Field dependence of magnon decay in yttrium iron garnet thin films

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Field-dependence of magnon decay in YIG thin films

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We discuss threshold field-dependence of the decay rate of the uniform magnon mode in yttrium-iron garnet (YIG) thin films. We demonstrate that decays must cease to exist in YIG films of thickness less than $1\,\mu$m, the lengthscale defined by the exchange length. We show that due to symmetry of the three-magnon coupling decay rate is linear in $\Delta H=(H_c-H)$ in the vicinity of the threshold field H_c instead of the step-like $\Gamma_{\mathbf{k}=0}\propto\Theta\left(\Delta H\right)$ expected from the two-dimensional character of magnon excitations in such films. For thicker films, decay rate should exhibit multiple steps due to thresholds for decays into a sequence of the 2D magnon bands. For yet thicker films, such thresholds merge and crossover to the 3D single-mode behavior: $\Gamma_{\mathbf{k}=0}\propto|\Delta H|^{3/2}$.

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Introduction.—Extensive experimental and theoretical research in a ferromagnetic insulator, yttrium-iron garnet [$Y_3Fe_2(FeO_4)_3$ or YIG] that has started more than half a century ago has benefited from this material's exceptional purity, high Curie temperature, and relative simplicity of the low-energy magnon spectrum.[1] Recent discovery of the Bose-Einstein condensation of the highly occupied magnon states created by microwave pumping in YIG thin films has attracted substantial interest.[2] Recently, threshold effects due to the so-called three-magnon splitting have been reported in a quasi-2D thin-films of YIG under microwave pumping and as a function of external field.[3] In this, as well as in the other recent works,[4] control over the spin current enhancement in layered metal-ferromagnet structures by the three-magnon processes is sought. This calls for a deeper theoretical insight in the decay dynamics of such structures. Fundamentally, given its outstanding properties,[1] YIG may offer a fertile playground in the studies of threshold phenomena as the decay conditions in it can be varied continuously by both film thickness and external magnetic field.

Decays.—In this work, we discuss decay rate of the uniform mode ($\mathbf{k}=0$ magnon) in an insulating ferromagnetic thin film as a function of external magnetic field and film thickness. In particular, using magnon dispersion in the lowest-mode approximation, we outline the ranges of the field and film thickness that favor decays in YIG. Specifically, we show that kinematic conditions for three-magnon decays cannot be met for the YIG films of thickness $d_{\rm min}\approx 1\,\mu$m or less and for the fields $H_c^{\rm max}\approx 600$ Oe or higher. The upper limit on the external field is defined solely by the magnetization of a ferromagnet, in agreement with Ref. 3. Less obvious is the existence of the limit on the film thickness, which is fixed by another fundamental characteristics of a ferromagnet: its exchange length. The physical reason for that limit is the decreasing role of the long-range dipolar interactions with the decrease of the film thickness. The presence of such a limit must be important for the control of relaxation and transfer of the spin current in layered structures, which rely on the three-magnon processes in YIG.[4]

We find that the threshold field-dependence of the decay rate for the $\mathbf{k}=0$ magnon near the threshold field H_c is $\Gamma_{\mathbf{k}=0}\propto|\Delta H|\cdot\Theta(\Delta H)$, where $\Delta H=(H_c-H)$, because the three-magnon interaction vanishes along the direction of the film magnetization, which is also precisely the $\mathbf{k}$-direction where magnon band minima are located. This leads to a reduction of the phase space for decays and results in a vanishing decay rate near the threshold, contrary the naïve expectation of the step-like increase of $\Gamma_{\mathbf{k}=0}\propto\Theta(\Delta H)$, when the symmetry of the three-magnon interaction is ignored.[5] We have supported our consideration by explicit calculations of both $T=0$ and $T=300$K relaxation rate dependencies on the field for several representative YIG film thicknesses.

