Comment on “Ginzburg-Landau theory of two-band superconductors: Absence of type-1.5 superconductivity”

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Comment on "Ginzburg-Landau theory of two-band superconductors: Absence of type-1.5 superconductivity" by V. G. Kogan and J. Schmalian

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The recent paper by V. G. Kogan and J. Schmalian Phys. Rev. B **83**, 054515 (2011) argues that the widely used two-component Ginzburg-Landau (GL) models are not correct, and further concludes that in the regime which is described by a GL theory there could be no disparity in the coherence lengths of two superconducting components. This would in particular imply that (in contrast to $U(1)\times U(1)$ superconductors), there could be no "type-1.5" superconducting regime in $U(1)$ multiband systems for any finite interband coupling strength. We point out that these claims are incorrect and based on an erroneous scheme of reduction of a two-component GL theory.

I. INTRODUCTION

The recent works Refs. 1 and 2 claim that the two-component Ginzburg-Landau (TCGL) theories can not be used to address any properties of two-component superconductors which involve disparity of density variations, in particular to describe type-1.5 superconducting state where $\xi_1 < \sqrt{2}\lambda < \xi_2$[3–18]. Here we point out several crucial errors in the analysis[1,2] thus demonstrating that the following main points in Ref.1 are incorrect,

- attempts to employ the GL functionals, on the one hand, and to assume different length scales, on the other, cannot be justified.
- the idea of 1.5-type superconductivity is not warranted by the GL theory
- $\Delta_1(\mathbf{r},T)/\Delta_2(\mathbf{r},T) = const$, ... this ratio remains the same at any T in the GL domain

First let us note that Refs. 1 and 2 fail to distinguish between two classes of systems where the type-1.5 state was previously discussed (i) $U(1)\times U(1)$ and (ii) two-band superconductors where interband coupling explicitly breaks symmetry down to $U(1)$. Namely the Refs. 1 and 2 mix up various aspects of physics specific to $U(1)\times U(1)$[3] (such as the very definition of the coherence lengths) with the different in several respects physics of $U(1)$ systems. The definitions of coherence lengths and type-1.5 regime in systems with non-zero interband coupling were explicitly discussed in detail in Ref. 6, long before the appearance of Refs. 1 and 2. Thus claims in Refs. 1 and 2 that this coupling was neglected in works on type-1.5 superconductivity are factually incorrect. Note that $U(1)\times U(1)$ symmetry is also possible in superconductors[19,20] which represents the most straightforward example of systems which cannot be characterized by a single universal GL parameter (physical examples can be found in Refs.3 and 10). However in what follows we focus exclusively on $U(1)$ two-band superconductors.

Two-component GL (TCGL) model were derived microscopically in Refs. 21–23. However indeed the conditions under which two-component GL expansions for two-band superconductors are formally justified, were not known (to the best of our knowledge) at the time of publication of Refs.1 and 2 but were rigorously established recently[15]. Therefore the aforementioned claim that TCGL expansion is injustifiable[1,2] is incorrect. In this comment we discuss which incorrect assumptions and technical errors led the authors of Refs.1 and 2 to opposite conclusions.

II. DEFINITIONS OF THE GL REGIME IN APPLICATION TO $U(1)$ MULTIBAND SYSTEMS.

Let us start with definitions. The Refs. 1 and 2 defines "GL theory" as the free energy proportional to τ^2 and the modules of the fields varying as $\tau^{1/2}$ where $\tau = (1 - T/T_c)$. Such simplistic definition indeed can be encountered in books on superconductivity which consider simplest single-component systems. However unfortunately such a definition does not work in general. The Ginzburg-Landau theory is a more general concept of a classical field theory description of a system, which in many physical cases does not necessarily appears in the leading order τ-expansion. In particular such a definition contradicts all existing literature on multicomponent GL theories in two band superconductors, which in contrast adopts the more general definition of GL expansion of the free energy by powers of gap amplitudes and spatial gradients[21–23,25]. It should be noted that it most obviously follows from the $U(1)$ symmetry of two-band superconductors, that the leading order expansion in the parameter τ yields a *single* order parameter field characterized by a single coherence length *by construction* (see e.g. a standard textbook Ref.34). The works[21–23], as well as more recent paper[15] use more general expansion in powers of gradients and amplitudes of two gap functions, which most obviously yields a more complicated temperature dependence, and cannot be expected to be obtainable in leading order expansion in τ. The TCGL expansion for two-band superconductor is thus an example of an expansion in several small parameters (small gaps and gradients). However this fact does not make it unjustifiable as claimed[1]. Indeed recently it was justified on formal grounds for a wide range of parameters[15] (see also remark[28]).

