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A Phase Transition arising from the Underscreened Anderson Lattice Model: A Candidate Concept for Explaining the Hidden Order in URu₂Si₂.

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Abstract

We analyze a novel type of phase transition that appears in the spin-rotationally invariant form of the underscreened Anderson Lattice Model and we obtain, with decreasing temperature, a continuous transition with opening of a gap. We suggest that this model might describe the “Hidden Order” transition in URu₂Si₂. We also examine the gaps that appear in the electronic dispersion relations of the bands of different orbital character and compare our results with those found through photo-electron spectroscopy.

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Keywords: underscreened Anderson Lattice, Hidden-Order, URu₂Si₂

1 Introduction

In 1985 it was found[1] that URu₂Si₂ became superconducting below 0.8K, and also that there was a large jump in the specific heat at 17.5 K. In 1986, transport, thermal and magnetic measurements[2] on URu₂Si₂ produced compelling evidence that the transition at 17.5 K produced a gap ~ 10 meV which spread across about 40 % of the Fermi-surface. Far-infrared reflectance measurements[3] provided direct evidence for a gap with a magnitude between 5.6 and 7.5 meV which formed below 17.5 K. Recent ultra-fast spectroscopy measurements[4] provided corroborating evidence for the

existence of gaps of 5 and 10 meV, and STM measurements[5] have shown the existence of 5 meV gaps. Initially, the transition was assumed to be of magnetic origin. Inelastic neutron scattering measurements[6] showed evidence for the existence of tiny ordered magnetic moments (of the order $10^{-2} \mu_B$). However, subsequent pressure measurements[7] showed that a transition to an antiferromagnetic state with well-defined ordered moments (of the order $0.4 \mu_B$) occurs above $P=0.5$ GPa. The application of pressure was found to have only a minimal effect on the transition temperature. NMR measurements[8] indicated that, at zero pressure, the system was inhomogeneous, containing both paramagnetic regions and regions of antiferromagnetism. It was confirmed that the patches of antiferromagnetism can be created either by impurity doping[9], or by stress fields in pure URu_2Si_2 [10]. Hence, it is now thought that the origin of the transition at 17.5 K is not due to the appearance of small moment antiferromagnetism, although there is significant evidence that the transition is produced by Fermi-surface nesting which is similar to that found in the high pressure Neel state[11, 12, 13, 14]. Since experiments were unable to identify the nature of the order parameter, the transition has come to be known as a “Hidden-Order” Transition. Over the twenty-seven years that have elapsed since its first discovery, there have been many theoretical[15, 16, 17, 18] and experimental attempts to uncover the nature of the transition. Some recent theories include descriptions of states with rotational spin-currents that break spin-rotational invariance, but not time-reversal invariance[15], modulated spin liquid states that break the C_4 rotational invariance[16], states with incommensurate hybridization density waves promoted by the spin-independent coulomb interaction between the 5f and conduction electrons[17], or unconventional spin-density wave states where the order parameter has d-wave symmetry[18]. The present status of the field has been comprehensively reviewed in reference[19].

In this paper, we shall examine the underscreened Anderson Lattice Model which describes two itinerant 5f bands which resonantly couple to a single conduction band. Like the Anderson Lattice Model[20, 21] which is a generalization of a single impurity model introduced to describe a magnetic impurity in a metal[22], the underscreened Anderson Lattice Model[23, 24] is a generalization of a model introduced to describe the single-impurity underscreened Kondo effect[25, 26]. The model is generic and is not specifically tailored to the electronic structure of URu_2Si_2 [27, 28].

The underscreened Kondo or Anderson Lattice Model were recently studied to describe the competition between ferromagnetism and Kondo effect, which has been observed in uranium and neptunium compounds [23, 24, 29]. But, in the URu_2Si_2 compound, the Kondo effect is not involved in the transition occurring at 17.5 K and we can use a simplified mean-field treatment of the Anderson Hamiltonian at the Hartree-Fock level. Moreover, as it will be explained later, we can simplify the problem by taking a zero value for one of the two d-f hybridization terms, without changing the physical results. Thus in the present model, the

f electrons interact via local (spin-rotationally-invariant) Coulomb and exchange interactions. We find that the system exhibits a competition between magnetic ordering and a novel type of ordering. The novel state corresponds to an inhomogeneous state with wave vector \underline{Q} and in which the 5f bands are mixed, in contrast the normal state where the 5f bands have pure orbital characters. However, the admixture is spin-dependent and, in certain circumstances, corresponds to a broken gauge invariance of the Hamiltonian. The type of correlation is best illustrated in the limit of zero hybridization, where the correlation can be described simply in terms of the 5f electron creation operators corresponding to the two bands (labeled by α and β). The correlation can be seen in the even-parity, broken time-reversal symmetry state constructed from products of operators of the type

