap and $\tau_{\rm LZ}$ should be independent on the energy splitting Δ:

$$\tau_{\rm LZ} = \mathcal{L}(\alpha). \tag{2.22}$$

The slow drive in the opposite extreme limit produces oscillations of very weak amplitudes so that one can assume a single transition; Stückelberg's fringes of the former sort could be absent on the interferometer. The advantage being the possibility to probe spectroscopic information about the coherent evolution of the system[60]. As the system feels the gap,

$$\tau_{\rm LZ} = \mathcal{L}(\alpha, \Delta). \tag{2.23}$$

Semiclassically, $\tau_{\rm LZ}$ is a time necessary to reach a turning point on imaginary time axis of integration contour[61].

This mechanism of slowing down the sweep rate in order to collect spectroscopic information about a quantum system was recently employed by Berns and co-workers in their experiment of spectroscopy analysis of solid-state artificial atom[60]. In that experiment, they pointed out that for Stückelberg's interference to occur, the time interval between two consecutive LZ transitions should be much more smaller than the relevant decoherence times.

An alternative way to tackle the traditional LZ problem consists on transforming the equation (2.14) to a differential equation. It can be achieved by applying a second order time derivative to both sides of equation (2.14) and excluding integral term with $\sin\left[\alpha(t^2 - t_1^2\right]$. Hence, we show that $\hat{\rho}^{(0)}(t)$ satisfies the third order differential equation,

$$\frac{d^3}{d\tau^3}\hat{\rho}^{(0)}(\tau) - \frac{1}{\tau}\frac{d^2}{d\tau^2}\hat{\rho}^{(0)}(\tau) - 4\Big[\frac{2\lambda}{\tau}\hat{\rho}^{(0)}(\tau) - (\tau^2 + 2\lambda)\frac{d}{d\tau}\hat{\rho}^{(0)}(\tau)\Big] = 0, \tag{2.24}$$

which can be interpreted as a differential equation for the probability. Here, we performed the time scaled transformation $\tau = t\sqrt{\alpha}$. While amplitudes found from the linear Schrödinger's equation satisfy a second order linear differential equation, probabilities from the von-Neumann equation rather satisfy a third order linear differential equation. Similar equation was written in Refs.[54, 62–64] in the same context of LZ theory and numerically solved in Ref.[54] with the aid of a Runge-Kutta algorithm. Here, using a correspondence between

Schrödinger and Bloch approaches we propose an analytic and exact solution to this kind of equations.

The natural initial condition $\hat{\rho}^{(0)}(-\infty)=1$ was gradually translated at each step of derivations and we solve Eq.(2.24) with the conditions:

$$\frac{d^2}{d\tau^2}\hat{\rho}^{(0)}(\tau)\mid_{\tau=-\infty}=-8\lambda,\quad \frac{d}{d\tau}\hat{\rho}^{(0)}(\tau)\mid_{\tau=-\infty}=0. \tag{2.25}$$

As $\hat{\mathcal{H}}(t)$ in the Schrödinger's equation (2.1) realizes $SU(2)$ symmetry operations, the amplitudes $C_1^{(1/2)}(\tau)$ and $C_2^{(1/2)}(\tau)$ also realize the same set of operations. This is due to the temporal linearity of the Schrödinger's equation which preserves symmetry properties. While a Schrödinger's equation describes dynamics of wave functions ($SU(2)$ spinors), the Bloch's equation deals with evolution of probability densities combined into a vector on unit sphere (Bloch's vector on 2-sphere). However, a local isomorphism between $SU(2)$ and $SO(3)$ establishes relations between these two objects.

In the population difference, the occupation probabilities are expressed in terms of transition amplitudes for half-spin in Eqs.(2.6) and (2.7) i.e.: $\hat{\rho}_{11}^{(0)}(\tau)=|C_1^{(1/2)}(\tau)|^2$ and $\hat{\rho}_{22}^{(0)}(\tau)=|C_2^{(1/2)}(\tau)|^2$ and this realizes the isomorphism we talked about. Thus, the solution of Eq.(2.24) reads,

$$\hat{\rho}^{(0)}(\tau)=-\lambda e^{-\pi\lambda/2}\Big[|D_{-i\lambda-1}(-i\mu_0\tau)|^2 -\frac{1}{\lambda}|D_{-i\lambda}(-i\mu_0\tau)|^2\Big]. \tag{2.26}$$

Here, $\mu_0=\mu/\sqrt{\alpha}$. We may deduce, from the same technique, an integral relation between Weber's functions. From Eq.(2.12) one may notice that $\hat{\rho}_{12}^{(0)}(t)=C_1^{(1/2)}(t)C_2^{(1/2)*}(t)$ and finds

$$D_{-i\lambda}(-i\mu t)D^*_{-i\lambda-1}(-i\mu t)=-\lambda\mu\int_{-\infty}^{t}\exp\Big(\frac{\mu^2}{2}[t^2-t_1^2]\Big)\Big(|D_{-i\lambda-1}(-i\mu t_1)|^2-\frac{1}{\lambda}|D_{-i\lambda}(-i\mu t_1)|^2\Big)dt_1. \tag{2.27}$$

Similar relation for $D^*_{-i\lambda}(-i\mu t)D_{-i\lambda-1}(-i\mu t)$ can be derived from $\hat{\rho}_{21}^{(0)}(t)=C_1^{(1/2)*}(t)C_2^{(1/2)}(t)$. Similar matching procedure was recently employed in Ref.[43] to establish an integral relation between Weber's functions not from $\hat{\rho}_{12}^{(0)}(t)$ as we did here but from $\hat{\rho}^{(0)}(t)$ in Eq.(2.11).

One can easily check that the limit $\tau\to\infty$ applied to (2.26) brings us back automatically to Eq.(2.9). Thus, the former represents the population difference at any given time τ. The solution of Eq.(2.24) gives an information about a time dependence of population difference directly measurable in the flux qubits experiments in a micromaser[65–67]. It might serve for transfer of population between two states at any time τ. For instance by measuring the LZ transition probability between two states, it provides information about the strength Δ of the coupling between states. It could also offer great advantages in experiments with atoms transfer, having only one parameter for control.

In the domain $\tau\leq 0$, the projection of Bloch's vector on z-axis is positive. The system remains in the state where it has been set initially. Passing now through the resonance, $\hat{\rho}^{(0)}(\tau)$ abruptly changes its concavity becoming either greater or less than zero. Just around the anticrossing region, the sharp drop of $\hat{\rho}^{(0)}(\tau)$ shows that $|1\rangle$ has started to feed $|2\rangle$ via the LZ mechanism.

In the domain $\tau>0$, one has $\rho^{(0)}(\tau)<0$, the two-level system experiences decaying oscillations while the population difference saturates to finite value. The oscillations correspond to an interference between states $|1\rangle$ and $|2\rangle$. This last remark tells us that population difference tends to maintain the majority of the system into the excited state rather than the ground state.

III. TRANSVERSE NOISE IN THE SPIN-1/2 LANDAU-ZENER THEORY

We now turn into a situation where LZ transitions are noise induced. Basically, the coupling between level positions fluctuates due to a transverse noise with Gaussian realizations.

LZ effects in presence of transverse classical noise including inter-level transitions are specified by the prototype Hamiltonian (2.2) considering

$$\Theta^x(t)=2f_x(t),\quad \Theta^y(t)=2f_y(t)\quad\text{and}\quad \Theta^z(t)=2\alpha t. \tag{3.1}$$

These definitions are also valid for the case $S=1$ we study below. The mean-zero stochastic functions $f_i(t)(i=x,y)$ in Eq.(3.1) are characterized by their first and second order moments,

$$\langle f_i(t)\rangle=0,\quad \langle f_i(t)f_j(t')\rangle=\eta^2\delta_{ij}\exp(-\gamma|t-t'|). \tag{3.2}$$

Here, η stands for the noise intensity that might be related to the absolute temperature via the universal *fluctuation dissipation theorem*[68] (see detailed discussion below). The parameter $\gamma=1/t_{noise}$ defines a time scale associated with the noise. Comparison of t_{noise} with characteristic time scales of LZ problem gives us a definition

of fast and slow noise limits. The dynamics of the system is governed by Eq.(2.13) for the occupation difference:

$$\frac{d\hat{\rho}(t)}{dt} = -4\int_{-\infty}^{t} \cos\Big[\alpha(t^2 - t_1^2)\Big] f_+(t) f_-(t_1)\hat{\rho}(t_1) dt_1, \tag{3.3}$$

where $f_\pm(t) = f_x(t) \pm i f_y(t)$. The solution of this equation is to be averaged over all possible realizations of the two-level system (ensemble average). The result of this averaging is different for the two limits of fast and slow noise.

A. Fast noise, spin-1/2

If noise is fast, the characteristic noise time $t_{noise} \ll \tau_{\rm LZ}$, one can average equation Eq.(3.3) directly and decouple the product $\langle f_+(t) f_-(t_1)\hat{\rho}(t_1)\rangle$ as $\langle f_+(t) f_-(t_1)\rangle\langle\hat{\rho}(t_1)\rangle$. The resulting master equation for the average $\langle\rho(t)\rangle$ gives the conventional equation for the transition probability as the average $\langle\hat{\rho}(t_1)\rangle$ doesn't really change in the exceedingly short time interval $\xi = t_1 - t$:

$$\frac{d}{dt}\langle\hat{\rho}(t)\rangle = -\hat{\Omega}(t)\langle\hat{\rho}(t)\rangle. \tag{3.4}$$

Here, the functional $\hat{\Omega}(t) \equiv \hat{\Omega}[\tilde{\omega}(t)]$ of the frequency $\tilde{\omega}(t) = 2\alpha t$ is defined through the two-time correlation function $\hat{\mathcal{R}}(|t - t_1|) = \langle f_+(t) f_-(t_1)\rangle$ as $\hat{\Omega}(t) = \hat{\Omega}^{(+)}(t) + \hat{\Omega}^{(-)}(t)$, where

$$\hat{\Omega}^{(\pm)}(t) = \int_{-\infty}^{\infty} \exp\Big[\pm i\tilde{\omega}(t)\xi\Big]\hat{\mathcal{R}}(|\xi|) d\xi, \tag{3.5}$$

are the power spectral densities of noise capturing information about environmental effects. For the Gaussian model we considered, Eq.(3.5) result in a Lorentzian. The frequency $\tilde{\omega}(t)$ is antisymmetric $\tilde{\omega}(-t) = -\tilde{\omega}(t)$, the Lorentzian spectral density is symmetric in the Fourier space leading thus to $\hat{\Omega}^{(+)}(t) = \hat{\Omega}^{(-)}(t)$. Eq.(3.4) is thus readily solved to give:

$$\langle\hat{\rho}(t)\rangle = \langle\hat{\rho}(-\infty)\rangle \exp\Big[-\int_{-\infty}^{t}\hat{\Omega}(t')dt'\Big], \tag{3.6}$$

and $\hat{\Omega}(t)$ is readily integrated accounting for the spectral density. As a result, we obtained the phase $\vartheta(t)$ accumulated during an interval of time $(-\infty, t]$:

$$\vartheta(t) = \int_{-\infty}^{t}\hat{\Omega}(t')dt' = \frac{\pi\hat{\mathcal{R}}(0)}{\alpha}\Big[1 + \frac{2}{\pi}\arctan(\frac{2\alpha}{\gamma}t)\Big]. \tag{3.7}$$

and $\theta(t) = 2\vartheta(t)$. By putting as usual $t = \infty$ one gets

$$\theta(\infty) = 2\int_{-\infty}^{\infty}\hat{\Omega}(t')dt' = \frac{4\pi}{\alpha}\hat{\mathcal{R}}(0). \tag{3.8}$$

So, the transition probability results from Eqs.(3.6) and (3.7) or (3.8) as a combination of the initial condition $\hat{\rho}(-\infty) = 1$ and the conservation law $\mathrm{Tr}\hat{\tilde{\rho}}(t) = 1$:

$$P_{fn} = \frac{1}{2}\Big[1 - e^{-\theta/2}\Big]. \tag{3.9}$$

Here, $\theta = \theta(\infty)$. Eq.(3.9) is generalized by Pokrovsky[27,28] to arbitrary correlation function. It demonstrates an equal distribution of the system between the ground and excited states after passing the crossing time for large noise $\eta \to \infty$. By taking the limits $\eta \to \infty$ and $\gamma \to \infty$ while keeping $\eta^2/\gamma = const$, white noise limit can be obtained from (3.2). Note, that $\theta \sim \eta^2$ does not depend on γ in that limit.

