Interaction-enhanced magnetically ordered insulating state at the edge of a two-dimensional topological insulator

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Interaction-enhanced magnetically ordered insulating state at the edge of a two-dimensional topological insulator

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We develop a theory of the correlated magnetically ordered insulating state at the edge of a two-dimensional topological insulator. We demonstrate that the gapped spin-polarized state, induced by the application of the magnetic field B, is naturally facilitated by electron interactions, which drive the critical easy-plane ferromagnetic correlations in the helical liquid. As the key manifestation, the gap Δ in the spectrum of collective excitations, which carry both spin and charge, is enhanced and exhibits a scaling dependence $\Delta \propto B^{1/(2-K)}$, controlled by the Luttinger liquid parameter K. This scaling dependence could be probed through the activation behavior $G \sim (e^2/h)\exp(-\Delta/T)$ of the longitudinal conductance of a Hall-bar device at lower temperatures, providing a straightforward way to extract the parameter K experimentally. Our findings thus suggest that the signatures of the interaction-driven quantum criticality of the helical liquid could be revealed already in a standard Hall-bar measurement.

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INTRODUCTION

Topological insulators [1–15] form a new class of materials with nontrivial band structure caused by spin-orbit interactions. The key physical feature that distinguishes a topological insulator (TI) from a conventional, nontopological, one is the presence of gapless surface or edge electron states. The edge of a two-dimensional (2D) topological insulator [1–3, 7, 9] supports two branches of gapless counter-propagating helical states with opposite spin projections on the axis perpendicular to the plane of the sample (Fig. 1). Protected by the time-reversal symmetry against single-particle nonmagnetic backscattering [16, 17], these edge modes serve as nearly ideal conducting channels that give rise to the quantum spin Hall effect. So far, a 2D topological insulator was realized in HgTe-CdTe quantum wells, which was first predicted theoretically [9] and shortly after confirmed experimentally [10, 11].

Interactions between electrons in the counter-propagating states lead to a one-dimensional helical Luttinger liquid (LL) phase [16–27], which hosts a number of remarkable physical properties, such as quantum criticality, bonding of the spin and charge degrees of freedom, and charge fractionalization. However, interaction effects in a LL are generally known to be quite elusive to experimental probes. In particular, for negligible single-particle backscattering, the longitudinal conductance e^2/h of a LL remains essentially unaffected by the interactions [28, 29]. In a helical LL, this holds as long as time-reversal symmetry is preserved and the system remains gapless. Probing interactions in this regime by a transport measurement generally requires creating a tunneling setup of some kind [20–24, 27].

In this paper, we demonstrate that electron interactions in a helical liquid reveal themselves in an interesting fashion once the time-reversal symmetry is broken by the application of an external magnetic field. Indeed, on the one hand, in the noninteracting picture, the magnetic field couples the counter-propagating edge states, opens a gap in the single-particle spectrum, and spin-polarizes the edge. On the other hand, in the absence of the magnetic field, interactions in a helical LL result in a tendency towards easy-plane ferromagnetism, manifested in a critical power-law decay of the spin correlations. Therefore, once the magnetic field is applied, one can naturally expect electron interactions to facilitate the formation of the spin-polarized state.

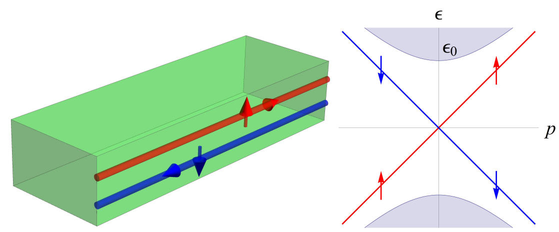

FIG. 1: (Color online) Helical edge states of a 2D topological insulator. (Left) The states propagating in the opposite directions have opposite spin projections on the direction perpendicular to the plane of the sample. (Right) In the absence of the magnetic field the counter-propagating states are gapless. Shaded regions depict the continuum of the extended bulk states with the insulating gap ϵ_0.

