Topologically protected surface Majorana arcs and bulk Weyl fermions in ferromagnetic superconductors

Jay D. Sau and Sumanta Tewari

Phys. Rev. B **86**, 104509 — Published 13 September 2012
DOI: 10.1103/PhysRevB.86.104509

Topologically protected surface Majorana arcs and bulk Weyl fermions in ferromagnetic superconductors

Jay D. Sau[1] and Sumanta Tewari[2]
[1]Department of Physics, Harvard University, Cambridge, MA 02138
[2]Department of Physics and Astronomy, Clemson University, Clemson, SC 29634

A number of ferromagnetic superconductors have been recently discovered which are believed to be in the so-called "equal spin pairing" (ESP) state. In the equal spin pairing state the Cooper pairs condense forming order parameters $\Delta_{\uparrow\uparrow}, \Delta_{\downarrow\downarrow}$ which are decoupled in the spin-sector. We show that these three-dimensional systems should generically support topologically protected surface Majorana arcs and bulk Weyl fermions as gapless excitations. Similar protected low-energy exotic quasiparticles should also appear in the recently discovered non-centrosymmteric superconductors in the presence of a Zeeman splitting. The protected surface arcs can be probed by angle-resolved photoemission (ARPES) as well as fourier transform scanning tunneling spectroscopty (FT-STS) experiments.

PACS numbers: 74.20.Rp, 03.65.Vf, 71.10.Pm

I. INTRODUCTION

Ferromagnetic superconductors display a remarkable coexistence of the mutually exclusive order parameters of ferromagnetism and superconductivity[1,2]. In the recently discovered Uranium-based Ising ferromagnetic superconductors UGe_2[3], URhGe[4], and UCoGe[5], it is believed that superconductivity appears in the spin-triplet p-wave equal-spin-pairing channel[6]. In this channel the superconducting order parameters $\Delta_{\uparrow\uparrow}$ and $\Delta_{\downarrow\downarrow}$ are decoupled in the spin space. In ferromagnetic superconductors typically the ferromagnetic transition temperature T^* far exceeds the superconducting transition temperature T_c. In this paper we show that in 3D the superconductivity in these materials supports protected chiral Dirac or Weyl nodes in the bulk and topologically robust open Majorana fermion arcs on the surface. Similar quasiparticle spectrum, but without the Majorana properties, have been recently predicted in some non-superconducting systems[7–9]. They have also been predicted to occur in superfluid He3-A[10,11] and the recently discovered systems known as non-centrosymmetric superconductors[12–14] in the presence of time reversal (TR) invariance[15]. As we show below, the ferromagnetic superconductor systems are another candidate providing one of the simplest platforms for the realization and detection of 3D gapless topological superconductivity (TS) with protected surface Majorana quasiparticles. Such modes on the surface should be accessible to surface sensitive probes such as Fourier transform scanning tunneling spectroscopy (FT-STS)[16–18].

The Weyl nodes in the ferromagnetic superconductors are characterized by an energy dispersion linearly proportional to momentum and come with a specific handedness or chirality. There are an even number of such nodes with each pair consisting of nodes with opposite chiralities. Such chiral Dirac, or Weyl, nodes are topologically protected in 3D by an invariant[19] which takes the values ± 1, and hence can only be removed when a pair of such nodes with opposite signs of the invariant collide in the momentum space. From the bulk-boundary correspondence such topologically protected gapless bulk spectrum leads to open Majorana fermion arcs on the surface which should be detectable in surface tunneling experiments.

Recently, there has been a lot of excitement about 2D and 1D TS systems supporting Majorana fermion quasiparticles defined by hermitian operators $\gamma_0^\dagger = \gamma_0$[20]. The canonical example of these systems is the 2D spin-less $p_x + ip_y$ superconductor[21] which is gapped in the bulk and has a single zero-energy Majorana fermion mode localized at order parameter defects such as vortices. Additionally, this system has gapless chiral Majorana fermion modes (defined by operators $\gamma_k^\dagger = \gamma_{-k}$) on the sample boundary. The robustness of such non-trivial topological properties of a 2D spin-less $p_x + ip_y$ superconductor can be understood in terms of a Z_2 topological invariant which is the parity of the Chern number of the corresponding 2D Bogoliubov-de Gennes (BdG) Hamiltonian[22,23]. Practical realizations of these exotic properties have recently been proposed in strong topological insulators (TI)[24] and, remarkably, even in ordinary semiconductors with spin-orbit coupling and a suitably directed Zeeman splitting[25–27].

