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LiBeB, a predicted phase with structural and electronic peculiarities

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Abstract

Beginning an in-depth analysis of binaries and ternaries in the Li/Be/B system, we examine the static structures and electronic properties of LiBeB (i.e 1:1:1) over a range of pressures. This as yet unknown compound is predicted to possess a stable ground state at 1 atm and some higher pressures. As the pressure rises, LiBeB goes through a diverse series of structures, beginning with metallic structures which feature chains and layers of atoms, progressing to structures built on "colorings" of the Laves phases, and containing helical arrangements of boron atoms, on to high pressure phases that are ternary variants of a bcc lattice. The density of states (DOS) at the Fermi level consistently falls in a pseudogap (sometimes a real gap is predicted); LiBeB is unlikely to be a good metal or superconductor. The distribution of the DOS follows what electronegativity would predict – Li electrons are

transferred to B. Some curious features of the LiBeB structures emerge, these include near-icosahedral coordination, independent of atom type; in a range of pressures a resemblance of the total DOS to that of metallic Be; and also a Dirac surface.

Introduction

In principle the upper left corner of the Periodic Table should provide an auspicious prospecting ground for superconducting materials: some of the elements themselves are free electron-like metals and thus feature high electronic densities of states at the Fermi level. And their light masses guarantee high phonon frequencies, and thus high Debye temperatures.

The quest for possible metallization and high temperature superconductivity of hydrogen itself under pressure has already been a long and challenging journey for both theoreticians and experimentalists [1–3]. Helium, the most unreactive of the inert gases, metallizes only at very high pressures [4–8]. Elsewhere, we have studied the Li/H system [9]; we and others have investigated BH₃ under pressure [10,11], and a variety of Li/B/H compounds [12–17]. Li and Be, immiscible at one atmosphere and ordinary temperatures, are predicted to form stable alloys of varying stoichiometry under pressure. Both Li and Be were added as dopants in a computational study of β -rhombohedral boron, to explain the electronic properties of the latter [18]. Layered structures of LiB were predicted to be stable and superconducting [19]; their synthesis has been attempted [20].

The search for new superconductors was in fact the impetus for the exploration of the ternary Li-Be-B system, the first chapter of which, on the simple 1:1:1 stoichiometry, is reported here. As we will learn, the allure of superconductivity proved to be just that, and is not supported by the computations.

However, as will become clear, we were much rewarded in our exploration with a fascinating array of structural and electronic features in this system: a predicted stable ground state compound at P=1 atm, changing structure drastically with increased pressure; unusual geometrical features such as helices and high coordination numbers reminiscent of transition metal intermetallics; a Dirac surface; an entry point to the combinatorics of bcc, hcp, and fcc lattices; and an isoelectronic principle for understanding the electronic structure of such ternary phases. And our work has already stimulated synthetic efforts on binaries and ternaries of Li, Be, and B.

Join us in the exploration of LiBeB. The computational details are given in the Appendix to this paper.

Searching for structures of LiBeB

Driven by nothing more than an urge for simplicity, we began our search for stable ternary phases with the Li:Be:B stoichiometry of 1:1:1. Other stoichiometries, such as Li₂BeB, will be the subject of a subsequent publication.

There is a multitude of ABC structures one could consider for LiBeB. To get oriented, we began with two attractive possibilities – the standard structure of the Half-Heusler compounds, and ternary variants of the MgB₂ structure. The former is known as a diverse family of ternary ABC compounds (they usually contain a transition metal), with a wide range of properties and applications [21,22]. The latter is a layered binary alloy and an extraordinary superconductor [23]; in it one could imagine replacing the magnesium and every second boron by either Li or Be. There are several ways to do this, and the Supporting Information (SI, see also references [24,25] therein) gives more details on some of these.

However, neither of these alternatives, in any isomeric form, turned out to be enthalpically competitive at P=1atm or any other pressure studied, when compared with the results from structure searches that we will discuss now.

The genetic algorithm structure searches that we employed (these implemented in the static approximation, see Appendix for details) then led us to a number of low enthalpy structures for LiBeB, with three structures emerging as stable in various pressure regions: a structure of $P2_1/m$ symmetry, stable at low pressures up to 15 GPa; an *Ama2* structure favored between 15 GPa and 70 GPa; and a *P3m1* structure stable at pressures higher than 70 GPa. Figure 1 shows the enthalpy data of these three and other candidate structures, with the zero of enthalpy taken as that of the respective stable atomic crystal structures. We find that (i) at a pressure of one atmosphere there are several LiBeB structures with negative enthalpies of formation, and (ii) the pressure variable leads to a significant stabilization of other structures.

But, as we discuss next, enthalpic ground state stability with respect to the elements is not necessarily sufficient to guarantee the overall stability of a ternary compound phase.



