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Comment on: "Vortex-assisted photon count and their magnetic field dependence in single-photon superconducting detectors"

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We discuss the importance of the vortex core energy and the realistic boundary conditions to the Fokker-Plank equation for the calculation of thermally-activated hopping of vortices across narrow superconducting films. Disregarding these issues in the paper by L. N. Bulaevskii, M. J. Graf and V. G. Kogan, Phys. Rev. B **85**, 014505 (2012), in which an uncertain London vortex core cutoff was used, can produced large numerical errors and a significant discrepancy between the results of Refs.[1,2] and[3]. These issues can be essential for the interpretation of experimental data on thin film photon detectors and other superconducting nanostructures.

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Recently L. N. Bulaevskii, M. J. Graf and V. G. Kogan published two papers on the theoretical description of experiments on NbN thin film-based photon detectors[1,2]. The central part of both works constituted the calculation of the voltage produced by thermally-activated hopping of vortices across a thin film strip. The authors of Ref.[1,2] used the approach developed in Ref. [3], which, in turn, was based on the earlier work by Ambegaokar et al.[4] describing the dynamics of a vortex in a film in terms of the Langevin equation $\eta\dot{x}+U'(x)=\zeta(t)$. Here x is the position of the vortex across the strip $(0<x<w)$, $\zeta(t)$ is the thermal noise source, and η is the viscous drag coefficient. The energy $U(x)$ of a vortex in a film of the width $w<\Lambda=\lambda^2/d$ and the thickness d in the presence of the dc current I and the magnetic field H perpendicular to the film is given by[3]

$$U(x)=\epsilon\ln\left[\frac{w}{\pi\xi_1}\sin\frac{\pi x}{w}\right]+\frac{\phi_0 Ix}{cw}+\frac{\phi_0 H}{8\pi\Lambda}x(w-x)\,, \quad (1)$$

where $\epsilon=\phi_0^2/16\pi^2\Lambda$, $\xi_1=0.34\xi$, ξ is the coherence length, λ is the London penetration depth, ϕ_0 is the flux quantum, and c is the speed of light. The voltage $V(T,H,I)$ caused by uncorrelated jumps of vortices is calculated using the Fokker-Planck equation for the probability density $f(x,t)$[4,5]:

$$\dot{f}=\partial_x[U'(x)f+Tf']. \quad (2)$$

The length $\xi_1=0.34\xi$ in Eq. (1) of Ref.[3] was calculated using a self-consistent solution of the Ginzburg-Landau (GL) equations which take into account the energy of the vortex core[6,7]. By contrast, the authors of Ref.[1,2] took $\xi_1=\xi/2$ by imposing an arbitrary London core cutoff and disregarded the vortex core energy. This model assumption of Ref.[1,2] can manifest itself in huge uncertainties in the thermally-activated vortex hopping rate, as will be shown below.

The results of Refs.[1,2] essentially reproduce those of Ref.[3], however the vortex hopping rate R_v derived in Ref.[3] differs by the factor $F=2^\nu(\nu-1)w/\xi$ from that of Ref.[1,2]. The authors of Refs.[1,2] asserted that, for $\nu=\epsilon/T=110$ used in Ref.[1] to fit the experimental data, R_v of Ref.[3] was overestimated by the factor $F=2^\nu(\nu-1)w/\xi\simeq 3.5\times 10^{36}$ because:

1. The factor 2^ν in F comes from the factor 2 under the logarithm in Eq. (1) which, according to Ref.[1,2], was missed in Ref.[3].

2. The factor $\nu-1$ in F results from the use of the periodic boundary conditions for Eq. (2) in Ref.[3] as opposed to a 'realistic' boundary condition of Ref.[1].

3. The factor w/ξ in F results from the vortex interaction which, according to Ref.[1,2], leads to the statistical weight of a vortex $P\sim L/w$, as opposed to $P\sim L/\xi$ used in[3], where L is the length of the strip.

