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Linewidth of the electromagnetic radiation from Josephson junctions near cavity resonances

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The powerful terahertz emission from intrinsic Josephson junctions in high-T_c cuprate superconductors has been detected recently. The synchronization of different junctions is enhanced by excitation of the geometrical cavity resonance. A key characteristics of the radiation is its linewidth. In this work, we study the intrinsic linewidth of the radiation near the internal cavity resonance. Surprisingly, this problem was never considered before neither for a single Josephson junction nor for a stack of the intrinsic Josephson junctions realized in cuprate superconductors. The linewidth appears due to the slow phase diffusion, which is determined by the dissipation and amplitude of the noise. We found that both these parameters are resonantly enhanced when the cavity mode is excited but enhancement of the dissipation dominates leading to the net suppression of diffusion and dramatic narrowing of the linewidth. The line shape changes from Lorentzian to Gaussian when either the Josephson frequency shifted away from the resonance or the temperature is increased.

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In a Josephson junction (JJ) biased by a dc voltage V the supercurrent oscillates with the angular frequency $\omega_J = 2eV/\hbar$. This allows to use the JJs as high-frequency electromagnetic (EM) generators. The radiation from a single JJ however is weak, only several picowatts. The radiation power can be enhanced using arrays of JJs. [1, 2] In 2007, coherent and strong terahertz (THz) radiations from intrinsic Josephson junctions (IJJs) of $Bi_2Sr_2CaCu_2O_8$ (BSCCO) [3] has been observed experimentally [4]. In this experiment, the radiation power was estimated as 0.5 μW which is several orders of magnitude stronger than that from a single junction. The frequency has ranged from 0.4 to 0.8 THz and inversely proportional to the mesa width. Such an observation has led the authors of Ref. 4 to conclusion that the strong radiation is due to the excitation of cavity modes insides the mesa.

Significant progress has been made in the last several years [5–18] and the radiation power is enhanced by two orders of magnitude. Recently, much attention has been paid to achieve frequency tunability [11, 12] and to enhance radiation power by using mesa arrays. [13] These developments suggest that the IJJs in high-T_c superconductors are extremely promising for development of efficient sources of THz EM waves. Such sources would have wide applications in different areas such as medical imaging, security, and new spectroscopy where progress is limited by lack of compact solid state generators. [19]

Besides the radiation power, another figure of merit of the THz radiation is the linewidth. The linewidth from a single point junction has been investigate about half century ago [20–22]. Fluctuations of Cooper pairs [21] and later the fluctuations of quasiparticles [22] are taken into account in theoretical calculations of the linewidth and a satisfactory consistency between theory and experiments was achieved. The radiation linewidth from a tall stack of IJJs was calculated in Ref. 23 assuming that the main source of damping is coming from external radiation. An extremely narrow relative linewidth (defined as the ratio of linewidth to radiation frequency) of order 10^{-9} was obtained. The line shape of radiation coming from BSCCO mesas at cavity resonance has been measured recently. [10, 24] The narrowest lines with width 20MHz are found in the high bias regime [24] while at low bias regime a typical linewidth is about 0.5 GHz. [10, 24] As excitation of the cavity mode is essential for synchronization of IJJs, it is important to understand its influence of the radiation line shape. Surprisingly, no theory exists on the linewidth of the radiation from JJs near cavity resonances neither for a long JJ nor for a stack of IJJs.

Here we present both analytical and numerical study on the linewidth of high frequency radiation from a JJ or a stack of IJJs near cavity resonances due to thermal fluctuations. The linewidth broadening is caused by the diffusion of the phase at wavenumber $\mathbf{k} = 0$. The line shape changes from Lorentzian to Gaussian when temperature is increased. Fluctuations with nonzero wave vectors lead to the suppression of the radiation power. As voltage is tuned close to the cavity resonance, the line width is sharpening significantly and being inverse proportional to the volume of the system. For typical parameters, the line shape is Lorentzian and the linewidth can be expressed in terms of IV characteristics. We give an theoretical limit for the linewidth using typical parameters for BSCCO.