The present paper is devoted to the theory of this correlated magnetically ordered insulating state, induced by the magnetic field and enhanced by the interactions, at the edge of a 2D topological insulator. Our key finding is that the gap Δ in the spectrum of collective excitations is enhanced by the interactions and exhibits a critical scaling dependence $\Delta \propto B^{1/(2-K)}$ on the magnetic field B. Its exponent is controlled by the LL parameter K, which characterizes the interaction strength. Crucially, this critical scaling should reveal itself in the low-temperature activation behavior $G \sim (e^2/h)\exp(-\Delta/T)$ of the longitudinal conductance of a Hall-bar device, which allows one to extract the LL parameter K and infer about the strength of interactions in a real system. Our work suggests that the interaction-driven quantum criticality of the helical liquid at the edge of a 2D topological insulator could be accessed already via a standard Hall-bar measurement.

The suppression of the longitudinal conductance with the applied magnetic field was already observed experimentally in HgTe quantum wells [10, 11]. However, two factors preclude direct comparison of the present prediction with that data: (i) the magnetic-field data were provided for a large sample of size $20 \times 13\mu\mathrm{m}^2$, for which backscattering was substantial; (ii) the temperature dependence of the conductance, necessary to extract the transport gap Δ, was not provided.

MODEL AND HAMILTONIAN

The effective low-energy Hamiltonian for the interacting electrons in the counter-propagating edge states of a 2D topological insulator in the presence of a magnetic field [11, 18] may be written down in the helical basis of right-moving (with respect to the x direction along the edge) spin-up ($\uparrow$) and left-moving spin-down ($\downarrow$) states as

$$\hat{H} = \hat{H}_0 + \hat{H}_\mathrm{m} + \hat{H}_\mathrm{i},\ \hat{H}_0 = \int \mathrm{d}x\, \psi^\dagger(x)\, v\hat{p}\, \sigma_z\, \psi(x), \quad (1)$$

$$\hat{H}_\mathrm{m} = -\Delta_0 \int \mathrm{d}x\, \psi^\dagger(x)(\sigma_x \cos\varphi_0 + \sigma_y \sin\varphi_0)\psi(x), \quad (2)$$

$$\hat{H}_\mathrm{i} = \frac{1}{2}\int \mathrm{d}x\, \mathrm{d}x'\, \psi^\dagger_\sigma(x)\psi^\dagger_{\sigma'}(x')V(x-x')\psi_{\sigma'}(x')\psi_\sigma(x). \quad (3)$$

Here, $\psi = (\psi_\uparrow, \psi_\downarrow)^\mathrm{t}$ is the two-component fermionic field operator, $\hat{p} = -\mathrm{i}\hbar\partial_x$, and $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices in the helical basis. The part $\hat{H}_\mathrm{m}$ describes the effect of the external magnetic field. For the in-plane orientation, $\mathbf{B} = B(\cos\varphi_0, \sin\varphi_0, 0)$, only the Zeeman effect is present, whereas the orbital effect vanishes; the angle φ_0 correspond to the direction of the field in the xy plane of the 2D sample and the gap is given by the Zeeman energy $\Delta_{0\parallel} \sim \mu_B B$. In case of the perpendicular orientation of the field, $\mathbf{B} = (0, 0, B)$, the Zeeman effect does not affect the dynamics and only the orbital effect remains. The orbital effect of the perpendicular field is estimated [11] to be stronger than the in-plane Zeeman effect, $\Delta_{0\perp} \sim 10\Delta_{0\parallel}$; $\Delta_{0\parallel} \approx 3$K and $\Delta_{0\perp} \approx 30$K at $B = 1$T. For arbitrary field orientation, the single-particle gap Δ_0 scales linearly with the magnetic field, $\Delta_0 \propto B$.

We consider the case of Coulomb interactions, $V(x) = e_*^2/|x|$ in Eq. (3), possibly screened by the nearby metallic electrodes beyond some length l_s; the charge $e_* = e/\sqrt{\varkappa}$ incorporates the effects of screening by the dielectric environment. This allows us to consider both unscreened and screened interactions, the latter modeling practically any finite-range interactions. The short-scale spatial cutoff α of the theory [Eqs.(1), (2), and (3)] and of the potential $V(x)$ is set by the decay scale of the edge states into the bulk. For simplicity, it is assumed that the chemical potential is exactly at the branch crossing $\epsilon = 0$ of the unperturbed edge spectrum $\epsilon_p = \pm vp$, where the correlation effects are strongest. This can be achieved by tuning the gate voltage to the minimum of the longitudinal conductance.