In this Comment we show that these statements are incorrect because they result from model artifacts of Refs.[1,2]. Below we address the issues taken into account in Ref.[3] but neglected in Refs.[1,2] and discuss their importance for a more consistent theory of thermally-activated hoping of vortices in thin films and the interpretation of experimental data.

A. 1. Core contribution

The authors of Ref.[1] apparently overlooked that $\xi_1=0.34\xi$ in Eq. (1) of Ref.[3] absorbs both the factor 2 under the logarithm[1,2,7,8] and the *vortex core energy* disregarded in Ref.[1,2]. Here Eq. (1) results from $U(x)=\epsilon[\ln[(2w/\pi\xi)\sin(\pi x/w)]+\beta]$ obtained in Ref. [7], where $\beta=0.38$ accounts for the vortex core energy, so that $\xi_1=0.34\xi=e^{-\beta}\xi/2$ in Eq. (1), unlike $\xi_1=\xi/2$ used in[1,2]. We emphasize that ξ in Eq. (1) used in Ref.[3] is a true coherence length but not an uncertain London cutoff factor $\sim\xi$ like in Refs.[1,2]. The value $\beta=0.38$ was extracted by comparing the lower critical field $H_{c1}=(\phi_0/4\pi\lambda^2)[\ln(\lambda/\xi)+0.497]$ calculated from the GL theory[6] with the London result $H_{c1}=(\phi_0/4\pi\lambda^2)[K_0(\xi/\lambda)+\beta]$, where β is the core contribution. Here the core energy results from the spatial variation of the order parameter not accounted for by the London cutoff, and $K_0(\xi/\lambda)\approx\ln(2\lambda/\xi)-0.577$ for $\lambda\gg\xi$. Matching these formulas for H_{c1} yields $\beta = 0.497 + 0.577 - \ln 2 = 0.38$. In the limit of $\kappa = \lambda/\xi \gg 1$, the core line energy is independent of the sample geometry so $\beta \approx 0.38$ is the same both in bulk samples and films with $w \gg \xi$, except for vortices spaced by $x \sim \xi$ from the film edge.

The incorporation of the vortex core energy in $U(x)$ of Ref.[3] eliminates the uncertainty of the London core cutoff and assures that the activation barrier U_m coincides with the exact numerical GL result for the vortex self-energy. Indeed, Eq. (1) with $\beta = 0.38$ used in Ref.[3] was fully confirmed by recent numerical simulations of the GL equations describing vortices in thin film strips for $J < 0.6J_d$ where J_d is the GL depairing current density. These calculations gave $\beta = 0.37, 0.38, 0.38$, and 0.38 for strips of widths $w/\xi = 7, 10, 15$, and 30, respectively. By contrast, $U(x)$ and particularly the vortex hopping rate of Ref.[1] are very sensitive to the arbitrary core cutoff which was *a-priori* chosen at $r = \xi$.

Taking β into account significantly decreases $R_v(\beta) \simeq \tilde{R}_v e^{-\beta\epsilon/T}$ as compared to $\tilde{R}_v$ calculated without the core contribution, while the variation of the core energy at the film edge affects the pre-exponential factor in R_v, as will be discussed below. The core contribution $\beta = 0.38$ is no less important than the London numerical correction $\ln(2/\pi) = -0.45$ in $U(x)$, so taking β into account is essential when comparing the model of Ref.[1,2] with experiment. Indeed, neglecting the vortex core energy in Ref.[1] overestimates R_v by $\sim \exp(\beta\epsilon/T) \sim 10^{18}$ for $\nu = \epsilon/T = 110$. In turn, changing the core cutoff from ξ to $\sqrt{2}\xi$ (which would be more consistent with the GL theory) in the model of Ref.[1] increases R_v by $2^{\nu/2} \simeq 3.6 \times 10^{16}$. This shows that the London model of Refs.[1,2] is hardly adequate for the calculation of the vortex activation energy and the more consistent GL theory should be used to calculate the parameters of $U(x)$ in Eq. (1)[3]. The importance of the vortex core contribution in the vortex-related dynamic phenomena has been extensively discussed in the literature (see, for example, the recent work[10] on the effect of the vortex core energy on the Berezinskii-Kosterlitz-Thouless transition).