For simplicity, let us first consider a single JJ with spatial modulation of the critical current [25]. The modulation of the critical current may be due to the defects in the junction, also can be introduced intentionally. A single junction with uniform external magnetic fields and the π phase kink state in a stack of IJJs also reduce to this model [26]. The equation of motion in dimensionless units can be written as [25, 27]

$$\partial_t^2\theta + \beta\partial_t\theta + g(x)\sin\theta - \nabla_{2d}^2\theta = J_n(\mathbf{r}, t) + J_{\mathrm{ext}}, \qquad (1)$$

where β is the damping do to the quasiparticle conductivity, $\nabla_{2d}^2 \equiv \partial_x^2 + \partial_y^2$ and $\mathbf{r} = (x, y)$. The radiation is weak and the boundary condition can be approximated as $\partial_{\mathbf{n}}\theta = 0$, where $\mathbf{n}$ is a unit vector normal to the surface. The spatial modulation is assumed along the x direction. $J_n(\mathbf{r}, t)$ is the white-noise current satisfying the fluctuation dissipation theorem (FDT) valid in equilibrium

$$\langle J_n \rangle = 0, \quad \langle J_n(\mathbf{r}, t) J_n(\mathbf{r}', t') \rangle = 2T\beta\delta(t - t')\delta(\mathbf{r} - \mathbf{r}'). \quad (2)$$

When the JJ is driven into the voltage state where the phase rotates following the ac Josephson relation, the FDT is violated as demonstrated below.

The Josephson junction is characterized by the intrinsic cavity modes with wavenumbers $\mathbf{k}_{nm} = (k_{xn}, \ k_{ym}) = (n\pi/L_x, \ m\pi/L_y)$ and frequencies $\omega_{nm} = \sqrt{(n\pi/L_x)^2 + (m\pi/L_y)^2}$. The cavity mode may be selected by the voltage of the junction, which determines the Josephson frequency. The x-modulated Josephson current couples the Josephson oscillations to the cavity modes with wavenumbers $\mathbf{k}_{n0}$. Without loss of generality, we consider the thermal fluctuations around the mode $(\pi/L_x, 0)$. In the voltage state without noise $J_n = 0$, the phase is described by $\theta_0 = \omega_J t + \mathrm{Re}[A \exp(i\omega_J t)]$ with $A = iF/(-\omega_J^2 + i\beta\omega_J + k_{x1}^2)$ and $F = \frac{2}{L_x}\int_0^{L_x} dx \cos(k_{x1}x) g(x)$. Here ω_J is the angular frequency determined by the dc voltage V, $\omega_J = V$. We restrict to the analytically tractable region $A \ll 1$. In this case, the IV curve $J_{\mathrm{ext}} = \omega_J t + \langle \sin\theta_0(\mathbf{r}, t)\rangle_{\mathbf{r},t}$ is given by

$$J_{\mathrm{ext}} = \beta\omega_J + \frac{F^2}{4}\frac{\beta\omega_J}{(\omega_J^2 - k_{x1}^2)^2 + \beta^2\omega_J^2}, \quad (3)$$

where $\langle \cdots \rangle_{\mathbf{r},t}$ is the spatial and temporal average. We introduce the dynamic conductivity $\beta_d \equiv dJ_{\mathrm{ext}}/d\omega_J$

$$\beta_d = \beta + \frac{\beta F^2}{4}\frac{\left(\omega_J^2 - k_{x1}^2\right)^2 - \beta^2\omega_J^2 - 4\omega_J^2\left(\omega_J^2 - k_{x1}^2\right)}{\left[\left(\omega_J^2 - k_{x1}^2\right)^2 + \beta^2\omega_J^2\right]^2}. \quad (4)$$

The first part is due to the usual conductivity β and the second part is due to resonant contribution, which sharply increases as $\omega_J \to k_{x1}$. As will reveal later, the linewidth is determined by β_d. In Fig. 1, the typical IV curve and β_d are shown, both of which are enhanced at the resonance.