Figure 1 (Color online). Enthalpies of formation for various ground state and static LiBeB structures, up to pressures of P=320GPa. Enthalpies are relative to the enthalpies of elemental crystals. Insert shows low pressure region.

Enthalpic stability

The LiBeB phases mentioned above are stable with respect to decomposition into crystals of the respective elements, and increasingly so under pressure, as Figure 1 shows. That means that the reaction LiBeB(s) \rightarrow Li(s) + Be(s) + B(s) has a positive Δ H ("(s)" indicates the solid phase). However, other escape routes must be considered, for example decomposition into binary or other ternary phases. The reaction LiBeB(s) \rightarrow Li(s) + BeB(s) could have a negative Δ H, and so could a more complicated one, such as LiBeB(s) \rightarrow ¼ Li(s) + ¼ LiBe(s) + ¼ Li₂BeB(s) + ½ BeB(s) + ¼ B(s). For binary mixtures, an elegant way to present stability of mixtures $A_{1-x}B_x$ is to use "tieline diagrams", where the convex hull of the enthalpies of formation of $A_{1-x}B_x$, plotted against the atomic ratio x of component B, is constructed. All binary stoichiometries that are part of this convex hull (also called the "global stability line") are stable in their ground states with respect to decomposition into other binary stoichiometries. Similarly, for ternary phase diagrams, the convex hull of the enthalpies of formations of all stoichiometries $A_{1-x-y}B_xC_y$ has to be determined for all atomic ratios x and y of components B and C [26]. Only the compositions defining this convex hull are truly stable compounds (though nothing can be said about kinetic persistence of metastable stoichiometries) [27–29].

For LiBeB, we construct the ternary convex hull by considering all possible binary mixtures Li_{1-x}Be_x, Li_{1-x}B_x, and Be_{1-x}B_x, at all pressures studied. For the LiBe mixtures, which are stabilized under pressure, we took relative stabilities from a recent computational study [30]. For the Li-B and Be-B mixtures, structural data from the ASM Alloy Phase Diagrams Center [31–33] was combined with results from structure searches by us to obtain binary tieline diagrams at the pressures of interest. Work on other ternary stoichiometries in the Li-Be-B phase diagram is currently in progress (as is experimental work in the Li-B and Be-B phase diagrams).

At all pressures, binary compounds of varying stoichiometry are points of stability, as indicated by the vertices along the phase diagram edges in Figure 2.



Figure 2. Ternary phase diagrams for ground state structures in the Li-Be-B system at various pressures. All labeled vertices indicate stable stoichiometries.

Is the ternary LiBeB phase stable with respect to decomposition to *all* possible products? At atmospheric pressure, the ternary $P2_1/m-6$ phase (described below) is indeed stable in this sense, as indicated by the existence of the vertex in the center of the ternary phase diagram in Figure 2, a corner of the hull of enthalpy.

At p=20GPa, the most stable ternary phase at that pressure is also enthalpically stable. At p=80GPa, however, the low-enthalpy ternary P3m1 LiBeB phase is *not* stable. It is part of the phase separation line connecting pure beryllium and 1:1 stoichiometric LiB. To put it another way, the reaction LiBeB(s) \rightarrow Be(s) + LiB (s) has a negative Δ H at high pressures – Δ H=-150 meV per formula unit at p=80 GPa, and Δ H=-160 meV per formula unit at p=160GPa. As mentioned above, this does not exclude the possibility of preparing metastable, kinetically persistent ternary LiBeB phases; also the gain in total enthalpy upon decomposition is relatively small. The reason for the relative instability of ternary LiBeB is the formation of very stable binary LiB phases under high pressures, which we will report on elsewhere.

LiBeB at Low Pressures: Low Dimensional Metallic Structures

At low pressures, a *P2*₁/*m*-6 phase with six atoms per unit cell is enthalpically favored (at atmospheric pressure by about -0.37eV per unit of LiBeB). The structure (see Figure 3, and the SI for lattice parameters of this and all other relevant structures) features buckled triangular nets of beryllium and boron, interspersed by lithium; within the BeB sheets, both boron and beryllium form one-dimensional chains. Not only can the crystal structure be dissected into two- and one-dimensional sub-lattices (see the SI for geometrical justification), but the consequences of these features arise in the electronic structure as well, as we will show below. Similar sandwich structures of BeB sheets alternating with Li layers are found in other low-enthalpy structures at atmospheric pressure, roughly those with

a negative enthalpy of formation in Figure 1. Higher enthalpy structures feature more close packed structures, which are in turn stabilized at higher pressures.



Figure 3 (Color online). Crystal structure of the $P2_1/m-6$ phase of LiBeB at P=1atm: side on view (left) and top view (right) of the buckled triangular BeB layers or sheets. Big red/medium grey/small green (dark gray/light gray/medium gray) spheres denote Li/Be/B atoms, respectively. A monoclinic unit cell of six atoms is indicated. Atoms less than 2.2Å apart are connected.