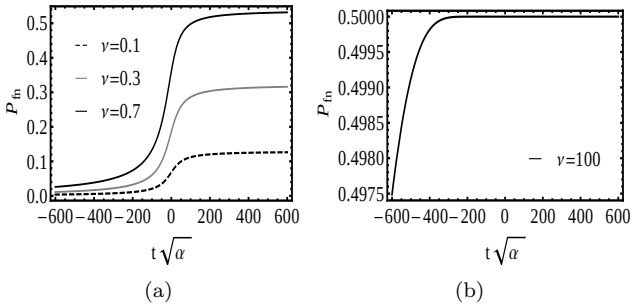

FIG. 2. Time evolution of the LZ transition probability in the diabatic basis of a two-level system in presence of a fast transverse noise for the rapid (a) and slow (b) passages. The amplitude of the fast noise is fixed. The noise is characterized by a dimensionless parameter $\nu = \pi\eta^2/\alpha$ and dimensionless frequency $\gamma_0 = \gamma/\sqrt{\alpha}$. For all calculations $\gamma_0 = 100$.

Here, we defined the dimensionless frequency $\gamma_0 = \gamma/\sqrt{\alpha}$ and dimensionless parameter $\nu = \pi\eta^2/\alpha$. If noise is directed on either the transverse direction (X-noise) or the two-components transverse noise (XY-noise), Eq.(3.9) reduces respectively to

$$P_{fn}^{x} = \frac{1}{2}\Big[1 - \exp\Big(-\frac{2\pi}{\alpha}\langle f_x(t) f_x(t)\rangle\Big)\Big] \tag{3.10}$$

and

$$P_{fn}^{xy} = \frac{1}{2}\Big[1 - \exp\Big(-\frac{2\pi}{\alpha}\Big[\langle f_x(t) f_x(t)\rangle + \langle f_y(t) f_y(t)\rangle\Big]\Big)\Big]. \tag{3.11}$$

Hence, in order to sum up noises in X and Y-directions it just suffices to do that in the argument of the exponential in Eq.(3.9). What happens if noise is colored in one direction and white in another? The answer to this question is provided by the argument of the exponential in Eq.(3.11). Obviously, white noise will dominate the colored one and there will not be a complete transfer of population: both states remaining constantly coupled.

The solution (3.7) generalizing (3.8) to finite times coincides at very large times with the results obtained

in Refs.[27 and 28]. Similar results were discussed in Ref.[29] for nonlinear drive with telegraph noise. A relevant note similar to the fast telegraph noise in a two-level system for this Gaussian model is that, noise fluctuations are averaged out as there are no fluctuations (see Fig.2).

We specify for further purposes that $\eta^2/\alpha \ll 1$ and $\eta^2/\alpha \gg 1$ correspond respectively hereafter to the sudden and adiabatic limits of transitions. In the adiabatic limit then, the transition probability depends non-analytically on the sweep velocity. Thus far, for $t_{noise} \ll \tau_{\rm LZ}$, there is no complete transfer of population; the two states are constantly occupied.

Spin-1/2, in a constant off-diagonal field and a fast transverse random field

Let us consider a spin coupled to a constant off-diagonal and sweeping magnetic field and a transverse noise source. Such pattern corresponds for instance to spin frustrated by hyperfine field or Overhauser's field and protected by a constant magnetic field. With reference to the recent experimental work in Ref.[42], we present an alternative way to protect spin propagation in spin-transistor (see Introduction). In this frame, LZ transitions are noise assisted and noise fields are no longer centered at the origin in the X-direction:

$$f_x(t) = \Delta + \tilde{f}_x(t), \quad f_y(t) = \tilde{f}_y(t). \tag{3.12}$$

Noise correlation functions for $\tilde{f}_i(t)$ are given by Eq.(3.2). The model (3.1) with Eq.(3.12) can also assume a spin weakly interacting with an environment, for example a nuclear spin bath. Assuming the spin-bath interaction as weak enough as bath relaxation is much faster than the inverse interaction energy, we may treat the bath as a fast noisy magnetic field[28].

As far as noise is no longer centered at the origin, we call $\hat{\rho}^{(\rm SF)}(t)$ the average of the total density matrix for the non-zero mean problem labeled by Eq.(3.12). Straightforward calculations for spin-1/2 suggest a governing equation of the form:

$$\frac{d}{dt}\hat{\rho}^{(\rm SF)}(t) = -4\Delta^2 \int_{-\infty}^{t} dt_1 \cos[\alpha(t^2 - t_1^2)]\hat{\rho}^{(\rm SF)}(t_1) \\ -4\int_{-\infty}^{t} dt_1 \cos[\alpha(t^2 - t_1^2)]\tilde{f}_+(t)\tilde{f}_-(t_1)\hat{\rho}^{(\rm SF)}(t_1). \tag{3.13}$$

Linear terms of noise have been dropped since after averaging and use of fast noise requirements they vanish. Hereafter, we adopt the label $\mathcal{P}$ to denote transition probabilities related to the non-centered noise.

As noise is fast, we can readily average Eq.(3.13) as we did before and apply the decoupling procedure associated with the other arguments of fast noise. As a result, the differential equation casts a form:

$$\frac{d}{dt}\langle\hat{\rho}^{(\rm SF)}(t)\rangle = -4\Delta^2 \int_{-\infty}^{t} dt_1 \cos[\alpha(t^2 - t_1^2)]\ \langle\hat{\rho}^{(\rm SF)}(t_1)\rangle \\ -\hat{\Omega}(t)\langle\hat{\rho}^{(\rm SF)}(t)\rangle. \tag{3.14}$$

We obtained the conventional master equation for the transition probability in which noise appears as a perturbing source. Noise essentially modifies the standard occupation difference $\hat{\rho}^{(0)}(t)$ by a decaying random phase factor. Mainly, noise produces dephasing during the transfer of population. The phase accumulated during the short time interval $t_1 - t \sim 1/\gamma$ is small enough such that:

$$\int_{-\infty}^{t_1} \Omega(t')dt' \approx \int_{-\infty}^{t} \Omega(t')dt'. \tag{3.15}$$

Indeed, as the characteristic frequency $\gamma \to \infty$ it appears that $t_1 \approx t$, justifying the approximation Eq.(3.15). Thus, the solution of Eq.(3.14) can be formally expressed as follows:

$$\langle\hat{\rho}^{(\rm SF)}(t)\rangle = \exp\Big[-\int_{-\infty}^{t} \hat{\Omega}(t')dt'\Big]\hat{\rho}^{(0)}(t). \tag{3.16}$$

For the solution at $t = \infty$, we derive $\hat{\rho}^{(0)}(\infty) = 2e^{-2\pi\lambda} - 1$ from Eq.(2.26) and the finding transition probability is obtained as follows:

$$\mathcal{P}_{fn}^{(\rm SF)} = \frac{1}{2}\Big[1 - e^{-\theta/2}(2e^{-2\pi\lambda} - 1)\Big]. \tag{3.17}$$

Eq.(3.17) describing the probability to remain in the same adiabatic state is fully consistent with the one obtained in Refs. 28. Namely, equation (42) in Ref. 28 describes a system initially set in the diabatic state $|1\rangle$ and conserves the same state while Eq.(3.17), describes a spin-flip between two different diabatic states.

As it is discussed in Ref. 27, the fast noise can lead to full equilibration depending on the time-scales involved (see Section VI of Ref. 27 for detailed analysis). In Eq.(3.17), assuming an adiabatic addition of noise $\theta \to \infty$ ($\alpha \to 0$), the probability achieves the value 1/2. Such a system loses its memory. It becomes obvious that by setting $\theta = 0$ we recover the LZ formula Eq.(2.9). Likewise, the requirement $\lambda = 0$ leads to the solution for fast noise centered at origin. Spin state evolution in spin-transistors might be protected during the transport by adiabatically applying a homogeneous magnetic field ($\lambda \gg 1$). This technique was employed in Ref.[42].

On Fig.3 the abrupt decay of the population difference $\langle\hat{\rho}^{(\rm SF)}(t)\rangle$ around the anti-crossing region characterizes a rapid transfer of population. However, as the condition $\langle\hat{\rho}^{(\rm SF)}(t)\rangle > 0$ is always fulfilled, there is no way to expect a complete transfer from one of the diabatic states to another with the fast noise. Fast noise being characterized by short time memory, essentially creates a dephasing between the states of a two-level system.

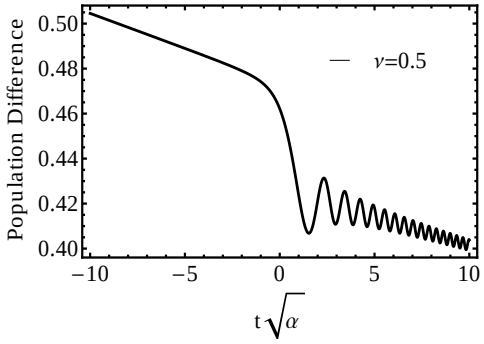

FIG. 3. Dynamical evolution of the population difference $\langle\hat{\rho}^{(\mathrm{SF})}(t)\rangle$ Eq.(3.14) in the presence of a fast transverse noise with characteristic decay rate $\gamma_0 = \gamma/\sqrt{\alpha}$ and amplitude $\nu = \pi\eta^2/\alpha$ for a rapid LZ drive $\lambda = \Delta^2/2\alpha \ll 1$. The numerical calculations are performed with the parameters $\gamma_0 = 100$, $\nu = 0.5$ and $\lambda = 0.05$.

B. Slow noise, spin-1/2

If now noise is slow ($t_{noise} \gg \tau_{\mathrm{LZ}}$), the decoupling procedure is not applicable, thus the density matrix equation of motion (3.3) cannot be reduced to a master equation. Instead one has to solve Eq.(3.3) and perform ensemble average over the distribution Q of noise:

$$\langle ...\rangle = \frac{1}{\sqrt{2\pi}\eta}\int_{-\infty}^{\infty} dQ ... \exp\Big(-\frac{Q^2}{2\eta^2}\Big). \tag{3.18}$$

The brackets $\langle ...\rangle$ indicate as usual the ensemble average. In a given realization of classical field Q, the LZ probability is given by standard equation

$$P_{\mathrm{LZ}}(Q) = 1 - \exp\Big(-\frac{\pi Q^2}{\alpha}\Big). \tag{3.19}$$

If the noise in transverse direction is single-component, we can always rotate our coordinate frame such a way that the fluctuations occur along X-direction. The noise - averaged LZ probability is defined as

$$P^x_{sn} = \frac{1}{\sqrt{2\pi}\eta}\int_{-\infty}^{\infty} dx P_{\mathrm{LZ}}(x)\exp\Big(-\frac{x^2}{2\eta^2}\Big), \tag{3.20}$$

and after straightforward calculation is given by

$$P^x_{sn} = 1 - \frac{1}{\sqrt{1+\dfrac{2\pi\eta^2}{\alpha}}}. \tag{3.21}$$

If the transverse noise is described by two orthogonal non-correlated components, the transition probability is averaged with a two-dimensional Gaussian distribution:

$$P^{xy}_{sn} = \frac{1}{2\pi\eta^2}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy P_{\mathrm{LZ}}(x,y)\exp\Big(-\frac{x^2+y^2}{2\eta^2}\Big), \tag{3.22}$$

which after calculations acquires the form

$$P^{xy}_{sn} = 1 - \frac{1}{1+\dfrac{2\pi\eta^2}{\alpha}}. \tag{3.23}$$

The difference between Eqs.(3.21) and Eq.(3.23) is a consequence of the effective two-dimensional character of noise fluctuation spectrum in the latter case and its one-dimensionality in the former case (see also Ref.53).