To calculate the linewidth, we need to know the response of the phase to the noise current. The phase is $\theta = \theta_0 + \tilde{\theta}$, with the phase due to the noise $\tilde{\theta}$ being governed by

$$\partial_t^2\tilde{\theta} + \beta\partial_t\tilde{\theta} + g(x)\cos(\theta_0)\tilde{\theta} - \nabla_{2d}^2\tilde{\theta} = J_n. \quad (5)$$

Phase diffusion is determined by slow phase dynamics corresponding to small frequencies $\Omega \ll 1$. Due to the Josephson oscillations, the modes with different frequencies are mixed and the slow mode with frequency Ω is coupled to the fast modes with frequencies $\Omega \pm \omega_J$. Near the cavity resonances the fast modes are resonantly enhanced and one can neglect coupling to the higher-frequency modes. Therefore the dominant contribution is given by $(k_x \approx 0, k_{ym}, \Omega)$ and $(k_{x1}, k_{ym}, \Omega \pm \omega_J)$. [26] The solution can be written as

$$\tilde{\theta}(\mathrm{r}, t) = \sum_{p=-1,0,1}\sum_{m=0}^{\infty} a_p(x, k_{ym})\cos(k_{ym}y)\exp[i(\Omega + p\omega_J)t], \quad (6)$$

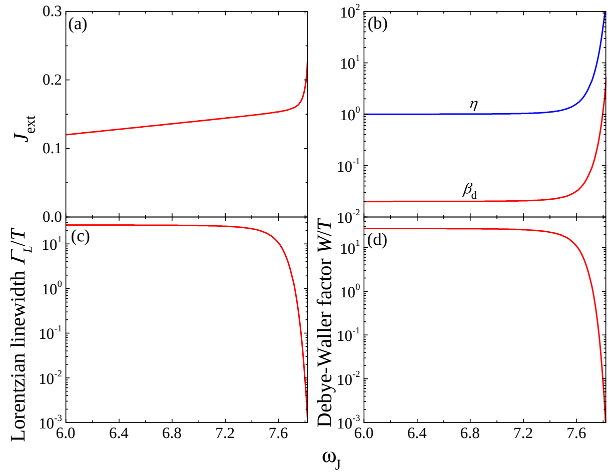

FIG. 1. (color online) (a) IV curve (b) the dynamic conductivity β_d and the inertial term η (c) the Lorentzian linewidth, and (d) the Debye-Waller factor normalized by temperature T. Parameters are: $\beta = 0.02$, $L_x = 0.4$, $L_y = 1.5$ and $F = \pi/4$ for a step modulation of critical current $g(x) = -\mathrm{sign}(x - L_x/2)$.

with $a_0(x, k_{ym}) \approx a_0(k_{ym})$ and $a_{\pm1}(x, k_{ym}) \approx a_{\pm1}(k_{ym})\cos(k_{x1}x)$. Substituting Eq. (6) into Eq. (5) and separating each frequency component, we obtain coupled equations for the slow and fast components. Excluding the fast components leads to equation for the slow component

$$(-\eta\Omega^2 + i\beta_d\Omega + c_\Omega^2 k_{ym}^2)a_0(k_x = 0, k_{ym}, \Omega) = \tilde{J}(\omega_J), \quad (7)$$

All parameters of this equation have the regular and resonance contributions. In particular, the dissipation parameter β_d coincides with the reduced differential conductivity Eq. (4). The parameters c_Ω^2 and η are given by, $c_\Omega^2 = 1 + F^2\mathrm{Re}\left[(\omega_J^2 + i\beta\omega_J - k_{x1}^2)^{-2}\right]/4$ and

$$\eta = 1 + \frac{F^2}{4}\mathrm{Re}\left[\frac{k_{x1}^2 + 3\omega_J^2 - 3i\beta\omega_J + \beta^2}{\left(k_{x1}^2 - \omega_J^2 + i\beta\omega_J\right)^3}\right]. \quad (8)$$

