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# The pressure impact on the structure, elasticity and electron density distribution of $CaSi_2O_5$

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Ab initio molecular dynamics simulations were used to reveal the mechanism of the five-fold to six-fold transition in Si coordination numbers of  $CaSi_2O_5$ . The longest first-neighbor Si-O distance drops from 2.8 to 1.8 Å upon the triclinic to monoclinic transition. We find significant bulk modulus softening during the structure crossover, which is due to appearance of intermediate Si-O connections in the triclinic phase under slightly non-hydrostatic stress. Nonetheless, no soft phonon modes were found in either structure, indicating that both structures are dynamically stable. Across the transition,  $c_{33}$  doubles and  $c_{35}$  increases by six-fold in magnitude due to the formation of new Si-O bonds. Chemical bonding analysis reveals distinctions in the electron localization function and bond ellipticity between the regular (1.8 Å) and the dangling Si-O bonds (2.8 Å), both of which suggest an impending disassociation of the dangling Si-O bond.

Keywords: pressure-induced transition; amorphization; pentacoordinate silicon; SiO<sub>5</sub>; elasticity; spontaneous

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strain; bulk softening, electron density distribution; electron localization function

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I. INTRODUCTION

The discovery of coexisting five-fold and six-fold co- 48 15 ordinated silicon atoms in the triclinic polymorph of 49 16  $CaSi_2O_5^{-1}$  opened the opportunity to explore in a crys- <sup>50</sup> 17 talline material one of the critical steps in the pro-<sup>51</sup> 18 cess of pressure-induced amorphization, a phenomenon <sup>52</sup> 19 of wide interest found in materials including ice, silica 53 20  $(SiO_2)^{2,3}$ , AlPO<sub>4</sub><sup>4</sup> and zeolites. An amorphous phase, <sup>54</sup> 21 which lacks long range atomic order (LRO), is thought to 55 22 be metastable and non-ergodic because its energy land- 56 23 scape exhibits multiple local minima<sup>5</sup>. One viable path <sup>57</sup> 24 for amorphization is elastic and/or vibrational instabili- 58 25 ties that destabilize crystal structures; for example, close 59 26 to amorphization, LRO sharply reduces to zero and a <sup>60</sup> 27 wide range of low frequency vibrational modes arise in 61 28 phonon spectra. As a result, sharp contrasts may occur 62 29 in amorphous solids including changes in volume, density, 63 30 mechanical properties (elasticity), electrical (resistivity) 64 31 and optical properties. Amorphous materials, such as 65 32 AlPO<sub>4</sub>, have been shown to possess a "memory" that <sup>66</sup> 33 preserves its crystalline structure and lattice orientation 67 34 when decompressed from its high-pressure amorphous 68 35 state<sup>4</sup>. More recently phase change materials (PCM) like <sup>69</sup> 36 Ge<sub>2</sub>Sb<sub>2</sub>Te<sub>5</sub> (GST) have found interesting applications as <sup>70</sup> 37 programmable memory bits, capable of reversibly switch-71 38 ing between amorphous and crystalline structures at a 72 39 remarkably low electric power consumption<sup>6</sup>. In struc- <sup>73</sup> 40 tural biology, organisms can utilize biogenic amorphous 74 41 calcium carbonate (ACC) as a transient precursor phase 75 42 to build crystals of desired shapes and polymorphs (cal- 76 43 cite and aragonite), such as for mollusk shells<sup>7</sup>. Amor-77 44

phous materials are ubiquitous in meteorites in which they are formed by shock-induced amorphization, and can be produced in laboratory controlled high-pressure devices such as diamond-anvil cells. There were once speculations associating the amorphization of serpentine, a hydrous mineral in subducting lithosphere, with deepfocus earthquakes (100 to 660 km depth) in the mantle<sup>8</sup>.

The fascinating phenomena arising from amorphization transformations have stimulated intensive studies in the field, including structural characterizations using xray and neutron diffraction, Raman and infrared spectroscopy, and large scale molecular dynamics simulations. Although previous studies on  $ZrW_2O_8^9$ , a well-known flexible framework structure that contracts when heated, have shed light on the problem by suggesting the connection between amorphization and negative thermal expansion, the detailed mechanism of amorphisation is in general still not well understood. In particular, while it is clear that the incipient step of the amorphization in which LRO breaks must involve the breakdown and reformation of chemical bonds, we do not know how this happens. How this incipient process proceeds is also critical for understanding the precursor steps for melting, crystallization and vitrification as well as atom diffusion in melts.