The Hamiltonian $\hat{H}$ [Eqs. (1), (2), and (3)] describes one-dimensional interacting Dirac fermions, which are massive in the presence of the magnetic field; for point interactions, this is known as the Thirring model [32, 33]. This fermionic model can be mapped a bosonic one by mean of the bosonization procedure [32, 33]. One relates the fermion fields $\psi_{\uparrow,\downarrow}(x)$ of the right and left movers to the bosonic ones $\varphi_{\uparrow,\downarrow}(x)$ as

$$\psi_{\uparrow,\downarrow}(x) = \frac{1}{\sqrt{2\pi\alpha}}\mathrm{e}^{\pm\mathrm{i}\varphi_{\uparrow,\downarrow}(x)}, \quad (4)$$

where the Klein factors are omitted. The operators $\varphi(x) = \frac{1}{2}[\varphi_\uparrow(x) + \varphi_\downarrow(x)]$ and $\theta(x) = \frac{1}{2}[\varphi_\uparrow(x) - \varphi_\downarrow(x)]$ satisfy the canonical (up to a coefficient) commutation relations $[\varphi(x), \partial_{x'}\theta(x')] = -\mathrm{i}\pi\delta(x-x')$ and are related to the coordinate and momentum variables of the collective excitations. In terms of $\varphi(x)$ and $\theta(x)$, the Hamiltonian $\hat{H}$ [Eqs. (1), (2), and (3)] can be expressed as

$$\hat{H}_0 = \frac{\hbar}{2\pi}\int \mathrm{d}x\, v[(\partial_x\theta)^2 + (\partial_x\varphi)^2], \quad (5)$$

$$\hat{H}_\mathrm{m} = -\frac{\Delta_0}{\pi\alpha}\int \mathrm{d}x\, \cos[2\varphi(x) + \varphi_0], \quad (6)$$

$$\hat{H}_\mathrm{i} = \frac{1}{2\pi^2}\int \mathrm{d}x\, \mathrm{d}x'\, \partial_x\varphi(x)V(x-x')\partial_{x'}\varphi(x'). \quad (7)$$

The Hamiltonian (5), (7), and (6) describes the dynamics of the collective edge excitations of a 2D topological insulator in the presence of a magnetic field. This is the sine-Gordon model [32, 33] for point interactions and its nonlocal generalization for finite-range interactions. Below we analyze the properties of this model.

COLLECTIVE SPIN-CHARGE EXCITATIONS

To visualize the collective excitations described by Eqs. (5), (6), and (7), let us link the fields $\varphi(x)$ and

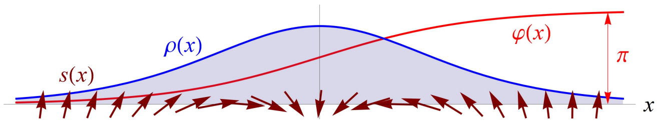

FIG. 2: (Color online) Collective spin-charge excitations of the edge of a 2D topological insulator. Excitations are described by the phase variable $\varphi(x)$, which determines both the in-plane spin polarization [Eq. (8)] and charge density [Eq. (9)]. As a specific illustrative example, a kink of height π in $\varphi(x)$ rotates the spin polarization in the xy plane of the sample by 2π and accumulates a unit charge in the region of variation of $\varphi(x)$.

$\theta(x)$ to the physical observables. From the relation (4), one obtains

$$\begin{pmatrix} s_x(x) \\ s_y(x) \end{pmatrix} = \frac{1}{2\pi\alpha} \begin{pmatrix} \cos(-2\varphi(x)) \\ \sin(-2\varphi(x)) \end{pmatrix} \tag{8}$$

for the x and y components of the spin density operator $\mathbf{s}(x) = \psi^\dagger_\sigma(x)\boldsymbol{\sigma}_{\sigma\sigma'}\psi_{\sigma'}(x)$ (defined without 1/2 factor) and

$$s_z(x) = \frac{1}{\pi}\partial_x\theta(x), \;\; \rho(x) = \frac{1}{\pi}\partial_x\varphi(x) \tag{9}$$