In this paper we extend the concepts of time reversal breaking TS states to 3D solid state materials and investigate possible experimentally accessible condensed matter systems where such physics can be realized. We find that ferromagnetic superconductor systems are ideal candidates in 3D where very similar physics is realized *even in the absence of a spin-orbit coupling and an external Zeeman splitting*. Interestingly, we find that in ferromagnetic superconductors the analogous states are no-longer gapped in the bulk, but have gapless points in the momentum space with a Weyl spectrum of the BdG quasiparticles. Even more interestingly, the existence of the bulk Weyl nodes directly corresponds to the existence of open gapless Majorana fermion arcs on some suitable surfaces of the 3D system. The open surface Majorana fermion arcs offer the tantalizing possibility of detecting gapless Majorana excitations using the available surface sensitive probes such as ARPES and STM tunneling experiments.

II. TOPOLOGICAL PHASES IN FERROMAGNETIC SUPERCONDUCTORS

Ferromagnetic superconductors are characterized by a spin-triplet pairing potential which separates in the spin-sector (i.e., $\Delta_{\uparrow\uparrow}$ and $\Delta_{\downarrow\downarrow}$ are the relevant order parameters). Moreover, because of the existence of a strong internal Zeeman splitting (owing to ferromagnetism) enough to suppress spin-singlet pairing, the superconducting order has been proposed to be[28] the so-called non-unitary p-wave type[6]. We will take below the relevant representative order parameter as

$$\Delta_{\sigma\sigma'}(\boldsymbol{k})=\delta_{\sigma\sigma'}\Delta_{\sigma}(k_z)\frac{(k_x+ik_y)}{k_F}, \tag{1}$$

with $\Delta_{\uparrow}\neq\Delta_{\downarrow}$, which is the order parameter of the A_2 phase of He3[6]. This form of the dependence on k_x, k_y implies that Δ_σ must be an even function of k_z because overall a spin-triplet superconducting order parameter must be odd in momentum space. The general results in this paper (i.e., the predictions of the bulk Weyl modes and surface Majorana arcs) will follow simply from the equal spin pairing structure of the superconductivity and that the order parameter is an even function in momentum space. The orbital form of the order parameter, Eq. (1), will be used at the end for numerical calculations. In practice a ferromagnetic superconductor may break into several domains in which the magnetization may point into different directions. Our theoretical calculations in this paper will apply within a single domain. The proposed experimental signatures of the surface Majorana arcs are in terms of the scanning tunneling microscopy experiments which, because of the atomic scale resolution, can in principle access the system within a single domain.

The mean-field Hamiltonian describing an equal spin pairing state for a p-wave superconductor is written as

$$\begin{aligned}H_{BCS}=&\sum_{\sigma}\int d^2\boldsymbol{k}\Big(\frac{k_x^2+k_y^2+k_z^2}{2m^*}-\varepsilon_{F,\sigma}\Big)f^{\dagger}_{\sigma\boldsymbol{k}}f_{\sigma\boldsymbol{k}}\\&+[\Delta_{\sigma}(\boldsymbol{k})f^{\dagger}_{\sigma\boldsymbol{k}}f^{\dagger}_{\sigma,-\boldsymbol{k}}+h.c]\end{aligned} \tag{2}$$

where $\sigma=\uparrow,\downarrow$ labels the spin index of the electron operators. Here we have assumed that the pairing potential is of the equal spin pairing form i.e. $\Delta_{\sigma\sigma'}(\boldsymbol{k})=\delta_{\sigma\sigma'}\Delta_{\sigma}(\boldsymbol{k})$ and the pairing potential is odd in momentum space so that $\Delta_{\sigma}(\boldsymbol{k})=-\Delta_{\sigma}(-\boldsymbol{k})$. The magnetization of the ferromagnetic superconductor is accounted for by the difference in the Fermi energies $(\varepsilon_{F,\uparrow}-\varepsilon_{F,\downarrow})$. Defining the Nambu spinor $\Psi(\boldsymbol{k})=(f^{\dagger}_{\uparrow}(\boldsymbol{k}),f^{\dagger}_{\downarrow}(\boldsymbol{k}),f_{\uparrow}(-\boldsymbol{k}),f_{\downarrow}(-\boldsymbol{k}))$, the BdG Hamiltonian for the ferromagnetic superconductor is written as

$$\begin{aligned}H_b(k_x,k_y,k_z)=&\Big(\frac{k_x^2+k_y^2+k_z^2}{2m^*}-\varepsilon_{F,av}-MN(0)\sigma_z\Big)\tau_z\\&+[\boldsymbol{\Delta}(\boldsymbol{k})\tau_{+}+h.c]\end{aligned} \tag{3}$$

