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Magnetic field induced color change in α -Fe₂O₃ single crystals

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We investigated the magneto-optical properties of α -Fe₂O₃ in order to understand the interplay between charge and magnetism in a model transition metal oxide. We discovered that hematite appears more red in applied magnetic field than in zero field conditions, an effect that is amplified by the presence of the spin flop transition. Analysis of the exciton pattern on the edge of the *d*-*d* color band reveals *C*₂ monoclinic symmetry in the high field phase. These findings advance our understanding of magnetoelectric coupling away from the static limit and motivate spectroscopic work on other iron-based materials under extreme conditions.

The interplay between charge, structure, and magnetism is at the heart of the rich tunability in transition metal oxides¹⁻³. Color properties in particular reveal crystal field environments and electronic transition around and between metal centers 4,5 . At the same time, the optical properties of a material often display collective excitations like excitons and magnon sidebands. They are induced by exchange coupling and therefore very sensitive to variations in magnetic order 6,7 . When combined with external stimuli like temperature or magnetic field, optical spectroscopy can reveal transition mechanisms, local symmetry breaking, and microscopic insight into fundamental mixing $processes^{8-10}$. In this work, we focus on α -Fe₂O₃, a model antiferromagnet with large exchange couplings and experimentally-realizable critical fields¹¹⁻¹³ and the parent compound from which other functional oxides (like multiferroic $BiFeO_3$ and $LuFe_2O_4$) derive.

 α -Fe₂O₃ is commonly known as hematite¹⁴. It crystallizes in the rhombohedral corundum structure $(R\bar{3}c)$ at ambient conditions¹⁵. The system is antiferromagnetic below the 260 K Morin transition $(T_M)^{16}$, with spins lying along the [111] axis of the trigonal unit cell (inset, Fig. 1(a)). Both temperature (T_M) and magnetic field $(B_{C\parallel})$ = 6.8 T, $B_{C\perp} = 16.0 \text{ T})^{12,13,17}$ drive a spin flop to the basal plane¹⁸, and in the high temperature/field phase, α -Fe₂O₃ is weakly ferromagnetic due to a slight (~10⁻⁴ degree) spin canting^{19,20}. Previous spectroscopic work revealed the electronic structure and identified the exciton and associated magnon sideband on the leading edge of the ${}^{6}A_{1g} \rightarrow {}^{4}T_{1g}$ on-site excitation^{21,22}. No exciton fine structure was resolved, which prevented an analysis of magnetic symmetry, and magnetic field effects in this iconic material have not been explored from an optical properties point of view.

Here we report the discovery that applied magnetic field drives a color change in α -Fe₂O₃ such that it appears more red in the high field phase. This chameleonic effect arises because the Fe³⁺ on-site excitations are intrinsically coupled to magnetic ordering. Spin-orbit coupling naturally mixes charge and spin excitations²³, but the effect is amplified here by the presence of the spin flop transition. Analysis of the exciton pattern superimposed on the color band reveals C2 monoclinic symmetry in the high field phase, providing a powerful and general illustration of how this type of analysis can be used for magnetic symmetry determination in the absence of neutron data. The discovery of magnetochromism in this model system extends our understanding of charge-spin coupling in iron-based solids^{8,24–26} and the functional oxides that derive from hematite, and it provides a strategy for the development of next-generation magneto-optical materials. This strategy recommends incorporation of collective magnetic transitions in intrinsically colored compounds.

High quality α -Fe₂O₃ single crystals were grown using the flux method. Most work on hematite has been carried out on samples of mineralogical origin, so the preparation of large crystals with high purity provides additional motivation for this work. X-ray shows no impurity phases and a single oxidation state. We employed five independent crystals of varying thicknesses and either (111) or $(\overline{110})$ orientation for our optical properties work²⁸. This combination allowed us to obtain complete results over the full frequency range of interest and optimal sensitivity to the small features on the leading edge of the ${}^{6}A_{1q}$ $\rightarrow {}^{4}T_{1q}$ on-site excitation. Spectra were collected using a Bruker Equinox 55 Fourier transform infrared spectrometer equipped with a microscope attachment (600–17000 $\rm cm^{-1}$). Temperature control was achieved with an openflow cryostat. Magneto-optical measurements (0.1 cm^{-1}) resolution) were carried out at the NHMFL using the 35 T resistive magnet system (B \parallel [111], B \perp [111]). We calculated the optical constants using a Glover-Tinkham analysis^{31,32}. Traditional peak fitting and group theoretical methods were employed as appropriate. The magnon density of states was calculated numerically²⁸.