for the z component of the spin density and the particle density $\rho(x) = \psi^\dagger_\sigma(x)\psi_\sigma(x)$ operators. As seen from Eq. (8), the angle $-2\varphi(x)$ corresponds to the direction of the spin polarization in the xy plane and the field $\varphi(x)$ is thus directly related to the spin degrees of freedom. At the same time, according to Eq. (9), the charge density is determined by the gradient of $\varphi(x)$. Therefore, the collective excitations carry *both* charge and spin, which is a direct consequence of the coupling between the spin and orbital degrees of freedom in the single-particle states. As a specific illustrative example of this property, a kink of height π in $\varphi(x)$ rotates the spin polarization in the xy plane by 2π and simultaneously accumulates a unit charge in the region of variation of $\varphi(x)$, Fig. 2. It was suggested in Ref. [18] to exploit this bonding of spin and charge degrees of freedom to observe charge fractionalization effects in domain-wall structures with inhomogeneous magnetization.

GAPLESS HELICAL LIQUID AT $B = 0$

Let us first consider the system in the absence of the magnetic field, $\hat{H}_\mathrm{m} = 0$, when the edge is in the helical LL phase, and obtain the excitation spectrum and basic correlations. The calculations can be conveniently performed in the Langrange finite-temperature formalism. From Eqs. (5) and (7), the action for the Fourier transformation $\varphi(\omega_n, q) = \int_0^{\hbar/T} \mathrm{d}\tau \int \mathrm{d}x\, \mathrm{e}^{\mathrm{i}\omega_n\tau - \mathrm{i}qx}\varphi(\tau, x)$ ($\hbar\omega_n = 2\pi T n$, $n \in \mathbb{Z}$) of the phase field takes the form

$$S_0[\varphi] + S_\mathrm{i}[\varphi] = T\sum_{\omega_n}\int \frac{\mathrm{d}q}{2\pi}\left(\frac{1}{u_q}\omega_n^2 + u_q q^2\right)\frac{|\varphi(\omega_n, q)|^2}{2\pi K_q}. \tag{10}$$

The momentum-dependent velocity u_q and LL interaction parameter K_q are given by

$$u_q/v = 1/K_q = \sqrt{1 + V(q)/(\pi\hbar v)} = \sqrt{r_s \ln[1/(q_*\alpha_*)]}, \tag{11}$$

where $V(q) = 2e_*^2\ln[1/(q_*\alpha)]$ is the Fourier transform of the potential $V(x)$, $r_s = 2e_*^2/(\pi\hbar v)$ is the Coulomb parameter, $q_* = \max(|q|, 1/l_s)$, and $\alpha_* \sim \alpha \mathrm{e}^{-1/r_s}$.

From Eqs. (10) and (11), one obtains the excitation spectrum $\omega(q) = u_q|q|$ of the collective edge excitations of a 2D topological insulator. For unscreened Coulomb interactions $V(q) = 2e_*^2\ln[1/(|q|\alpha)]$ at $ql_s \gtrsim 1$, u_q and K_q depend logarithmically on q and the excitations have a 1D plasmon-type spectrum $\omega(q) \propto q\sqrt{\ln(1/q)}$. At spatial scales exceeding the screening length l_s, $ql_s \lesssim 1$, the interactions become effectively short-range with $V(q)$ saturating to the value $V(q \lesssim 1/l_s) = 2e_*^2\ln(l_s/\alpha)$. The velocity $u_q = u$ and interaction parameter $K_q = K$ become q-independent, $u/v = 1/K = \sqrt{r_s\ln(l_s/\alpha_*)}$, and the spectrum $\omega(q) = u|q|$ linear. In the absence of the magnetic field the spectrum is gapless, but for unscreened Coulomb interactions the log-dependence of the velocity u_q signals of a strong tendency towards gap opening.