where $M=\frac{\varepsilon_{F,\uparrow}-\varepsilon_{F,\downarrow}}{2N(0)}$ is proportional to the magnetization, $N(0)$ is the density of states at the fermi level, and $\varepsilon_{F,av}=\frac{(\varepsilon_{F,\uparrow}+\varepsilon_{F,\downarrow})}{2}$ is the average fermi energy of the two spin components. The Nambu spinor $\Psi(\boldsymbol{k})$ is a four-component spinor that can be thought of as a vector in the tensor product space of a two-component spin-degree of freedom consisting of the labels $\sigma_z=\uparrow,\downarrow$ in the spinor and a particle-hole degree of freedom consisting of the upper pair of components and lower pair of components. The Pauli matrices $\tau_{x,y,z}$ in Eq. 3 act on the particle-hole degree of freedom while σ_z acts on the spin degree of freedom. Also $\tau_{\pm}=\tau_x\pm i\tau_y$. Here $\boldsymbol{\Delta}(\boldsymbol{k})$ is the pairing potential matrix with matrix-elements $\delta_{\sigma\sigma'}\Delta_{\sigma}(\boldsymbol{k})$.

The properties of the 3D phase described by Eq. 3 can be understood by dimensional reduction in momentum space to classes of 2D topological phases. The idea of dimensional reduction is to consider the Hamiltonian $H_b(k_x,k_y,k_z)$ for the translationally invariant 3D bulk system as a set of 2D systems i.e.,

$$H^{(2D)}_{k_z}(k_x,k_y)=H_b(k_x,k_y,k_z) \tag{4}$$

where $H^{(2D)}_{k_z}(k_x,k_y)$ are effective 2D Hamiltonians parametrized by k_z. The topological properties of such 2D Hamiltonians, which break the time reversal symmetry, are characterized by the Chern number associated with the occupied states[29]. The Chern number of a 2D topological Hamiltonian counts the number of chiral edge states at a given k_z. The edge states of the 2D system can be understood as surface states of the 3D system for surfaces which are parallel to the z-axis. For such surface states, the wave-vector along z, k_z is a good quantum number and the dispersion of the surface states can be obtained from the dispersion of the edge states.

III. TOPOLOGICAL INVARIANTS

The 3D BdG Hamiltonian H_b of the ferromagnetic superconductor systems has a particle-hole symmetry $\Lambda=i\tau_yK$, which anti-commutes with H_b. Here K is the complex conjugation operator. To make use of this symmetry, we will assume from here onwards that H_b is an even function of k_z so that $H^{(2D)}_{k_z}$ is also particle-hole symmetric. In this case, one can use the so-called Pfaffian topological invariant, $\mathrm{sgn}(Pf(i\tau_yH^{(2D)}_{k_z}(k_x=k_y=0)))$[23] which determines the parity of the Chern number. The Chern number is the integer topological invariant of a 2D quadratic Hamiltonian which gives the number of chiral edge states (equal to the value of the invariant)[29].

For BdG Hamiltonians the chiral edge states are Majorana edge states (i.e., the satisfy $\gamma_k^\dagger = \gamma_{-k}$)[21]. By parity of the Chern number we mean if the Chern number is even (even parity) or odd (odd parity). The Pfaffian of the anti-symmetrized BdG Hamiltonian, in 2D, is related to the Chern number by $Pf(i\tau_y H_{k_z}^{(2D)}(k_x = k_y = 0)) = \exp(i\pi C_1)$ (with C_1 the Chern number)[23]. Thus, the Pfaffian topological invariant $\mathrm{sgn}(Pf(i\tau_y H_{k_z}^{(2D)}(k_x = k_y = 0)))$ gives the parity of C_1; it is negative when C_1 is odd, positive when C_1 is even. As a function of k_z when the Pfaffian invariant changes sign it indicates a jump in the integer invariant C_1. A jump in the value of C_1 can only happen if the spectral gap closes at the corresponding value of k_z (with no closing of the spectral gap the 2D systems parametrized by k_z, see Eq. (4), must have the same values for C_1 since it can only be an integer and thus must be the same for 2D systems which can be adiabatically connected to each other).

As mentioned above the Pfaffian invariant is non-trivial (i.e, is negative), the Chern parity is odd, otherwise the latter is even[23]. Since the pairing potentials vanish at $(k_x = k_y = 0, k_z)$, the Pfaffian topological invariant is found to be

$$\mathrm{sgn}(Pf(i\tau_y H_{k_z}^{(2D)}(k_x, k_y = 0))) = \mathrm{sgn}(\varepsilon_{F,\uparrow} - k_z^2)\mathrm{sgn}(\varepsilon_{F,\downarrow} - k_z^2). \tag{5}$$

The above topological invariant is non-trivial only in the restricted range of values of k_z which satisfy:

$$k_{z,c,\downarrow} = \sqrt{2m^* \varepsilon_{F,\downarrow}} < |k_z| < k_{z,c,\uparrow} = \sqrt{2m^* \varepsilon_{F,\uparrow}}. \tag{6}$$

For values of k_z outside this range, the system has even Chern parity.