In this study we approach this problem from a different angle. There exists in nature a crystalline material CaSi<sub>2</sub>O<sub>5</sub> which is a unique system for studying the incipience of amorphization, because the two polymorphs of CaSi<sub>2</sub>O<sub>5</sub> convert into each other reversibly at moderate pressure (0.2 GPa) and stress conditions and, in conjunction, a silicon-oxygen (Si-O) bond switches on (~ 1.8 Å) and off (~ 2.8 Å) when conversions occur.

This remarkable material permits a comprehensive the-1 oretical investigation on the Si-O bond formation and 2 rupture in crystalline phases, their influence on elasticity 3 and structural stability, the quantum mechanical nature 4 of the regular and elongated Si-O bonds, and the impli-5 cations for amorphization. It also allows us to explore 6 whether the assumption of conventional crystal chem-7 istry that such a long Si-O distance is completely non-8 bonded is valid. However, since their first unambiguous 9 characterization through single-crystal x-ray diffraction 10 experiments<sup>1</sup>, only limited efforts have been devoted to 11 understanding this system. For example, Warren et al.<sup>10</sup> 12 have analyzed covalent bond populations through Mul-13 liken analysis and postulated a phase transition path 14 that interpolates between the five-fold and six-fold co-15 ordinated structures. Downs et al.<sup>11</sup> have studied bond 16 critical point properties in CaSi<sub>2</sub>O<sub>5</sub> and pointed out a 17 large bond ellipticity in the lengthened Si-O bond (2.8 18 Å), denoting the impending rupture of this bond. The en-19 thalpy of formation for the transition was measured from 20 calorimetry<sup>12</sup>. Here we focus on the transition mecha-21 nism at the atomic scale. We have performed a com-22 prehensive first-principles molecular dynamics study on 23 CaSi<sub>2</sub>O<sub>5</sub>, including high-pressure structural simulations, 24 lattice dynamics, elasticity, and electron density analysis. 25

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# II. METHOD

Our calculation is based on the density functional 27 calculation<sup>13</sup> using the Perdew-Burke-Ernzerhof (PBE) 28 GGA<sup>14</sup> form for the exchange correlation functional 29 in combination with the plane-wave pseudopotential 30 method as implemented in the Quantum ESPRESSO 31 package<sup>15</sup>. The Troullier and Martins<sup>16</sup> type norm-32 conserving pseudopotentials were used for Si, O and Ca. 33 The accuracy of the Si and O pseudopotentials has been 34 demonstrated in previous high-pressure studies on the 56 35  $Mg_2SiO_4$  olivine-wadsleyite-ringwoodite system<sup>17</sup> and on 36 the  $MgSiO_3$  majorite-perovskite-ilmenite system<sup>18</sup>. The 37 PBE-type Ca pseudopotential was generated using a ref-  $^{57}$  erence configuration  $3s^23p^{4.8}3d^{0.2}~(r_{\rm c}=2.0~{\rm a.u.}$  for s,~p,38 39 and d channels, p local), similar to the recipe of Karki 58 40 and Wentzcovitch<sup>19</sup>. The non-linear core correction was <sup>59</sup> 41 not used because of the negligible effect on the equation  $_{60}$ 42 of state parameters, a 0.5% increase in volume and a  $_{61}$ 43 1% decrease in bulk modulus for CaO. The kinetic en- <sub>62</sub> 44 ergy cutoff for plane waves  $(E_{\rm cut})$  was chosen to be 80  $_{63}^{63}$  Ry, and a 2 × 2 × 2 (with  $(\frac{1}{2}\frac{1}{2}\frac{1}{2})$  shift from the origin)  $_{64}^{64}$  Monkhorst-Pack k-point mesh<sup>20</sup> was used for Brillouin  $_{65}^{65}$ 45 46 47 zone samplings of the electronic states. Increasing the k- 66 48 point mesh to  $4 \times 4 \times 4$  and  $E_{\rm cut}$  to 90 Ry does not alter 67 49 equation of state parameters or the triclinic to the mono- 68 50 clinic transition pressure. Crystal structures were relaxed 69 51 under hydrostatic pressures using the variable-cell shape 70 52 molecular dynamics<sup>21,22</sup>. The dynamical matrices were 71</sup> 53 calculated on a  $2 \times 2 \times 2$  *q*-point mesh, and then interpo- 72 54 lated to a denser q-point mesh  $(4 \times 4 \times 4)$  to obtain the <sup>73</sup> 55