$$\left(\alpha_{\underline{k}} f_{\underline{k}+\underline{Q},\uparrow}^{\dagger,\alpha} + \beta_{\underline{k}} f_{\underline{k},\uparrow}^{\dagger,\beta} \right) \left(\beta_{\underline{k}}^* f_{-\underline{k}-\underline{Q},\downarrow}^{\dagger,\alpha} - \alpha_{\underline{k}}^* f_{-\underline{k},\downarrow}^{\dagger,\beta} \right) \quad (1)$$

acting on a vacuum state $|0\rangle$, where the coefficients of the operators are normalized to unity

$$|\alpha_{\underline{k}}|^2 + |\beta_{\underline{k}}|^2 = 1 \quad (2)$$

It is seen that the mixed two-particle states and the corresponding unmixed two-particle states ($\alpha_{\underline{k}} = 1$, $\beta_{\underline{k}} = 0$) have precisely the same type of expectation values for spin-conserving 5f orbital single-particle operators, and likewise for the 5f spin operators. The interference terms can only be measured by a combined spin and orbital sensitive measurement.

2 The Model

The model describes two degenerate localized 5f bands (labeled by $\chi = \alpha, \beta$), which acquire itinerant character by direct hopping between neighboring 5f shells and the mixing with one itinerant conduction band. The Hamiltonian is written as

$$\hat{H} = \hat{H}_f + \hat{H}_d + \hat{H}_{fd} \quad (3)$$

where \hat{H}_f describes the f electrons, \hat{H}_d describes the itinerant conduction band and \hat{H}_{fd} describes the hybridization. The Hamiltonian \hat{H}_f is given by

$$\begin{aligned} \hat{H}_f = & \sum_{\underline{k},\sigma,\chi} E_f^\chi(\underline{k}) n_{f,\underline{k},\sigma}^\chi + \left(\frac{U}{2N} \right) \sum_{\underline{k},\underline{k}',\underline{q},\sigma,\sigma',\chi,\chi'} f_{\underline{k}+\underline{q},\sigma}^{\dagger,\chi} f_{\underline{k}'-\underline{q},\sigma'}^{\dagger,\chi'} f_{\underline{k},\sigma'}^\chi f_{\underline{k}',\sigma}^\chi \\ & + \left(\frac{J}{2N} \right) \sum_{\underline{k},\underline{k}',\underline{q},\sigma,\sigma',\chi,\chi'} f_{\underline{k}+\underline{q},\sigma}^{\dagger,\chi} f_{\underline{k}'-\underline{q},\sigma'}^{\dagger,\chi'} f_{\underline{k}',\sigma'}^\chi f_{\underline{k},\sigma}^\chi \end{aligned} \quad (4)$$

in which the first term, proportional to $E_f^\chi(\underline{k})$, describes the dispersion relation for the χ -th 5f band while the second and third terms describe the screened Coulomb interaction between the 5f electrons in the same

5f-shell. The third term contains an inter-orbital exchange interaction which is required to make the model spin rotationally invariant[30]. Since we have selected only two 5f bands, the model is not invariant under spatial rotations[31]. The conduction electron Hamiltonian \hat{H}_d can be expressed as

$$\hat{H}_d = \sum_{\underline{k}, \sigma} \epsilon(\underline{k}) d_{\underline{k}, \sigma}^\dagger d_{\underline{k}, \sigma} \quad (5)$$

where $\epsilon(\underline{k})$ describes the dispersion relation of conduction electrons labeled by the Bloch wave vector \underline{k} . The Hamiltonian describing the on-site hybridization process is given, as usual, by

$$\hat{H}_{fd} = \sum_{\underline{k}, \sigma, \chi} \left(V_\chi(\underline{k}) f_{\underline{k}, \sigma}^{\dagger, \chi} d_{\underline{k}, \sigma} + V_\chi^*(\underline{k}) d_{\underline{k}, \sigma}^\dagger f_{\underline{k}, \sigma}^\chi \right) \quad (6)$$