III. COHERENCE LENGTHS IN TCGL MODEL AND REDUCTION TO THE SINGLE COMPONENT GL THEORY IN THE $T \to T_c$ LIMIT.

A. The reduction argument in Ref. 1

As we discussed above in two band superconductors the broken symmetry is only $U(1)$[29] thus, by symmetry, *in the limit* $T \to T_c$, TCGL expansion should be reduced to the conventional text-book single-component GL theory. However the reduction derivation presented in[1] is principally incorrect. The crux of the argument presented in Ref. 1 is that, the TCGL field equations [Eqs. (3,4) in Ref.1]

$$a_1\Delta_1 + b_1\Delta_1|\Delta_1|^2 - \gamma\Delta_2 - K_1\Pi^2\Delta_1 = 0 \qquad (1)$$
$$a_2\Delta_2 + b_2\Delta_2|\Delta_2|^2 - \gamma\Delta_1 - K_2\Pi^2\Delta_2 = 0$$
$$\nabla^2 A - \nabla(\nabla \cdot A)$$
$$+\frac{16\pi^2}{\phi_0}\sum_{\nu=1,2} K_\nu(\Delta_\nu^*\Pi\Delta_\nu - \Delta_\nu(\Pi\Delta_\nu)^*) = 0 \qquad (2)$$

are *generically*, i.e. irrespectively of intercomponent coupling strength γ are well-approximated near T_c by the simpler system describing condensates with equal coherence length *coupled only by a vector potential* [Eqs. (7) and (8) in Ref.1]

$$\alpha\tau\Delta_1 + \beta_1\Delta_1|\Delta_1|^2 - K\Pi^2\Delta_1 = 0 \qquad (3)$$
$$\alpha\tau\Delta_2 + \beta_2\Delta_2|\Delta_2|^2 - K\Pi^2\Delta_2 = 0$$
$$\nabla^2 A - \nabla(\nabla \cdot A)$$
$$+\frac{16\pi^2}{\phi_0}\sum_{\nu=1,2} K_\nu(\Delta_\nu^*\Pi\Delta_\nu - \Delta_\nu(\Pi\Delta_\nu)^*) = 0, \qquad (4)$$

where $\Pi = \nabla - ie\mathbf{A}$ and the new parameters α, K, β_ν are related to the coefficients in starting Eqs.(1) and GL parameter τ.

Below we present a generic argument that this result and therefore the reduction procedure are incorrect at any temperatures. First we comment that the obtained reduced system of Eqs.(3) contradicts the principles of Landau theory. That is, the initial set of equations corresponds to the system with broken $U(1)$ symmetry. Equations from the second set (3) are coupled only through $\mathbf{A}$ and thus corresponds to independently conserved condensates. Thus Eqs.(3) are the field equations corresponding to a free energy functional with spontaneously broken $U(1) \times U(1)$ symmetry. Note that the interband Josephson coupling in the initial set of equations breaks the symmetry of the system down to $U(1)$ symmetry, but no phase locking terms are present in the reduced system of equations. Therefore the reduced theory fails to account to this effect and is wrong already on symmetry grounds. Furthermore Landau theory for $U(1)$ systems dictates that there is only one diverging coherence length *in the limit* $T \to T_c$ associated with a single complex field (but not two degenerate coherence lengths associated with two fields coupled by vector potential only).

B. Coherence lengths in two-band superconductors

The work 1 (and the recent follow up cited therein 27) claim that the gap fields $\Delta_{1,2}$ have two independently diverging in the limit $T \to T_c$ coherence lengths, which become degenerate in "GL domain" at small τ, where the system is claimed to be described by two equations coupled only by vector potential.

Such incorrect conclusion regarding the evolution of the length scales in the $T \to T_c$ limit is based on misunderstanding of how coherence lengths are defined in two-band systems. The erroneous claim that two coherence lengths are attributed directly to $\Delta_{1,2}$ and that they become identical near T_c) originates in the attempt[2] to assess coherence lengths though a comparison of the gap function profiles in the 1D boundary problem. From the observation that the overall profiles become identical in the $T \to T_c$ limit the authors of Refs.1 and 2 concluded that the two gap functions are characterized by the similar coherence lengths. Such approach is technically incorrect because one cannot extract information of coherence length from a naive inspection of an overall density profiles in a nonlinear theory. Instead the correct analysis of coherence lengths in two-band superconductor requires an accurate consideration of the asymptotic solutions of linearized field equations for the gap functions[10,14]. As shown[10,14,15] in the wide range of parameters for finite interband Josephson coupling there exist two asymptotical normal modes with different coherence lengths (or inverse masses of the normal modes). The two distinct coherence lengths appear as a result of hybridization of the superconducting gap fields, and *cannot* be directly attributed to the $\Delta_{1,2}$ fields at any finite Josephson coupling and at any temperature. Instead normal modes are associated with linear combinations of $\Delta_{1,2}$ and thus coherence lengths are hybridized[6,10,14,15]. Moreover one of the two coherence lengths does not diverge in the limit $\tau \to 0$.