The finite-size quantization in the film thickness direction leads to the formation of multiple magnon bands.[6] We argue that the field-dependence for the decay rate should exhibit multiple steps linear in $|H_{c_i}-H|$, corresponding to the H_{c_i} thresholds for decays into a sequence of magnon bands. For thick films, these multiple thresholds should merge in a continuum and decay rate will crossover to a 3D single-band behavior: $\Gamma_{\mathbf{k}=0}\propto|\Delta H|^{3/2}$. These results should be helpful in finding an optimal set of parameters for spin-current enhancement.[3]

Dispersion, density of states.—Despite its fairly complicated crystal structure, at low energies YIG can be described with great success as an effective large-spin Heisenberg ferromagnet on a cubic lattice with nearest-neighbor exchange, long-range dipolar interactions, and negligible spin anisotropy.[1,7] Thus, at long-wavelength, magnon energy in YIG is determined by a competition between three couplings: exchange, dipolar, and Zeeman. For a ferromagnetic crystal of the film geometry and external field directed in-plane where it co-aligns with the magnetization direction, as is done most commonly in experiments, the lowest-mode approximation for the magnon energy yields:[5,6,8,9]

$$E_{\mathbf{k}}=\sqrt{\Big(h+\rho\mathbf{k}^2+\widetilde{\Delta}_{\mathbf{k}}\sin^2\theta_{\mathbf{k}}\Big)\Big(h+\rho\mathbf{k}^2+\Delta_{\mathbf{k}}\Big)},\quad(1)$$

where $\Delta_{\mathbf{k}}=f_{\mathbf{k}}\Delta$, $\widetilde{\Delta}_{\mathbf{k}}=(1-f_{\mathbf{k}})\Delta$ and $h=\mu H$, $\rho=JSa^2$, and $\Delta=4\pi\mu M$ are the energy scale parametriza-

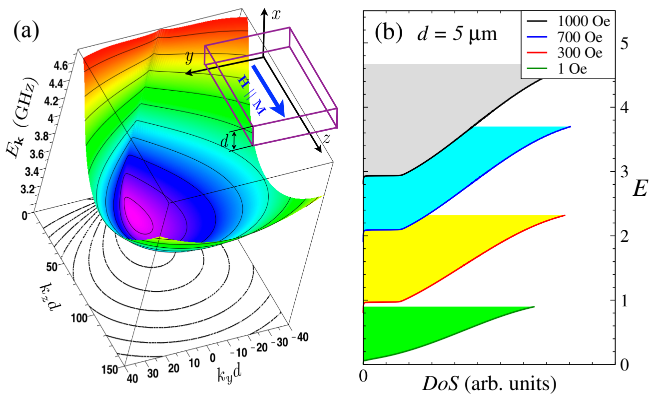

FIG. 1. (Color online) (a) 2D plot of magnon dispersion in YIG in the lowest-mode approximation, Eq. (1), for $H = 1000$ Oe, $d = 5$ μm, and in $k_z > 0$ sector. Inset: axes conventions relative to the film and the field/magnetization direction. (b) 2D magnon density of states (DoS) for several representative field values, E is in GHz.

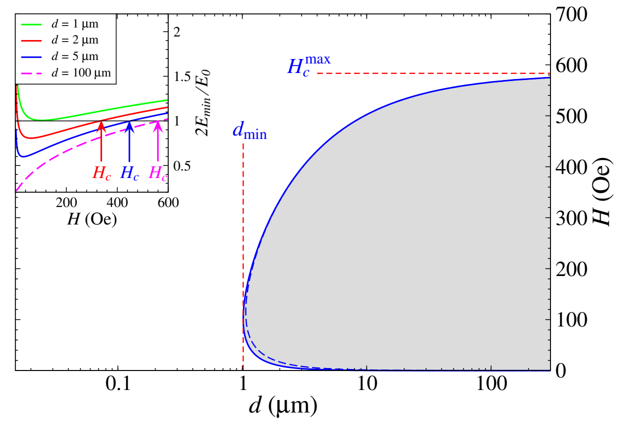

FIG. 2. (Color online) $d-H$ decay diagram for YIG, shaded area is where decays are allowed. Solid (dashed) boundary is the numerical (analytical) solution for the threshold boundary, see text. $H_{\max}$ and $d_{\min}$ are shown. Inset: $2E_{\min}/E_0$ vs H for several d's, upper threshold fields are indicated.