The Coulomb interaction can be re-written in the form

$$\begin{aligned} \hat{H}_{int} &= \left(\frac{U - J}{2N} \right) \sum_{\underline{k}, \underline{k}', \underline{q}, \sigma, \chi \neq \chi'} f_{\underline{k} + \underline{q}, \sigma}^{\dagger, \chi} f_{\underline{k}, \sigma}^\chi f_{\underline{k}' - \underline{q}, \sigma}^{\dagger, \chi'} f_{\underline{k}', \sigma}^{\chi'} \\ &+ \left(\frac{U}{2N} \right) \sum_{\underline{k}, \underline{k}', \underline{q}, \sigma, \chi, \chi'} f_{\underline{k} + \underline{q}, \sigma}^{\dagger, \chi} f_{\underline{k}, \sigma}^\chi f_{\underline{k}' - \underline{q}, -\sigma}^{\dagger, \chi'} f_{\underline{k}', -\sigma}^{\chi'} \\ &+ \left(\frac{J}{2N} \right) \sum_{\underline{k}, \underline{k}', \underline{q}, \sigma, \chi \neq \chi'} f_{\underline{k} + \underline{q}, \sigma}^{\dagger, \chi} f_{\underline{k}, \sigma}^{\chi'} f_{\underline{k}' - \underline{q}, -\sigma}^{\dagger, \chi'} f_{\underline{k}', -\sigma}^\chi \quad (7) \end{aligned}$$

To aid the analysis, we shall introduce the normalized non-Hermitian operator $\hat{z}_{\underline{q}, \sigma}$

$$\hat{z}_{\underline{q}, \sigma} = \frac{1}{N} \sum_{\underline{k}} f_{\underline{k} + \underline{q}, \sigma}^{\dagger, \beta} f_{\underline{k}, \sigma}^\alpha \quad (8)$$

This product of operators provides a measure of the coupling between the two types of f bands. The last line in the interaction of eqn.(7) originates from the spin-flip term which was required from considerations of spin-rotational invariance[30]. We shall also introduce the 5f orbital charge density operator via

$$\hat{\rho}_{\underline{q}, \sigma}^\chi = \frac{1}{N} \sum_{\underline{k}} f_{\underline{k} + \underline{q}, \sigma}^{\dagger, \chi} f_{\underline{k}, \sigma}^\chi \quad (9)$$

It should be noted that the first term in the interaction eqn.(7) can be expressed in terms of products of either $\hat{z}_{\underline{q}, \sigma}$ or of the orbital density operators $\hat{\rho}_{\underline{q}, \sigma}^\chi$.

We shall assume that the α and β bands are degenerate and that $V_\beta(\underline{k}) = 0$. In this case, one can see that the Hamiltonian is invariant under a gauge transformation of the β electrons, which is independent of the gauge invariance of the α and conduction electrons. This gauge symmetry is analogous to the chiral gauge symmetry present in the massless

limit of the Dirac equation. It is important to note that taking $V_\beta(\underline{k}) = 0$ does not change the physical results but only simplifies the calculations.

3 The Mean-Field Approximation

In the mean-field approximation, the interaction term in the Hamiltonian is expanded in powers of the fluctuations of bi-linear products of operators.

$$\begin{aligned}\Delta \hat{n}_{f,\sigma}^x &= \left(\hat{n}_{f,\sigma}^x - n_{f,\sigma}^x \right) \\ \Delta \hat{z}_{q,\sigma} &= \left(\hat{z}_{q,\sigma} - z_{q,\sigma} \right)\end{aligned}\quad (10)$$

where the hats have been dropped for the averaged quantities. We have assumed that the average f electron occupation numbers are translationally invariant (i.e. $n_{f,i,\sigma}^x = n_{f,\sigma}^x$), but we have retained the momentum-dependence of the expectation values of the non-Hermitian operators. The terms in the Hamiltonian quadratic in the fluctuations are neglected. This approximation reduces the Hamiltonian to an expression quadratic in fermionic operators that can be diagonalized. We shall assume that the average $z_{q,\sigma}$ is a non-zero complex number for commensurate momentum transfers \underline{Q} , where \underline{Q} could be any vector that is both on and normal to the Brillouin zone boundary. The assumption of a finite momentum transfer corresponds to having an inhomogeneous state. However, the inhomogeneous nature of this state is masked in either purely spin sensitive or purely orbital sensitive measurements.