In the $P2_1/m-6$ structure at atmospheric pressure, the shortest separations between like elements are comparable to (but slightly shorter than) their respective bulk materials: B-B separations of 1.74Å, compared to 1.79Å in α -boron; Be-Be distances of 2.16Å, compared to 2.21Å in hcp-beryllium; and Li-Li separations of 2.92Å, compared to 3.04Å in hR9-lithium. Drawing the coordination polyhedra and neighbor distance histograms for all three elements (see Figure S4 in the SI) allows us to recognize the $P2_1/m-6$ phase as a layered structure. This can also be seen by monitoring the energy needed to separate the BeB sheets: starting from the P=1atm geometry, we elongated the *a* axis while keeping the BeB sheets intact and reoptimizing the Li atoms. The energy needed for complete separation is only 0.3eV per LiBeB unit. We are not aware of any other ternary ABC compound with this structure type. The substantially negative enthalpy of formation of LiBeB, and its stability relative to all binary decompositions (see Figure 2) augurs well for the eventual synthesis of this phase.

The $P2_1/m-6$ phase is metallic. Its band structure and electronic density of states (DOS) per electron are shown in Figure 4. The DOS has, between -12 and - 8.5eV below the Fermi energy, a typical feature of weakly interacting one-dimensional chains (a square-root decrease of the DOS after an initial sharp onset); and between -8.5 and -6eV a typical two-dimensional structure (plateau of DOS with a square onset). Indeed, if one performs a charge integration restricted to those energy intervals, one obtains the charge densities shown in the lower panel of Figure 4, clearly indicating the 2D features within the BeB sheets, and the 1D feature along the B chains within these sheets.

The 1D feature at the bottom of the valence band of LiBeB is dominated by boron 2s electrons. That this makes sense follows from the energies of the 2s and 2p electrons of isolated boron, beryllium, and lithium atoms, shown by the bars at the right in Figure 4. For both 2s and 2p levels the energy orders B<Be<Li, an order consistent with a higher effective charge for boron, due to ineffective screening by the other electrons in the valence shell. We note that a similar electronic two-dimensionality was found by us before in LiBe and LiBe₂ which, however, are only stable under pressures of at least 50 GPa and 15 GPa, respectively [30].

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A feature more relevant to the potential superconductivity of LiBeB is the value of the DOS at the Fermi energy, which is about 0.06 states/eV/electron; this is about twice the value for hcp-beryllium, but only about 12% of the value of bcc-lithium. Or, put another way, in a free electron gas of the same valence bandwidth as LiBeB (about 12.5eV), the DOS at the Fermi energy would be 0.12 states/eV/electron. Chemistry burns a hole into the DOS around the Fermi energy: not as deep as in pure beryllium, but enough to render this compound far from a free electron-like metal near the Fermi energy.



Figure 4 (Color online). Upper panel: Electronic band structure (left) and DOS per electron (right) of the $P2_1/m-6$ phase of LiBeB. The DOS plot also indicates atomic energy levels of Li, Be, and B, normalized to the solid's Fermi energy. Lower panel: charge density isosurfaces ($\rho=0.01e/Å^3$) of 2D (left) and 1D features (right), integrated over the respective energy ranges indicated in the DOS plot above.

Intermediate Pressure Phases: Helices

Under pressure, more compact structures than the one we just examined are stabilized in their ground states. Between P=15GPa and P=70GPa, the structure searches reveal an *Ama2* phase shown in Figure 5 as most stable. It can be seen as a hexagonal network of lithium atoms (in an approximation to the hexagonal diamond structure, space group $P6_3/mmc$), with linear beryllium chains filling its cavities along the [1120] and [1210] directions, and buckled boron chains along the [0001] direction. The latter feature is reminiscent of the structure of the known binary compound LiB, which has a hexagonal packing of Li atoms and linear chains of B atoms in cavities along the *c* axis [34,35]. Only slightly higher in enthalpy than the *Ama2* phase (by about 60meV per formula unit at p=40GPa) is a *Cmcm-12* phase with similar features, but having all beryllium chains parallel to each other, as illustrated in Figure 5.



Figure 5 (Color online). Crystal structure of the intermediate pressure phases *Ama2* (top) and *Cmcm-12* (bottom) of LiBeB, at P=40GPa. Side on views (left) and top views along the c axes (right) are shown. The connectivity shown follows the structural interpretation given in the text.

There is a slightly more efficient way to incorporate the features described above – linear beryllium chains, buckled boron chains, and a hexagonal lithium network – into a crystal structure of LiBeB. By stacking three unit cells of the *Ama2* phase along the *c* axis and rotating them by 120 degrees with respect to each other, one obtains a $P6_{1}22$ phase with 36 atoms in the unit cell that is slightly lower in enthalpy than the *Ama2* phase, by about 5meV per formula unit at P=40GPa. Constructed in this way, the $P6_122$ phase has buckled boron chains with a regular precession of the buckling direction around the *c* axis; the off-axis boron atoms form a 6_1 helix along the *c* axis, see Figure 6. The feature of chiral 6_1 chains of face- or edge-sharing polyhedra has been found previously, e.g., in the crystal structures of CsCuCl₃ [36,37] or AuF₃ [38].