The overall gap function profiles are determined by nonlinearities and thus not only by masses of the normal modes but also by their amplitudes[15]. Therefore it is not possible to extract the information about the coherence lengths just analyzing the overall profile of the gap functions[2]. The correct reduction of TCGL model to single-component GL theory takes place because in the limit $T \to T_c$ the mode with a non-diverging coherence length looses its *amplitude*[15], but not because two coherence lengths gradually become degenerate.

IV. MISCONCEPTIONS

i For unclear reasons, the Ref. 1 criticises previous works on type-1.5 superconductivity for *"assumption of two different penetration lengths* $\lambda_{1,2}$*"*. We are not aware of any papers on two-band superconductivity where such assumptions were made. As far as we know the notations $\lambda_{1,2}$ were used in literature on type-1.5 superconductivity only as characteristic constants, parameterizing GL free energy while the physical magnetic field penetration length was always determined self-consistently.

ii In contrast to what was claimed in Ref.2 no assumptions of having zero interband coupling but equal T_c for all components in two-band systems were made[3,6,10,14].

iii The work Ref.1 claims that it is not possible to obtain fractional vortices in multicomponent superconductors discussed in the Refs.[32]. Obviously *in the limit* $T \to T_c$ in two-band systems fractional vortex excitations are suppressed. However fractional vortices can exist in $U(1) \times U(1)$ systems and in two-band $U(1)$ theories at finite-τ. The authors of Ref.1 missed that the papers in Ref.32 deal with fractional vortex solutions not in the $T \to T_c$ limit and not even in the GL model but exclusively in the London theory. In fact in a GL model the fractional vortex solutions are quite different (see corresponding discussion for $U(1) \times U(1)$ systems in Ref.31). Moreover Refs. 32 primarily focuses on the $U(1) \times U(1)$ systems. Thus the results in Ref.32 are entirely unrelated to the arguments on $T \to T_c$ limit in two-band systems. However we mention that occurrence of fractional vortices in two-component GL models with intercomponent Josephson coupling in mesoscopic samples was investigated by other groups[33].

iiii The work[1] also contains mutually exclusive claims. On one hand from the incorrect derivation of Eqs.(3) it would follow that in the limit $T \to T_c$ the $U(1)$ TCGL theory is reduced to the $U(1) \times U(1)$ theory when the gap functions are coupled only by the vector potential (and not by phase-locking terms). On the other hand Ref.[1] claims that in two-band superconductors the GL theory can only describe the gap functions having the same phase. If the former of these claims contradicts the Landau theory (see the discussion above), the latter statement also yields unreasonable conclusions negating for example the existence of the phase difference excitations[30]. At finite-τ when two-band GL theory is well justified, the appearance of the gradients of the phase difference between component at finite τ is in fact a quite generic effect[11] because the mass of the phase-difference mode does not diverge in the $T \to T_c$ limit[15].

V. CONCLUSIONS

We discussed the errors in the treatment of $T \to T_c$ limit in two-component superconductors in Ref.[1], which led to the incorrect (at any temperatures) system of field equations (3) for the gap fields and incorrect conclusions on the behavior of coherence lengths. We also pointed out that contrary to the claims in Ref.1 TCGL expansion is justified and can be used to describe systems with disparity in coherence lengths as was demonstrated on formal grounds in[15] and does allow type-1.5 regimes.

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26 Note that this situation is not specific to just two-band systems but also occurs in e.g. theories of exotic superconductors. For example phenomenological multicomponent GL theories where one field component scales with temperature as $(1 - T/T_c)^{1/2}$ while another component as $(1 - T/T_c)^{3/2}$ are also widely used in connections with exotic non-s-wave superconductors[25].

27 A. A. Shanenko, M. V. Milosevic, F. M. Peeters, and A. V. Vagov. Phys. Rev. Lett. **106**, 047005 (2011). This paper adopts from[1] the incorrect system of eqs. (3) in the leading order in τ. It also raises the question whether the phenomenological GL functional used in[10,21–23] is complete enough to define two distinct density-rated length scales. The recent microscopic work[15] validates the usage of the two-band GL model like (1), in a certain parameter range although this is not a τ-based expansion.

28 In two-band systems where intercomponent coupling breaks symmetry down to $U(1)$, by symmetry, it is obvious that at mean-field level, GL theory should reduce to a single-component one *in the limit* $\tau \to 0$. However the limiting $\tau \to 0$ analysis unfortunately does not have any physical significance in a generic two-band system. Essentially for any finite τ it misses the qualitative effect of the presence of two coherence lengths for the superconducting density variation realized by "heavy" and "light" asymptotical modes[14,15]. The type-1.5 regime in such systems is associated with the interplay of two density scales and the magnetic field penetration depth. Furthermore as shown in microscopic calculations the masses of the fields (inverse length scales) in two-band models can change rapidly and in a non-trivial way with decreasing temperature[14,15]. Thus a limiting $\tau \to 0$ analysis in multiband system, in general, cannot give even an approximate physical picture even at very small τ.

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