FIG. 1: Crystal structures for the two polymorphs of CaSi<sub>2</sub>O<sub>5</sub>: triclinic I $\overline{I}$  (a) and monoclinic A2/a (b). The SiO<sub>5</sub> and SiO<sub>6</sub> units are shown in yellow; SiO<sub>4</sub> tetrahedra are shown in blue; Ca-centered polyhedra are shown in light green; oxygen atoms are shown in red spheres. The dotted lines in (a) denote the hypothetical bonding between Si and O at a distance 2.8 Å. The Ca-O bonds shown here range from 2.2 to 2.5 Å.

phonon density of states.

## **III. RESULTS AND DISCUSSION**

The triclinic  $CaSi_2O_5$  polymorph (space group II) was first unambiguously identified by Angel et al.<sup>1</sup> through single-crystal x-ray diffraction. The crystals were obtained by synthesis at high pressures and temperatures. and then quenched to ambient conditions to obtain the triclinic phase. This triclinic structure [Fig. 1(a)] is distinctive for the coexisting four-, five- and six-fold coordinated silicon atoms by oxygen. The tetrahedrallycoordinated silicon atoms cross-link the silicate chains formed by corner-sharing SiO<sub>5</sub> and SiO<sub>6</sub> polyhedra. The pentacoordinate Si atoms are separated 2.8 Å away from the dangling oxygen atoms, in contrast with regular Si-O bonds (1.6 Å in tetrahedral and 1.8 Å in octahedral environments). An experimental  $study^{23}$  showed that upon compression to 0.2 GPa, each dangling O atom is recaptured by a pentacoordinate Si atom, thus forming



FIG. 2: Polyhedral connectivity in  $CaSi_2O_5$  at 0 GPa from the simulation: (a) the  $I\bar{I}$  phase and (b) the A2/a phase. The bond lengths that are directly involved in the formation of the 'sixth' Si-O bond are indicated. The green spheres represent Ca atoms, blue Si, and red O.

an  $SiO_6$  unit. The resulting phase has a monoclinic, 1 titanite (CaTiSiO<sub>5</sub>)-type structure (space group A2/a) 2 comprising chains of corner-sharing SiO<sub>6</sub> octahedra in-3 terconnected through  $SiO_4$  tetrahedra [Fig. 1(b)]. The 4 reverse transition (six-fold to five-fold coordinate) was 5 observed in multi-anvil but not in diamond-anvil exper-6 iments, which was attributed to the presence of non-7 hydrostatic stress in the multi-anvil  $cell^{23}$ . As the rup-8 ture of the Si-O bond in  $CaSi_2O_5$  is representative of the <sup>37</sup> 9 incipient process for amorphization which is commonly <sup>38</sup> 10 observed in NMR studies<sup>24</sup>, we use ab initio molecular 11 dynamics simulations to reveal the microscopic process 12 underlying the transition which is not accessible experi-13 42 mentally. 14 43

# 15 A. Structural and compressional properties

Our simulations confirm that the triclinic phase is ener-48 16 getically more stable than the monoclinic phase at room 49 17 pressure. The predicted crystal structure parameters 50 18 for the two polymorphs at room pressure are compared 51 19 with results from x-ray diffraction measurements in Ta- 52 20 ble I. The predicted unit cell lengths and cell angles are 53 21 mostly within one-half percent from experimental val- 54 22 ues, although GGA overestimates the volume by 1.7 % 55 23 and underestimates the bulk modulus by  $\sim 10$  %, which  $_{56}$ 24 is well known from previous DFT studies on thermody- 57 25 namic properties of upper mantle minerals $^{25}$ . 58 26