Let us now study the correlations. The operators that describe coupling between the counter-propagating helical modes are given by the "spin-flip" components $s_\pm(x) = s_x(x) \pm \mathrm{i}s_y(x)$ of the spin density (8),

$$s_+(x) = \psi^\dagger_\uparrow(x)\psi_\downarrow(x) = \frac{\mathrm{e}^{-2\mathrm{i}\varphi(x)}}{2\pi\alpha}. \tag{12}$$

The tendency towards gap opening is thus directly related to the spin polarization in the xy plane of the sample. Calculating the correlation function of $s_\pm(x)$ with respect to the action (10) at zero temperature $T = 0$, we obtain

$$\langle s_+(x)s_-(0)\rangle \propto \begin{cases} \exp\left[-4\sqrt{\ln\left(|x|/\alpha_*\right)/r_s}\right], & |x| \lesssim l_s, \\ \left(l_s/|x|\right)^{2K}, & |x| \gtrsim l_s. \end{cases} \tag{13}$$

For screened Coulomb interactions at $|x| \gtrsim l_s$ the correlations (13) of the in-plane spin density $s_{x,y}(x)$ have a LL power-law decay. For unscreened Coulomb interactions at $|x| \lesssim l_s$, the decay is slower than any power law. The interactions in the helical liquid thus result in the tendency towards easy-plane ferromagnetic ordering. However, due to strong quantum fluctuations in a 1D system the long-range order is not formed, $\langle\mathbf{s}(x)\rangle = 0$. For unscreened Coulomb interactions, the tendency towards ferromagnetism is as strong as that towards Wigner crystallization in a conventional one-dimensional electron system [30, 31]. Note that numerical factors in the spectrum

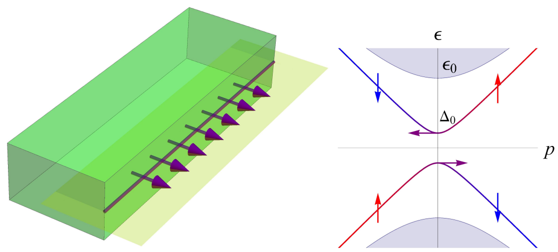

FIG. 3: (Color online) Magnetically ordered insulating state at the edge of a 2D topological insulator. Magnetic field couples the spin up and down helical states, opens a gap Δ_0 in the single-particle edge spectrum (right), and polarizes the electron spins in the plane of the sample (left). The many-body gap Δ [Eq. (14)] is enhanced by the interactions compared to the bare gap Δ_0.

$\omega(q) = u_q|q|$ [Eq. (11)] and correlation function (13) differ from those of Refs. [30, 31] because in our case electrons are single-flavored.

In the massless LL phase, the edge conductance $G_{\rm edge} = e^2/h$ is essentially unaffected by the interactions and the edge remains a perfect conducting channel [28, 29]. Therefore, in the absence of perturbations that break time-reversal symmetry, the interactions do not reveal themselves in the transport measurement of either the two-terminal or Hall-bar longitudinal conductance $G = 2G_{\rm edge} = 2e^2/h$, where the factor 2 is due to two edges in the former case and due to the mode equilibration in the contacts in the latter case.

GAPPED MAGNETICALLY ORDERED PHASE AT $B > 0$

The situation changes, if the magnetic field is applied, $\Delta_0 > 0$ in $\hat{H}_{\rm m}$ [Eq. (2)]. Even in the absence of interactions, the magnetic field couples the helical counter-propagating states [11, 18] according to Eq. (2) and opens a gap Δ_0 in the single-particle spectrum $\epsilon_p = \pm\sqrt{(vp)^2 + \Delta_0^2}$ of the Hamiltonian $\hat{H}_0 + \hat{H}_{\rm m}$, Fig. 3. In the ground state, the edge becomes spin polarized in the plane of the sample in the direction φ_0, $\langle \mathbf{s}(x)\rangle \propto (\cos\varphi_0, \sin\varphi_0, 0)$.

Opening of the single-particle gap Δ_0 has a direct consequence on transport. For the noninteracting electrons, the edge conductance can be calculated using the Landauer formula and for long enough edge of length $L \gg \hbar v/\Delta_0$ it is given by

$$G_{\rm edge}(T) = \frac{2e^2/h}{\exp(\Delta_0/T) + 1}.$$

The presence of the gap makes the edge insulating at temperatures $T \ll \Delta_0$, where the conductance follows the Arrhenius activation law $G_{\rm edge}(T) \approx 2(e^2/h)\exp(-\Delta_0/T)$.

