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# Spin and charge distribution symmetry dependence of stripe phases in two-dimensional electron systems confined to wide quantum wells

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Measurements in clean two-dimensional electron systems confined to wide GaAs quantum wells in which two electric subbands are occupied reveal an unexpected rotation of the orientation of the stripe phase observed at a half-filled Landau level. Remarkably, the reorientation is sensitive to the spin of the half-filled Landau level and the symmetry of the charge distribution in the quantum well.

## I. INTRODUCTION

A low-disorder two-dimensional electron system (2DES) subjected to a strong perpendicular magnetic field ( $B$ ) displays a variety of novel quantum phases. At high  $B$ , when the Fermi energy ( $E_F$ ) resides in the lowest ( $N = 0$  and  $1$ ) Landau levels (LLs), electrons typically condense into incompressible liquid states and exhibit the fractional quantum Hall effect<sup>1</sup>. At lower  $B$ , when  $E_F$  lies in the higher LLs ( $N \geq 2$ ), phases with non-uniform density are often the ground states. More specifically, when a spin-split  $N \geq 2$  LL is half filled, the 2DES is expected to break the rotational symmetry by forming a unidirectional charge density wave, the so-called stripe phase<sup>2,3</sup>. Experimentally, strong anisotropy is seen in in-plane transport coefficients at LL filling factors  $\nu = 9/2, 11/2, 13/2$ , and  $15/2$ : the longitudinal resistance commonly vanishes along the  $[110]$  crystal direction along which the stripes form ("easy" axis), but exhibits a strong peak along the  $[1\bar{1}0]$  direction ("hard" axis)<sup>4,5</sup>. It is believed that a native symmetry-breaking field, which is still unidentified after more than a decade of research, is responsible for orienting the stripe phases along  $[110]$ <sup>6,7</sup>. The associated anisotropy energy is estimated to be a few mK per electron<sup>8,9</sup> from the fact that an in-plane magnetic field of  $\sim 1$  T can overcome this energy and re-align the stripes to be perpendicular to the field direction<sup>10,11</sup>. Even without any external symmetry-breaking field, the stripes are known to rotate from the "normal" ( $[110]$ ) direction to the "abnormal" ( $[1\bar{1}0]$ ) direction when the 2DES density is raised above a critical density  $\sim 2.9 \times 10^{11} \text{ cm}^{-2}$ <sup>12</sup>. The density-induced rotation occurs at very similar densities for 2DESs confined to either GaAs/AlGaAs hetero-junctions or GaAs quantum wells; also it does not depend on the LL spin orientation as it happens for both filling factors  $\nu = 9/2$  and  $11/2$ <sup>12,13</sup>.

Here we report a study of stripe phases in wide GaAs quantum wells (QWs) where two electric subbands are occupied. Our main focus is on the evolution of the orientation of stripe phases as we increase the density while keeping the QW charge distribution symmetric (balanced). More precisely, we monitor the magnetoresistance near LL filling factors  $\nu = 13/2$  and  $15/2$  when  $E_F$  lies in the two, spin-split,  $N = 2$  LLs of the symmetric (S) subband (the  $S2\uparrow$  and  $S2\downarrow$  levels) while the  $N = 0$

LLs of the antisymmetric subband ( $A0\uparrow$  and  $A0\downarrow$  levels) are fully occupied<sup>14</sup>. We find that when  $E_F$  lies in  $S2\downarrow$  the stripes are always formed along the "normal" ( $[110]$ ) direction. But, when  $E_F$  lies in the  $S2\uparrow$  level, the orientation of the stripes can rotate to be along the "abnormal" ( $[1\bar{1}0]$ ) direction at high densities. At a density where the stripe phase at  $\nu = 13/2$  is along the abnormal direction, we can rotate it back to the normal direction by making the QW charge distribution asymmetric while keeping the density fixed. Our observations therefore reveal that the symmetry-breaking mechanism that determines the direction of the stripe phases depends not only on the 2DES density but also on the *spin* orientation of the LL in which  $E_F$  resides, and on the symmetry of the charge distribution in the QW.