Figure 1(a) displays the optical response of α -Fe₂O₃. We assign the two strong, broad bands centered at ~11550 and 15300 cm⁻¹ as ${}^{6}A_{1g} \rightarrow {}^{4}T_{1g}$ and ${}^{6}A_{1g} \rightarrow {}^{4}T_{2g}$ on-site excitations^{21,22}. These d-d excitations are formally forbidden although they appear in many oxides due to spin-orbit coupling, exchange interaction, and odd parity phonons that hybridize states and break inver-



FIG. 1: (a) (Color online) Absorption coefficient, $\alpha(\omega)$, of α -Fe₂O₃ in the range of on-site Fe³⁺ d-d excitations at 300 and 4.2 K. Inset: rhombohedral lattice²⁷ and low temperature four sublattice spin structure¹⁸. (b) Close up view of the fine structure on the leading edge of the ${}^{6}A_{1q} \rightarrow {}^{4}T_{1q}$ on-site excitation at 4.2 K in the $\alpha,\,\sigma$ and π polarizations. Inset: closeup view of the excitons. (c) Comparison of the σ -polarized magnon sideband absorption spectrum with the calculated density of states. Inset: Calculated magnon density of states using optimized exchange constants $J_1 = 7.6$ K, $J_2 = 2.0$ K, $J_3 = -27.7$ K, and $J_4 = -22.2$ K²⁸. The frequency channel width was taken to be the same as the spectral resolution (1 cm^{-1}). The nature of the critical points in Brillouin zone are indicated²⁹. (d) Temperature dependence of E1 (π) exciton and magnon sideband (MS1, α) peak positions, expressed as the ratio of $\frac{\omega_e(T)}{\omega_e(0)}$ and $\frac{\omega_m(T)}{\omega_m(0)}$, respectively. The latter compares well with the calculated sublattice magnetization^{11,30} using the same exchange constants listed above. Inset: $\alpha(\omega)$ of the σ -polarized magnon sideband at different temperatures.

sion symmetry 9,21,35 . Low temperature reveals a great deal of fine structure on the leading edge of the ${}^{6}A_{1q} \rightarrow$ ${}^{4}T_{1q}$ band (Fig 1(b)). These strongly polarized features are collective excitations. Their properties are summarized in Table I. For the purposes of our discussion, the fundamental magnetic- and electronic-dipole excitations, M1 and E1, are most important. Interactions between four translationally inequivalent Fe³⁺ sites in the magnetic structure give rise to Davydov splitting between the $(M1\sigma, E1\pi)$ and $(M2\sigma, E2\pi)$ exciton pairs²⁸. These splittings are 38.6 and 14.0 cm⁻¹, respectively, an order of magnitude larger than in $YCrO_3$ $(2 \text{ cm}^{-1})^{36}$ and Cr_2O_3 $(3.5 \text{ cm}^{-1})^{37}$. This is a consequence of the larger inter-sublattice coupling energy in α -Fe₂O₃. The feature labeled MS1 is assigned as the magnon sideband²². Based on the position, shape, and polarization behavior, we associate it with the $M1\sigma$ exciton³⁸. The in-

TABLE I: Properties of ${}^{6}A_{1g} \rightarrow {}^{4}T_{1g}$ fine structure in α -Fe₂O₃ at 4.2 K. In the line notation, M represents magnetic dipole character, E is electric dipole and MS is magnon sideband. In the polarization notation, α ($\tilde{E} \perp c, \tilde{B} \perp c$), σ ($\tilde{E} \perp c, \tilde{B} \parallel c$), π ($\tilde{E} \parallel c, \tilde{B} \perp c$), \tilde{E} and \tilde{B} are the electric and magnetic vector of light, c is [111] axis of the crystal. $f \equiv \frac{2c}{N_{e}\pi\omega_{p}^{2}} \int_{\omega_{1}}^{\omega_{2}} n\alpha(\omega, B) d\omega$, here, $N_{e} = 5$ is the number of electrons per Fe site, $n\simeq 2.23$ is the refractive index, ω_{p} is the plasma frequency $\equiv \sqrt{\frac{e^{2}\rho}{m\epsilon_{0}}}$, e and m are the charge and mass of an electron, ϵ_{0} is the vacuum dielectric constant, ρ is the density of Fe sites, c is the speed of light, ω_{1} and ω_{2} are the frequency limits of integration³³.