tions of the external field, exchange, and dipolar interactions, respectively. The form-factor $f_{\mathbf{k}} = (1-e^{-|\mathbf{k}|d})/|\mathbf{k}|d$ is from the dipolar sums in the direction of the film thickness $d \gg a$, and $\theta_{\mathbf{k}}$ is the angle between ferromagnet's magnetization (directed in-plane, z-axis) and magnon's in-plane 2D wavevector $\mathbf{k}$, see Fig. 1(a). We use $\mu = g\mu_B$ where $g = 2$ is an effective g-factor and μ_B is Bohr magneton. In this work we adhere to the notations and units of Ref. 6, which has provided a detailed microscopic spin-wave theory of YIG in $1/S$ approximation, and use experimental parameters for YIG, magnetization $4\pi M = 1750$ G, exchange stiffness $\rho/\mu = 5.17 \cdot 10^{-13}$ Oe m^2, and lattice constant $a = 12.376$ Å. In Fig. 1(a), magnon dispersion from Eq. (1) for a representative field $H = 1000$ Oe and film thickness $d = 5$ μm is shown.

It should be noted that the dipolar interactions are responsible for the non-trivial structure of magnon band with the minima at finite wave-vectors, which allow the decay conditions to occur. Although approximate, Eq. (1) provides a close quantitative description of the lowest 2D magnon energy band,[6] quantized due to finite film thickness d in the x-direction (Fig. 1(a), inset). At small $\mathbf{k}$, dipolar interactions dominate over the exchange and result in a steep decrease from the energy of the uniform mode, $E_0 = \sqrt{h(h+\Delta)}$, for $\mathbf{k}$'s along the magnetization direction. At larger $\mathbf{k}$, exchange energy dominates, giving $E_{\mathbf{k}} \approx h + \rho\mathbf{k}^2$, while at intermediate $\mathbf{k}$ competition between exchange and dipolar terms results in peculiar-shaped minima at $\pm\mathbf{k}_m = (0, \pm k_m)$, see Fig. 1(a).

The 2D density of magnon states from Eq. (1) is shown in Fig. 1(b) for several representative field values. Predictably, 2D DoS exhibits a step-like increase at the band minimum, which is followed by an unusual, almost linear increase, reminiscent of the similar behavior for the relativistic dispersion. The latter is due to a nonparabolicity of the long-wavelength magnon dispersion, see Fig. 1(a).

"Decay diagram".—For the decays to take place the kinematic conditions must be fulfilled. For the two-magnon decay (three-magnon splitting) of the uniform mode, the condition to be satisfied is simply $E_0 = 2E_{\mathbf{q}}$. With the microscopic parameters, such as exchange stiffness and magnetization, fixed, some other parameters can be varied to allow or to forbid decays altogether. As is clear from Eq. (1), external field increases the energies of both the uniform mode and the minimum, making decays kinematically impossible at some higher field value.[3] Another parameter is the film thickness d, which enters Eq. (1) through the formfactor $f_{\mathbf{k}}$. While the manner in which d influences decays is not a priori clear, both trends, vs field and vs thickness, can be examined numerically. Such an examination is exemplified in Fig. 2, which shows $2E_{\min}/E_0$ vs field for several film thicknesses. Clearly, when the plotted quantity is < 1, decays are allowed, and the crossing of 1 corresponds to a threshold field for decays.

Our Fig. 2 gives the complete d–H "decay diagram" for YIG with the shaded area showing the parameter space where decays are allowed. Two key results are clear in Fig. 2: (i) the upper threshold field does not exceed some $H_c^{\max}$ even for large values of d, (ii) there exists a lower limit on the film thickness $d_{\min}$, below which the decays of the uniform mode may not occur at all. Solid boundary is the numerical solution of Eq. (1) for the energy minimum and the decay conditions. Dashed line is an approximate analytical solution, which turns out to be very precise. The latter also gives us a deeper insight into the nature of $H_c^{\max}$ and $d_{\min}$ discussed next.