## II. METHOD

Our samples were grown by molecular beam epitaxy, and each consists of a wide GaAs QW bounded on either side by undoped  $\text{Al}_{0.24}\text{Ga}_{0.76}\text{As}$  spacer layers and Si  $\delta$ -doped layers. We report here data for two samples, with QW widths  $W = 42$  and  $51$  nm, and as-grown densities of  $n \simeq 2.8$  and  $2.5 \times 10^{11} \text{ cm}^{-2}$ , respectively. The low-temperature ( $T = 0.3$  K) mobilities of these samples are  $\mu \simeq 600 \text{ m}^2/\text{Vs}$ . Each sample has a van der Pauw geometry, with alloyed InSn contacts at the four corners of a  $4 \times 4 \text{ mm}^2$  piece cleaved from a wafer. We solder the sample onto a thin film of In which serves as a back-gate electrode, and deposit Ti/Au on top of the sample to use a front-gate. The front-gate covers more than 90% of the top surface, but leaves bare the four small contact regions near the four corners ( $\sim 0.5 \times 0.5 \text{ mm}^2$ )<sup>15</sup>. We carefully control the density and the charge distribution symmetry in the QW by applying voltage biases to these gates<sup>16-18</sup>. The measurements were carried out in a dilution refrigerator with base temperature  $T \simeq 30$  mK, and we used low-frequency ( $< 20$  Hz) lock-in techniques to measure the transport coefficients.

Throughout this article, the longitudinal resistances measured along the  $[110]$  direction ( $R_{xx}$ ) are shown in red, and those measured along the  $[1\bar{1}0]$  direction ( $R_{yy}$ ) are shown in black. With this notation, a black trace showing a much larger resistance than a red trace corresponds to the "normal" stripe orientation ( $[110]$ ), i.e., the

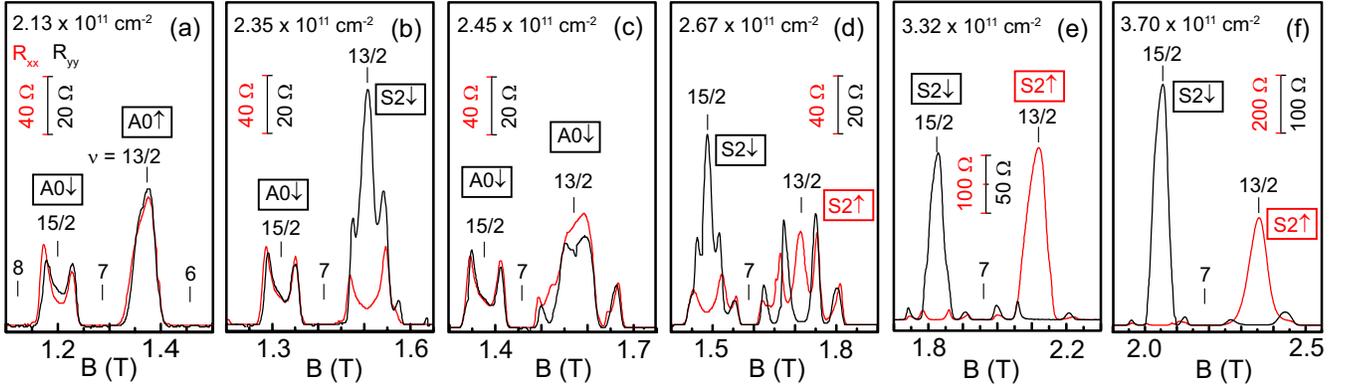


FIG. 1. (color online) Longitudinal magneto-resistance measured at  $T = 30$  mK in a 42-nm-wide QW along the  $[110]$  ( $R_{xx}$ , red traces) and  $[1\bar{1}0]$  ( $R_{yy}$ , black traces) directions. Data are shown in the filling factor range  $6 < \nu < 8$  at different electron densities as indicated. The field positions of half-filled LLs ( $\nu = 13/2$  and  $15/2$ ) are marked by vertical lines, and the LLs in which  $E_F$  resides at the different half-fillings are indicated in boxes.