$\eta \approx 1$ and $c_\Omega^2 \approx 1$ off the resonance and are enhanced near the resonance as shown in Fig. 1 (b). It is important to note that the noise amplitude is also enhanced near the resonance and is proportional to the total current, Eq. (3),

$$\left\langle |\tilde{J}(\omega_J)|^2 \right\rangle = \frac{2T\beta}{L_x L_y}\left(1 + \frac{F^2}{4}\frac{1}{(\omega_J^2 - k_{x1}^2)^2 + \beta^2\omega_J^2}\right) = \frac{2T}{L_x L_y}\frac{J_{\mathrm{ext}}}{V}. \quad (9)$$

The phase diffusion constant D_0 is given by $D_0 = \left\langle |\tilde{J}(\omega_J)|^2 \right\rangle / \beta_d^2$ and can be represented as

$$D_0 = 2T\frac{IR_d^2}{V}, \quad (10)$$

where $I = J_{\mathrm{ext}} L_x L_y$ is the current and $R_d = 1/(\beta_d L_x L_y) = dV/dI$ is the differential resistance. It is important to emphasize that in this nonequilibrium regime the FDT is violated for the slow mode $\langle \tilde{J}(t)\tilde{J}(t')\rangle \neq 2T\beta_d\delta(t-t')$. The spectrum for the a_0 mode is $\Omega^2(k_{ym}) = (i\beta_d\Omega + c_\Omega^2 k_{ym}^2)/\eta$, which becomes gapless when $k_{ym} \to 0$ as a consequence of the invariance with respect to constant phase shift. Thus this diffusive mode is most important for the linewidth broadening, and we will consider this mode in the following calculations of the linewidth.

In the presence of slow fluctuating phase,

$$\tilde{\theta}_0 = \int \frac{d\Omega}{2\pi} \sum_m a_0 \cos(k_{ym}y) \exp(i\Omega t),$$

the supercurrent density $J_s(x,y) = g(x)\sin\left[\omega_J t + \tilde{\theta}_0\right]$ is also fluctuating which gives rise to the nonzero linewidth. Here we have neglected the weak plasma oscillation inside the sine function. The fluctuating plasma oscillation $\tilde{\phi}(k_{x1}, k_{ym}, \omega)$ is given by

$$\tilde{\phi} = \frac{-F\int dtdy \sin(\omega_J t + \tilde{\theta}_0)\exp(-i\omega t)}{L_y\left(k_{x1}^2 - \omega_J^2 + i\beta\omega_J\right)}. \quad (11)$$

The linewidth is determined by the spectrum density $S = \langle \tilde{\phi}(k_{x1}, 0, \omega)\tilde{\phi}(-k_{x1}, 0, -\omega)\rangle$

$$S = \frac{F^2\int dtdy \cos(\omega_J t)\exp(-i\omega t)\exp[-K_-(y,t)/2]}{(k_{x1}-\omega_J)^2 + (\beta\omega_J)^2}. \quad (12)$$

where $K_-(y,t) = \langle[\theta_0(y,t) - \theta(0,0)]^2\rangle$ is the fluctuation phase correlation function which can be approximately evaluated as

$$K_-(y,t) \approx 2W + D_0\left[t - \frac{\eta}{\beta_d}\left(1 - \exp(-t\beta_d/\eta)\right)\right]. \quad (13)$$