Figure 6 (Color online). Crystal structure of the $P6_122$ phase of LiBeB. Side view (left) and top view (bottom right) are shown. Be atoms are omitted, and are on the empty vertices of the boron-centered trigonal antiprisms (shaded green/gray). The boron helix around the *c* axis is drawn as a guide to the eye. Unit cell is indicated. Top right shows the electronic DOS per electron at P=40GPa.

Electronically, these intermediate pressure structures are metallic; however,

the density of states at the Fermi energy is significantly depleted compared to the low-pressure $P2_1/m$ -6 phase, see Figure 6. The electronic structure reflects the low-

dimensional structural features – chains of Be and B atoms – discussed above (see SI for more details).

High Coordination and Coloring a Laves Phase

As Figure 1 shows, the intermediate pressure regime is dominated by several close-packed phases, which are similar in enthalpy and move in a similar way with pressure. They share the structural features of the hexagonal Laves phases. For these phases, even more than for the previously discussed $P2_i/m-6$ phase, the division into such sub-lattices can seem arbitrary (even though electronic features we discuss below support this division). In fact, however, they can alternatively be seen as ordered ternary variants of the hexagonal Friauf-Laves phase C14 (prototype MgZn₂) [39–41], with lithium occupying the *4f* Wyckoff position, beryllium partially occupying the *6h* position, and boron the remaining *6h* and the *2a* positions. Accordingly, the coordination polyhedra of the atoms in these phases (see Figure S2 in the SI) are slightly distorted icosahedra for both boron and beryllium (one of these, for one boron site, is shown below in Figure 7) and nominally a Z16 Frank-Kasper polyhedron [42] for lithium.



Figure 7 (Color online). Coordination polyhedron of one of two unique boron sites in the *Ama2* phase, at P=40GPa. All distances from central boron atoms are between 1.9Å and 2.3Å.

Given the very unequal size (i.e., covalent or ionic radii) of the atoms, coordination numbers are to a degree arbitrary (here, we arrive at them by observing natural gaps in neighbor distance histogram plots which are included in the SI). But these high coordination numbers, and quasi-icosahedral coordinations, lead us to think that we could equally well describe these phases as having high coordination at every atomic site. Note that while lithium is found in other Laves phases, most notably CaLi₂ [43–46], this seems to be the first structure where it occupies the "large atom" lattice site of the Laves phase, i.e., the site the X atom occupies in a typical Laves phase XY₂.

Other phases with a hexagonal lithium network intercalated by lowdimensional boron and beryllium structures were found in the structure searches, but with higher enthalpies of formation. There are 70 ways to distribute two different kinds of atoms in 1:1 stoichiometry over the eight *6h* and *2a* Wyckoff sites in the primitive unit cell of the C14 structure type – or, in other words, to "color" the *2a* and *6h* sites with two colors [47–49]. Nine of these are of unique symmetry [50]. We optimized all of these combinations at P=40GPa, and found that the *Ama2* and *Cmcm-12* phases described above are indeed the two with the lowest enthalpy; structures and relative enthalpies of all of these ternary Laves phase variants are listed in the SI.

There are 12870 ways to distribute two kinds of atoms in the *doubled* unit cell of the C14 structure (doubled along the *c* axis, and containing 24 atoms), 351 of which are symmetry-unique. We systematically optimized these as well and found one phase (also of *Ama2* symmetry) with an enthalpy at P=40GPa slightly lower than the *Ama2* structure discussed above – like the latter, it features linear beryllium and buckled boron chains, but the buckled boron atoms go off into different directions off the *c* axis, in an attempt to get as close to a helical arrangement as is possible within the unit cell constraint (see the SI for details). This ground state *Ama2* phase is slightly less stable then the *P6₁22* structure constructed above.

Going further, there are 2704156 ways to distribute two atoms in the *tripled* unit cell of the C14 structure – but we could not check how many of these are related by symmetry, nor could we calculate their energies. One of these structures is the $P6_{1}22$ phase we described above, and extrapolating from the structural trends we saw in the single and double C14 unit cell, it is a plausible candidate for the global enthalpy minimum.

Lastly, one might wonder whether the cubic C15 Laves phase (prototype MgCu₂, space group Fd3m, 24 atoms in the unit cell) would provide competitive structural candidates for LiBeB [51]. In this phase, the "large" atoms occupy the *8a* Wyckoff sites and form a diamond-like sub-lattice, whereas the "small" atoms on *16d* sites form tetrahedra fused at each vertex and staggered along the *c* direction. Of the 12870 different ways to distribute Be and B amongst the *16d* sites, 97 are unique; and all of these are at least 100meV per formula unit higher in ground state enthalpy (at P=40GPa) than the *P6₁22* structure based on the C14 Laves phase.