As expected, renormalization of the inter-level distance by a stochastic function considerably affects the generic picture of LZ transitions with off-diagonal coupling. Fast noise pours a large energy into the system during the crossing, destroys the memory of the system in the domain of strong couplings identically distributing the system between the ground and excited states. We note, that in contrast to effects of the fast noise which does not change analytical properties of the LZ probability at extreme adiabatic limit $\alpha \to 0$, the two-component slow noise transfers the property of the LZ probability from Gaussian to Lorentzian, thus making it analytic function of α in this limit.

The expansion method we exposed may allow one to formulate transition probabilities for finite times. For the case of X-noise, for example, the solution Eq.(2.17) yields:

$$P^x_{sn}(t) = 1 - \frac{1}{\sqrt{1+\dfrac{2\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}}. \tag{3.24}$$

In the limit $t \to \infty$, where $F(\infty) = 1$ and $\ln W(\infty) \to 0$, we return to Eq.(3.21). One can do the same with the two-components transverse noise (XY-noise model) and get

$$P^{xy}_{sn}(t) = 1 - \frac{1}{1+\dfrac{2\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}. \tag{3.25}$$

Similarly, the limit $t \to \infty$ brings us back to Eq.(3.23). These solutions may now be interpreted in an interference pattern as they involve Fresnel-like integrals via $F(t)$.

P^x_{sn} and P^{xy}_{sn} have the same shape but P^{xy}_{sn} is greater than P^x_{sn}.

Spin-1/2, in a constant off-diagonal field and a slow transverse random field

We dealt up to this point with LZ transitions induced by the slow noise. This approach can be generalized straightforwardly to the case where LZ transitions are induced by an external magnetic field (in the case of spin systems) or by an effective "field" associated with the finite transparency of the inter-well barrier in a double-well potential in cold gases. To describe this effect, one should take into account the value of the stochastic field

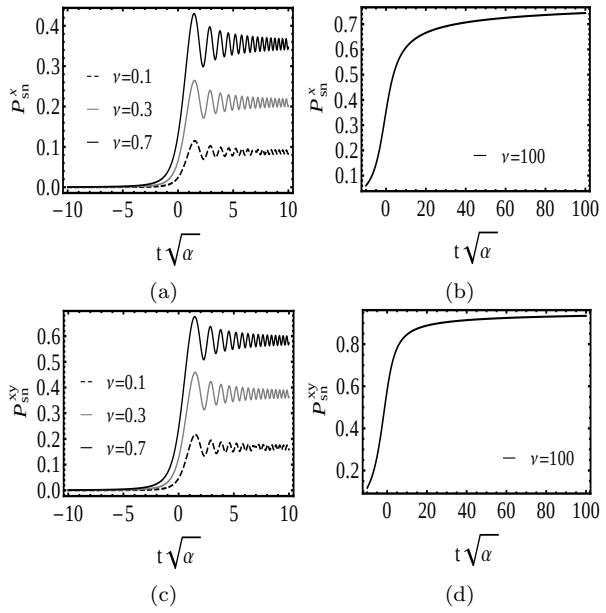

FIG. 4. Time evolution of the LZ transition probability in the diabatic basis of the two-level system in the presence of slow one- (panels (a) and (b)) and two- (panels (c) and (d)) component transverse noises (see discussion in the text). Panels (a) and (c) represent the results of numerical calculations for the small- amplitude noise. The data for the large-amplitude noise are shown on the panels (b) and (d).

$f_i(t)$ as defined in Eq.(3.12). Then, averaging over slow one-component noise fluctuation, results in

$$\mathcal{P}^x_{sn}(t) = 1 - \frac{\exp(-2\pi\lambda\Phi_1(t))}{\sqrt{1+\dfrac{2\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}}, \qquad (3.26)$$

where the phase $\Phi_\upsilon(t)$ with $\upsilon = 1, 2$, expressed as

$$\Phi_\upsilon(t) = \frac{\Big[F(t)+\ln W(t)\Big]}{1+\dfrac{2\pi\upsilon\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}. \qquad (3.27)$$

is due the local deviation of noise created in the X-direction. If noise was also shifted along the Y-direction, we would have an additional phase such that the argument of the exponential in Eq.(3.26) would be $\Phi_\upsilon(t) = \Phi^x_\upsilon(t) + \Phi^y_\upsilon(t)$. This would offer an access to sum noises. Nonetheless, the choice we adopted has a great advantage in controlling noise fluctuations in a two-level system. For the meantime, the infinite time limit of Eq.(3.26) suggests that:

$$\mathcal{P}^x_{sn}(\infty) = 1 - \frac{1}{\sqrt{1+\dfrac{2\pi\eta^2}{\alpha}}}\exp\Big\{-\frac{2\pi\lambda}{1+2\pi\eta^2/\alpha}\Big\}. \qquad (3.28)$$

In the sudden limit,

$$\mathcal{P}^x_{sn}(\infty) \approx P_{\rm LZ}(\infty). \qquad (3.29)$$

In the adiabatic limit,

$$\mathcal{P}^x_{sn}(\infty) = 1 - \sqrt{\frac{\alpha}{2\pi\eta^2}}\exp\Big\{-\frac{\Delta^2}{2\eta^2}\Big\}. \qquad (3.30)$$

Thus, the argument of the exponent does not depend on the velocity. Similarly, slow XY-noise in the presence of a constant magnetic field results in

$$\mathcal{P}^{xy}_{sn}(t) = 1 - \frac{\exp(-2\pi\lambda\Phi_1(t))}{1+\dfrac{2\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}. \qquad (3.31)$$

Therefore,

$$\mathcal{P}^{xy}_{sn}(\infty) = 1 - \frac{1}{1+2\pi\eta^2/\alpha}\exp\Big\{-\frac{2\pi\lambda}{1+2\pi\eta^2/\alpha}\Big\}, \qquad (3.32)$$

and in the sudden limit,

$$\mathcal{P}^{xy}_{sn}(\infty) \approx P_{\rm LZ}(\infty), \qquad (3.33)$$

while the adiabatic limit reads:

$$\mathcal{P}^{xy}_{sn}(\infty) = 1 - \frac{\alpha}{2\pi\eta^2}\exp\Big\{-\frac{\Delta^2}{2\eta^2}\Big\}. \qquad (3.34)$$

A two-level system subjected to a small-amplitude ($\eta^2/\alpha \ll 1$) slow noise in the presence of a magnetic field is insensitive to noise structure for long time asymptotic of transition probability. For such a setup, the magnetic field effects prevail on the noise and a pre-exponential factor is close to one. These effects are supported by Fig.5 where we plotted Eq.(3.28) for $t = \infty$. We essentially show on Fig.5 that adiabatic addition of noise, considerably suppressed the previous tendency.

Besides, by putting $\eta \to 0$ in Eq.(3.28) one immediately comes back to (2.17), the LZ formula for finite times.

The averaging procedure described in details allows to calculate the LZ probability if, for example, the noise is fast in one of transverse directions and slow in another orthogonal direction. In that case, one should first average the Bloch's equation over fast realization and after solve the "effective" Bloch's equation in given realization of slow fields. As a result, the fast noise contributes only to the argument of LZ exponent, while the slow noise appears both in exponential and pre-exponential factors. Therefore, numerical fit of experimental data[44–46] could provide an information for both kind of noises without requiring additional measurements.

By applying the *fluctuation dissipation theorem* one can associate some effective temperature with equal time two-point correlation function as follows

$$\langle f_i(t)^2\rangle = A\cdot T, \qquad (3.35)$$

where A and T are respectively the coupling constant with the environment (model dependent) and the absolute temperature in the units $k_B = 1$. In this frame, the solution (3.34) acquires the Arrhenius[70] form

$$P_{sn}^{xy}(\infty) = 1 - \frac{\alpha}{2\pi A \cdot T} \exp\Big\{-\frac{E}{T}\Big\}, \tag{3.36}$$

where $E = \Delta^2/2A$ is the activation energy[24]. The pre-exponential factor provides proper normalization of the distribution.

The theory of noise induced LZ effect may be extended to the multilevel LZ problems, where more complicated patterns of transient oscillations in the tunneling probability of transition from initial to final state of a nanosystem with nontrivial dynamical symmetry is expected.

IV. BASIC RELATIONS FOR THREE-LEVEL SYSTEMS

A. Schrödinger spin-1 picture

Consider the LZ transition in a three-level spin-1 system with the upper and lower levels described respectively by $C_1^{(1)}(t)$ and $C_2^{(1)}(t)$ that characterize respectively the eigenenergy states $E_\pm(t) = \pm 2(\alpha^2 t^2 + \Delta^2)^{1/2}$. The wave function $C_0^{(1)}(t)$ characterizing the middle level corresponds to the eigenenergy state $E(t) = 0$. This eigenenergy state does not evolve in time and the transitions between neighboring energy levels are allowed.

The operator S^z is diagonal in its eigen representation and has the eigen values -1, 0 and $+1$ as diagonal elements that match respectively the states $|1\rangle$, $|0\rangle$ and $|2\rangle$ that form avoided-level crossing points. From Eqs.(2.1) and (2.3), we arrive at a system of three decoupled differentials equations for the states $C_1^{(1)}(t)$, $C_0^{(1)}(t)$ and $C_2^{(1)}(t)$. The first pair of equations for the states with

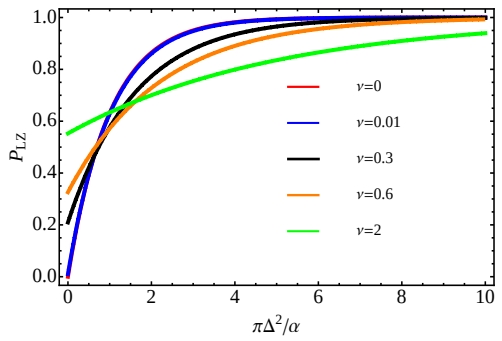

FIG. 5. (Color online) Landau-Zener transitions probabilities for the two-level system at infinite time as a function of dimensionless parameter $\pi\Delta^2/\alpha$ in the presence of a one-component slow transverse noise. The parameter $\nu = \pi\eta^2/\alpha$ characterizes the noise amplitude.

minimal/maximal projection to z-axis is:

$$\frac{d^3}{dz^3}C_1^{(1)}(z) + (4i\lambda - 2 - z^2)\frac{d}{dz}C_1^{(1)}(z) - zC_1^{(1)}(z) = 0, \tag{4.1}$$

$$\frac{d^3}{dz^3}C_2^{(1)}(z) + (4i\lambda + 2 - z^2)\frac{d}{dz}C_2^{(1)}(z) - zC_2^{(1)}(z) = 0. \tag{4.2}$$

The dynamics of the middle level is independently derived and is governed by a third order linear differential equation of the form (2.24). For sake of consistency it is presented here as follows:

$$\frac{d^3}{dz^3}C_0^{(1)}(z) - \frac{1}{z}\frac{d^2}{dz^2}C_0^{(1)}(z) - 4\Big[\frac{i\lambda}{z}C_0^{(1)}(z) + (\frac{z^2}{4} - i\lambda)\frac{d}{dz}C_0^{(1)}(z)\Big] = 0. \tag{4.3}$$

Obtaining these equations, no assumptions on the initial preparation of the system have been adopted. One may arbitrary select a particular ground state and ask questions about the probability to find the system on the excited states.