one that is commonly seen in standard, single-subband QWs at low densities. Conversely, a black trace showing a much smaller resistance than a red trace signals that the stripes are formed along  $[1\bar{1}0]$ , which we refer to as the "abnormal" orientation.

### III. RESULTS AND DISCUSSION

Figure 1 illustrates one of our main findings. It shows  $R_{xx}$  and  $R_{yy}$  traces, in the filling range  $6 < \nu < 8$ , for a symmetric (balanced) 42-nm-wide QW at six different densities  $n = 2.13, 2.35, 2.45, 2.67, 3.32,$  and  $3.70 \times 10^{11} \text{ cm}^{-2}$ . At  $\nu = 13/2$ , as a function of increasing  $n$ , transport is first isotropic (Fig. 1(a)), shows a "normal" anisotropy (Fig. 1(b)), becomes isotropic again (Fig. 1(c)), and then exhibits anisotropy but now along the "abnormal" direction (Figs. 1(d-f)). The behavior at  $\nu = 15/2$ , however, is markedly different; it is isotropic in Figs. 1(a-c) and then shows a "normal" anisotropy at higher  $n$  (Figs. 1(d-f)). The traces shown in Figs. 1(d-f) are particularly noteworthy: transport is anisotropic at both  $\nu = 13/2$  and  $15/2$ , but the orientation of the anisotropy in a *single trace* is different at these two fillings.

In order to understand the data of Fig. 1, we present in Fig. 2 a schematic LL fan diagram for this 42-nm-wide QW sample as a function of  $n$ , or equivalently the magnetic field position of  $\nu = 13/2$  ( $B_{\nu=13/2}$ )<sup>19</sup>. We show the LLs for the symmetric (S) and antisymmetric (A) electric subbands. The index (0, 1, or 2) following S and A is the LL orbital quantum number ( $N$ ), and the up- ( $\uparrow$ ) and down-spin ( $\downarrow$ ) levels are represented by solid and dashed lines, respectively. The relevant energies are the subband separation ( $\Delta$ ), the cyclotron energy ( $\hbar\omega_c$ ), and the Zeeman energy ( $E_Z$ ). As we increase  $n$  while keeping the QW balanced,  $\hbar\omega_c$  and  $E_Z$  increase but  $\Delta$  decreases<sup>16–18</sup>, causing crossings of the S2 and A0 levels. As we discuss below, these crossings are consistent with the evolution seen in Fig. 1. We emphasize that, although the LL fan diagram shown in Fig. 2 is schematic<sup>19</sup>, it is closely based on the parameters of our sample. For example, we measured  $\Delta$  from Fourier transforms of the Shubnikov-de Haas oscillations at low magnetic fields<sup>17</sup>. These measured  $\Delta$  are also consistent with all the parallel-spin LL crossings we observe in this sample<sup>17</sup>; these crossings occur