Let us move on to higher pressures.

High pressure phases of LiBeB: Interpenetrating Cubic Sublattices

At pressures higher than P=70GPa (relative volume compression about 1.4 for the $P6_122$ phase), still more compact structures are stabilized. The most stable phases at high pressures are P3m1, Imm2, $P2_1/m$ -12 and Cmcm-6 (see Figure 1; note that the high-pressure $P2_1/m$ -12 and Cmcm-6 phases differ from the phases $P2_1/m$ -6 and Cmcm-12 described in the previous paragraphs). All of these are ternary variants of a body-centered cubic (bcc) lattice: all atoms occupy (approximately) bcc lattice sites, with different local arrangements of the two simple cubic sub-lattices. Under pressure, the constituents seem to resolve for themselves the question of what the optimal "coloring" of bcc lattice sites is, with equal contribution from three different "colors" [47–49]. Indeed, seeing the structures as colorings of the bcc

lattice helps us to appreciate the complexity of structural choices available to this ternary.

Studying the cubic sub-lattices of these phases separately yields some insight into what comprises the enthalpically favorable structures. For the lowest enthalpy *P3m1* and *Imm2* structures, one cubic sub-lattice contains boron and lithium atoms in a NaCl-type crystal structure, whereas the other sub-lattice is more complex, featuring all beryllium atoms and the remaining boron and lithium atoms, see Figure 8. If *all* atoms of the second sub-lattice were beryllium atoms (this would correspond to the stoichiometry LiBe₂B) the structure would be of the well-known Heusler type. Incidentally, in a structure search at that stoichiometry, we found the Heusler phase to be indeed the most stable crystal structure for LiBe₂B at a pressure of P=80GPa.



Figure 8 (Color online). Crystal structure of *P3m1* (top) and *Imm2* (bottom) phases of LiBeB at p=160GPa, taken apart into their sub-lattices. Left: unit cell of complete structure; middle: NaCl-type sublattices of LiB stoichiometry; right: the respective second sub-lattice of LiBe₄B stoichiometry. The composite structure contains 3 LiB and 1 LiBe₄B sublattices.

The *P3m1* and *Imm2* phases differ only by little in enthalpy (about 15....30meV per formula unit over the pressure range 10....320GPa), because they only differ with respect to the lithium-boron arrangement in the beryllium-dominated sub-lattice: Li and B form a bilayer in the *P3m1* structure, and isolated linear chains in the *Imm2* phase, see Figure 8. The conventional cubic unit cell of the *P3m1* phase then contains no less than 432 atoms, or 36 primitive unit cells.

The *Cmcm-6* and $P2_1/m-12$ phases, somewhat higher in ground state enthalpy than the two structures just analyzed, exhibit more symmetry. Their conventional cell (the smallest quasi-orthorhombic cell) contains 24 atoms, or 4 primitive unit cells. In Figure 9 we show their primitive unit cells together with the two simple cubic sub-lattices of their approximate bcc structures. In both phases, the two sub-lattices are identical but shifted with respect to each other along the crystalline *c* axis. They contain all three elements in LiBeB stoichiometry.



Figure 9 (Color online). Crystal structures of the *Cmcm-6* (top) and $P2_1/m-12$ (bottom) phases of LiBeB. Left: primitive unit cell; middle: conventional unit cell with simple cubic sub-lattice 1; right: conventional unit cell with sub-lattice 2.

For the *Cmcm-6* phase, both sub-lattices contain a two-dimensional square lattice of beryllium atoms that alternates with a simple cubic boron-lithium bilayer. Superimposing the two sub-lattices, all beryllium atoms are then to be found at the center of a cubic boron-lithium cage. Note that the low-pressure $P2_1/m-6$ phase transforms into this *Cmcm-6* phase at about p=40GPa: upon compression of the

former, the initially well-separated beryllium-boron sheets of the $P2_1/m-6$ phase begin to penetrate the lithium layers and interact, and in the process form the threedimensional network that is the *Cmcm-6* phase.

For the high-pressure $P2_1/m$ -12 phase, both sub-lattices also contain all three elements, where layers of beryllium alternate with bilayers of lithium-boron. These layers are orthogonal to the [110] direction of the simple cubic lattice, see Figure 9 (in the *Cmcm*-6 phase, they are orthogonal to the [100] direction). And as in the *Cmcm*-6 phase, every beryllium atom in the final structure is found at the center of a lithium-boron cube.