IV. BULK WEYL FERMIONS

The family of topological superconductors described by the Hamiltonian $H_{k_z}^{(2D)}$ undergoes a quantum phase transition from the topological to the non-topological phase when the Chern parity changes at the values of k_z where the conditions in Eq. 6 are saturated. Such topological quantum phase transitions are accompanied by a closing of the topological gap at $k_x = k_y = 0$. The set of energy eigenvalues of the 3D Hamiltonian $H_b(k_x, k_y, k_z)$ in Eq. 3 is a union of the energy eigenvalues of the entire family of 2D topological superconductors described by the Hamiltonians $H_{k_z}^{(2D)}$. Therefore the Hamiltonian $H_b(k_x, k_y, k_z)$ must have gapless points at $\boldsymbol{K}_\sigma = (k_x = 0, k_y = 0, k_z = \pm k_{z,c,\sigma})$, with a two-fold degeneracy of eigenstates, $|\tau_z = \pm 1, \sigma_z = \sigma\rangle$, where $\sigma = \uparrow, \downarrow$. The dispersion of the pair of degenerate states ($|\tau_z = \pm 1, \sigma_z = \sigma\rangle$) around the degeneracy points $\boldsymbol{K}$ can be obtained by expanding the Hamiltonian in Eq. 3 as $\boldsymbol{k} = \boldsymbol{K} + \delta\boldsymbol{k}$ to linear order in $\delta\boldsymbol{k}$. The resulting Hamiltonian then describes a spectrum resembling a three-dimensional Dirac cone,

$$H_b(\boldsymbol{K}_\sigma + \delta\boldsymbol{k}) = \boldsymbol{\delta k} \cdot \boldsymbol{\nabla}\Delta_{R,\sigma}(\boldsymbol{K}_\sigma)\tau_x + \boldsymbol{\delta k} \cdot \boldsymbol{\nabla}\Delta_{I,\sigma}(\boldsymbol{K}_\sigma)\tau_y + \delta k_z \frac{2k_{z,c,\sigma}}{m^*}\tau_z + o(\delta\boldsymbol{k}^2), \tag{7}$$

where $\Delta_{R,I,\sigma}(\boldsymbol{K}_\sigma)$ are the real and imaginary parts of $\Delta_\sigma(\boldsymbol{K}_\sigma)$ in Eq. 2. Such Dirac cone spectra in 3D are protected (by 'protected' we mean that the spectrum remains gapless) because any perturbation of the bulk Hamiltonian $H_b(\boldsymbol{k})$ which does not couple two Dirac cones (which are separated in the momentum space by a finite extent in k_z) can only shift the position of the Dirac points and, in particular, cannot create a gap in the spectrum. A perturbation to Eq. 3 can only remove Dirac cones from the spectrum by merging them in pairs[19]. This kind of protection of Dirac cones in $D = 3$ can be further represented by associating them with a topological invariant[19] which, for brevity, we do not discuss here. Bulk 3D semi-metals with such Dirac-like point Fermi surfaces are referred to as Weyl semimetals. Thus H_b in Eq. 3 represents a nodal superconductor with four isolated Dirac cones which we call a Weyl superconductor.

V. SURFACE MAJORANA ARCS

The 2D topological superconductor Hamiltonians $H_{k_z}^{(2D)}(k_x, k_y)$, which are parameterized by k_z, have an odd Chern number in the range Eq. 6 and are therefore characterized by chiral Majorana edge states that are confined to the edge of the system. A surface along the $x - z$ plane for the original 3D Hamiltonian Eq. 3 is translationally invariant along the z and x directions and therefore has well-defined k_z and k_x momenta. Therefore the surface state with a fixed k_z and k_x of the 3D Hamiltonian Eq. 3 is identical to the edge state with momentum k_x of the 2D Hamiltonian in Eq. 4 with k_z as a parameter value. The energy of such a chiral Majorana mode vanishes for $k_x = 0$ at any value of k_z in the range in Eq. 6 and therefore appears on the surface ARPES spectrum as a Majorana arc.