Line	Polarization	Position	f	Representation	
		(cm^{-1})	10^{-10}	C_3	$R\bar{3}C$
M1	σ	9727.9	4.8	А	Γ_2^+
E1	π	9766.4	80	А	Γ_2^{-34}
	$lpha,\sigma$		8.6	$\mathrm{C}_1/\mathrm{C}_2$	Γ_3^-
E2	π	9776.1	4.3	А	Γ_2^{-34}
	$lpha,\sigma$		23	$\mathrm{C}_1/\mathrm{C}_2$	Γ_3^-
M2	σ	9790.1	2.5	А	Γ_2^+
MS1	α,σ,π	10471.5	1400		

tensity is stronger than that of M1 σ because this electric dipole-allowed excitation involves pairs of ions and thus breaks the parity selection rule^{9,39}. The magnon sideband frequency can be expressed as $\omega = \omega_e + \omega_m^{40}$, where ω_e is the exciton frequency, and ω_m is the magnon frequency. We find $\omega_m = 743.7 \pm 0.1 \text{ cm}^{-1}$ in excellent agreement with the magnon energy obtained by neutron scattering¹¹. Other collective excitations including phonon sidebands and the two magnon sideband are also observed²⁸.

We analyzed the magnon sideband selection rules⁴¹ at different points in the Brillioun zone and calculated the density of states using the magnon dispersion of Samuelsen^{28,42}. The results are in good agreement with the measured absorption spectrum (Fig. 1(c)) and explain the directionally-dependent magnon sideband shape²⁸. The exciton and magnon features display characteristic temperature dependence. As shown in Fig. 1(d), expressing the magnon sideband frequency as $\frac{\omega_m(T)}{\omega_m(0)}$ reveals high temperature softening, whereas the reduced frequency of the $E1\pi$ exciton is relatively constant. The former compares well with a numerical calculation of the normalized sublattice magnetization $\left(\frac{M(T)}{M(0)}\right)$ using the random phase approximation²⁸. The discrepancy in the high temperature range is because magnetization measures the thermal average population of magnons and is less sensitive to the zone boundary (A point) dispersion^{40,43}. Similar effects are observed in MnF_2^{40} and $Cr_2O_3^{10}$.

Figure 2(a) displays the field-induced absorption difference spectra of α -Fe₂O₃ for the α -polarization and



FIG. 2: (Color online) (a) Absorption difference spectra $[\alpha(B) - \alpha(B = 0 T)]$ of α -Fe₂O₃ at 4.2 K for the α -polarization and B \parallel [111] at selected magnetic fields: 5, 10, 15, 35 T. The corresponding low temperature zero field absorption spectrum is shown as a blue solid curve for comparison. Inset: oscillator strength change in the range of the magnon sideband and ${}^{6}A_{1g} \rightarrow {}^{4}T_{1g}$ on-site excitation as a function of magnetic field. (b, c) Close-up view of the excitons and magnon sideband at selected fields. (d) Peak position vs. magnetic field for the exciton and magnon sideband features. Error bars of these positions are smaller than the symbol size and not shown.

B \parallel [111]. The magnetochromic response reveals that the on-site excitations are intrinsically coupled to the microscopic spin structure. We can quantify the fieldinduced color change with the partial sum rule (inset. Fig 2(a))³³. Oscillator strength changes (Δf) show a first order transition at $B_{C\parallel} = 6.6 \pm 0.2$ T when B \parallel [111], and a broad transition at $B_{C\perp} = 16.2 \pm 0.2$ T when $B \perp [111]^{28}$. These critical fields are easily assigned as field-induced spin flop transitions in agreement with magnetization¹² and ultrasonic attenuation measurements¹³. Importantly, the d-d on site excitations are in the visible range and responsible for the reddish color of α -Fe₂O₃. As shown in the absorption difference plot, this system absorbs fewer photons in the red color range in high magnetic field (on the order of 2% less, the maximum transmittance change is $\sim 16\%^{28}$). α -Fe₂O₃ thus appears more red in the magnetically reoriented phase⁴⁴. This color change is observed because the coherent spin transition amplifies the charge-spin interaction beyond what one might expect from traditional spin-orbit processes alone. The field-induced reduction of oscillator strength of the ${}^{6}A_{1g} \rightarrow {}^{4}T_{1g}$ band is partially recovered in the magnon sideband (inset, Fig. 2(a)). Optical property changes driven by magnetic order reorientation have been observed in other functional materials including FeCO₃ (color contrast at the 13 T spin-flop transition due to a "double" magnon sideband mechanism)⁶ and BiFeO₃ (color change at 20 T due to shifted magnon sideband excitations caused by spin spiral quenching)⁸. Here, we report the discovery of such an effect in α -Fe₂O₃, although it occurs by a fundamentally different mechanism.