One might argue that ternary "colorings" of other close-packed structures could also yield competitive high-pressure structures for LiBeB. We therefore optimized all the unique crystal structures that are created by placing Li, Be, and B on the lattice sites of an fcc lattice (there are, in fact, 11 unique ways to do this for the 6-atom unit cell we chose) and on the lattice sites of an hcp lattice (there are 1515 unique ways to do this for our 12-atom unit cell). Neither of these structure types provides competitive structures in the high-pressure region (P \geq 100GPa): all fcc-based structures were at least 1.2eV per formula unit higher in ground state enthalpy, and the hcp-based structures at least 0.75eV per formula unit higher than the global minimum, the *P3m1* structure.

Viewing these structures as interpenetrating cubic sublattices should not lead us to think that these are not compact structures. As a histogram of neighbor distances shows (see the SI for these), all the atoms in these structures are close to 14 (8+6) coordinate.

Electronic Structure of the High Pressure Phases of LiBeB

The detailed arrangement of the atoms in these close-packed structures determines their electronic properties. And here we get a surprise. While P3m1 and Imm2 are semiconductors, the Cmcm-6 and $P2_1/m-12$ phases are metallic. However, neither is as far from the other as far as that categorization might imply: the band gaps in the P3m1 and Imm2 phases at p=160GPa are only about 0.3eV and 0.7eV, respectively; and the Fermi level in the metallic Cmcm-6 and $P2_1/m-12$ phases is positioned at a pseudo-gap and again severely depleted in states, see Figure 10 and Figure 11.



Figure 10. Band structures and electronic DOS per electron of the *P3m1* (top) and *Imm2* (bottom) phases. All energies are given relative to the valence band maximum (VBM). The orthorhombic unit cell (24 atoms) was used for the *Imm2* phase. A gap above the highest occupied level is quite apparent.

The electronic band gaps in the two semiconducting phases are small over the entire pressure range (see SI for the pressure dependence of the band gaps): their maxima are 0.36eV at P=70GPa for the *P3m1* phase, and 0.7eV at P=120GPa for the *Imm2* phase. The gap in the *P3m1* structure seems to close at the upper end of the pressure range we investigated, at about P=350GPa – note that for this pressure, band gaps obtained from normal DFT calculations, might be underestimated.



Figure 11. Band structures and electronic DOS per electron of the *Cmcm-6* (top) and $P2_1/m-12$ (bottom) phases at P=160GPa. On the top right, the DOS of bcc-Be at P=160GPa is shown. All energies are given relative to the Fermi level. Though the band gap has closed the DOS per electron is low.

The detailed features in the band structures and DOS's of the semiconducting phases depend on the microscopic atomic arrangement. For example, the DOS of the P3m1 phase has several flat bands and corresponding peaks in the DOS – these arise from combinations of localized boron 2s orbitals; the lowest three valence bands at E=-16...-11eV show only weak dispersion, these are boron-boron bonding orbitals of 2s and 2p character. These can also be seen in the other close packed phases, all of which exhibit boron-boron interactions which we classify as bonding based on the

interatomic separations and plots of the energy-projected charge densities for these phases (given in the SI).

Is LiBeB at High Pressure Similar to Beryllium?

The DOS of the two high pressure metallic phases of LiBeB is very similar to the DOS of pure beryllium (which of course is isoelectronic to LiBeB) in a hypothetical body-centered cubic phase. In the top panel of Figure 11, we show the DOS of bcc-Be at a pressure of 160GPa next to the DOS of the *Cmcm-6* phase: the total bandwidth and general features such as the pronounced pseudogap at the Fermi level and maxima at -7eV and 3eV are well reproduced. Other details of the electronic structure, such as the peak at -15eV in the DOS of the *Cmcm-6* phase (which is due to low-lying boron *2s* states) are, of course, unique to the LiBeB system. Note that pure beryllium has a high pressure bcc phase that is predicted to be stabilized at around 400GPa [52], and has not been found up to 200GPa [53]. For the isoelectronic LiBeB, the bcc variants discussed here are stabilized at much lower pressures. Since *P3m1* is the most stable of these high-pressure phases, we expect a metal-insulator transition for LiBeB to occur at the stabilization pressure of *P3m1* with respect to *Ama2*, at about P=70GPa in the static approximation.

As Figure 13 shows, the Fermi level in LiBeB falls in the deep trough of a pseudogap, something which also happened in the intermediate pressure phases. This is a not uncommon feature of intermetallic compounds, from the Laves phases to the γ -brasses. If one wants to design a material with a higher DOS, potentially a superconductor, one might want to add or subtract electrons without affecting the

underlying crystal structure. For electron doping, and given the electronegativity order of the elements in LiBeB, lithium "doping" comes to mind, for instance in a hypothetical Li₂BeB. We have studied this hypothetical material in some detail, and other Li_xBe_yB_z stoichiometries as well. With respect to the issue at hand, insertion of additional Li into the LiBeB structure leads to a major change in geometry, and a simple "doping" view will not work – even though we do indeed find enhanced DOS at the Fermi level in Li₂BeB; these results will be reported separately.