The typical extent in k_z of such a non-degenerate Majorana arc is limited to the range given in Eq. 6. Below, as mentioned earlier, we take the order parameter of the ferromagnetic superconductor systems to be of the non-unitary equal spin pairing type with an orbital structure given by $\Delta_{\sigma\sigma'}(\boldsymbol{k}) = \delta_{\sigma\sigma'}\Delta_\sigma \frac{(k_x + ik_y)}{k_F}$ with $\Delta_\uparrow \neq \Delta_\downarrow$. In this case, the pairing potential $\Delta_{\sigma\sigma'}(\boldsymbol{k})$ is associated with a Chern number 2, therefore one would have a pair of chiral surface modes propagating along the surface in the range $|k_z| < k_{z,\downarrow}$. This could in general appear as a pair of fermi arcs of Bogolibov quasiparticles in the ARPES or STM spectrum. Since the BdG Hamiltonian now decouples into a spin-$\uparrow$ and spin-$\downarrow$ sector, the 2D Hamiltonian $H_{k_z,\sigma}^{(2D)}(k_x, k_y)$ for each spin σ can be thought of as independent odd Chern number topological superconductors which have a Majorana chiral edge mode in the ranges

$|k_z| < k_{z,c,\uparrow}$ for spin-up electrons and $|k_z| < k_{z,c,\downarrow}$ for spin-down electrons. Therefore we find that the surface arcs exist over a much larger range in k_z i.e between $-k_{z,c,\uparrow}$ and $k_{z,c,\uparrow}$.

These chiral edge modes exist for k_z satisfying $|k_{z,\sigma}| < k_{z,c,\sigma}$ and have a dispersion of the form $\varepsilon_{k_z}(k_x) = v(k_z)k_x$, where $\sigma =\uparrow,\downarrow$ is the S_z spin-sector along which the modes are polarized. Therefore the dispersion of the corresponding surface mode is given by

$$\epsilon(k_x, k_z, \sigma) = v(k_z, \sigma)k_x \sim \frac{\Delta_\sigma(k_z)}{\sqrt{2m^*(\varepsilon_{F,\sigma} - k_z^2)}}k_x \quad (8)$$

for $|k_z| < k_{z,c,\sigma}$ and $|\epsilon(k_x, k_z, \sigma)| > \epsilon_g$ where ϵ_g is a finite positive gap for $|k_z| > k_{z,c,\sigma}$. The dispersion of the surface modes given in Eq. 8 has the special property that the energy vanishes on a pair of lines $k_x = 0$ which terminates at $k_z = \pm k_{z,c,\sigma}$. These lines are referred to as Majorana arcs. We refer to these lines in energy as Majorana arcs because the operators γ_{n,k_x,k_z} associated with these zero-energy states satisfy the Majorana constraint

$$\gamma^\dagger_{n,k_x=0,k_z} = \gamma_{n,-k_x,-k_z}. \quad (9)$$

As mentioned before the Majorana character of the surface arcs is only protected in the restricted range in Eq. 6. In the rest of the k_z range, the dispersion arcs are spin-degenerate Majorana fermions corresponding to a spin-label in Eq. 9 of $n =\uparrow,\downarrow$, which in principle can be split into more conventional Dirac fermions. In particular an in-plane Zeeman splitting along x, which we call $V_x^{(Z)}$, can lead to a mixing of the $\sigma =\uparrow,\downarrow$ states so that the surface Majorana arcs now split into a pair of fermi arcs, indexed by $s = \pm 1$, with dispersion

$$\begin{aligned}\epsilon(k_x, k_z, s) \sim & \frac{v(k_z,\uparrow) + v(k_z,\downarrow)}{2}k_x \\ & + s\sqrt{\left(\frac{v(k_z,\uparrow) - v(k_z,\downarrow)}{2}\right)^2 k_x^2 + V_x^{(Z)2}}\end{aligned} \quad (10)$$

for $|k_z| < k_{z,c,\downarrow}$. The fermi-arcs have wave-functions

$$f_{s=1,k_x,k_z} = \gamma_{\uparrow,k_x,k_z} + i\gamma_{\downarrow,k_x,k_z} \quad (11)$$
$$f_{s=-1,k_x,k_z} = \gamma_{\uparrow,k_x,k_z} - i\gamma_{\downarrow,k_x,k_z}, \quad (12)$$

which no longer satisfy the constraint Eq. 9 because $f^\dagger_{s,k_x,k_z} \neq f_{s,-k_x,-k_z}$. Therefore we refer to them as Fermi arcs rather than Majorana arcs.