Magnon sideband formation is controlled by exchange coupled pairs^{39,45}, and as a consequence, field-induced magnetic order reorientation will significantly impact the behavior. Strikingly, the magnon sideband displays a 27% increase in oscillator strength at 35 T compared to its zero field value (inset, Fig. 2(a))²⁸. Magnon sideband intensity is dependent upon (i) exciton + magnon joint density of states and (ii) collinearity of the magnetic structure^{6,7}. It will decrease dramatically if spin collinearity is violated as in RbMnF_3^7 and FeCO_3^6 . Based on the observed intensity increase, we conclude that spin collinearity is, for all practical purposes, conserved through the spin-flop transition in α -Fe₂O₃⁴⁶. Figure 2(c) shows a close-up view of the magnon sideband excitation at selected magnetic fields. The fieldinduced frequency shifts are summarized in Fig. 2(d), and like the oscillator strength, place the critical spin flop fields at $B_{C\parallel} = 6.6 \pm 0.2$ T and $B_{C\perp} = 16.2 \pm 0.2$ T^{28} . Importantly, the magnon sideband softens by ~ 6 $\rm cm^{-1}$ through the 6.6 T transition. Assuming that the magnon dispersion does not change substantially⁴², we can estimate changes in the exchange constants $\left(\frac{\Delta J}{I}\right)$ using a least squares fit of the magnon density of states to the 35 T σ -polarized spectrum²⁸. The results show that ferromagnetic couplings J_1 and J_2 increase by $\sim 15\%$ and 8% respectively, whereas antiferromagnetic couplings $|J_3|$ and $|J_4|$ decrease by ~1%. These results indicate larger direct exchange¹¹ and smaller superexchange in the high field phase. The modified exchange interactions impact the electronic structure through spin-orbit processes.

Exciton behavior can be used to reveal magnetic symmetry in the high field phase of α -Fe₂O₃. At B_{C||}, the Zeeman splitting disappears, and a new pattern emerges (Fig. 2(d)). This more complicated exciton pattern arises from the transition to a different magnetic ordering. Above B_{C||}, lower symmetry lifts the degeneracy of the E1 α excition and gives rise to a new peak at ~9748 cm⁻¹⁴⁷. Two magnetic-dipole excitons, M1 σ and M2 σ , are also activated due to the new coordinate system (and selection rules) above 6.2 T. M1 σ hardens by 4.6 cm⁻¹, whereas M2 σ softens by 0.5 cm⁻¹. To explore the symmetry of the high field phase, we make use of the observed exciton pattern in the α -, σ -, and π -polarizations²⁸ and



FIG. 3: (a) Schematic view of the high temperature¹⁸/field magnetic structure of α -Fe₂O₃. The C2 rotational axis is indicated. (b) View of the magnetic ordering pattern looking down the [111] axis showing only the Fe centers. Blue arrows denote the collinear spin arrangement, and purple arrows schematically show the non-collinear state. The canting angle is on the order of $\sim 10^{-4}$ degrees and arises due to Dzyaloshinskii–Moriya interactions^{19,20}. Canting induces a weak ferromagnetic moment along the direction of the C2 rotational axis (red arrow). This symmetry is clarified on the central Fe site, where the spin vector on the site of interest is shown along with that on the site below.

invoke two additional constraints: (i) the new magnetic order symmetry must be a subgroup of the original $R\bar{3}c$ structure and (ii) spin collinearity is conserved above $B_{C\parallel}$. The latter is based upon the increase in magnon side band intensity with field and requires that inversion and C2 rotational operations be maintained. Several symmetry candidates emerge from this analysis⁴⁸. They include P1, $P\bar{1}$, C2, Cc, C2/c, R3, $R\bar{3}$, R32, and R3c. Only the C2/c monoclinic structure contains the necessary inversion center and C2 rotational axis. We thus infer that the field-induced magnetic ordering is monoclinic with C2/c symmetry - at least to first order. We know, however, that weak canting due to the Dzyaloshinskii-Moriya effect formally eliminates the inversion center^{19,20} (Fig. 3(b)). This higher order effect places additional constrains on the system. Reevaluating the magnetic symmetry with the surviving C2 operation (and without the inversion center), we find that only the C2 monoclinic structure meets our criteria²⁸. We therefore conclude that the formal symmetry of the field-induced ordered phase of α -Fe₂O₃ is C2. Switching the applied field to the (111) plane, the exciton pattern in the new magnetic order is consistent with the aforementioned group theory predictions for C2/c or C2 magnetic symmetry (depending upon whether spin canting-induced symmetry breaking is ignored or invoked)²⁸, demonstrating that magnetic fields in different directions drive to identical high field phases in α -Fe₂O₃.

Summarizing, we report the discovery of magnetochromism in one of the world's oldest and most iconic antiferromagnetic materials, α -Fe₂O₃. In addition to a field-induced color change due to the reorientation of magnetic order, the exciton pattern reveals C2 monoclinic symmetry in the high field phase. This work advances our understanding of the interplay between charge and magnetism, identifies a strategy for the development of next-generation color change materials, and motivates fundamental research on other iron-based materials under extreme conditions and away from the static limit.

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