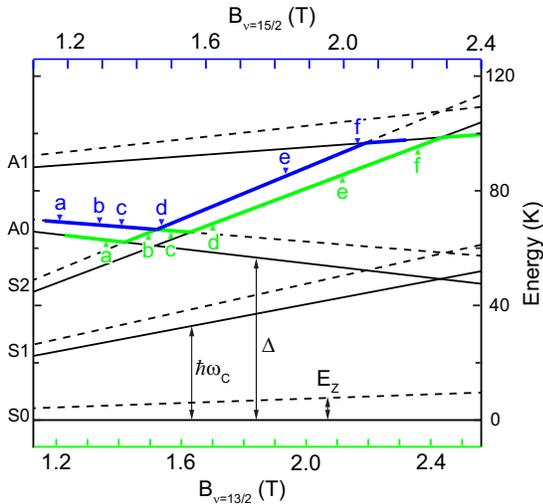


FIG. 2. (color online) Schematic Landau level energy diagram as a function of density for the 42-nm-wide QW. The relevant energies are the subband separation ( $\Delta$ ), and the cyclotron and Zeeman energies ( $\hbar\omega_c$  and  $E_Z$ ); the up- ( $\uparrow$ ) and down-spin ( $\downarrow$ ) levels are represented by solid and dashed lines, respectively. The energies are plotted as a function of the field position of filling factor  $\nu = 13/2$  (bottom axis) and  $\nu = 15/2$  (top axis). The positions of  $E_F$  at  $\nu = 13/2$  and  $15/2$  are marked by the green and blue lines, respectively. The triangles labeled *a* to *f* point to the positions of  $\nu = 13/2$  and  $15/2$  for the densities at which traces are shown in Figs. 1(a-f).

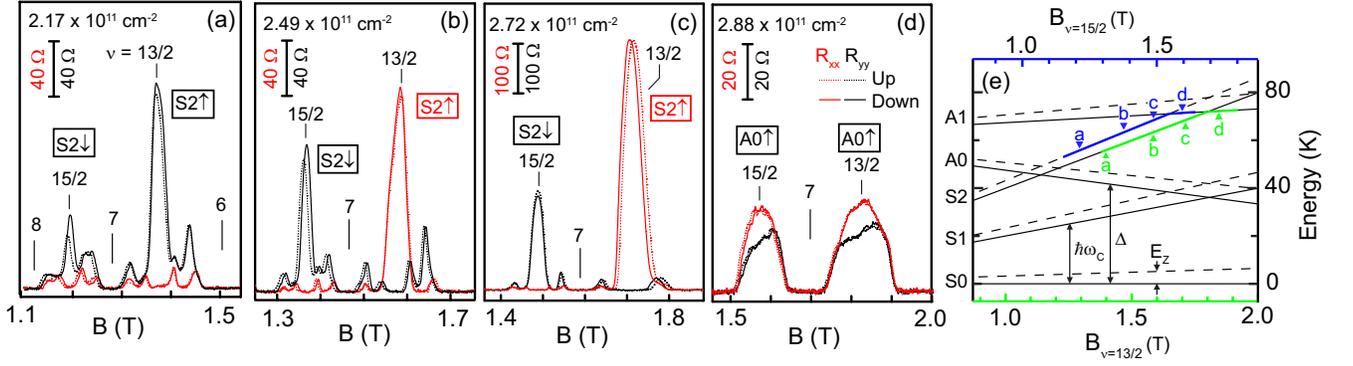


FIG. 3. (color online) (a-d)  $R_{xx}$  and  $R_{yy}$  measured in a 51-nm-wide QW at four different densities as indicated. Solid (dotted) traces were taken while the field was swept down (up). All traces were taken at a sweep rate of 1 T/hour. (e) The LL energy fan diagram corresponding to the data shown in (a-d); the notations used are the same as those in Fig. 2.

at  $\Delta = j \cdot \hbar\omega_c$  where  $j = 1, 2, 3, \dots$ . We found that the expression  $\Delta = 80 - 8.2 \cdot n$ , which we use in Fig. 2 plot, accurately describes the dependence of  $\Delta$  on  $n$  in the range of densities reached in our experiments ( $\Delta$  has units of K and  $n$  is given in units of  $10^{11} \text{ cm}^{-2}$ ). For  $E_Z$  we used an effective  $g$ -factor of  $g^* = 3.5$  which is 8-fold enhanced relative to the GaAs band  $g$ -factor (0.44); this  $E_Z$  is consistent with all the observed crossings between LLs of antiparallel-spin, which are signaled by spikes in the longitudinal resistance<sup>18</sup>.