Large-$|\mathbf{k}_m|d$ approximation—At large enough d the wavevector of the magnon energy minimum satisfies $|\mathbf{k}_m|d \gg 1$. Then the formfactor $f_{\mathbf{k}_m} \approx 1/|\mathbf{k}_m|d$ is small, reflecting the reduced role of dipolar interactions in the energy of the $\mathbf{k}_m$-magnon. One can then show[10] that both the exchange and the dipolar energies for the $\mathbf{k}_m$-

magnon scale as $\propto d^{-2/3}$ and thus are $\ll h$ for any reasonable field. This implies that the energy of the magnon band minimum is $E_{\min} \approx h$, expected for the uniform mode in the absence of dipolar interactions. Then, the decay threshold equation, $2E_{\min} = E_0$, trivially gives $h_c = \Delta/3$, relating saturated value of the decay threshold field to the material's magnetization: $H_c^{\max} = 4\pi M/3$ (= 583 Oe for YIG in Fig. 2(b)), see also Ref. 3.

The physical question is: what parameter of the ferromagnet defines $d_{\min}$? One can extend the large-$|\mathbf{k}_m|d$ approach and find[10] that the energies of the dipolar and exchange interactions must be related by $|\mathbf{k}_m|d = \left(\Delta d^2/4\rho\right)^{1/3}$. With this, the decay threshold condition $2E_{\min} = E_0$ leads to an algebraic equation in H_c vs d, which can be resolved in a compact form.[10] It is plotted as a dashed line in Fig. 2(b), which coincides almost exactly with the decay boundary obtained from Eq. (1) numerically. From the same equation we find the minimal thickness to be $d_{\min} \approx C\sqrt{\rho/\Delta}$, explicitly related to the exchange length of the ferromagnet, $\ell_{ex} = \sqrt{\rho/\Delta}$, albeit with a large numerical coefficient $C \approx 62.04$.[10] Using parameters for YIG, the exchange length is $\ell_{ex} \approx 13.9a$ and the minimum thickness $d_{\min} = 1.067$ μm, remarkably close to the numerical result $d_{\min} = 1.017$ μm.

The physical reason for the very existence of $d_{\min}$ is the decreasing role of long-range dipolar interactions in magnon's energy with the decrease of film thickness, as the relation of $d_{\min}$ to exchange length implies. The fact that the decay boundary in d exceeds exchange length by a large numerical factor is due to a rather stringent requirements of the decay conditions. We emphasize that the provided $d-H$ diagram should apply equally to the other thin film ferromagnets.

Decay rate.—Transitions that involve changing the number of magnons, such as decays, recombination or coalescence, originate from the dipolar interactions that couple longitudinal and transverse spin components and therefore do not conserve magnetic[1,7] as well as mechanical angular momentum.[3] Microscopically, dipolar interactions result in anharmonic couplings of magnons, which, for the decay processes of the $\mathbf{k} = 0$ uniform mode into two magnons at $\mathbf{q}$ and $-\mathbf{q}$, can be written as

$$\mathcal{H}^{(3)} = \frac{1}{2}\sum_{\mathbf{q}} V^{(3)}_{0;\mathbf{q},-\mathbf{q}}\left(a^\dagger_{\mathbf{q}}a^\dagger_{-\mathbf{q}}a_0 + \text{H.c.}\right). \qquad (2)$$

The three-magnon coupling in (2) has an angular dependence: $V^{(3)}_{0;\mathbf{q},-\mathbf{q}} = V_0 \sin 2\theta_{\mathbf{q}}$, with $V_0 = \Delta/\sqrt{2S}$.[1,11,12] This angular dependence is essential as it reflects the symmetry of dipolar coupling of the transverse, S^x (S^y), and longitudinal, S^z, spin components: $V^{xz} \propto xz/r^3$ ($V^{yz} \propto yz/r^3$). In particular, it is natural for this coupling to vanish for the spin-wave propagating with the momentum $\mathbf{q}$ along the direction of magnetization $\mathbf{M}$ (z-axis, Fig. 1(a)).[11] We would like to point out that it is also precisely the direction along which the minima of the magnon band are located.[13] Therefore, the amplitude of the decay of $\mathbf{k} = 0$ magnon into two magnons at