The evolution of the triclinic structure towards mon- 59 27 oclinic symmetry is simulated using molecular dynam- 60 28 ics. This transition is associated with one quarter of sil- 61 29 icon atoms changing their formal or apparent coordina- 62 30 tion number from five to six. A close view of the local 63 31 environment surrounding the dangling oxygen atom at 64 32 room pressure (Fig. 2a) shows that it is 2.8 Å away from 65 33 Si<sup>V</sup> (the five-fold coordinated Si) but forms a regular O- 66 34



FIG. 3: Variation with pressure of cell parameters — (a) cell lengths, (b) cell angles and (d) cell volumes — across the triclinic (symbols joined by dashed lines) to the monoclinic (solid lines) transition. Also shown is (c) the simulation for the Si-O bond length during its formation process.

Si bond (1.61 Å) with Si<sup>IV</sup> (tetracoordinate Si) and at the same time it is connected to two Ca atoms in bond lengths of 2.3 and 2.4 Å, respectively. Upon compression, the elongated Si-O bond shrinks from 2.8 to 1.8 Å but one of the Ca-O distances increases from 2.4 to 3.1 Å (the dashed line in Fig. 2b). This can be interpreted as the breakdown of the Ca-O bond in favor of forming the short Si-O bond (1.8 Å). In addition, the Si-O bond formation is affected not merely from the hopping of the dangling O atom but involves a translation of the whole SiO<sub>4</sub> unit towards the pentahedron (see Fig. 2b), indicating the nature of a collaborative transition.

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Shown in Fig. 3 are the pressure evolution of the crystal structure parameters and that of the elongated Si-O bond length involved in the transition from GGA calculations. The open symbols connected by the dashed lines represent the simulation starting with a triclinic structure (-4 GPa), whereas the solid straight lines are results starting from a monoclinic structure at high pressures (10 GPa). When compressed from 0 to 2 GPa, the cell-parameters of the triclinic phase vary smoothly with pressure, including cell lengths and cell angles, meanwhile the lengthened Si-O distance decreases from 2.8 Å to 2.6 Å (Fig. 3c). A sudden structural variation occurs, however, between 2 and 2.7 GPa, because multiple competing structures were encountered in the simulation that differ in internal shear stress but have very close enthalpy, which apparently have prevented the code from identifying optimal structures that satisfy hydrostatic conditions. This reveals the inability of the triclinic structure to maintain hydrostatic stress with increasing pressures. In addition, no soft phonon modes

were found in the triclinic structure even at 2 GPa (see 35 1 phonon dispersions in the appendix) in the regime in 36 2 which the structure is showing bulk softening (see below), 37 3 indicating that the structure with 5-fold coordinated Si 38 4 is dynamically stable at 2 GPa. This together with the 39 5 lack of soft phonon modes in the monoclinic structure  $_{40}$ 6 (the high-pressure phase) from 0 to 5 GPa from this cal- 41 7 culation indicates that this transition is not driven by  $_{\rm 42}$ 8 soft phonon modes. What these simulations therefore 43 9 show, in contrast to the constrained simulations of War- 44 10 ren et al.<sup>10</sup>, is that the process is completely reversible 45 11 and that intermediate structural states, at least up to 2 46 12 GPa in these simulations, are stable. 13 47



FIG. 4: Spontaneous strains for the low-pressure triclinic structure relative to the high-pressure monoclinic structure in  $CaSi_2O_5$  (for details see appendix).

When the pressure exceeds 2.7 GPa, the triclinic phase 14 rapidly converged to the monoclinic titanite structure. 15 The structural parameters at 2.7 GPa are significantly 16 different from those obtained at 2 GPa: the length of the 17 c axis drops from 6.48 to 6.34 Å and that of the a axis 18 increases from 13.0 to 13.15 Å, but the change in the b19 axis is minute, from 8.40 to 8.37 Å (Fig. 3a). In addition, 20 the  $\beta$ -angle increases abruptly from 112.1° to 113.5°, and 21 the other two angles,  $\alpha$  and  $\gamma$ , tend to approach 90° 22 (Fig. 3b). The cause of this dramatic structural change 23 is not just the recovery of the regular Si-O bond  $(1.8 \text{ \AA})$ 24 from the dangling bond state (2.8 Å). The Si-O vector 25 has significant components along both the b and c axes, 26 and so both of these cell parameters decrease. But the  $^{\rm 55}$ 27 formation of the short Si-O bond also increases the intra-28 chain repulsion resulting in an expansion in the chain 56 29 length (a axis) and further changes in the cell parameters 5730 result from the cooperative rotations of all components 58 31 of the structure. 32