The Majorana arcs are obtained by solving $\epsilon(k_x, k_z, s) = 0$. This leads to the equation for the Majorana arc

$$k_x = \pm\frac{V_x^{(Z)}}{\sqrt{v(k_z,\uparrow)v(k_z,\downarrow)}} \approx \pm\frac{V_x^{(Z)}\sqrt{\epsilon_F - k_z^2}}{\sqrt{\Delta_\uparrow\Delta_\downarrow}} \quad (13)$$

where we have assumed the magnetization to be small compared to the total density (i.e. $|\varepsilon_\uparrow - \varepsilon_\downarrow| \ll \varepsilon_{F,av}$) and the expression $\Delta_\sigma(k_z) = \Delta_\sigma$. The resulting Majorana arcs are ellipses in the (k_x, k_z) plane between $|k_z| < k_{z,c,\downarrow}$ as plotted in Fig. 1.

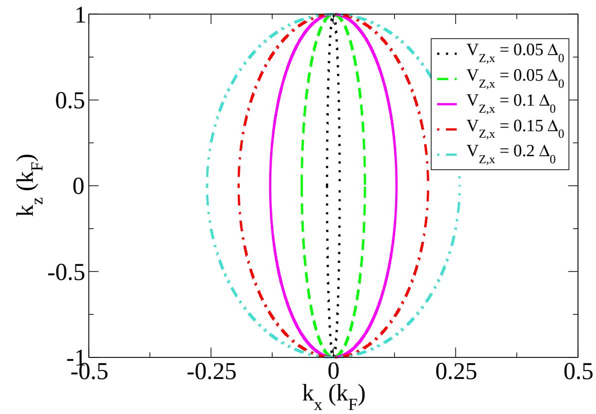

FIG. 1. $V_x^{(Z)}$-dependent surface fermi contour plotted according to Eq. 13 for $\Delta_\downarrow = 0.6\Delta_\uparrow, \Delta_\uparrow = \Delta_0$. The width of the contour in the k_x direction is proportional to $V_x^{(Z)}$. As $V_x^{(Z)} \to 0$, the spin-$\uparrow$ and spin-$\downarrow$ sectors decouple and the fermi contour evolves into a pair of surface Majorana arcs.

The Majorana arcs obtained in these systems are similar to Majorana surface states[33] and non-chiral Majorana modes[24] and require the wave-vector (k_x, k_y) to be a good quantum number, thus requiring translational invariance. Additionally, similar to the case of the proximity induced superconductivity from the high-T_c cuprate superconductors[34], the breaking of translational invariance such as by impurities or domain walls would also scatter the Majorana fermions into the bulk quasiparticles leading to a finite life-time. However, the resulting Majorana arcs should still have interesting observable consequences as long as the mean scattering time is small.

VI. QUASIPARTICLE INTERFERENCE OF MAJORANA ARCS IN STM:

The surface Majorana arcs shown in Fig. 1 should in principle be directly visible as arcs in momentum space at the Fermi energy in the ARPES spectrum. However, since these arcs are separated from the bulk states by a relatively small superconducting gap ($\sim 1K \sim 0.1$ meV), it is not clear if the typical energy resolution achieved in ARPES is sufficient to resolve the Majorana arcs. On the other hand, tunneling experiments[31] provide a very convenient way to detect Majorana fermion systems. While one might worry that normal electrons might have a vanishingly small matrix element for tunneling into Majorana states, theoretical studies of proposals for such experiments[31] have shown that tunneling of normal electrons into states containing Majorana fermions is an efficient way to detect such states. In fact, recent point contact tunneling experiments[32] on $Cu_xBi_2Se_3$ accompanied by theoretical studies[33] have shown evidence for

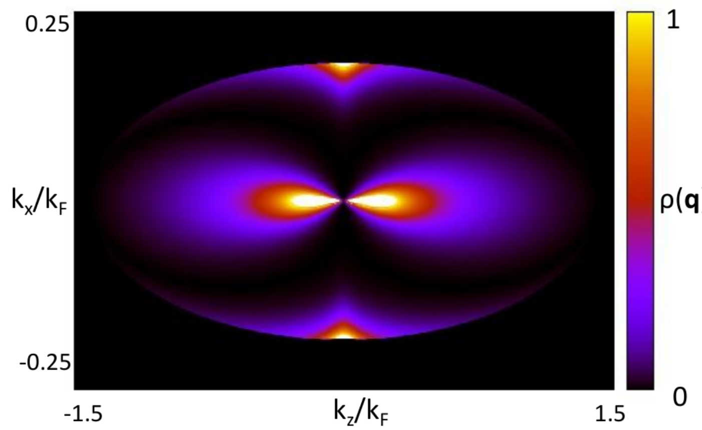

FIG. 2. (color online) The joint density of states $\rho(\boldsymbol{q})$ (in arbitrary units) at the fermi surface, which, as explained in the text, would be measured by FT-STS on the surfaces of ferromagnetic superconductor systems. We have used Eq. 14, and the parameter values $V_x^{(Z)} = 0.1\Delta_\uparrow$ and $\Delta_\downarrow = 0.6\Delta_\downarrow$ (i.e. same as Fig. 1).