Considering equations (2.14) and (2.24) then equation (4.3) may take the form:

$$\frac{d}{dz}C_0^{(1)}(z) = -4i\lambda\int_{-\infty}^{z} dz_1 \cosh\Big[\frac{1}{2}(z^2 - z_1^2)\Big]C_0^{(1)}(z_1). \tag{4.4}$$

To find the solutions of Eqs.(4.1)-(4.2), it would be instructive to know that the square of the Weber's functions in Eqs.(2.6) and (2.7) satisfies the third order differential equation[69]:

$$\frac{d^3}{dz^3}\Big[C_1^{(1/2)}(z)\Big]^2 + (4i\lambda - 2 - z^2)\frac{d}{dz}\Big[C_1^{(1/2)}(z)\Big]^2 - z\Big[C_1^{(1/2)}(z)\Big]^2 = 0, \tag{4.5}$$

$$\frac{d^3}{dz^3}\Big[C_2^{(1/2)}(z)\Big]^2 + (4i\lambda + 2 - z^2)\frac{d}{dz}\Big[C_2^{(1/2)}(z)\Big]^2 - z\Big[C_2^{(1/2)}(z)\Big]^2 = 0. \tag{4.6}$$

For consistency, we will expressed all our solutions through the Weber's function. The solutions to these equations are, $D^2_{i\lambda}(z)$, $D^2_{-i\lambda-1}(iz)$, $D^2_{-i\lambda-1}(-iz)$ or any product of any of the functions $D_{i\lambda}(z)$, $D_{-i\lambda-1}(iz)$, and $D_{-i\lambda-1}(-iz)$[69]. With given initial conditions, these give the basis to the solutions of Eqs.(4.1) and (4.2).

Consider the case when the initial conditions are:

$$C_1^{(1)}(-\infty) = 1, \quad C_0^{(1)}(-\infty) = 0 \quad \text{and} \quad C_2^{(1)}(-\infty) = 0, \tag{4.7}$$

then $C_1^{(1)}(t) = B_- D^2_{-i\lambda}(-iz)$ and $C_2^{(1)}(t) = B_+ D^2_{-i\lambda-1}(-iz)$. Here, $B_\pm$ are normalization factors. We establish relations between the wave functions of the triplet $C_1^{(1)}(t)$, $C_0^{(1)}(t)$, $C_2^{(1)}(t)$ and the doublet $C_1^{(1/2)}(t)$, $C_2^{(1/2)}(t)$ states considering in addition the normalization condition $\sum_{m=-S}^{S}|C_m^{(S)}(t)|^2 = 1$:

$$C_1^{(1)}(t) = \Big[C_1^{(1/2)}(t)\Big]^2, \tag{4.8}$$

$$C_0^{(1)}(t) = \sqrt{2}C_1^{(1/2)}(t)C_2^{(1/2)}(t), \tag{4.9}$$

$$C_2^{(1)}(t) = \Big[C_2^{(1/2)}(t)\Big]^2. \tag{4.10}$$

Considering conditions

$$C_1^{(1)}(-\infty) = 0, \quad C_0^{(1)}(-\infty) = 1 \quad \text{and} \quad C_2^{(1)}(-\infty) = 0, \tag{4.11}$$

then, Eq.(4.3) or its integral-differential form (4.4) is isomorphic to (2.24):

$$C_0^{(1)}(t) = -\lambda e^{-\pi\lambda/2}\Big[|D_{-i\lambda-1}(-i\mu t)|^2 - \frac{1}{\lambda}|D_{-i\lambda}(-i\mu t)|^2\Big]. \tag{4.12}$$

From conditions $C_1^{(1)}(-\infty) = C_2^{(1)}(-\infty) = 0$, the solutions of Eqs.(4.1) and (4.2) satisfy the following equations:

$$C_1^{(1)}(t) = \sqrt{2\lambda}e^{-i\pi/4}\int_{-\infty}^{z}\exp\Big(\frac{1}{2}[z^2 - z_1^2]\Big)C_0^{(1)}(z_1)dz_1, \tag{4.13}$$

$$C_2^{(1)}(t) = \sqrt{2\lambda}e^{-i\pi/4}\int_{-\infty}^{z}\exp\Big(-\frac{1}{2}[z^2 - z_1^2]\Big)C_0^{(1)}(z_1)dz_1, \tag{4.14}$$

and

$$\frac{d}{dz}C_0^{(1)}(z) = \sqrt{2\lambda}\Big(C_1^{(1)}(z) + C_2^{(1)}(z)\Big)e^{-i\pi/4}. \tag{4.15}$$

Substituting Eq.(4.12) into Eqs.(4.13) and (4.14) considering (2.27) then this yields

$$C_1^{(1)}(t) = \sqrt{2\lambda}\exp\Big(i\varphi' - \frac{i\pi}{4} - \frac{\pi\lambda}{2}\Big) D_{-i\lambda}(-i\mu t)[D_{-i\lambda-1}(-i\mu t)]^*, \tag{4.16}$$

$$C_2^{(1)}(t) = -\sqrt{2\lambda}\exp\Big(i\varphi' + \frac{i\pi}{4} - \frac{\pi\lambda}{2}\Big) [D_{-i\lambda}(-i\mu t)]^* D_{-i\lambda-1}(-i\mu t). \tag{4.17}$$

Here, φ' is an arbitrary phase factor. The above permits to achieve the LZ transition probabilities expressed through the following :

$$C_1^{(1)}(t) = -\sqrt{2}C_1^{(1/2)}(t)C_2^{(1/2)*}(t), \tag{4.18}$$

$$C_0^{(1)}(t) = |C_1^{(1/2)}(t)|^2 - |C_2^{(1/2)}(t)|^2, \tag{4.19}$$

$$C_2^{(1)}(t) = \sqrt{2}C_1^{(1/2)*}(t)C_2^{(1/2)}(t). \tag{4.20}$$

For completeness, the solution of (4.1)-(4.3) with initial conditions

$$C_1^{(1)}(-\infty) = 0, \quad C_0^{(1)}(-\infty) = 0 \quad \text{and} \quad C_2^{(1)}(-\infty) = 1 \tag{4.21}$$

can be found with the help of Eqs.(4.18)-(4.20). It is instructive to know that the three-level system for $S = 1$ possesses an additional symmetry level that imitates a particle-hole $SU(2)$ symmetry group[53]. A transition matrix for $S = 1$ is then constructed as follows:

$$\hat{\mathcal{U}}^{\rm LZ}(t) = \begin{bmatrix} \Big[C_1^{(1/2)}(t)\Big]^2 & \sqrt{2}C_1^{(1/2)}(t)C_2^{(1/2)}(t) & \Big[C_2^{(1/2)}(t)\Big]^2 \\ -\sqrt{2}C_1^{(1/2)}(t)C_2^{(1/2)*}(t) & |C_1^{(1/2)}(t)|^2 - |C_2^{(1/2)}(t)|^2 & \sqrt{2}C_1^{(1/2)*}(t)C_2^{(1/2)}(t) \\ \Big[C_2^{(1/2)*}(t)\Big]^2 & -\sqrt{2}C_1^{(1/2)*}(t)C_2^{(1/2)*}(t) & \Big[C_1^{(1/2)*}(t)\Big]^2 \end{bmatrix} \tag{4.22}$$

The transition matrix in (4.22) is thus a generalization of the result in Ref. 28. Here, the matrix element $\mathcal{U}_{nm}^{\rm LZ}(t)$

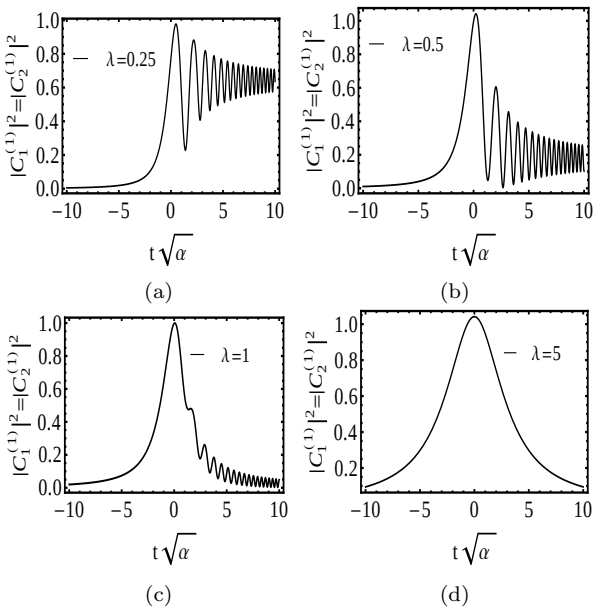

FIG. 6. Time evolution of the LZ transition probability in the diabatic basis of the three-level system. Panels (a) and (d) show the numerical results for rapid and slow LZ drive respectively. The results on intermediate regime are presented on panels (b) and (c). The LZ parameter $\lambda = \Delta^2/2\alpha$. The time is in the units of $1/\sqrt{\alpha}$.

Initial occupation for $t=-\infty$	Final occupation for $t=\infty$
1	$e^{-4\pi\lambda}$
0	$2[e^{-2\pi\lambda}-e^{-4\pi\lambda}]$
0	$[1-e^{-2\pi\lambda}]^2$
0	$2[e^{-2\pi\lambda}-e^{-4\pi\lambda}]$
1	$[1-2e^{-2\pi\lambda}]^2$
0	$2[e^{-2\pi\lambda}-e^{-4\pi\lambda}]$
0	$[1-e^{-2\pi\lambda}]^2$
0	$2[e^{-2\pi\lambda}-e^{-4\pi\lambda}]$
1	$e^{-4\pi\lambda}$

TABLE I. Landau-Zener transition probabilities in the three-level system.

is the transition amplitude for the transition from the diabatic state $|n\rangle$ to $|m\rangle$. Applying the condition $t=\infty$ to our generalized results yields exactly those in Ref. 28 for all transition matrix elements (see TABLE.I).

Compute numerically the dynamical evolution of the model (2.2) for $S=1$ by solving the Schrödinger equation for the amplitudes $C_n^{(1)}(t)$, $(n=1,0,2)$ then we plot the population $|C_n^{(1)}(t)|^2$ considering conditions (4.6). These results are depicted on FIG.6. As foreseen, the populations $|C_1^{(1)}(t)|^2$ and $|C_2^{(1)}(t)|^2$ are identically distributed on levels defined by the states $|1\rangle$ and $|2\rangle$.

Figures 6(a)-6(d) show how population of each of the levels changes with the LZ parameter. Taking limits in Eq.(4.22) as $t\to\infty$ achieves the results in Ref. 72 (see TABLE.I).

B. Bloch spin-1 picture

Focus in this heading is exclusively on $P_{01}(t) = |\mathcal{U}_{01}^{\rm LZ}(t)|^2$ and $P_{00}(t)=|\mathcal{U}_{00}^{\rm LZ}(t)|^2$. Here $\mathcal{U}_{01}^{\rm LZ}(t)$ and $\mathcal{U}_{00}^{\rm LZ}(t)$ are matrix elements of the time evolution operator 3×3 matrix defined in (4.22).

The dynamics of the system is governed by 9 equations for the components of the 3×3 density matrix. This is reduced to a set of 6 equations due to symmetry of the levels. Knowledge of two of the matrix elements is enough to compute the other matrix elements considering the condition for the probability conservation. In this regard, we reduce the problem to a set of two equations for $\hat\rho_+(t)=\hat\rho_{11}(t)-\hat\rho_{00}(t)$ and $\hat\rho_-(t)=\hat\rho_{22}(t)-\hat\rho_{00}(t)$. Here, the indices $1,0$ and 2 denote the three crossing levels. It would be convenient to express the probabilities $P_{01}(t)$ and $P_{00}(t)$ in exponential form as we did in the preceding section for LZ spin-1/2 probabilities. Eq.(4.12) establishes a relation between the occupation probability $P_{00}(t)$ and the LZ transition probability $P_{\rm LZ}(t)$:

$$P_{00}(t)=(2P_{\rm LZ}(t)-1)^2. \tag{4.23}$$

The normalization of probabilities helps one to express the finite tunneling time probabilities as:

$$P_{01}(t)=2\Big[\exp\Big\{-2\pi\lambda[F(t)+\ln W(t)]\Big\}- \exp\Big\{-4\pi\lambda[F(t)+\ln W(t)]\Big\}\Big], \tag{4.24}$$

and

$$P_{00}(t)=\Big[1-2\exp\Big\{-2\pi\lambda[F(t)+\ln W(t)]\Big\}\Big]^2. \tag{4.25}$$

This will aid to derive the LZ transition probabilities in the slow noise approximation. This paper considers the transition probabilities in the fast and slow noise approximations. Detailed calculations for relevant equations are found in Appendix B.