A deeper understanding of the transition mechanism 60 33 can be gained by plotting the pressure dependence of 61 34

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the components of the spontaneous-strain tensor (e.g., the GGA results) associated with the symmetry-breaking transition (from monoclinic to triclinic, as defined by Eq. 38 to 42  $in^{26}$ ). The results from Fig. 4 shows the development of spontaneous strain under decompression. The non-symmetry-breaking strains  $e_{11}$ ,  $e_{22}$ ,  $e_{33}$ ,  $e_{13}$  and  $e_{23}$ all evolve linearly with one another, as required by symmetry. At 2.7 GPa, the appearance of the symmetrybreaking strain  $e_{12}$  (a non-zero value) corresponds to the discontinuous jump in the  $\gamma$  angle from the simulation (Fig. 3b). At lower pressures, the presence of large axial strains,  $e_{33}$  (positive) and  $e_{11}$  (negative), correlate with the lengthening of the Si-O bond parallel to the caxis (Fig. 1a), which expands the c axis and shrinks the length of the silicate chains (along the a axis). Similarly the large negative shear-strain  $e_{23}$  develops due to the increase in  $\alpha$  angle and c axis. The stronger decrease of  $e_{22}$  and especially  $e_{33}$  compared to the increase in  $e_{11}$  as the transition is approached results in a softening of the structure.



FIG. 5: The calculated bulk modulus of CaSi<sub>2</sub>O<sub>5</sub> across the triclinic to monoclinic phase transition. The symbols represent the bulk modulus calculated by finite difference between consecutive P, V data points obtained in simulations. Away from the transition, the lines are the EoS fit to the P-V data of the two phases. In the neighborhood of the transition the line is an interpolation between the data points.

#### **Elastic** properties B.

By analyzing the pressure evolution of unit cell volume, we clearly see a structure softening (Fig. 5) accompanying the structure crossover. While at pressures well away from the phase transition the bulk modulus can be described by finite strain equation of state (EoS), there is no adequate method to fit the variation of bulk modulus

in the neighbourhood of the transition. We have there-1 fore calculated the bulk modulus in this regime by fi-2 nite difference between P-V calculated by simulations at 3 consecutive pressure points. Such a phenomenon is typ-4 ical of softening preceeding structural phase transitions 5 at high pressures (e.g.<sup>27,28</sup>), including softening caused by spin transitions (e.g.<sup>29</sup>). As pressure is increased to-6 7 wards the transition, the bulk modulus of the triclinic 8 phase suddenly drops from 95 GPa to nearly zero, before 9 it bounces back to 163 GPa, the value of the monoclinic 10 phase. This clearly correlates with the contraction of 11 the dangling Si-O bond from 2.8 to 1.8 Å. In fact, this 12 softening is obviously the combined effect of the rapid 13 contraction and softening of the crystallographic b and 14 c directions [Fig. 3(a)], and thus the decrease in  $e_{22}$  and 15  $e_{33}$  (Fig. 4), as a result of the cooperative rearrangement 16 of the structure described above. 17