Majorana surface states in such materials by tunneling. On the other hand, low temperature and low noise STM measurements often have sub-100 mK energy resolution allowing one access to the energy scale of the Majorana arcs. This energy resolution is crucial to be able to separate out the relatively small phase space associated with the Majorana arcs from the bulk states. At low biases the current in the STM will be composed of a contribution from the Majorana arcs together with the gapless bulk states near the Dirac points of the Weyl semi-metals. Because STM is a surface probe and the bulk states are delocalized over the bulk of the system and have a vanishingly small density of states near zero-energy, one can expect the bulk contribution to be small.

While conventional STM is able to detect the Majorana arcs as gapless modes on the surface, it does not provide any information about the structure (finite extent in momentum, curvature, etc.) of the Majorana arcs in momentum space. Such information can be obtained by FT-STS[16–18]. FT-STS relies on the fact that impurity-scattering at the surface leads to a spatially varying local quasiparticle density of states, $n(\boldsymbol{r})$ at the surface, which can be determined from the spatial variation of the tunneling current $I_t(\boldsymbol{r}) \propto n(\boldsymbol{r})$ at the surface. The resulting current map obtained from STM can be Fourier transformed to obtain $I_t(\boldsymbol{q}) = \int d\boldsymbol{r} I_t(\boldsymbol{r}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \propto \int d\boldsymbol{r} n(\boldsymbol{r}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$. The disorder averaged square of the Fourier transform $\langle |I(\boldsymbol{q})|^2\rangle$ can be shown to be related to the joint density of states $\rho(\boldsymbol{q})$ (i.e. $\rho(\boldsymbol{q}) \propto \langle |I(\boldsymbol{q})|^2\rangle$) defined by

$$\rho(\boldsymbol{q}) = \int d\boldsymbol{k} \delta(\epsilon(\boldsymbol{k}))\delta(\epsilon(\boldsymbol{k}+\boldsymbol{q})), \tag{14}$$

where $\epsilon(\boldsymbol{k})$ is the surface mode dispersion in Eq. 11. The application of the FT-STS method[16–18] outlined above for the weakly disordered surface of a ferromagnetic semiconductor is expected to lead to the characteristic structure that is plotted in Fig. 2.

VII. CONCLUSION:

We propose the recently discovered ferromagnetic superconductors[1–5] as experimentally accessible 3D systems supporting topologically protected chiral Weyl fermions in the bulk and open Majorana fermion arcs on suitably oriented surfaces. The Weyl nodes in the bulk are topologically protected because they arise from topologically unavoidable closing of the quasiparticle gap at isolated points in the momentum space. The existence of the bulk Weyl nodes directly corresponds to the existence of open gapless Majorana fermion arcs (Fig. 1) on suitable surfaces of the 3D system. The surface Majorana fermion arcs offer the tantalizing possibility of detecting gapless Majorana excitations using the available surface sensitive probes such as ARPES and spectroscopic STM experiments (Fig. 2).

The ferromagnetic superconductor systems are not the only materials which can support 3D gapless TS states with broken time reversal symmetry. The newly discovered 3D non-centrosymmetric superconducting materials[12–14] can also support such states in the presence of a sufficiently large Zeeman splitting. The Zeeman splitting, however, should not be accompanied by a large orbital depairing field which may destroy the superconductivity itself. This can be ensured by choosing materials with a large enough g-factor. so that a relatively small magnetic field can still create a Zeeman splitting larger than the superconducting order parameter.

J.S. thanks the Harvard Quantum Optics Center for support. S.T. thanks Grants DARPA-MTO (FA9550-10-1-0497) and NSF (PHY-1104527) for support.