We show the correspondence between the Schrödinger and Bloch approaches. Schrödinger dynamics of $\mathcal{N}$-level

systems describing spin $S = (\mathcal{N}-1)/2$ may be expressed through a set of $\mathcal{N}$ coupled first order linear differential equations (LDE). These equations represent $\mathcal{N}$ independent $\mathcal{N}$-th order LDE.

The time evolution operator expressed through Jacobi $\mathcal{N}-1$ order polynomials[61,71] may be constructed on the basis of the $SU(2)$ group with fundamental spinors. Therefore, the solution of $\mathcal{N}$-th order LDE is expressed through the $(\mathcal{N}-1)$-fold product of Weber's functions. The Bloch dynamics of the spin S is based on one vector and $2S-1$ tensor Bloch equations. This is due to the fact that the density matrix has $2S$ conservable values[28].

V. TRANSVERSE NOISE IN THE SPIN-1 LANDAU-ZENER THEORY

In this section we evaluate the tunneling probabilities for the case when the inter-level distance between the states of a three-level system is renormalized by a random classical field. Similar studies were performed by Pokrovsky[27] with restriction to fast noise. To the best of our knowledge the slow noise approximation has not yet been investigated for three-level systems. So, we study the LZ transition probabilities for the three-level system in the slow noise approximation by applying an ensemble averaging over all possible noise realizations. The procedure to obtain the equation of motion for the density matrix describing transitions in three-level systems imitate that of $S = 1/2$ in section II of this paper (details of the procedure can be found in Appendix B). From equations (B1)-(B5) one gets:

$$\frac{d\hat{\rho}_+(t)}{dt} = -4\int_{-\infty}^{t}\Theta_+(t)\Theta_-(t_1)\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\rho}_+(t_1)dt_1 - 2\int_{-\infty}^{t}\Theta_+(t)\Theta_-(t_1)\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\rho}_-(t_1)dt_1 + \Phi, \quad (5.1)$$

$$\frac{d\hat{\rho}_-(t)}{dt} = -2\int_{-\infty}^{t}\Theta_+(t)\Theta_-(t_1)\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\rho}_+(t_1)dt_1 - 4\int_{-\infty}^{t}\Theta_+(t)\Theta_-(t_1)\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\rho}_-(t_1)dt_1 + \Psi. \quad (5.2)$$

Here, $\Theta_\pm(t) = (\Theta^x(t) \pm i\Theta^y(t))/\sqrt{2}$. Equations (5.1) and (5.2) are obtained considering the functions $\Theta_+(t)$ and $\Theta_-(t)$ to be noise fields. Φ and Ψ are functions of $\Theta_+(t)\Theta_+(t_1)$ and $\Theta_-(t)\Theta_-(t_1)$. It is instructive to note that the averages $\langle\Theta_+(t)\Theta_+(t_1)\rangle$ and $\langle\Theta_-(t)\Theta_-(t_1)\rangle$ vanish as Gaussian correlators and consequently Φ and Ψ will not contribute to the transition probabilities. If $\Theta_+(t)$ and $\Theta_-(t)$ are not noise correlated then this is not applicable and the components $\hat{\rho}_{12}(t)$ and $\hat{\rho}_{21}(t)$ will enter the expression of the density matrix.

We verify if the method employed in Section II for the spin-1/2 LZ transition relates a third order differential equation considering $\hat{\rho}_{12}^{(0)}(t)$ and $\hat{\rho}_{21}^{(0)}(t)$:

$$\begin{aligned}\frac{d}{dt}\hat{\rho}_+^{(0)}(t) = &-8\Delta^2\int_{-\infty}^{t}\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\rho}_+^{(0)}(t_1)dt_1 - 4\Delta^2\int_{-\infty}^{t}\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\rho}_-^{(0)}(t_1)dt_1\\ &-6\Delta^2\int_{-\infty}^{t}\exp\Big[i\alpha(t^2-t_1^2)\Big]\hat{\rho}_{21}^{(0)}(t_1)dt_1 - 6\Delta^2\int_{-\infty}^{t}\exp\Big[-i\alpha(t^2-t_1^2)\Big]\hat{\rho}_{12}^{(0)}(t_1)dt_1, \quad (5.3)\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\hat{\rho}_-^{(0)}(t) = &-4\Delta^2\int_{-\infty}^{t}\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\rho}_+^{(0)}(t_1)dt_1 - 8\Delta^2\int_{-\infty}^{t}\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\rho}_-^{(0)}(t_1)dt_1\\ &-6\Delta^2\int_{-\infty}^{t}\exp\Big[i\alpha(t^2-t_1^2)\Big]\hat{\rho}_{21}^{(0)}(t_1)dt_1 - 6\Delta^2\int_{-\infty}^{t}\exp\Big[-i\alpha(t^2-t_1^2)\Big]\hat{\rho}_{12}^{(0)}(t_1)dt_1. \quad (5.4)\end{aligned}$$

It is less obvious to derive a third order differential equation for the matrices:

$$\hat{\rho}^{(0)}(t) = \begin{bmatrix}\hat{\rho}_+^{(0)}(t)\\ \hat{\rho}_-^{(0)}(t)\end{bmatrix} \quad \text{and} \quad \hat{\mathcal{Q}}^{(0)}(t) = \begin{bmatrix}\hat{\rho}_{21}^{(0)}(t)\\ \hat{\rho}_{12}^{(0)}(t)\end{bmatrix}, \quad (5.5)$$

as we did for the spin $S = 1/2$. The matrix elements in the second matrix of equation (5.5) generated by the

last two terms in Eqs.(5.3) and (5.4) might be viewed as external sources for a homogeneous matrix element equation of the form (2.14). However, an appropriate choice of variables leads to the non homogeneous equation:

$$\frac{d}{dt}\hat{\rho}^{(0)}(t) = -4\Delta_M^2 \int_{-\infty}^{t} \cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\rho}^{(0)}(t_1)dt_1 \\ -6\Delta^2\hat{\mathcal{J}}(t)\hat{\mathcal{Q}}^{(0)}(t). \quad (5.6)$$

Obtaining this, we define corresponding functions in the absence of noise as follows as in Eq.(5.5). In equation (5.6), the square of the inter-level distance is given by the following matrix

$$\Delta_M^2 = 2\Delta^2 \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}. \quad (5.7)$$

The operator $\hat{\mathcal{J}}(t)$ acts onto the subspace $\mathcal{S}_b$ of the vector $\hat{\mathcal{Q}}^{(0)}(t)$ and generates the last two terms in Eqs.(5.3) and (5.4). The Lie algebra associated with the time derivative properties of $\hat{\mathcal{J}}(t)$ can be defined from the relation,

$$\hat{\mathcal{J}}(t)\hat{\rho}_{12}^{(0)}(t) = \int_{-\infty}^{t} \cos\Big[\alpha(t^2-t_1^2)\Big]\mathrm{Re}[\hat{\rho}_{12}^{(0)}(t_1)]dt_1 \\ -i\int_{-\infty}^{t} \sin\Big[\alpha(t^2-t_1^2)\Big]\mathrm{Im}[\hat{\rho}_{12}^{(0)}(t_1)]dt_1. \quad (5.8)$$

Eq.(5.8) shows $\hat{\mathcal{J}}(t)$ in action into $\mathcal{S}_b$. The action of $\hat{\mathcal{J}}(t)$ is symmetric so that $\hat{\mathcal{J}}(t)\hat{\rho}_{21}^{(0)}(t) = \hat{\mathcal{J}}(t)\hat{\rho}_{12}^{(0)}(t)$. Obviously, the operators d/dt and its higher orders d^q/dt^q $(q>1)$ have non-zero actions in $\mathcal{S}_b$. The simultaneous actions $\frac{d}{dt}\circ\hat{\mathcal{J}}(t)$ and $\frac{d^2}{dt^2}\circ\hat{\mathcal{J}}(t)$ including $\frac{d^q}{dt^q}\circ\hat{\mathcal{J}}(t)$ are then also defined in $\mathcal{S}_b$.

By evaluating $\frac{d}{dt}\hat{\mathcal{J}}(t)\hat{\rho}_{12}^{(0)}(t)$ and $\frac{d^2}{dt^2}\hat{\mathcal{J}}(t)\hat{\rho}_{12}^{(0)}(t)$ and considering the initial condition $\hat{\rho}_{12}^{(0)}(-\infty)=0$ we define equivalent initial conditions helpful for further purposes. It can be verified that,

$$\frac{d}{dt}\hat{\mathcal{J}}(t)\hat{\rho}_{12}^{(0)}(t)\mid_{t=-\infty} = \hat{\mathcal{J}}(t)\hat{\rho}_{12}^{(0)}(t)\mid_{t=-\infty} = 0. \quad (5.9)$$

These properties are directly applicable to $\hat{\rho}_{21}^{(0)}(t)$ as $\hat{\rho}_{21}^{(0)}(-\infty)=0$. We omitted the symbol $\circ$ keeping in mind the ordering of actions in $\mathcal{S}_b$, namely, $\hat{\mathcal{J}}(t)$ first passes and d^q/dt^q follows.

Eq.(5.6) imitates Eq.(2.14) for the column matrix $\hat{\rho}^{(0)}(t)$ of one variable and may be transformed to a non homogeneous linear third order differential equation:

$$\frac{d^3}{d\tau^3}\hat{\rho}^{(0)}(\tau) - \frac{1}{\tau}\frac{d^2}{d\tau^2}\hat{\rho}^{(0)}(\tau) - 4\Big[\frac{2\lambda_M}{\tau}\hat{\rho}^{(0)}(\tau) \\ -(\tau^2+2\lambda_M)\frac{d}{d\tau}\hat{\rho}^{(0)}(\tau)\Big] = -6\Delta^2\hat{\mathcal{X}}(\tau), \quad (5.10)$$

where

$$\hat{\mathcal{X}}(\tau) = \frac{1}{\sqrt{\alpha}}\frac{d^2}{d\tau^2}\hat{\mathcal{J}}(\tau)\hat{\mathcal{Q}}^{(0)}(\tau) - \frac{1}{\sqrt{\alpha}}\frac{d}{d\tau}\hat{\mathcal{J}}(\tau)\hat{\mathcal{Q}}^{(0)}(\tau) \\ +\frac{4\tau^2}{\sqrt{\alpha}}\hat{\mathcal{J}}(\tau)\hat{\mathcal{Q}}^{(0)}(\tau) \quad (5.11)$$

with $\lambda_M = \Delta_M^2/2\alpha$ being the LZ parameter in matrix form. The physical sense of $\hat{\mathcal{X}}(\tau)$ may be achieved by expressing $\hat{\rho}_{12}^{(0)}(\tau) = C_1^{(1)}(\tau)C_2^{(1)*}(\tau)$ and $\hat{\rho}_{21}^{(0)}(\tau) = C_1^{(1)*}(\tau)C_2^{(1)}(\tau)$ through Weber's functions.