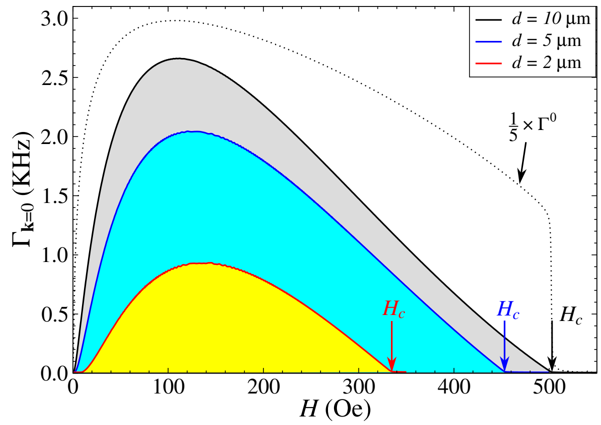

FIG. 3. (Color online) The $T = 0$ decay rate $\Gamma_{\mathbf{k}=0}$ vs H for d = 2, 5, and 10 μm. Dotted line is $\frac{1}{5}\Gamma_{\mathbf{k}=0}$ for d = 10 μm with the angular-dependence of the three-magnon coupling omitted, $V^{(3)}_{0;\mathbf{q},-\mathbf{q}} \Rightarrow V_0$.

the band minima $\mathbf{q}_m$ and $-\mathbf{q}_m$ is zero. This will lead to a rather spectacular violation of the naïve expectation: while kinematic conditions for the decay of $\mathbf{k} = 0$ magnon into $\pm\mathbf{q}_m$ are just met at H_c, the corresponding decay amplitude is vanishing. Thus, the decay rate must increase gradually from the threshold, not in a jump-like fashion as in the DoS, Fig. 1(b).

At $T = 0$ only spontaneous magnon decays are allowed.[14] The three-magnon recombination processes have to obey the same kinematic constraints as the decay, having therefore the same threshold conditions. The three-magnon coalescence processes involving $\mathbf{k} = 0$ mode correspond to the "vertical" transitions, which are forbidden either kinematically as in the single mode case or by the quantum number of the inerband transition in the multi-band situation. The four-magnon scattering amplitude from the exchange interaction vanishes identically for the uniform mode, while the remaining four-magnon interactions from the dipole-dipole interaction together with impurity scattering should be providing a background with weak H and T dependence, distinct from the threshold behavior discussed here.

With this in mind, using kinetic approach,[1,7] which takes into account the balance between decay and recombination processes in the relaxation time approximation, the decay rate[15] of the uniform mode is

$$\Gamma_{\mathbf{k}=0} = \pi\sum_{\mathbf{q}}\left|V^{(3)}_{0;\mathbf{q},-\mathbf{q}}\right|^2\left[2n_{\mathbf{q}} + 1\right]\delta\left(E_0 - 2E_{\mathbf{q}}\right) \qquad (3)$$

where $n_{\mathbf{q}} = [e^{\hbar E_{\mathbf{q}}/k_b T} - 1]^{-1}$ is the Bose occupation factor. It is clear from Eq. (3) that while the magnitude of the decay rate at finite T can be substantially modified from the $T = 0$ result by the Bose-occupation factors, the qualitative threshold behavior must remain the same.

One can investigate the threshold behavior of Eq. (3) analytically and obtain $\Gamma_{\mathbf{k}=0} \propto |E_0 - 2E_{\min}|$ for $H \to H_c$.[10] Given the proportionality between (E_0–$2E_{\min}$) and

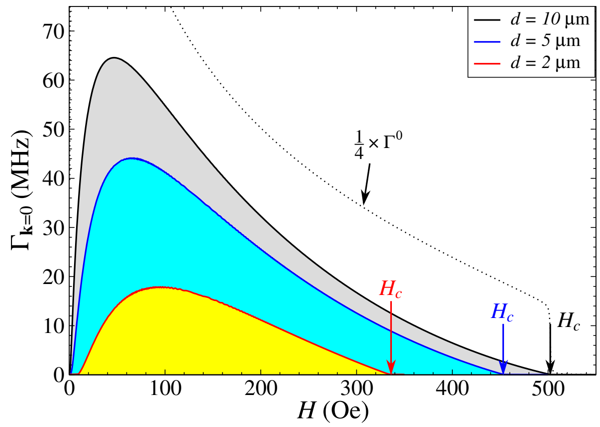

FIG. 4. (Color online) same as in Fig. 3, $T = 300$K.