To understand the influence of Si-O bond formation on 18 elasticity, we compare the calculated elastic constants of 19 the two CaSi<sub>2</sub>O<sub>5</sub> polymorphs (Table II). Due to struc-20 tural similarity, some elastic constants, such as  $c_{11}$ ,  $c_{55}$ 21 and  $c_{12}$ , are close or of the same order of magnitude for 22 the two polymorphs, as expected, but there are signifi-23 cant differences existing between some other elastic con-24 stants. For example, the value of  $c_{33}$  of the monoclinic <sub>56</sub> 25 phase is double that of the triclinic phase,  $c_{35}$  is six times 26 greater, and  $c_{44}$  is 58% greater. This is clearly due to the  $\frac{1}{58}$ 27 formation of the extra short Si-O bond ( $\sim 1.8$  Å) and thus 28  $SiO_6$  octahedra, which stiffens the crystal structure along  $\frac{1}{60}$ 29 the c axis resulting in a large  $c_{33}$ , meanwhile increases the  $\frac{1}{51}$ 30 response of the axial stress  $(\sigma_{33})$  to the shear strain in the 31 *a-c* plane  $(e_{13})$ , producing a much larger  $c_{35}$ . It is also 32 noted that (Table II) the Cauchy relations which hold 33 for atomic interactions with purely central forces and in  $\frac{1}{65}$ 34 a lattice with inversion symmetry<sup>30</sup> are better obeyed in  $\frac{65}{66}$ 35 the triclinic phase than in the monoclinic phase. For ex-  $\frac{1}{67}$ 36 ample, the ratios  $c_{23}: c_{44}, c_{13}: c_{55}$ , and  $c_{12}: c_{66}$ , which  $_{68}$ 37 equal one according to Cauchy relations, are 0.81 (0.23),  $_{_{69}}$ 38 0.83 (0.71), and 1.5 (2.7) in the triclinic (monoclinic) 39 case, respectively. 40 71

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# C. Electron density analysis

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Here we discuss features of electron density distri-<sup>76</sup> 42 butions around the regular octahedral and pentahedral <sup>77</sup> 43 sites, and correlate structure stability with chemical 78 44 bonding. The polarization of the Si atoms in the tri-79 45 clinic phase is revealed by the Born effective charge ten- <sup>80</sup> 46 sors. The  $Z_{33}$  components of Si in the tetrahedral, pen-<sup>81</sup> 47 tahedral, and octahedral environments are 3.18, 3.64, 82 48 and 4.17, respectively (Table III), suggesting that the 83 49 effective charge transfer amount from Si<sup>V</sup> to the dan- 84 50 gling oxygen atom is indeed much weaker than from Si<sup>VI</sup> 85 51 to its regular oxygen vertex. In the language of quan- <sup>86</sup> 52 tum chemistry, a bond path can be found along the 87 53 2.8 Å Si<sup>V</sup>-O connection, but it differs markedly from <sup>88</sup> 54 the regular Si<sup>VI</sup>-O bonds, as shown in Figs. 7(a,b) which <sup>89</sup> 55



FIG. 6: Comparison of the elastic constants for the triclinic phase (II) and the monoclinic phase (A2/a) in  $CaSi_2O_5$  at 0 GPa from the GGA calculation.

compares cross-sections of the valence electron density through the  $SiO_5$  pentahedron and through the regular  $SiO_6$  octahedron. The valence electron density map around Si<sup>VI</sup> shows a cube-shaped isosurface, which is isotropic towards the six O neighbors [Fig. 7 (b)]. The density around the center of the pentahedron, however, displays a bell-shaped isosurface pointing towards the dangling O atom [Fig. 7 (a)]. At the bond critical point, a local minimum in density along the elongated Si-O path (2.8 Å) is 0.1 e Å<sup>-3</sup>, which is much smaller than the minimum density along the regular Si-O bond ( $\sim 1.8$  Å) in the octahedral environment,  $0.3 \text{ e} \text{ Å}^{-3}$ . This is consistent with the previous Hartree-Fock all electron calculations<sup>11</sup>, which shows that the density values at the bond critical points are 0.85 and 0.1 e  $Å^{-3}$  for the regular and elongated Si-O bonds, respectively. Close to the critical point [Fig. 7 (a)], the density anisotropy perpendicular to the bond path is clearly seen from the asymmetric contour line at 0.1 e Å<sup>-3</sup>. A particular measure of this anisotropy is called the bond ellipticity, which is defined as  $\epsilon = \lambda_1/\lambda_2$ , with  $\lambda_1$  and  $\lambda_2$  being the two negative eigenvalues of the Hessian matrix (the second partial derivatives of density with respect to coordinates) at the critical point and in the order of decreasing magnitude. In regular Si-O bonds,  $\epsilon \approx 1$ , but in the elongated Si-O path it is much larger,  $3.55^{11}$ , indicating instability of the bond which is susceptible to rupture.