Close to a Dirac surface?

The band structure of the *Cmcm-6* phase (top panel of Figure 11) has an interesting feature: along the Γ -Y direction, which in our choice of unit vectors is along the path $(0,0,0) \rightarrow (0,0,1/2)$ in the Brillouin zone, two electronic bands with linear dispersion cross just above the Fermi energy. Is this close to a "Dirac point"?

Using two different cuts through the Brillouin zone, and plotting within the respective planes the two-dimensional dispersion of the two bands involved, we obtain the following picture (see Figure 12): perpendicular to the Γ -Y direction, and within the k_x=0 plane, the two bands cross along a line. In reciprocal coordinates, Y=(0,0,±0.164)Å⁻¹. Both bands show linear dispersion across a significant part of the Brillouin zone, and especially along their lines of intersection at k_z=±0.08Å⁻¹. While the effective mass is zero along the Γ -Y direction, it is infinite along the k_y direction. Within the k_y=0.08Å⁻¹ plane, we see the conical intersection of the two bands, at k_x=0 and k_z=±0.08Å⁻¹, see Figure 12. By varying k_y, this cone can be found along the

line of intersection of the two bands, throughout most of the Brillouin zone. This conical feature is therefore not a fully three-dimensional "Weyl point" [54], but two-dimensional in nature, similar to what is found in graphene [55], or in ternary Mn-Bi compounds [56,57].

Note that the conical intersection of the two bands is only about 0.2eV above the Fermi level – one could imagine that, through doping or uniaxial stress of the *Cmcm-6* unit cell, one might shift the Fermi energy up to coincide with the intersection line of these two bands, and thus the collective of intersection cones.



Figure 12 (Color online). Two-dimensional electronic band structure of the *Cmcm-6* phase at p=160GPa in the k_y =0.08Å⁻¹ plane (top) and the k_x =0 plane (bottom). Top left sketches the Brillouin zone and these two planes. Blue (dark gray) and orange (light gray) surfaces depict the electronic bands near the Fermi energy, the underlying gray rectangle and hexagon depict the Brillouin zone. The special path Γ -Y is indicated in both plots.

Superconductivity?

Despite a lack of a unifying microscopic theory of superconductivity, several macroscopic observables are found to correlate well with the measured transition temperature T_c : a high density of states at the Fermi level and large electron-phonon coupling are found in a wide variety of superconducting materials, and seem to be crucial properties. In the earliest form of the BCS theory of phonon-based superconductivity, T_c is proportional to the Debye temperature T_D of the material. Hence, in a qualitative survey, we may simply monitor N(0), the electronic density of states at the Fermi energy E_F , and T_D , the Debye temperature, for the metallic phases of ternary LiBeB.



Figure 13 (Color online). Electronic DOS at the Fermi level, for various metallic LiBeB phases and pressures. Here, as earlier, DOS is normalized per valence electron. "Global min" follows the lowenthalpy structures for various pressures. The electron gas DOS is calculated at all pressures for densities corresponding to the *Ama2* structure.

The density of states at the Fermi level N(0) is shown in Figure 13 for various low-enthalpy metallic structures. The layered $P2_1/m-6$ structure stable at low pressures has a significantly higher N(0) per valence electron than all other, more compact, structures. The latter do not differ much from the (quite depleted) N(0)value of pure beryllium in the hcp phase. Note that in MgB₂ at P=1atm, N(0)=0.09 states/eV/electron. For comparison, we have also drawn the DOS of a free electron gas, evaluated at all pressures at the r_s value of the *Ama2* structure: all compounds' DOS are significantly lower than this ideal case, as hinted at by the prominent pseudogaps in basically all structures. Furthermore, N(0) remains more or less constant over a wide range of pressures. One could not argue from such a flat DOSvs-pressure relationship for much of a pressure effect on superconductivity.

The Fermi surfaces of the metallic structures are given in Figure 14: they consist of two or three sheets that generally form elongated protrusions in the Brillouin zone. The "Dirac line" feature in the band structure of the *Cmcm-6* phase results in two parallel Fermi surface lines along the k_y direction, as seen in the blue (dark gray) sheet in Figure 14.



Figure 14 (Color online). Fermi surfaces of various LiBeB phases at different pressures. From top left to lower right: (i) $P2_1/m$ -6 at p=1atm (ii) Ama2-12 at p=40GPa (iii) Cmcm-12 at p=40GPa (iv) Cmcm-6 at p=160GPa (v) $P2_1/m$ -12 at p=160GPa. Brillouin zones are not drawn to scale. Blue (dark gray), green (light gray), and yellow (medium gray) sheets denote the first, second, and third band crossing the Fermi level, respectively.