$\Delta H = (H_c - H)$ demonstrated in Fig. 2, this yields *linear* dependence of the decay rate on the field relative to the threshold: $\Gamma_{\mathbf{k}=0} \propto |\Delta H| \cdot \Theta(\Delta H)$, see Figs. 3 and 4. Once again, this is the consequence of an effective suppression of the phase space for decays due to the discussed angular dependence of the three-magnon coupling.

In Figs. 3 and 4 we show $\Gamma_{\mathbf{k}=0}$ for three different film thicknesses and for $T = 0$ and $T = 300$K, respectively. While the overall scale in the two Figures is in a completely different frequency range,[10] the shapes of Γ vs H are qualitatively very similar, especially concerning their threshold behavior vs field, in agreement with the above analysis. The relative difference between the curves for different d's reflects the smaller phase space for decays in thinner films. In the same Figs. 3 and 4 we demonstrate a dramatic contrast with the results of Eq. (3) if the angular dependence of the three-magnon coupling in Eq. (2) is neglected, $V^{(3)}_{0;\mathbf{q},-\mathbf{q}} \Rightarrow V_0$. The latter results exhibit jumps at H_c's and a linear increase after that, similarly to the 2D magnon DoS in Fig. 1(b). One should also note that the overall decay rate is also markedly overestimated if the angular dependence of the three-magnon interaction is ignored.

In thicker films, the decay rate will be further modified by the multiple magnon bands that occur due to finite-size quantization in the film thickness direction.[6] It will exhibit a sequence of steps linear in $|H_{c_i} - H|$, where H_{c_i} is the threshold field for decay into an ith band, increasing the decay rate every time the corresponding kinematic conditions are met. Strictly speaking, the angle $\theta_{\mathbf{q}}$ in $V^{(3)}_{0;\mathbf{q},-\mathbf{q}}$ is between the 3D $\mathbf{q}$-vector and magnetization vector $\mathbf{M}$. In the quasi-2D geometry with the levels quantized in the x-directions, q_x has discrete values and the minimal value of $\theta^{\min}_{\mathbf{q}^i_{\mathrm{m}}} \approx q^{\min}_{x,i}/|\mathbf{q}|$ is zero only for the lowest magnon band. Thus, the thresholds in H_{c_i}'s will have some small step-like behavior. However, this effect must be negligibly smaller compared to the steps in Figs. 3 and 4 when the angular dependence in $V^{(3)}$ is ignored altogether. Extending our analysis to the limit of thicker films where multiple bands merge into a single 3D band, using Eq. (3) we obtain[10] that the decay rate near the threshold field will crossover to $\Gamma_{\mathbf{k}=0} \propto |\Delta H|^{3/2}$.

Conclusions.— In this work, we have discussed the field-dependence of the decay rate of the uniform magnon mode in YIG thin films and the effects of film thickness in it. As a result of our analysis, we have established that the two key characteristics of a ferromagnet, its magnetization and exchange length, define the extent of the $d-H$ parameter range that favors decays in thin film geometry. Our calculations of the decay rate should provide an important guidance for the experimentalists in designing the optimal conditions for the control of spin current and its relaxation in thin films. A particularly intriguing suggestion to pursue is to study the properties of a thin film with varying thickness, which may permit decays and, as a consequence, the spin current enhancement in one part of the film and forbid it in the other.

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[15] Our $\Gamma_{\mathbf{k}=0}$ corresponds to half-width at half-maximum, *i.e.*, $\Gamma_{\mathbf{k}=0} = \frac{1}{2}\tau^{-1}_{\mathbf{k}=0}$.