Here

$$W(y,t) = \frac{D_0\beta_d^2}{2}\sum_{m>0}\int d\Omega\left(\frac{1-\exp[i(k_{ym}y + \Omega t)]}{\left(\eta\Omega^2 - c_\Omega^2 k_{ym}^2\right)^2 + (\beta_d\Omega)^2}\right),$$

accounts for the contribution from the gapped modes with $k_{ym} > 0$ and the rest term in $K_-(y,t)$ accounts for the diffusive mode with $k_{ym} = 0$. In the interesting region where $\beta_d \ll D_0$ and $L_y \leq 4\pi c_\Omega\sqrt{\eta}/\beta_d$, we obtain

$$W = \pi D_0\beta_d L_y^2/(6c_\Omega^2), \quad (14)$$

which becomes independent on time and coordinate. W accounts for the suppression of the radiation and is known as the Debye-Waller factor. The Debye-Waller factor decreases near the resonance as shown in Fig. 1 (d). The broadening of the linewidth is due to the second term in K_-. In the region when $D_0/2 \gg \beta_d/\eta$, $K_-(t) = D_0\beta_d t^2/(2\eta)$ and the line shape is

$$S = \frac{F^2 L_y \exp(-W/2)}{\left(k_{x1}^2 - \omega_J^2\right)^2 + (\beta\omega_J)^2}\exp\left[\frac{-(\omega-\omega_J)^2}{D_0\beta_d/(2\eta)}\right]. \quad (15)$$

The line shape is Gaussian with the linewidth $\Gamma_G \equiv \Delta\omega/(2\pi) = D_0\beta_d/(8\pi\eta)$. In the other limit $D_0/2 \ll \beta_d/\eta$,

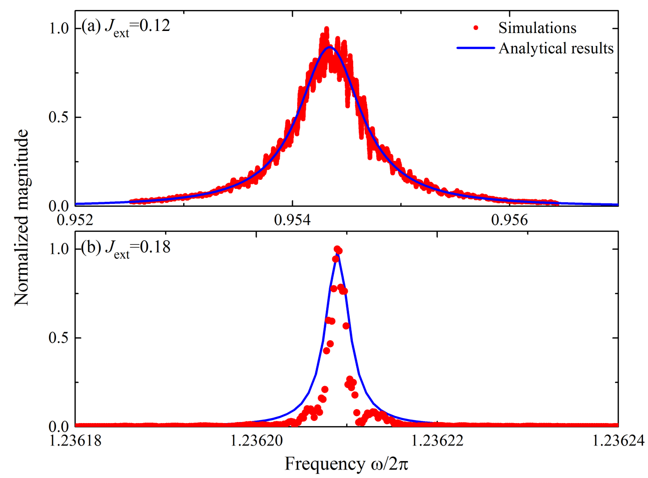

FIG. 2. (color online) Comparison of the line shape obtained analytically and numerically both off the resonance (a) and at the resonance (b). The linewidth sharpen significantly at the cavity resonance. Parameters are the same as those in Fig. 1 and $T = 1.778\times10^{-5}$.

the line shape is determined by slow phase diffusion at large times $K_-(t) \approx D_0 t$

$$S = \frac{D_0F^2L_y\exp(-D_0\eta/\beta_d)\exp(-W/2)}{2[\left(k_{x1}^2 - \omega_J^2\right)^2 + (\beta\omega_J)^2][(\omega-\omega_J)^2 + D_0^2/4]}. \quad (16)$$

The line has Lorentzian shape with the width $\Gamma_L = D_0/(2\pi)$. The diffusion of the gapless perturbation $a_0(k_{ym} = 0)$ with a diffusion constant D_0 caused by thermal fluctuations is responsible for the linewidth broadening. For typical parameters of Nb-Al/AlO$_x$-Nb JJs with $L_x = L_y = 10\lambda_J$ at low temperature 4.2 K, we have $\beta \approx 0.1$, $T \approx 10^{-3}$, where λ_J is the Josephson length $\lambda_J \approx 10\ \mu$m. [27] The line shape in this region is Lorentzian because $D_0/2 \ll \beta_d/\eta$. Approaching the resonance, the linewidth decreases significantly as shown in Fig. 1 (c). In both cases, the line shape is proportional to T and is inversely proportional to the lateral area of the junction L_xL_y. This behavior is very natural. Qualitatively, one may treat the JJ as a two-dimensional ensemble of oscillators. If these oscillators are synchronized, the linewidth is sharpened as the inverse of the population of oscillators, which is proportional to the junction area L_xL_y. [1] However as temperature is increased to close to T_c, the line shape evolves into Gaussian. The crossover from Lorentzian to Gaussian line shape occurs at the temperature $T^* = \beta_d^3\omega_J L_x L_y/(\eta J_{\rm ext})$. Such Lorentzian-to-Gaussian crossover in the line shape was predicted for a point junction [20] and for the Josephson flux flow region [28].