Let us now take the interactions into account. In terms of the collective excitations, the effect of the magnetic field is described by the cosine term (6) in the bosonized Hamiltonian. The fact that the ground state is spin polarized means that the phase field $\varphi(x)$ is locked in the minimum of the cosine term, $\langle\varphi(x)\rangle = -\varphi_0/2$. The collective excitations are now massive and for low energies described by the fluctuations of $\varphi(x)$ around this minimum. Since even without the magnetic field the interactions tend to order the edge ferromagnetically, naturally, the gap Δ in the spectrum of the collective excitations turns out to be enhanced compared to its bare single-particle value Δ_0. For screened Coulomb interactions we obtain

$$\Delta \sim \epsilon_0 \left(\frac{\Delta_0}{\epsilon_0}\right)^{\frac{1}{2-K}} \propto B^{\frac{1}{2-K}}, \qquad (14)$$

up to a numerical factor ~ 1. Here ϵ_0 is the bulk insulator gap, which determines the high energy cutoff of the edge spectrum and is assumed $\epsilon_0 \gg \Delta_0$. For HgTe quantum wells, it is estimated $\epsilon_0 \sim 100$K [11]. The result (14) can be obtained by several means, e.g., using the self-consistent harmonic approximation [33].

The gap (14) has a power-law dependence on the bare gap $\Delta_0 \sim \mu_B B$ and hence on the magnetic field B. The exponent $1/(2-K)$ of this dependence is controlled by the LL interaction parameter K, which varies between $K = 1$ in the noninteracting case and $K = 0$ for infinitely strong finite-range interactions; these cases give the lowest $\Delta_{\rm min} = \Delta_0$ and highest $\Delta_{\rm max} \sim \sqrt{\Delta_0\epsilon_0} \propto \sqrt{B}$ possible values of the many-body gap Δ, respectively. Due to the long-range nature of the Coulomb forces, for unscreened interactions the gap appears to be close to $\Delta_{\rm max}$ even for moderate interaction strength $r_s \sim 1$. Performing the harmonic approximation [33], we obtain

$$\Delta^2 \sim \Delta_0\epsilon_0 \exp[-\sqrt{2\ln(\epsilon_0/\Delta_0)/r_s}]. \qquad (15)$$

The gap (15) differs from the $K = 0$ limit $\Delta_{\rm max}$ of Eq. (14) only by a function of Δ_0/ϵ_0 that varies slower than any power law. The result (15) applies if the correlation length $l_\Delta = \hbar v/\Delta$ determined from Eq. (15) does not exceed the screening length, $l_\Delta \lesssim l_s$. Otherwise, what concerns the gap, the interactions are effectively screened and the gap is given by Eq. (14). For unscreened Coulomb interactions, the enhancement of the gap could thus be quite substantial: for $\Delta_0 \sim 1$K and $\epsilon_0 \sim 100$K one gets $\Delta_{\rm max} \sim 10$K. The enhancement of the gap means, in particular, that interactions should favor observation of the effects predicted in Ref. [18].

SUMMARY AND EXPERIMENTAL MANIFESTATION

Summarizing, we studied the correlated magnetically ordered insulating state at the edge a of 2D topological insulator. This spin-polarized state is induced by the application of the magnetic field and naturally facilitated by electron interactions, which drive the easy-plane ferromagnetic correlations in a helical liquid. The key manifestation of the correlations is that the gap $\Delta \propto B^{1/(2-K)}$ [Eq. (14)] in the spectrum of the collective spin-charge excitations exhibits a scaling dependence on the magnetic field B, controlled by the Luttinger liquid parameter K, reflecting the quantum criticality of the helical liquid.

The main experimental implication of our findings is that electron interactions should readily reveal themselves in the insulating transport behavior of the magnetically ordered phase in a standard Hall-bar setup: the gap Δ determines the activation dependence $G(T) \propto (e^2/h)\exp(-\Delta/T)$ of either two-terminal or longitudinal conductance at temperatures $T \ll \Delta$. This should allow one to extract the Luttinger liquid parameter K and infer about the strength of the interactions in the helical liquid via the scaling dependence $\Delta \propto B^{1/(2-K)}$ of the gap. Our findings thus suggest a Hall-bar device in an applied magnetic field as the minimal setup to access the interaction-driven quantum criticality of the helical liquid at the edge of a 2D topological insulator.

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