Focusing first on  $\nu = 13/2$ , in Fig. 2 we show the position of the  $E_F$  at this filling in green, and mark the densities (or  $B_{\nu=13/2}$ ) corresponding to the data of Fig. 1 with up-pointing triangles. At the lowest density  $n = 2.13 \times 10^{11} \text{ cm}^{-2}$ ,  $E_F$  lies in the  $A0\uparrow$  level at  $\nu = 13/2$  and the 2DES is isotropic as seen in Fig. 1(a). As we increase  $n$  to  $2.35 \times 10^{11} \text{ cm}^{-2}$ ,  $E_F$  moves to the  $S2\downarrow$  level at  $\nu = 13/2$ . Strong anisotropy is seen in the data (Fig. 1(b)), consistent with  $E_F$  now lying in an  $N = 2$  LL. The resistance peak in  $R_{yy}$  and minimum in  $R_{xx}$

indicate the stripe phase is along the "normal" direction. Further increasing  $n$  to  $2.45 \times 10^{11} \text{ cm}^{-2}$ , this stripe phase disappears and the 2DES becomes isotropic again (Fig. 1(c)) when  $E_F$  moves back to an  $N = 0$  LL, namely the  $A0\downarrow$  level. The anisotropy reappears as soon as  $E_F$  moves to the  $S2\uparrow$  level at  $n = 2.67 \times 10^{11} \text{ cm}^{-2}$  (Fig. 1(d)) and the 2DES remains anisotropic up to the highest  $n$  achievable in this sample. Remarkably, however, in Figs. 1(d-f) at  $\nu = 13/2$  we observe a resistance *peak* in  $R_{xx}$  and a *minimum* in  $R_{yy}$ , signaling that the stripe direction has rotated and is now along the "abnormal" direction.

At  $\nu = 15/2$ , transport is isotropic at the lowest three  $n$  (Figs. 1(a-c)). This is expected as  $E_F$  lies in the  $A0\downarrow$  level; see the blue lines and the down-pointing triangles in Fig. 2. When  $n$  is further increased,  $E_F$  moves to the  $S2\downarrow$  level and the 2DES becomes anisotropic (Figs. 1(d-f)) at  $\nu = 15/2$ . In sharp contrast to the  $\nu = 13/2$  case, however, the stripe phase at  $\nu = 15/2$  is oriented along the "normal" direction up to the highest  $n$  achievable in the sample. It is clear that at a given fixed density (e.g., Fig. 1(e)), the stripes' direction depends on the spin orientation of the LL where  $E_F$  resides ( $S2\uparrow$  for  $\nu = 13/2$  and  $S2\downarrow$  for  $\nu = 15/2$ )<sup>21</sup>.

Data taken in a 51-nm-wide QW (Fig. 3) qualitatively confirm the spin-dependent reorientation of the stripe phase. As we increase  $n$ , the stripe phase rotates from the normal to the abnormal direction if  $E_F$  lies in the  $S2\uparrow$  level at  $\nu = 13/2$ , but it never rotates when  $E_F$  is in the  $S2\downarrow$  level at  $\nu = 15/2$  (Figs. 3(b,c)). However, the reorientation at  $\nu = 13/2$  is not seen at the lowest  $n$  (Fig. 3(a)), suggesting that it depends on  $n$  also. Figure 3(d) indicates that, as expected, the 2DES becomes isotropic at the highest  $n = 2.9 \times 10^{11} \text{ cm}^{-2}$  when  $E_F$  moves to the  $A1$  LLs (see Fig. 3(e)). Note also that in Figs. 3(a-d) we are showing data for different magnetic field sweep directions. In contrast to previous observations near the stripe phase reorientations in single-subband 2DESs<sup>12,13</sup>, we observe no hysteresis in our data<sup>22</sup>.