For comparison, we performed numerical calculation of Eq. (1). We assumed that the system is uniform along the y direction meaning that the only $k_{ym} = 0$ mode is taken into account. We calculated the ac electric field at one edge of the JJ at $x = 0$ and then performed the Fourier transform to obtain the spectrum. The results for numerical calculations and analytical treatment are shown in Fig. 2 . Off the resonance there is a perfect agreement between two approaches. When the voltage $V = \omega_J$ is tuned close to the cavity resonance, the amplitude of the plasma oscillation A increases and our theory based on linear expansion becomes inaccurate in this strongly nonlinear region. There is a small discrepancy between the analytical and numerical results.

We next proceed to study the radiation linewidth for a stack of free-standing IJJs. The dynamics of the phase difference θ_l in the l-th junction are described by [29–35]

$$[(1+\beta_{ab}\partial_t)-\zeta\Delta^{(2)}][\sin\theta_l+\beta_c\partial_t\theta_l+\partial_t^2\theta_l+\tilde{j}_z(\mathbf{r},l,t)] = (1+\beta_{ab}\partial_t)\nabla_{2d}^2\theta_l+\nabla_{2d}\cdot[\tilde{\mathbf{j}}_{ab}(\mathbf{r},l+1,t)-\tilde{\mathbf{j}}_{ab}(\mathbf{r},l,t)], \quad (17)$$

where $\Delta^{(2)}h_l \equiv h_{l+1}+h_{l-1}-2h_l$ is the finite difference operator, $\zeta=(\lambda_{ab}/s)^2$ is the inductive coupling, β_c and β_{ab} are dissipation parameters due the out-of-plane and in-plane quasiparticle conductivities. [35] The out-of-plane $\tilde{j}_z$ and in-plane Gaussian noise current have the correlators

$$\langle\tilde{j}_z(\mathbf{r},l,t)\tilde{j}_z(\mathbf{r}',l',t')\rangle = 2T\beta_c\zeta^{-1}\delta_{ll'}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'), \quad (18)$$

$$\langle\tilde{j}_\mu(\mathbf{r},l,t)\tilde{j}_\upsilon(\mathbf{r}',l',t')\rangle = 2T\beta_{ab}\delta_{ll'}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')\delta_{\mu\upsilon}, \quad (19)$$

with $\mu,\upsilon = x,y$. When the Josephson frequency approaches the cavity mode $(k_{x1},0)$, the π kink state is stabilized [15, 16]

$$\theta_l = \omega_J t+\theta_{s,l}(x)+A\cos\left(\frac{\pi}{L_x}x\right)\exp(i\omega_J t), \quad (20)$$

where $\theta_{s,l}(x)$ runs abruptly from 0 to π at the center of the junction, which can be treated as a step function $\theta_{s,l}(x) = [\mathrm{sign}(x-L_x/2)+1]\pi/2$. [16, 36] In the absence of thermal fluctuations, Eq. (17) reduces to Eq. (1) with a step modulation $g(x) = -\mathrm{sign}(x-L_x/2)$ [26]. For the same reason, the linewidth is due to the phase diffusion with the gapless mode with $\mathbf{k}=0$, while the the gapped modes with $\mathbf{k}\neq 0$ contribute to the Debye-Waller factor. The most important difference is that the noise amplitude, Eq. (9) acquires additional $1/N$ factor because the effective noise current for the slow mode is given by averaging over independent noise currents in all synchronized junctions. Here N is the number of junctions. Correspondingly, the phase diffusion coefficient D_0 also becomes N times smaller. The linewidth in IJJs case is again given by Eqs. (15) and (16) with $D_0 \to D_0/N$, and with a different Debye-Waller factor. For the Lorentzian line shape when $D_0/2 \ll \beta_d/\eta$, the linewidth in terms of IV curve is $\Gamma_L = TIR_d^2/(\pi VN^2)$, where R_d and V are the total differential resistance and voltage over the whole stack. As R_d^2/VN corresponds to the contribution from one junction and does not depend on N, the linewidth of the stack contains an additional $1/N$ factor in comparison with a single junction, Eq. (10), due to the in-phase oscillations in different junctions.