Considering Eq.(5.9), the solution of Eq.(5.10) may be obtained with the aid of $\hat{\rho}^{(0)}(-\infty) = -[1\ \ 1]^{\mathrm{T}}$ and the following conditions

$$\frac{d^2}{d\tau^2}\hat{\rho}^{(0)}(\tau)\mid_{\tau=-\infty} = -8\lambda_M, \quad \frac{d}{d\tau}\hat{\rho}^{(0)}(\tau)\mid_{\tau=-\infty} = 0. \quad (5.12)$$

Then the solution of (5.10) is given by

$$\hat{\rho}_+^{(0)}(\tau) = \lambda e^{-\pi\lambda}\Big(2|D_{-i\lambda}(-i\mu_0\tau)|^2|D_{-i\lambda-1}(-i\mu_0\tau)|^2 \\ +\lambda\Big[|D_{-i\lambda-1}(-i\mu_0\tau)|^2 - \frac{1}{\lambda}|D_{-i\lambda}(-i\mu_0\tau)|^2\Big]^2\Big). \quad (5.13)$$

The nontrivial dynamics of the population difference for the three-level LZ transition at any given time τ is governed by Eq.(5.10). Considering the limit $\tau\to\infty$, we have the population difference

$$\hat{\rho}_+^{(0)}(\infty) = \hat{\rho}_-^{(0)}(\infty) = 12e^{-3\pi\lambda}\sinh(\pi\lambda) - 1. \quad (5.14)$$

Using Eq.(5.14) and the conservation of probability $\mathrm{Tr}\hat{\tilde{\rho}}(\tau)=1$, we arrive at the transition probabilities in the second part of TABLE.I.

A. Fast noise, spin-1

For the proper apprehension of the reader we review briefly the effects of fast noise on a three-level system. In the spirit of previous derivations, we transform Eqs.(5.1)-(5.2) to

$$\frac{d}{dt}\hat{\rho}(t) = -4\int_{-\infty}^{t}\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\mathcal{R}}_M(|t-t_1|)\hat{\rho}(t_1)dt_1. \quad (5.15)$$

Here,

$$\hat{\rho}(t) = \begin{bmatrix} \langle\hat{\rho}_+(t)\rangle \\ \langle\hat{\rho}_-(t)\rangle \end{bmatrix}, \quad (5.16)$$

and

$$\hat{\mathcal{R}}_M(|t-t_1|) = 2\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}\hat{\mathcal{R}}(|t-t_1|) \quad (5.17)$$

the matrix correlator. Equation (3.3) is structurally identical to Eq.(3.3). Similarly we define $\hat{\Omega}_M(t) = \hat{\Omega}_M^{(+)}(t) + \hat{\Omega}_M^{(-)}(t)$ as

$$\hat{\Omega}_M^{(\pm)}(t) = \int_{-\infty}^{\infty} \exp\Big[\pm i\tilde{\omega}(t)\xi\Big]\hat{\mathcal{R}}_M(|\xi|)d\xi. \qquad (5.18)$$

The solution of Eq.(5.15) can be found as

$$\langle\hat{\rho}_{11}(\infty)\rangle = \frac{1}{3}\Big(1 + \langle\hat{\rho}_+(-\infty)\rangle\Big[\sinh\frac{\theta}{2} + 2\cosh\frac{\theta}{2}\Big]e^{-\theta} - \langle\hat{\rho}_-(-\infty)\rangle\Big[\cosh\frac{\theta}{2} + 2\sinh\frac{\theta}{2}\Big]e^{-\theta}\Big), \qquad (5.19)$$

$$\langle\hat{\rho}_{00}(\infty)\rangle = \frac{1}{3}\Big(1 - \langle\hat{\rho}_+(-\infty)\rangle\Big[\cosh\frac{\theta}{2} - \sinh\frac{\theta}{2}\Big]e^{-\theta} + \langle\hat{\rho}_-(-\infty)\rangle\Big[\sinh\frac{\theta}{2} - \cosh\frac{\theta}{2}\Big]e^{-\theta}\Big), \qquad (5.20)$$

$$\langle\hat{\rho}_{22}(\infty)\rangle = \frac{1}{3}\Big(1 - \langle\hat{\rho}_+(-\infty)\rangle\Big[\cosh\frac{\theta}{2} + 2\sinh\frac{\theta}{2}\Big]e^{-\theta} + \langle\hat{\rho}_-(-\infty)\rangle\Big[\sinh\frac{\theta}{2} + 2\cosh\frac{\theta}{2}\Big]e^{-\theta}\Big). \qquad (5.21)$$

We considered the matrix transformation

$$\exp\Big[-\int_{-\infty}^{\infty}\hat{\Omega}_M(t')dt'\Big] = e^{-\theta}\begin{pmatrix}\cosh\frac{\theta}{2} & -\sinh\frac{\theta}{2}\\ -\sinh\frac{\theta}{2} & \cosh\frac{\theta}{2}\end{pmatrix}. \qquad (5.22)$$

The results agree with those of Pokrovsky[28]. The general form of these equations for arbitrary t can be obtained by $\theta \to \theta(t)$ (see TABLE.II), where $\theta(t)$ is defined similarly as in (3.7). On TABLE.II, we show infinite time transition probabilities for all possible initial occupation of the system. One can see that the transition probabilities for $S = 1$ have the same form as for $S = 1/2$. In the white noise approximation, we have the same probability distribution for all the triplet states.

Initial occupation for $t = -\infty$	Final occupation for $t = \infty$
1	$\frac{1}{3}(1 + \frac{3}{2}e^{-\theta/2} + \frac{1}{2}e^{-3\theta/2})$
0	$\frac{1}{3}(1 - e^{-3\theta/2})$
0	$\frac{1}{3}(1 - \frac{3}{2}e^{-\theta/2} + \frac{1}{2}e^{-3\theta/2})$
0	$\frac{1}{3}(1 - e^{-3\theta/2})$
1	$\frac{1}{3}(1 + 2e^{-3\theta/2})$
0	$\frac{1}{3}(1 - e^{-3\theta/2})$
0	$\frac{1}{3}(1 - \frac{3}{2}e^{-\theta/2} + \frac{1}{2}e^{-3\theta/2})$
0	$\frac{1}{3}(1 - e^{-3\theta/2})$
1	$\frac{1}{3}(1 + \frac{3}{2}e^{-\theta/2} + \frac{1}{2}e^{-3\theta/2})$

TABLE II. Fast noise transition probabilities in the three-level system.

Spin-1 in a constant off-diagonal field and a fast transverse random field

We investigate the LZ transition assisted by fast-noise. The two-component noise is defined by Eq.(3.12). The mean-value of the stochastic function describing noise in X-direction is non-zero:

$$\langle\Theta_\pm(t)\Theta_\pm(t_1)\rangle \neq 0. \qquad (5.23)$$

The matrix density describing the noise assisted transition may now be represented as follows:

$$\hat{\rho}^{(\mathrm{SF})} = \begin{bmatrix}\langle\hat{\rho}_+^{(\mathrm{SF})}(t)\rangle\\ \langle\hat{\rho}_-^{(\mathrm{SF})}(t)\rangle\end{bmatrix} \quad \text{and} \quad \mathcal{Q}^{(\mathrm{SF})}(t) = \begin{bmatrix}\langle\hat{\rho}_{21}^{(\mathrm{SF})}(t)\rangle\\ \langle\hat{\rho}_{12}^{(\mathrm{SF})}(t)\rangle\end{bmatrix}. \qquad (5.24)$$

Initial occupation for $t=-\infty$	Final occupation for $t=\infty$
1	$\frac{1}{3}(1-[\frac{3}{2}e^{-\theta/2}-\frac{1}{2}e^{-3\theta/2}][1-4e^{-2\pi\lambda}+3e^{-4\pi\lambda}]+[\frac{3}{2}e^{-\theta/2}+\frac{1}{2}e^{-3\theta/2}][3e^{-4\pi\lambda}-2e^{-2\pi\lambda}])$
0	$\frac{1}{3}(1-e^{-3\theta/2}[1-4e^{-2\pi\lambda}+3e^{-4\pi\lambda}]-e^{-3\theta/2}[3e^{-4\pi\lambda}-2e^{-2\pi\lambda}])$
0	$\frac{1}{3}(1+[\frac{3}{2}e^{-\theta/2}+\frac{1}{2}e^{-3\theta/2}][1-4e^{-2\pi\lambda}+3e^{-4\pi\lambda}]-[\frac{3}{2}e^{-\theta/2}-\frac{1}{2}e^{-3\theta/2}][3e^{-4\pi\lambda}-2e^{-2\pi\lambda}])$
0	$\frac{1}{3}(1+e^{-3\theta/2}[6e^{-2\pi\lambda}-6e^{-4\pi\lambda}-1])$
1	$\frac{1}{3}(1-2e^{-3\theta/2}[6e^{-2\pi\lambda}-6e^{-4\pi\lambda}-1])$
0	$\frac{1}{3}(1+e^{-3\theta/2}[6e^{-2\pi\lambda}-6e^{-4\pi\lambda}-1])$
0	$\frac{1}{3}(1+[\frac{3}{2}e^{-\theta/2}+\frac{1}{2}e^{-3\theta/2}][1-4e^{-2\pi\lambda}+3e^{-4\pi\lambda}]-[\frac{3}{2}e^{-\theta/2}-\frac{1}{2}e^{-3\theta/2}][3e^{-4\pi\lambda}-2e^{-2\pi\lambda}])$
0	$\frac{1}{3}(1-e^{-3\theta/2}[1-4e^{-2\pi\lambda}+3e^{-4\pi\lambda}]-e^{-3\theta/2}[3e^{-4\pi\lambda}-2e^{-2\pi\lambda}])$
1	$\frac{1}{3}(1-[\frac{3}{2}e^{-\theta/2}-\frac{1}{2}e^{-3\theta/2}][1-4e^{-2\pi\lambda}+3e^{-4\pi\lambda}]+[\frac{3}{2}e^{-\theta/2}+\frac{1}{2}e^{-3\theta/2}][3e^{-4\pi\lambda}-2e^{-2\pi\lambda}])$

TABLE III. Transition probabilities for the three-level system in both a constant magnetic field and a classical transverse noise.

The dynamics of the system is described by the equation:

$$\frac{d}{dt}\hat{\rho}^{(\mathrm{SF})}(t)=-4\Delta_M^2\int_{-\infty}^{t}\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\rho}^{(\mathrm{SF})}(t_1)dt_1 -4\int_{-\infty}^{t}\cos\Big[\alpha(t^2-t_1^2)\Big]\hat{\mathcal{R}}_M(|t-t_1|)\hat{\rho}^{(\mathrm{SF})}(t_1) -6\Delta^2\hat{\mathcal{J}}(t)\hat{\mathcal{Q}}^{(\mathrm{SF})}(t)dt_1. \tag{5.25}$$

Considering equations (3.14), (3.16) then this permits us to write the solution of equation (5.25). We consider the decaying factors inducing dephasing that enter the final transition probabilities:

$$\hat{\rho}^{(\mathrm{SF})}(t)=\exp\Big[-\int_{-\infty}^{t}\hat{\Omega}_M(t')dt'\Big]\hat{\rho}^{(0)}(t), \tag{5.26}$$

and

$$\hat{\mathcal{Q}}^{(\mathrm{SF})}(t)=\exp\Big[-\int_{-\infty}^{t}\hat{\Omega}_M(t')dt'\Big]\hat{\mathcal{Q}}^{(0)}(t), \tag{5.27}$$

Equations (5.26) and (5.27) permit us to have the following relations:

$$\langle\hat{\rho}_{11}^{(\mathrm{SF})}(\infty)\rangle=\frac{1}{3}\Big(1+\hat{\rho}_{+}^{(0)}(\infty)\Big[\sinh\frac{\theta}{2}+2\cosh\frac{\theta}{2}\Big]e^{-\theta}-\hat{\rho}_{-}^{(0)}(\infty)\Big[\cosh\frac{\theta}{2}+2\sinh\frac{\theta}{2}\Big]e^{-\theta}\Big), \tag{5.28}$$

$$\langle\hat{\rho}_{00}^{(\mathrm{SF})}(\infty)\rangle=\frac{1}{3}\Big(1-\hat{\rho}_{+}^{(0)}(\infty)\Big[\cosh\frac{\theta}{2}-\sinh\frac{\theta}{2}\Big]e^{-\theta}+\hat{\rho}_{-}^{(0)}(\infty)\Big[\sinh\frac{\theta}{2}-\cosh\frac{\theta}{2}\Big]e^{-\theta}\Big), \tag{5.29}$$

$$\langle\hat{\rho}_{22}^{(\mathrm{SF})}(\infty)\rangle=\frac{1}{3}\Big(1-\hat{\rho}_{+}^{(0)}(\infty)\Big[\cosh\frac{\theta}{2}+2\sinh\frac{\theta}{2}\Big]e^{-\theta}+\hat{\rho}_{-}^{(0)}(\infty)\Big[\sinh\frac{\theta}{2}+2\cosh\frac{\theta}{2}\Big]e^{-\theta}\Big). \tag{5.30}$$

From equations (5.28) and (5.30) considering the dynamics of the system from an initial occupation for $t=-\infty$ to a final occupation, for $t=\infty$ this permit

to write TABLE.III of the transition probabilities.