In addition, the isosurfaces of the electron localization function<sup>31</sup> calculated from the valence electron density are shown in Fig. 7c. The inner isosurfaces with iso-value 0.85 confines two types of domains: one is shared electron domains represented by small, sphere-shaped density accumulations between bonded Si and O atoms (shown in green lines), and the other is large cone-shaped and

torus-shaped domains near O atoms, representing elec- 20 1 tron lone-pair domains. Clearly, a shared density ac-21 2 cumulation domain exists between all Si<sup>VI</sup>-O bonds but <sup>22</sup> 3 there is no such feature close to the dangling oxygen atom <sup>23</sup> 4 (Fig. 7c). On the contrary, the localization of the  $SiO_5$  <sup>24</sup> 5 unit appears well defined with the dangling oxygen atom 25 6 excluded out of their domains as seen from cross-sections 26 7 of the localization function (Fig. 7d). 27 8



FIG. 7: Chemical bonding for the elongated and regular Si-O <sup>50</sup> bonds in the triclinic CaSi<sub>2</sub>O<sub>5</sub> — (a) and (b): the valence <sup>51</sup> electron density maps; (c) and (d): maps of electron local-<sup>52</sup> ization function. Si atoms are shown in green spheres and O <sup>53</sup> atoms in red spheres. (a) A cross-section through the SiO<sub>5</sub> <sup>54</sup> pentahedron containing the dangling Si-O bond of 2.8 Å; the <sup>55</sup> contour lines for electron density are at 0.1, 0.18, 0.25, 0.5, 0.8, and 1.7 e Å<sup>-3</sup>. (b) A cross-section through a regular SiO<sub>6</sub> <sup>56</sup> octahedron together with the density contour lines at 0.2, 0.3, <sup>57</sup> 0.4, 1.7, 3.35, and 5.0 e Å<sup>-3</sup>. (c) Isosurfaces of the electron lo-<sup>58</sup> calization function of the corner-sharing SiO<sub>5</sub> (left) and SiO<sub>6</sub> <sup>59</sup> (right), with the inner iso-value at 0.85 and the outer at 0.7. (d) The two cross-sections for the electron localization function fun

# IV. CONCLUSION

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We have studied in detail the phase transition mecha-10 nism associated with the triclinic to monoclinic transition  $_{61}$ 11 in CaSi<sub>2</sub>O<sub>5</sub> from first-principles calculations. Our simu- 62 12 lation revealed a transition path from the low pressure  $_{63}$ 13 phase to the high pressure phase, along which one quarter 64 14 of silicon atoms change their coordination number from 65 15 five to six. Accompanying the increase in the coordina- 66 16 tion number are a decrease in the Si-O distance from 2.8 67 17 to 1.8 Å and an increase in the Ca-O distance from  $2.4_{68}$ 18 to 3.1 Å. These bond-length variations are interpreted 69 19

as a result from the competition for the vertex-oxygen atom between the  $SiO_5$  and the  $CaO_8$  units (Fig. 2). In addition, we emphasize that not only is the transition accompanied by the displacement of a single oxygen vertex atom, but also by the collective adjustment of adjacent atoms around the  $SiO_5$  pentahedron, such as the  $SiO_4$  group. Although the experimental study shows that the triclinic to monoclinic transition is nearly instantaneous (within a pressure interval less than 0.04 GPa), the molecular dynamics simulation suggests the existence of intermediate structures in which the dangling Si-O distance can be 2.6 Å (at 2 GPa). The phonon calculations at 2 GPa show no signs of soft phonon modes. We also analyzed the spontaneous strain during the monoclinic to the triclinic transition, and find  $e_{12}$  being the only dominant symmetry-breaking strain, which corresponds to the discontinuity in the  $\gamma$ -angle upon the transition. The elasticity calculation shows that  $c_{33}$  and  $c_{35}$  of the monoclinic phase is much larger than those of the triclinic phase. This is well rationalized in terms of structure by the formation of the short Si-O bond (1.8 Å) in the monoclinic phase, in contrast the corresponding Si-O distance in the triclinic phase is 2.8 Å. To characterize chemical bonding properties around SiO<sub>5</sub> pentahedra and SiO<sub>6</sub> octahedra, we analyzed the valence-electron density and the electron localization function of the triclinic structure. The elongated Si-O bond is peculiar in that it has a small electron density, but an exceptionally large bond ellipticity at the bond-critical point, which indicates that the bond is weak, and anisotropic in the transverse direction, therefore susceptible to rupture. Clear distinction is observed from maps of the electron localization function between the elongated Si-O bond and the regular Si-O bond; a local maximum exist in the latter bond but not in the former. Nonetheless, despite the interpretation of conventional crystal chemistry that this long Si-O distance does not constitute a 'bond' our results show that in this case there is still weak residual interaction between the Si and O, that would presumably be broken on further expansion of the structure.