The linewidth can be expressed in term of IV characteristics. The same expression is also derived long time ago for a point junction. [22] For an ideal case when all junctions in the stack are synchronized, we estimate the intrinsic linewidth of the IJJs for $N=600$ to be $\Gamma=0.1$ MHz at $T=4.2$ K, which is a fundamental limit for the THz generator based on BSCCO. In experiments, the junctions are usually partially synchronized, and the linewidth is larger than that in the ideal case. Furthermore, the linewidth decreases with temperature if $R_d(T)$ drops fast when T increases.

For the mesa structures used in experiments, [4] there is an additional dissipation due to the radiation into the base crystal [37]. This dissipation can be described using an effective larger damping coefficient β'. Off the resonance, the linewidth sharpens due to the radiation into base crystal because D_0 decreases with β according to Eq. (10). However near the resonance, the linewidth increases since D_0 increases with β near the resonance.

Finally we discuss the relation between the derived linewidth and the quality factor of the cavity. The quality factor for the JJs in Eq. (1) is $Q=\omega_{nm}/\beta$, and the corresponding linewidth is $\Gamma_c = 2Q/\omega_J$. Γ_c is a property of the cavity and is independent on the gain medium (Josephson oscillations). Γ_c shall be interpreted as the upper bound for the linewidth, which is realized for the completely unsynchronized Josephson oscillations. In the case of synchronized oscillations as we considered here, the phase-diffusion linewidth is much smaller than Γ_c.

To summarize, we have studied the linewidth of the high frequency electromagnetic radiation from Josephson junctions and a stack of intrinsic Josephson junctions near cavity resonances. The linewidth is caused by the diffusion of the superconducting phase at the gapless mode with wavenumber $\mathbf{k}=0$. The gapped modes with nonzero wave vectors are responsible for the suppression of the radiation power. The linewidth is Lorentzian in low temperature region and can be calculated directly from the IV characteristics. We also predicted a lower bound for the linewidth of the strong terahertz radiation from $Bi_2Sr_2CaCu_2O_8$.

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[35] In Eq. (17), the inductive coupling is denfined as $\zeta = (\lambda_{ab}/s)^2$ and the renormalized conductivities along the c axis and ab plane are defined as $\beta_c = 4\pi\sigma_c/(\epsilon_c\omega_p)$ and $\beta_{ab} = 4\pi\sigma_{ab}\lambda_{ab}^2/(\lambda_c^2\epsilon_c\omega_p)$ with the Josephson plasma frequency $\omega_p = c/(\lambda_c\sqrt{\epsilon_c})$. Here λ_{ab} and λ_c are the London penetration depths and s is the period of the stack of IJJs. Frequency is in units of ω_p; length is in units of λ_c; current is in units of the Josephson critical current density J_c; temperature is in units of $\Phi_0^2 s/(16\pi^3\lambda_{ab}^2 k_B)$ and magnetic field is in units of $\Phi_0/(2\pi\lambda_c s)$ with $\Phi_0 = hc/(2e)$ the flux quantum. For BSCCO, $\omega_J/(2\pi) \approx 0.1$ THz, $\lambda_{ab} \approx 0.4\,\mu$m and $\lambda_c \approx 200\,\mu$m. The dimensionless electric field is given by $E_{z,l} = \partial_t\varphi_l$, with E in units of $\Phi_0\omega_p/(2\pi cs)$.

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