B. 2. Boundary condition

Here we show that the extra factor $\sim \nu^{-1}$ in R_v of Ref.[1] does not come from the different boundary conditions used in Refs.[1] and[3], but rather from artifacts of the model of Ref.[1]. The vortex crossing rate was obtained in Ref.[1] from the standard formula for a particle hopping between two potential wells[5]:

$$R_v^{-1} = D \int_0^{x_1} e^{-U(x)/T} dx \int_{x_0}^{w} e^{U(x)/T} dx, \qquad (3)$$

where $D = T/\eta$, $x_0 \sim \xi$, and x_1 is a length smaller than x_m at which $U(x)$ is maximum.

The authors of Ref.[1] assumed a model form of $U_{BGK}(x)$ in Eq. (3): $U_{BGK} = U(x)$ where $U(x)$ is given by Eq. (1) with $\xi_1 = \xi/2$ for $x > x_0 \sim \xi$, $U_{BGK}(x) = 0$ for $0 < x < x_0$, and $U_{BGK}(x) = \infty$ at $x = 0$. The infinite repulsive barrier at the film edge was introduced artificially to trap vortices in the film by imposing the boundary condition of zero probability current $S = \dot{x}f$ at $x = 0$ for Eq. (2). Vortex hopping in this model occurs as a 'pre vortex'[1] is somehow placed in the film past this barrier, but it is unclear how this model can describe penetration of vortices in the film.

The postulated form of $U_{BGK}(x) = 0$ at $0 < x \lesssim \xi$ significantly overestimates the first integral $Z = \int_0^{x_1} \exp[-U(x)/T]dx$ in Eq. (3). To see how it happens, it is instructive to juxtapose $U_{BGK}(x)$ with $U(x)$ obtained by numerical simulations of vortices using the GL equations, which take into account the vortex core energy and its change near the edge. These calculations have shown that the energy of a vortex, $U(x) = (b + ax/\xi)\epsilon$, increases linearly with the distance x of the core phase singularity from the film edge up to $x \sim \xi$[11,12]. This gives rise to a constant force $a\epsilon/\xi$ caused by a "string" of the suppressed order parameter between the core and the surface, where $a \sim 0.1 - 0.3$ and the constant $b \sim 0.05 - 0.1$ accounts for the fact that $U(x) > 0$ even at $x \to 0$ due to local superconductivity suppression around a vortex core as it emerges from the film edge[11]. These features are essential for the evaluation of R_v if $U(x) > T$ at $x < \xi$.

Substituting $U(x) = (b + ax/\xi)\epsilon$ in $Z = \int_0^{x_1} \exp[-U(x)/T]dx$ yields $Z = \xi e^{-b\nu}/a\nu$ for $e^{-a\nu} \ll 1$. As follows from Eqs. (1) and (3), the factor $e^{-b\nu}$ can be combined with $e^{\beta\nu}$ from the second integral in Eq. (3), so that the effect of the vortex core on the hopping rate $R_v \simeq \tilde{R}_v \exp[(b - \beta)\nu]$ is determined by the difference of core energies in the bulk and at the film edge. Here both a and b appear to be dependent of current[11], indicating that the London notion of the rigid vortex core becomes hardly adequate at $x \sim \xi$.

The calculation of $Z \sim \int_{\xi_1}^{\infty} (\xi_1/x)^\nu dx \simeq \xi_1/\nu$ in Ref.[3] is qualitatively consistent with the GL result. Here the cutoff $\sim \xi_1$ where the London theory becomes invalid was used, and the upper limit can be extended to infinity if $e^{-a\nu} \ll 1$ and $I \ll I_d$, where $I_d = c\phi_0/16\pi^2\Lambda\xi$ is of the order of the depairing current. By contrast, the potential, $U_{BGK}(x) = 0$ at $0 < x < x_0$, yields $Z = x_0 \sim \xi^1$, which overestimates Z by the factor $\sim \nu \gg 1$ as compared to both the GL results and Ref. [3]. Treating a vortex like a particle in the London model combined with the Fokker-Plank equation does bring uncertain factors ~ 1 in Z coming from the edge effects discussed above. Yet the simplified model of Ref.[1] does not capture the qualitative behavior of $Z \sim \xi/\nu$, which follows from the more consistent GL theory and the approach of Ref.[3] (also adopted in Ref.[2]). A more realistic model of the vortex core penetration would require solving the time-dependent GL equations[13].