Figure 4 illustrates yet another remarkable property of

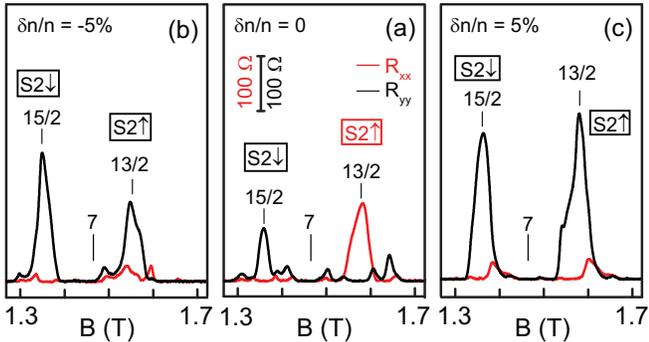


FIG. 4. (color online) (a)  $R_{xx}$  and  $R_{yy}$  taken in the 51-nm-wide QW at a fixed density of  $n = 2.5 \times 10^{11} \text{ cm}^{-2}$  when the QW charge distribution is symmetric. (b-c) Traces are shown as the charge distribution in the QW is made asymmetric by transferring  $\sim 5\%$  of total charge from the back side of the QW to the front side (b) or vice-versa (c).

the stripe phases in our samples. Here data are shown for the 51-nm-wide sample of Fig. 3 at a fixed density of  $n = 2.5 \times 10^{11} \text{ cm}^{-2}$  while we make the charge distribution in the QW asymmetric via applying front- and back-gate voltage biases with opposite polarity. When the charge distribution is symmetric (Fig. 4(a)) the stripe phase at  $\nu = 13/2$  is along the abnormal direction, but a small asymmetry in the charge distribution reorients the phase along the normal direction<sup>23</sup>.

The data presented in Figs. 1-4 provide evidence for additional subtleties and twists in the physics of stripe phases in 2DESs. While we do not have an explanation for the behaviors revealed in our wide QW data, some implications are noteworthy. First, in both the 42- and 51-nm-wide QWs, the reorientation at  $\nu = 13/2$  occurs at a very similar density,  $n \simeq 2.5 \times 10^{11} \text{ cm}^{-2}$ <sup>24</sup>. Therefore, we cannot rule out the possibility that our observed reorientation is density-induced. However, in single-subband, narrow QWs, the stripe phases at  $\nu = 9/2$  and  $11/2$  both rotate above the same threshold density ( $\sim 2.9 \times 10^{11} \text{ cm}^{-2}$ , see<sup>12</sup>), suggesting that the electron spin is not playing a role. In contrast, the rotation we report here in wide QWs appears to be spin-dependent: the stripe phase rotates at  $\nu = 13/2$  when  $E_F$  lies in the  $S2\uparrow$  level, but never rotates at  $\nu = 15/2$  when  $E_F$  is in the  $S2\downarrow$  level. Also, in our samples the stripe phase rotates at a density ( $n \simeq 2.5 \times 10^{11} \text{ cm}^{-2}$ ) which is smaller than the well-established critical density  $n \simeq 2.9 \times 10^{11} \text{ cm}^{-2}$  in hetero-junctions and narrow QW samples<sup>12,13</sup>. Moreover, the filling factors in our study ( $\nu = 13/2$  and  $15/2$ ) are larger than in previous reports ( $9/2$  and  $11/2$ ). Together with the lower threshold densities, this implies that the transition fields in our experiment are much smaller compared to previous measurements ( $\sim 1.5 \text{ T}$  vs.  $\sim 2.8 \text{ T}$ ).