In the Hartree-Fock Approximation, the temporal and spatial Fourier transform of the single-electron f-f Green's function satisfy the equations of motion

$$\begin{aligned}\left(\omega - E_{f,\sigma}^\alpha(\underline{k}) \right) G_{ff,\sigma}^{\alpha,\chi'}(\underline{k}, \underline{k}', \omega) &= \delta^{\alpha,\chi'} \delta_{\underline{k}, \underline{k}'} + V_\alpha(\underline{k}) G_{df,\sigma}^{\chi'}(\underline{k}, \underline{k}', \omega) \\ &+ \kappa_{-\underline{Q},\sigma} G_{ff,\sigma}^{\beta,\chi'}(\underline{k} + \underline{Q}, \underline{k}', \omega) \\ \left(\omega - E_{f,\sigma}^\beta(\underline{k}) \right) G_{ff,\sigma}^{\beta,\chi'}(\underline{k}, \underline{k}', \omega) &= \delta^{\beta,\chi'} \delta_{\underline{k}, \underline{k}'} + V_\beta(\underline{k}) G_{df,\sigma}^{\chi'}(\underline{k}, \underline{k}', \omega) \\ &+ \kappa_{\underline{Q},\sigma}^* G_{ff,\sigma}^{\alpha,\chi'}(\underline{k} - \underline{Q}, \underline{k}', \omega)\end{aligned}\quad (11)$$

where the Hartree-Fock f band dispersion relation $E_{f,\sigma}^X(\underline{k})$ is given by

$$E_{f,\sigma}^X(\underline{k}) = E_f^X(\underline{k}) + \sum_{\chi'} \left((U - J) n_{f,\sigma}^{\chi'} (1 - \delta^{\chi,\chi'}) + U n_{f,-\sigma}^{\chi'} \right) \quad (12)$$

and the gap parameter $\kappa_{\underline{Q},\sigma}$ is defined as the complex number

$$\kappa_{\underline{Q},\sigma} = J z_{-\underline{Q},-\sigma} - (U - J) z_{-\underline{Q},\sigma} \quad (13)$$

The mixed f-d Green's function is found to satisfy

$$\left(\omega - \epsilon(\underline{k}) \right) G_{df,\sigma}^{\chi'}(\underline{k}, \underline{k}', \omega) = V_\alpha(\underline{k})^* G_{ff,\sigma}^{\alpha,\chi'}(\underline{k}, \underline{k}', \omega) + V_\beta(\underline{k})^* G_{ff,\sigma}^{\beta,\chi'}(\underline{k}, \underline{k}', \omega) \quad (14)$$

The above equation form a closed set which are easily solved when $V_\beta(\underline{k}) = 0$ and \underline{Q} is commensurate with the lattice. The solutions are given by

$$\begin{aligned} G_{ff,\sigma}^{\alpha,\chi'}(\underline{k}, \underline{k}', \omega) &= \frac{(\omega - \epsilon(\underline{k}))}{D_\sigma(\underline{k}, \omega)} \left[(\omega - E_{f,\sigma}^\beta(\underline{k} + \underline{Q})) \delta^{\alpha,\chi'} \delta_{\underline{k},\underline{k}'} + \kappa_{\underline{Q},\sigma} \delta^{\beta,\chi'} \delta_{\underline{k}+\underline{Q},\underline{k}'} \right] \\ G_{ff,\sigma}^{\beta,\chi'}(\underline{k}, \underline{k}', \omega) &= D_\sigma(\underline{k} + \underline{Q}, \omega)^{-1} \left[\kappa_{\underline{Q},\sigma}^* (\omega - \epsilon(\underline{k} + \underline{Q})) \delta^{\alpha,\chi'} \delta_{\underline{k}+\underline{Q},\underline{k}'} \right. \\ &\quad \left. + \left((\omega - E_{f,\sigma}^\alpha(\underline{k} + \underline{Q}))(\omega - \epsilon(\underline{k} + \underline{Q})) - |V_\alpha(\underline{k} + \underline{Q})|^2 \right) \delta^{\beta,\chi'} \delta_{\underline{k},\underline{k}'} \right] \end{aligned} \quad (15)$$

where the denominator $D_\sigma(\underline{k}, \omega)$ is given by

$$\begin{aligned} D_\sigma(\underline{k}, \omega) &= \left[\left(\omega - E_{f,\sigma}^\beta(\underline{k} + \underline{Q}) \right) \left(\omega - E_{f,\sigma}^\alpha(\underline{k}) \right) - |\kappa_{\underline{Q},\sigma}|^2 \right] \left(\omega - \epsilon(\underline{k}) \right) \\ &\quad - |V_\alpha(\underline{k})|^2 \left(\omega - E_{f,\sigma}^\beta(\underline{k} + \underline{Q}) \right) \end{aligned} \quad (16)$$

For completeness, we give the d-electrons Green's function

$$G_{dd,\sigma}(\underline{k}, \underline{k}', \omega) = \frac{\delta_{\underline{k},\underline{k}'}}{D_\sigma(\underline{k}, \omega)} \left[\left(\omega - E_{f,\sigma}^\beta(\underline{k} + \underline{Q}) \right) \left(\omega - E_{f,\sigma}^\alpha(\underline{k}) \right) - |\kappa_{\underline{Q},\sigma}|^2 \right] \quad (17)$$

The zeros of the denominator $D_\sigma(\underline{k}, \omega)$ yield the Hartree-Fock quasi-particle dispersion relations for electrons of spin σ .