# V. ACKNOWLEDGMENTS

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FIG. 8: Phonon dispersions and vibrational density of states of the triclinic CaSi<sub>2</sub>O<sub>5</sub> at 2 GPa from GGA calculations. X, Y, and Z denote, respectively, the bisecting points of the three reciprocal vectors:  $b_1$ ,  $b_2$ , and  $b_3$ .

# VI. APPENDIX

A. Spontaneous strain

We have calculated the bulk and shear moduli for 3 the monoclinic  $CaSi_2O_5$  using the Reuss theory (see 4 e.g. <sup>32,33</sup>),  $K_R = [s_{11} + s_{22} + s_{33} + 2(s_{12} + s_{13} + s_{23})]^{-1}$ and  $G_R = 15[4(s_{11} + s_{22} + s_{33}) - 4(s_{12} + s_{13} + s_{23}) + 3(s_{44} + s_{55} + s_{66})]^{-1}$ , with the tensor notation denoted 5 6 7 in<sup>34</sup>. We have adopted the notion from Carpenter et al. <sup>26</sup> 8 to calculate the spontaneous-strain tensor related to the 9 symmetry-breaking transition from the monoclinic to the 10 triclinic phase (Eq.38 to  $42 \text{ in}^{26}$ ). 11

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# B. Phonon dispersions

Phonon dispersions and vibrational density of states of
the triclinic structure at 2 GPa from the GGA calculation
are shown in Fig. 8.

TABLE I: Crystal cell parameters and equation of states of the two polymorphs of  $CaSi_2O_5$ , the triclinic phase (space group II) and the monoclinic phase (space group A2/a), from the static GGA calculations compared with experiments at room conditions.

Triclinic	a(Å)	b(Å)	c(Å)	$\alpha(^{\circ})$	$\beta(^{\circ})$	$\gamma(^{\circ})$	Vol (Å <sup>3</sup> )	K (GPa)	K'
GGA	13.03	8.43	6.58	92.59	111.96	90.97	670.07	87.0	4.1
$Exp^1$	12.917	8.450	6.515	93.161	111.48	90.79	660.32		
Monoclinic	a(Å)	b(Å)	c(Å)	$\beta(^{\circ})$	Vol $(Å^3)$	K (GPa)	K'		
GGA	6.63	8.40	6.41	113.82	326.5	152.0	4.0		
$\operatorname{Exp}^{2,3}$	6.543	8.392	6.342	113.17	320.10	178.2	4		
1;1;2;23;3;3	35								

TABLE II: Elastic constants of the monoclinic (A2/a) and the triclinic  $(I\overline{I})$  CaSi<sub>2</sub>O<sub>5</sub> at 0 GPa from static GGA calculations, in unit of GPa. The dashed lines represent strictly zero owing to monoclinic symmetry.

		U	U								
C <sub>ij</sub>	11	22	33	44	55	66	12	13	14	15	16
A2/a	373.4	414.1	293.3	116.6	75.0	42.3	113.2	53.0	_	0.0	_
Im1	313.7	319.0	158.0	73.8	68.2	67.3	100.0	56.3	3.7	23.4	1.2
$C_{ij}$	23	24	25	26	34	35	36	45	46	56	
A2/a	26.8	_	0.0	-	-	90.0	_	-	0.0	_	
Im1	59.8	34.6	-3.1	-4.0	47.9	13.0	-11.1	3.0	10.3	1.2	

TABLE III: Diagonal elements of the Born effective charge tensor of silicon atoms in tetrahedral, pentahedral, and octahedral environment in  $CaSi_2O_5$  (see Fig. 1).

$Z_{ij}$	Si(IV)	Si(V)	Si(VI)
$Z_{11}$	3.08	3.90	4.11
$Z_{22}$	3.48	4.05	3.99
$Z_{33}$	3.18	3.64	4.17

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