The Debye temperature T_D is estimated using the relation

$$T_{D} = \hbar c k_{D}$$
, with $c = \sqrt{B_{0} / \rho}$ and $\frac{4\pi}{3} k_{D}^{3} = (2\pi)^{3} n$,

where *c* is the speed of sound, ρ the mass density, and *n* the atom density which determines the Debye wave vector k_D . The bulk modulus B_0 is obtained from equation-of-state fits to the energy-volume relation E(V) for each phase. The pressure dependence of these Debye temperatures is shown in Figure 15, together with the T_D values for the elemental phases of Li, Be, and B, and their experimental values at atmospheric pressure.



Figure 15 (Color online). Debye temperatures for various metallic LiBeB phases (solid lines with symbols), elemental crystals (dashed lines), and experimental values (diamond symbols and labels). Note that Be and B have on this scale indistinguishable experimental Debye temperatures.

All LiBeB phases show a rapid increase of T_D under pressure, driven by a reduced compressibility (i.e., increased bulk modulus) that outweighs the net inverse proportionality on the material density. The experimental values of T_D for elemental Li, Be, and B at atmospheric pressures are reproduced reasonably well, and up to p=40GPa the ternary compounds are "sandwiched" between the "soft" Li and the "hard" Be and B. Note that all compact LiBeB phases show very similar dynamical properties in this respect, independent of the detailed atomic arrangement. In contrast, the layered $P2_1/m-6$ structure shows a much more rapid increase of T_D with pressure, up to p=25GPa when it undergoes a phase transition to the *Cmcm-6* structure.

Estimated transition temperatures for these structures (using McMillan's formula as cited above, and setting $\mu^*=0.10$) are not very encouraging. We find rather small electron-phonon coupling parameters: $\lambda=0.23$ for $P2_1/m$ -6; $\lambda=0.18$ for *Ama2*; and $\lambda=0.23$ for *Cmcm*-6. Following the most stable metallic phases, we obtain $T_c=10$ mK for the $P2_1/m$ -6 phase at P=1atm; $T_c=0$ for the *Ama2* phase at P=40GPa; and $T_c=10$ mK for the *Cmcm*-6 phase at P=160GPa.

Summary

We present here calculations on 1:1:1 LiBeB, a so far unknown phase. By tuning the external pressure, we predict a sequence of stable phases characterized by very different structural motifs – ranging from layered structures, via ternary variants of hexagonal Laves phases (with helices in them), to intricately "colored" body-centered cubic structures. We expect in LiBeB a metal-to-insulator transition at a pressure of P=70GPa, where the body-centered structures first become stable. The material will return at still higher pressures to being metallic. Neither the DOS at the Fermi level, nor the electron-phonon coupling in these structures is encouraging for high Tc superconductivity.

The electron distribution in all phases follows electronegativity considerations – the lithiums effectively transfer electrons to the borons, and the states near the Fermi level are mostly on the beryllium atoms. In fact, some of the high pressure phases have a DOS quite similar in nature to that of Be metal. In one of the high pressure phases we see a Dirac point near the Fermi energy, actually a twodimensional surface of such points. Predicted to be stable thermodynamically at

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P=1 atm and some higher pressures, LiBeB, while not likely to be a superconductor, has some remarkable and instructive structural and electronic features.

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Appendix

Computational Details

Predicting crystal structures of systems of a given chemical composition is well recognized as an exceedingly difficult task [58]. This is even more complicated if one moves from the familiar conditions of P=1atm, amenable to chemical synthesis and intuition, to high applied pressures [59]. An efficient sampling of the rich crystalline configurational space must therefore be performed to detect potential ground state structures. Here this is achieved by utilizing an evolutionary algorithm approach [60–63], as implemented in the XtalOpt package [64]. Optimized structures and energies are obtained through density functional theory as programmed in a plane wave formalism in the VASP package [65], using the Perdew-Burke-Ernzerhof parameterization of the electronic exchange-correlation energy [66], and the

Projector Augmented Wave (PAW) method to describe the electron-ion interaction [67,68]. Structure searches were performed at p=1atm, 40 GPa, 80 GPa, 160 GPa, and 320 GPa, using standard PAW data sets for the atoms (which include 2s and 2p valence electrons) and a plane wave energy cutoff of E_c =400eV. Low energy candidate structures were then re-optimized across the whole pressure range using all-electron PAW data sets for lithium and beryllium, a "hard" data set for boron (cutoff radii 1.6, 1.8, and 1.1 a₀, respectively), and an energy cutoff of E_c =800eV. Enthalpies of formation of ternary structures were calculated with respect to known atomic structures of lithium, beryllium, and boron as well as the computed and known binary phases made up of the above.

Estimates of the transition temperatures to superconductivity were obtained using the Allen-Dynes modification of McMillan's formula [69,70] as implemented in the QUANTUM ESPRESSO code [71], and utilizing ultrasoft pseudopotentials [72] for Li, Be, and B from www.quantum-espresso.org.

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