Second, as illustrated in Fig. 4, the rotated stripe phase can be switched back to the normal direction when the QW charge distribution is made asymmetric. This observation has important implications for the possible origins of the symmetry-breaking potential. For example, Koduvayur *et al.*<sup>25</sup> recently reported that the appli-

cation of in-plane shear strain can alter the exchange potential and re-align the stripe direction in GaAs 2D hole systems. Thus they suggested that the residual strain due to surface charge induced fields is responsible for the symmetry-breaking potential of the stripe phases in both hole and electron 2D systems in GaAs. Our data of Fig. 4 do not agree with this conjecture as they show that the stripe phase can be made to lie along the same (normal) direction for electric fields of opposite polarity.

#### IV. CONCLUSION

The experimental observations reported here point to additional intricacies that determine how a GaAs 2DES chooses the direction of its anisotropic (stripe) phases at half-filled LLs. Besides the 2DES density, the spin orientation of the LL where  $E_F$  lies, as well as the symmetry of the charge distribution can both play roles in stabilizing the stripe phase direction. The spin-dependence is particularly puzzling because the energy of a stripe phase normally should not depend on the spin orientation of the carriers. It is possible that factors such as the mixing of the nearby LLs, particularly the A1 LLs, are responsible for the spin-dependence we observe. The details we present here, namely, our samples' parameters (well width, density, charge-distribution symmetry, and LL energy diagrams) should provide stimulus and quantitative input for future work aimed at understanding what determines the orientations of the stripe phases.

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- <sup>14</sup> When the A0 levels are fully occupied, e.g., case *e* in Fig. 2,  $\nu = 13/2$  and  $15/2$  in our samples are equivalent to  $9/2$  and  $11/2$  fillings in the standard, single-subband systems. In both cases  $E_F$  lies in the S2 levels.
- <sup>15</sup> The ungated regions near the contacts may cause artifacts in the measurements when their density differs from that of the gated region. We carefully examined all our data and saw no correlation between the rotation of the stripes and the polarity of such density difference.
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- <sup>19</sup> The LL diagrams of Figs. 2 and 3(e) are schematic as they do not take into account, e.g., variations of subband separation  $\Delta$  resulting from the discrete LL occupancy at the level crossings. However, we expect such variations to be very small when the QW is symmetric; see, e.g., Davies *et al.*<sup>20</sup> and Yang Liu *et al.*<sup>18</sup>. We emphasize that our main conclusion, the spin-dependent reorientation of the stripes, holds as we observe such reorientation even when the 2DES is far from any LL crossing (see, e.g., Fig. 3).
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- <sup>21</sup> It is worth remarking that we do not see any unexpected behaviors in our samples at lower fillings. Referring to the Fig. 2 LL diagram, at the lowest densities, when  $E_F$  lies in either of the S2 levels at  $\nu = 9/2$  and  $11/2$ , we observe stripe phases along the "normal" direction. (Note that in this low density regime, up to  $\nu = 6$ , our 2DES is similar to a standard, single-subband system.) At higher densities, when  $E_F$  moves to the A0 LLs at  $\nu = 9/2$  and  $11/2$ , the 2DES becomes isotropic and fractional quantum Hall states appear around these fillings; see, e.g., Ref.<sup>17</sup>. At yet higher densities, as the A0 and S1 levels cross, we observe a rich evolution of the fractional quantum Hall states, very similar to Fig. 1 of Ref.<sup>18</sup>.
- <sup>22</sup> To be more precise, our bidirectional field sweep data on the 51-nm-wide sample were taken at densities which are about 5% away from the transition density. We cannot rule out that hysteresis might exist closer to the transition density.
- <sup>23</sup> At a higher density of  $n = 2.7 \times 10^{11} \text{ cm}^{-2}$ , the stripe phase at  $\nu = 13/2$  in the 51-nm-wide QW sample remains along the abnormal direction when  $\delta n/n \simeq 4\%$ .
- <sup>24</sup> Also, in a 65-nm-wide QW, we do not see any rotation of the stripe phase at  $\nu = 13/2$  up to the highest achievable density of  $n = 2.0 \times 10^{11} \text{ cm}^{-2}$ .
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