[1] Dai Aoki, Frederic Hardy, Atsushi Miyake, Valentin Taufour, Tatsuma D. Matsuda, Jacques Flouquet, C. R. Physique **12**, 573 (2011).
[2] V.P.Mineev, arXiv:0812.2171 (2008).
[3] S. S. Saxena, P. Agarwal, K. Ahilan, F. M. Grosche, R. K. W. Haselwimmer, M. J. Steiner, E. Pugh, I. R. Walker, S. R. Julian, P. Monthoux, G. G. Lonzarich, A. Huxley, I. Sheikin, D. Braithwaite and J. Flouquet, Nature **406** (2000) 587.
[4] D. Aoki, A. Huxley, E. Ressouche, D. Braithwaite, J. Flouquet, J.-P. Brison, E. Lhotel and C. Paulsen, Nature **413** (2001) 613.
[5] N. T. Huy, A. Gasparini, D. E. de Nijs, Y. Huang, J. C. P. Klaasse, T. Gortenmulder, A. de Visser, A. Hamann, T. Gorlach and H. v. Lohneysen, Phys. Rev. Lett. **99** (2007) 067006.
[6] D. Vollhardt and P. Wolfle, *The Superfluid Phases of Helium 3* (Taylor & Francis, 1990).
[7] X. Wan, A.M. Turner, A. Vishwanath, and S.Y. Savrasov, Phys. Rev. B **83**, 205101 (2011).
[8] G. Xu, H. Weng, Z. Wang, X. Dai, Z. Fang, arXiv:1106.3125.
[9] A.A. Burkov, L. Balents, arXiv:1105.5138.
[10] T.T. Heikkila, N.B. Kopnin, G.E. Volovik, Pis'ma ZhETF **94**, 252 (2011).
[11] Y. Tsutsumi, M. Ichioka, and K. Machida, Phys. Rev. B **83**, 094510 (2011).
[12] K. Togano et al., Phys. Rev. Lett. **93**, 247004 (2004).
[13] P. Badica, T. Kondo, and K. Togano, J. Phys. Soc. Jpn. **74**, 1014 (2005).
[14] G. Amano et al., J. Phys. Soc. Jpn. **73**, 530 (2004).
[15] A. P. Schnyder, S. Ryu, Phys. Rev. B **84**, 060504(R) (2011).
[16] J. E. Hoffman et al., Science **297**, 1148 (2002).
[17] A. V. Balatsky, I. Vekhter, and Jian-Xin Zhu, Rev. Mod. Phys. **78**, 373 (2006).
[18] H. Beidenkopf, P. Roushan, J. Seo, L. Gorman, I. Drozdov, Y. S. Hor, R. J. Cava, A. Yazdani arXiv:1108.2089 (2011).
[19] F.R. Klinkhamer, G.E. Volovik, JETP Lett. **80**, 343 (2004).
[20] Nayak, C., Simon, S. H., Stern, A., Freedman, M. & Das Sarma, S. Non-Abelian anyons and topological quantum computation, *Rev. Mod. Phys.* **80**, 1083 (2008).
[21] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
[22] A. Kitaev, *Physics-Uspekhi* **44**, 131 (2001).
[23] P. Ghosh, J. D. Sau, S. Tewari, S. Das Sarma, *Phys. Rev. B* **82**, 184525 (2010).
[24] L. Fu and C. L. Kane, Phys. Rev. Lett. **100**, 096407 (2008).
[25] Jay D. Sau, R. M. Lutchyn, S. Tewari, S. Das Sarma, Phys. Rev. Lett. **104**, 040502 (2010).
[26] J. D. Sau, S. Tewari, R. Lutchyn, T. Stanescu and S. Das Sarma, Phys. Rev. B **82**, 214509 (2010).
[27] Y. Oreg, G. Refael, F. V. Oppen, Phys. Rev. Lett. **105**, 177002 (2010).
[28] K. Machida and T. Ohmi, Phys. Rev. Lett. **86**, 850 (2001).
[29] S. Ryu, A. Schnyder, A. Furusaki, A. Ludwig, New J. Phys. **12**, 065010 (2010).
[30] K. Sengupta, I. Zutic, H-J. Kwon, V. M. Yakovenko, and S. Das Sarma, Phys. Rev. B 63, 144531 (2001).
[31] Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. **74**, 3451 (1995); C. J. Bolech, E. Demler, Phys. Rev. Lett. 98, 237002 (2007); K. T. Law, Patrick A. Lee, and T. K. Ng, Phys. Rev. Lett. **103**, 237001 (2009); K. Flensberg, Phys. Rev. B **82**, 180516 (2010); M. Wimmer, A.R. Akhmerov, J.P. Dahlhaus, C.W.J. Beenakker, New J. Phys. **13**, 053016 (2011).
[32] S. Sasaki et al., Phys. Rev. Lett. **107**, 217001 (2011).
[33] L. Fu and E. Berg,Phys. Rev. Lett. 105, 097001 (2010); T. H. Hsieh, L. Fu Phys. Rev. Lett. 108, 107005 (2012).
[34] J. Linder, Y. Tanaka, T. Yokoyama, A. Sudbo, and N. Nagaosa, Phys. Rev. Lett. **104**, 067001 (2010); S. Takei, B. M. Fregoso, V. Galitski, S. Das Sarma, arXiv:1206.3226 (2012).