The above consideration shows that the claim of Ref.[1] that the extra factor $\sim \nu$ in R_v^{-1} comes from the 'realistic' boundary conditions as opposed to the periodic $U(x)$ of Ref.[3] is misleading. In fact, the solution of Eq. (2)

adopted in Ref.[3] is only defined inside the film $0 < x < w$ and does not require any unphysical barriers at the film edges. Here the use of periodic $U(x)$ in Eq. (2) is a standard method of satisfying the boundary conditions of a fixed probability flux S of vortices entering and exiting the film, which is equivalent to the method of images for solving the Laplace or diffusion equations. For example, Eq. (1) can be obtained by either finding a proper analytical function or summing up potentials of an infinite chain of vortex-antivortex images outside the film. Moreover, if only the forward jumps of vortices are taken into account in the limit of $e^{-\nu} \ll 1$ considered in Ref.[1], Eq. (3) reduces to Eq. (7) of Ref.[3]. This is not surprising because the exponentially small probability current S is mostly determined here by narrow vicinities of neighboring minimum and maximum of $U(x)$, so the boundary condition of fixed S[3] appears to be very close to the boundary condition $S = 0$ of Ref.[1].

C. 3. Correlation effects

Finally we comment on the statement of Ref.[2] that the statistical weight of a single vortex penetrating through the film edge should be $P \sim L/w$, instead of $P \sim L/\xi$ used in[3]. It is noteworthy that the models of Refs.[1–3] only hold in the limit of exponentially low density of vortices, thus P should coincide with its value in the thermodynamic limit. The assumption of $P \sim L/w$ is therefore inconsistent with the thermodynamics of vortices in thin films[4,14] used to obtain $P \sim L/\xi$ in Ref.[3]. Here $P \sim L/\xi$ is the 1D analog of the statistical weight $P = C(L/\xi)^2$ of a single vortex in the film of area L^2 where $C \sim 1$ depends on the distribution of the order parameter in the vortex core[14].

The assumption $P \sim L/w^2$ was based on the fact that two vortices in the middle of the strip strongly repel each other if they are separated by distances smaller than the interaction radius w/π. However, uncorrelated hopping of vortices described by Eq. (2) implies that they enter the film at random times and are separated by distances larger than w at any given moment. Taking vortex correlations into account requires solving coupled equations for the higher order correlation functions which cannot be described by Eq. (2). Basically, the authors of Ref.[2] selected rare events when two vortices enter the film nearly simultaneously and ascribed their statistical weight $P \sim L/w$ to all vortex jumps. However, repulsion of vortices suppresses their simultaneous entering the film, forcing them to go one by one so that a vortex can enter at any of L/ξ edge sites after the preceding vortex in the area $\sim w$ has already crossed the film. Such uncorrelated jumps[3] have a much higher probability than the correlated jumps assumed in Refs.[1,2]. In addition, the interaction radius of vortices at the film edge ($x \sim \xi$) is much smaller than w because currents of two vortices spaced by the distance s along the edge are nearly extinguished by their antivortex images, resulting in the dipole interaction $U(\xi, s) \sim \epsilon(\xi/s)^2$ which does not extend well beyond $s > \xi$.

In conclusion, we show the importance of the vortex core energy, the realistic behavior of $U(x)$ at the film edge, and the physical boundary conditions to Eq. (2) for the calculation of thermally-activated hopping of vortices across narrow films. Disregarding these issues in Refs.[1,2] has produced large numerical errors and a significant discrepancy between the results of Refs.[1,2] and[3]. This can also be essential for the interpretation of experimental data on thin film photon detectors.

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