The quantities $\hat{\rho}_{+}^{(0)}(\infty)$ and $\hat{\rho}_{-}^{(0)}(\infty)$ can be obtained from TABLE.I. The corresponding transition probabilities are reported in TABLE.III.

B. Slow noise, spin-1

The transition probabilities for $S = 1$ subjected to slow noise are obtained in the same spirit as was discussed for $S = 1/2$. In a given realization Q of noise, the system of equations for the population differences (Eq.(5.3)-Eq.(5.4)) is reduced to Eq. (5.10). The solutions of this problem for $\tau = \infty$ are derived via Eq.(5.14).

The LZ solutions for the case of one-component slow transverse noise are given by

$$\mathcal{P}_{sn}^{x}[0\to 1](t) = 2\Big[\frac{\exp(-2\pi\lambda\Phi_1(t))}{\sqrt{1+\frac{2\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}} - \frac{\exp(-4\pi\lambda\Phi_2(t))}{\sqrt{1+\frac{4\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}}\Big], \tag{5.31}$$

$$\mathcal{P}_{sn}^{x}[0\to 0](t) = 1 - \frac{4\exp(-2\pi\lambda\Phi_1(t))}{\sqrt{1+\frac{2\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}} + \frac{4\exp(-4\pi\lambda\Phi_2(t))}{\sqrt{1+\frac{4\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}}. \tag{5.32}$$

For the two-component transverse noise, the transition probabilities read:

$$\mathcal{P}_{sn}^{xy}[0\to 1](t) = 2\Big[\frac{\exp(-2\pi\lambda\Phi_1(t))}{1+\frac{2\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]} - \frac{\exp(-4\pi\lambda\Phi_2(t))}{1+\frac{4\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}\Big], \tag{5.33}$$

and

$$\mathcal{P}_{sn}^{xy}[0\to 0](t) = 1 - \frac{4\exp(-2\pi\lambda\Phi_1(t))}{1+\frac{2\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]} + \frac{4\exp(-4\pi\lambda\Phi_2(t))}{1+\frac{4\pi\eta^2}{\alpha}\Big[F(t)+\ln W(t)\Big]}. \tag{5.34}$$

In equations (5.31)-(5.34) the notation $a \to b$ denotes, transition from the diabatic state $|a\rangle$ to state $|b\rangle$.

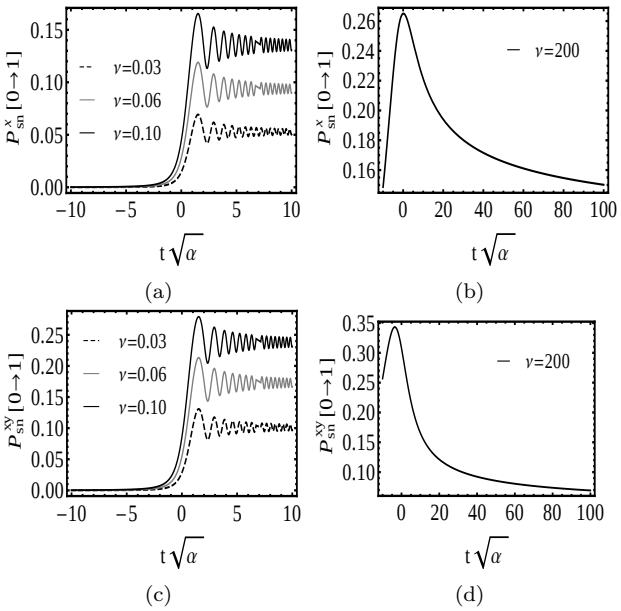

FIG. 7. Time evolution of the LZ transition probability in the diabatic basis of the three-level system in the presence of slow one- (panels (a) and (b)) and two- (panels (c) and (d)) component transverse noises (see discussion in the text). Panels (a) and (c) represent the results of numerical calculations for the small- amplitude noise. The data for the large-amplitude noise are shown on the panels (b) and (d).

$\mathcal{P}_{sn}^{\ell}[0\to 1] = \mathcal{P}_{sn}^{\ell}[0\to 2]$, with $\ell = x, xy$. Solutions (5.31)-(5.34) represent the general LZ transition probabilities for a three-level system in the presence of the slow one- and two- dimensional transverse noise. This is also relevant for both noise-induced and noise-assisted transitions. Letting $\lambda = 0$, in (5.31)-(5.34) we achieve slow-noise-induced LZ transition.

VI. DISCUSSION ON EFFECTS OF NOISE ON LANDAU ZENER TIMES

It is well-known that if one deals with a system of consequent Landau-Zener transitions, it is not sufficient to characterize a behavior of such system by an asymptotic values of probabilities. One also needs to define a tunnel time[54,73–78] in order to put a borderline between two cases when the consequent tunnel processes can or can not be considered independently.

There exist several ways to define the tunnel Landau-Zener time for two level system. Although we are not going to dwell onto detailed discussion of tunnel times in this paper, let us list a few physical definitions. One possible approach is so-called "internal clock" definition. It is based on analysis of LZ probability behavior at finite times. As it has been pointed out several times along our discussion, the finite time probability dynamics is characterized by monotonous function for slow adiabatic

passage, while for sudden (rapid) transition it oscillates before saturation at constant value. These oscillations correspond to interference processes and determine the population of two states. Therefore, the "internal clock" approach defines the Landau-Zener time as the width of transition to its asymptotic value (see Refs.[77 and 78] for detailed discussion).

An alternative approach to a definition of LZ times is based on "external clock" probe. In that case, the LZ Hamiltonian is perturbed by a periodic transverse field $\delta\hat{\mathcal{H}}(t) = \epsilon\sin(\omega t + \phi)$. Where, ω is the frequency of the field and ϕ its initial phase. The LZ time is determined through analysis of infinite time probability as a function of external field frequency (see details in Ref.[78]).

Both definitions consistently lead to estimation of LZ times as $\tau_{\rm LZ} = \Delta/\alpha$ for slow adiabatic passage and $\tau_{\rm LZ} = 1/\sqrt{\alpha}$ for rapid passage. Obviously, both definitions can be straightforwardly generalized for multi-level LZ transition.

Let us consider a slow noise as a special case of "external clock". We add a perturbation $\delta\hat{\mathcal{H}} = 2f_x(t)S^x$ to the system such that the coupling Δ is deviated as $\tilde{\Delta}(t) = \Delta + f_x(t)$. This case has been discussed in the Section III B and corresponds to a non-centered one-component transverse noise. The role of noise is to frustrate the spins in the direction of the Zeeman field. Let us consider a square fluctuation of the Bloch's vector as a probe for LZ time:

$$\langle(\delta\vec{b})^2\rangle = \langle\vec{b}^2\rangle - \langle\vec{b}\rangle^2. \tag{6.1}$$

Since the classical noise only dephases the system and does not create any dissipation in it, the condition $\vec{b}^2 = 1$ holds. In general, $\langle(\delta\vec{b})^2\rangle$ mixes the diagonal and off-diagonal components of the density matrix but for the two- and three- level systems subject to classical transverse noise we consider thus far, $\langle b_x\rangle = \langle b_y\rangle = 0$ and $b_z(t) = \hat{\rho}_{11}(t) - \hat{\rho}_{22}(t)$. Thus we write,

$$\langle(\delta\vec{b})^2\rangle = 4\mathcal{P}_{sn}(t)(1 - \mathcal{P}_{sn}(t)). \tag{6.2}$$

The subscript sn refers as usual to slow noise. It should however be noted that relation (6.2) works both for two- and three- level systems under the assumption that the system initially prepared in one of the (upper or lower) diabatic states.

With these ideas in mind, we check the numerical behavior of $\langle(\delta\vec{b})^2\rangle$ for these initial conditions of the spin. Essential of results are depicted by Fig.8. Interestingly, $\langle(\delta\vec{b})^2\rangle$ abruptly increases around the anticrossing region and saturates to its top value, confirming a spin flip transition. After the transition, the variance slightly fluctuates (slight decay of $\langle(\delta\vec{b})^2\rangle$) in the direction of the Zeeman field for adiabatic addition of noise (see Figs.8b):

$$\langle(\delta\vec{b})^2\rangle \leq \langle(\delta\vec{b})^2\rangle_{max}. \tag{6.3}$$

The two- and three- level systems, seems no longer sensitive to any addition of noise from certain value of ν:

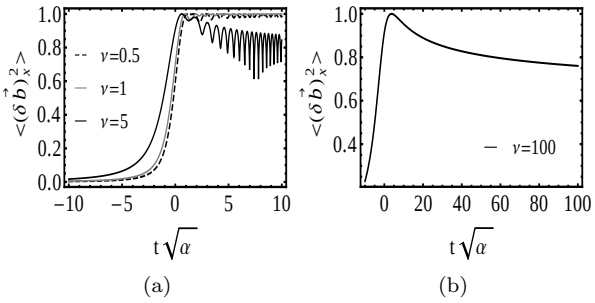

FIG. 8. Typical time evolution of the fluctuation of the square of the Bloch vector given by equation (6.2). Panel (a) shows the result for small and intermediate values of the noise amplitude. The large-amplitude noise results are presented on panel (b).

states are in thermal equilibrium. Transition time may then be defined as the particular moment where the square fluctuation of the Bloch's vector achieved its maximum value.

This qualitative definition can be experimentally probed. For the physical realization of this condition, we pose that $\langle(\delta\vec{b})^2\rangle$ is a function of the variable $\mathcal{P}_{sn}(t)$. Then, the latter behaves as the quadratic function $h(x) = 4x(1-x)$ defined in the real space. The mathematical requirement for a maximum of a function yields,

$$\mathcal{P}_{sn}(\tau_{\rm LZ}) = \frac{1}{2}. \tag{6.4}$$

This definition actually coincides with the half-width condition $\mathcal{P}_{sn}(\tau_{\rm LZ}) = \frac{1}{2}\mathcal{P}_{max}$, where $\mathcal{P}_{max}$ is the maximum value of probability. We emphasize in addition that the definition (6.4) holds both for two- and three-level systems.

VII. CONCLUSIONS

In this paper, we discuss the effects of a transverse colored noise on both two and three-level systems subject to LZ transitions. The approximate solution for the traditional LZ problem is written down in terms of Fresnel's integrals and appears to be useful for exploring the effects of slow noise at finite times. We demonstrated in the framework of von-Neumann equation, that the effects of noise on a two-level system were regulated by an integral-differential master equation of the form (2.13). We showed that for a fast Gaussian noise it is sufficient to average that equation while for the slow noise correct procedure is based on averaging the solutions over the Gaussian realization of the noise. These arguments have been found to be general for description of multi-level systems were complicated interference patterns are expected. The solution of Bloch's equation is generalized for the finite times LZ probabilities of two-and three-

level system models in the presence of slow noise. We have essentially shown that for any initial preparation of noise along one of transverse directions (X-noise) or two-component transverse noise (XY-noise) the probability is renormalized by new functions with shapes of standard LZ curves. The famous frequent exponentials appearing in LZ transition probabilities are considerably discriminated by an inverse square-root function of η^2/α.

In the absence of noise, we showed that population difference for two- and three- level system can be found as a solution of a third order linear differential equation. The solution of this equation is given in terms of products of the parabolic cylinder Weber's functions. We investigated solutions by evoking an isomorphism between Schrödinger's and Bloch's pictures. In the presence of noise in general, the equations for density matrix elements are integral-differential equations. Their solutions can be found through the averaging procedure discussed in the paper.

In conclusion, we would like to mention various realization of two- and three- level Landau-Zener transitions in recent quantum transport experiments[44–46]. The two-electron spin quantum bits are manipulated by the gate voltage applied to GaAs double quantum dot in the presence of external transverse magnetic field. The low energy two-electrons states in a double quantum well are given by three singlet and one triplet states. While the singlet states are not affected by the external magnetic field, the degeneracy of the triplet state is lifted out by the external Zeeman field. In addition, there is a fluctuating Overhauser's field appearing due to a hyperfine interaction of electrons and nuclear magnetic field of Ga and As sublattices of a host material. On one hand, the slowly fluctuating Overhauser's field is known to be responsible for both decoherence and dephasing[81]. On the other hand, if the double dot is not symmetric, the hyperfine magnetic field can result in transitions between singlet and triplet states. Therefore, in addition to three singlet states which form a three-level system, one of the triplet component should also be taken into account. Moreover, the transition between the singlet and triplet states provides a mechanism of nuclear spin polarization and effective cooling down the nuclear subsystem. Thus, the Overhauser's field leads to two competing effects of both nuclear polarization and depolarization due to relaxation and dephasing. The model we discuss in the paper does not account for the effects of relaxation only addressing the question of dephasing by classical fast- and slow- Gaussian noise. Nevertheless, the competition between the polarizing (due to two- and three- state transitions) and depolarizing (due to the dephasing) effects is fully taken into account. The suppression of the LZ transition by the Overhauser's field fluctuations characterizes the effective temperature associated with noise and can give a qualitative explanation for the nuclear spin depolarization mechanism.

Appendix A: Perturbative solution of the LZ problem

The integral-differential equation for conventional LZ problem (2.14) in the absence of noise can be solved iteratively by setting the perturbative series expansion of $\hat{\rho}^{(0)}(\tau)$ via the parameter Δ^2/α:

$$\hat{\rho}^{(0)}(\tau)=\sum_{k=0}^{\infty}\Big(-\frac{4\Delta^2}{\alpha}\Big)^k\hat{\rho}_k^{(0)}(\tau). \tag{A1}$$

Where $\hat{\rho}_0^{(0)}(\tau)=1$ and ,

$$\hat{\rho}_k^{(0)}(\tau)=\int_{-\infty}^{\tau}d\tau_1\int_{-\infty}^{\tau_1}d\tau_2\cos[\tau_1^2-\tau_2^2)]\times ...$$
$$...\times\int_{-\infty}^{\tau_{2k-2}}d\tau_{2k-1}\int_{-\infty}^{\tau_{2k-1}}d\tau_{2k}\cos[\tau_{2k-1}^2-\tau_{2k}^2]. \tag{A2}$$

(See Ref. 79 for details of calculation of sophisticated multiple integrals (A2) appearing in a classical-mechanical problem of a ball rolling on a Cornu spiral.)

In the presence of noise, we do $\Delta\to\eta$ in Eq.(A1) and the function $\hat{\rho}_k^{(0)}(\tau)$ is modified $\hat{\rho}_k(\tau)$:

$$\hat{\rho}_k(\tau)=\int_{-\infty}^{\tau}d\tau_1\int_{-\infty}^{\tau_1}d\tau_2\cos[\tau_1^2-\tau_2^2]\times ...$$
$$...\times\int_{-\infty}^{\tau_{2k-2}}d\tau_{2k-1}\int_{-\infty}^{\tau_{2k-1}}d\tau_{2k}\cos[\tau_{2k-1}^2-\tau_{2k}^2]\times$$
$$\times F^{(k)}(\tau_1,\tau_2,...,\tau_{2k}), \tag{A3}$$

where

$$F^{(k)}(\tau_1,\tau_2,...,\tau_{2k})=$$
$$\eta^{-2k}\langle f_+(\tau_1)f_-(\tau_2)...f_-(\tau_{2k-1})f_+(\tau_{2k})\rangle. \tag{A4}$$

To calculate the higher order correlation function, the Wick theorem is used. For the zero-mean random variables $f_+(\tau)$ and $f_-(\tau)$ this theorem suggests that:

$$F^{(k)}(\tau_1,\tau_2,...,\tau_{2k})=\eta^{-2k}$$
$$\begin{cases}\sum_{pairs}\prod_{n=1}^{k}\langle f_+(\tau_{2n-1})f_-(\tau_{2n})\rangle, & \text{for even}\quad k,\\ 0, & \text{for odd}\quad k.\end{cases}. \tag{A5}$$

The summation $\sum_{pairs}$ runs over all possible combinations of pairs out of the 2k variables $(\tau_1,\tau_2,...,\tau_{2k})$. Calculations for a one-component transverse noise lead to the Kayanuma result[26] ($\gamma_0=\gamma/\sqrt{\alpha}$):

$$F^{(k)}(\tau_1,\tau_2,...,\tau_{2k})=\sum_{pairs}\exp\Big(-\gamma_0\sum_{n=1}^{k}|\tau_{2n-1}-\tau_{2n}|\Big), \tag{A6}$$

while the two-component transverse noise (XY-noise), using combinatorics , Keldysh model[80]:

$$F^{(k)}(\tau_1,\tau_2,...,\tau_{2k}) = \sum_{pairs} 2^k \exp\Big(-\gamma_0 \sum_{n=1}^{k} |\tau_{2n-1}-\tau_{2n}|\Big). \tag{A7}$$

For slow or fast noise, we do respectively $\gamma \to 0$ and the former time-dependent function (A4) turns out to be a simple combinatorial factor. The long-time asymptotic value of $\hat{\rho}_k^{(0)}(\tau)$ becomes

$$\hat{\rho}_k^{(0)}(\infty) = \frac{\pi^k}{2^{2k-1}k!}, \quad k \geq 1. \tag{A8}$$

This helps to find (2.9) in a perturbative expansion:

$$P_{\rm LZ}(\infty) = -\sum_{k=1}^{\infty} a_k \Big(\frac{\pi\Delta^2}{\alpha}\Big)^k. \tag{A9}$$

Here, $a_k = (-1)^k/k!$ and the radius of convergence for Eq.(A9) equals infinity. Following considering the limit of slow or fast noise, the probability (A9) is modified by a coefficient in the perturbative expansion. We write the solution of the integral equation (3.3) for the cases of slow and fast noise driven LZ transition as

$$P_{\rm LZ}(\infty) = -\sum_{k=1}^{\infty} a_k b_k \Big(\frac{\pi\eta^2}{\alpha}\Big)^k, \tag{A10}$$

where b_k are the combinatorial factors which appear after averaging over noise realization and depend on either fast or slow noise.

1. Slow noise

It has been shown in Ref. 80 that for slow noise, the combinatorial factor b_k is expressed as:

$$b_k = \sum_{pairs} 1 = (2k-1)!!, \tag{A11}$$

for an X-noise model and

$$b_k = \sum_{pairs} 2^k = 2^k k!, \tag{A12}$$

for an XY-noise.

2. Fast noise

The case of fast noise is completely different. In contrast with the slow noise, as shown in Ref. 25 only a single term out of the $(2k-1)!!$ pairings in Eq.(A4) contributes to the summation for an X- noise:

$$b_k = 2^{k-1}. \tag{A13}$$

For an XY- model, noise contributes as

$$b_k = 2^{2k-1}. \tag{A14}$$

Equation (A10) can be viewed as a result of averaging the LZ probability over disorder noise realizations (the exponential function in LZ probability containing the fluctuating field is an ensemble average). This is in contrast to the fast noise case for which the argument of exponential function in the LZ probability is proportional to "two-point noise correlation function" (the argument is disorder average). Moreover, the statement concerning disorder averaging remains true for any finite time transition probability. Therefore, the same time dependent function $F(t) + \ln W(t)$ will enter the equation for the finite time slow noise driven LZ transition. Since the coefficient b_k strongly depends on k, the radius of convergence of the perturbative expansion must also be found. After summing up the perturbative series for the LZ probability within the circle of convergence, the function has to be analytically continued into the outer part of the circle. We will identify analytical functions describing slow noise driven LZ probability and consider finite time LZ transition. Proceeding, with b_k in Eqs.(A11)-(A14) we will recover exactly all the LZ probability found previously.

Appendix B: Spin-1 density matrix equations

The equation of motion for the density matrix describing transitions in three-level systems can be obtained in the same way as we discussed in details earlier for $S = 1/2$. In this Appendix we present the full set of these equations for completeness of the discussion about connections between Schrödinger and Bloch pictures:

$$\frac{d\hat{\rho}_+(t)}{dt} = i\Theta_+(t)(2\hat{\rho}_{10}(t) - \hat{\rho}_{02}(t)) - i\Theta_-(t)(2\hat{\rho}_{01}(t) - \hat{\rho}_{20}(t)) \tag{B1}$$

$$\frac{d\hat{\rho}_-(t)}{dt} = -i\Theta_+(t)(2\hat{\rho}_{02}(t) - \hat{\rho}_{10}(t)) + i\Theta_-(t)(2\hat{\rho}_{20}(t) - \hat{\rho}_{01}(t)). \tag{B2}$$

Here,

$$\hat{\rho}_{10}(t) = i\frac{\int_{t_0}^{t}\exp\left(i\int_{t_0}^{t_1}\Theta^z(\tau')d\tau'\right)\Theta_-(t_1)\hat{\rho}_+(t_1)dt_1}{\exp\left(i\int_{t_0}^{t}\Theta^z(\tau')d\tau'\right)} + i\frac{\int_{t_0}^{t}\exp\left(i\int_{t_0}^{t_1}\Theta^z(\tau')d\tau'\right)\Theta_+(t_1)\hat{\rho}_{12}(t_1)dt_1}{\exp\left(i\int_{t_0}^{t}\Theta^z(\tau')d\tau'\right)}, \tag{B3}$$

$$\hat{\rho}_{12}(t) = -i\frac{\int_{t_0}^{t}\exp\left(2i\int_{t_0}^{t_1}\Theta^z(\tau')d\tau'\right)\Theta_-(t_1)(\hat{\rho}_{02}(t_1) - \hat{\rho}_{10}(t_1))dt_1}{\exp\left(2i\int_{t_0}^{t}\Theta^z(\tau')d\tau'\right)}, \tag{B4}$$

$$\hat{\rho}_{02}(t) = -i\frac{\int_{t_0}^{t}\exp\left(i\int_{t_0}^{t_1}\Theta^z(\tau')d\tau'\right)\Theta_-(t_1)\hat{\rho}_-(t_1)dt_1}{\exp\left(i\int_{t_0}^{t}\Theta^z(\tau')d\tau'\right)} - i\frac{\int_{t_0}^{t}\exp\left(i\int_{t_0}^{t_1}\Theta^z(\tau')d\tau'\right)\Theta_+(t_1)\hat{\rho}_{12}(t_1)dt_1}{\exp\left(i\int_{t_0}^{t}\Theta^z(\tau')d\tau'\right)}. \tag{B5}$$

Here, $\hat{\rho}_{ij}(t) = \hat{\rho}^*_{ji}(t)$ and $\hat{\rho}_{\pm}(t) = \hat{\rho}^*_{\pm}(t)$.

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