The quantity $z_{\underline{Q},\sigma}$ is determined from the expectation value of the product of operators which are off-diagonal in the band indices

$$z_{\underline{Q},\sigma}^* = \frac{1}{N} \sum_k \langle f_{\underline{k},\sigma}^\dagger, \alpha f_{\underline{k}+\underline{Q},\sigma}^\beta \rangle \quad (18)$$

which can be related to the definition of the real-time Green's function for small negative times $t = -\eta$ where $\eta \rightarrow 0$. On expressing the real time Green's function in terms of its Fourier Transform and on closing the contour in the upper-half complex ω -plane, as well as noting the pole structure of the Green's function, one finally arrives at the result

$$z_{\underline{Q},\sigma}^* = -\frac{1}{N} \sum_{\underline{k}} \left[\int_C \frac{d\omega}{2\pi i} f(\omega) G_{ff,\sigma}^{\beta,\alpha}(\underline{k} + \underline{Q}, \underline{k}, \omega) \right] \quad (19)$$

where $f(\omega)$ is the Fermi-function and where the contour C encloses the real axis. Since the f-f Green's functions involve $z_{\underline{Q},-\sigma}$, the $z_{\underline{Q}}$'s must be determined self-consistently. Furthermore, since the Green's functions for

5f electrons of spin σ with mixed band-indices are odd functions of $z_{Q,-\sigma}$, the self-consistency equations have the trivial solution $z_{Q,\sigma} = 0 \forall \sigma$ which corresponds to the conservation of the number of electrons in the β -band. We shall first consider the trivial solution with $z_{Q,\sigma} = 0$.

3.1 The Normal State

The normal state is defined as that for which $z_{Q,\sigma} = 0 \forall \sigma$. The single-electron Green's functions for the 5f bands reduce to

$$\begin{aligned} G_{ff,\sigma}^{\beta}(\underline{k}, \underline{k}', \omega) &= \frac{\delta_{\underline{k}, \underline{k}'}}{\omega - E_{f,\sigma}^{\beta}(\underline{k})} \\ G_{ff,\sigma}^{\alpha}(\underline{k}, \underline{k}', \omega) &= \frac{(\omega - \epsilon(\underline{k})) \delta_{\underline{k}, \underline{k}'}}{(\omega - E_{f,\sigma}^{\alpha}(\underline{k})) (\omega - \epsilon(\underline{k})) - |V_{\alpha}(\underline{k})|^2} \end{aligned} \quad (20)$$

in which the dispersion relation for the β 5f Hartree-Fock quasi-particles is simply given by $E_{f,\sigma}^{\beta}(\underline{k})$. The conduction electron states are admixed with 5f states of α -character and their Green's functions reduce to

$$G_{dd,\sigma}(\underline{k}, \underline{k}', \omega) = \frac{(\omega - E_{f,\sigma}^{\alpha}(\underline{k})) \delta_{\underline{k}, \underline{k}'}}{(\omega - E_{f,\sigma}^{\alpha}(\underline{k})) (\omega - \epsilon(\underline{k})) - |V_{\alpha}(\underline{k})|^2} \quad (21)$$

The Green's functions for the α -th band can be re-cast in the form

$$\begin{aligned} G_{ff,\sigma}^{\alpha}(\underline{k}, \underline{k}, \omega) &= \frac{|A_{\sigma}^{+}(\underline{k})|^2}{\omega - E_{\sigma}^{+}(\underline{k})} + \frac{|A_{\sigma}^{-}(\underline{k})|^2}{\omega - E_{\sigma}^{-}(\underline{k})} \\ G_{dd,\sigma}(\underline{k}, \underline{k}, \omega) &= \frac{|B_{\sigma}^{+}(\underline{k})|^2}{\omega - E_{\sigma}^{+}(\underline{k})} + \frac{|B_{\sigma}^{-}(\underline{k})|^2}{\omega - E_{\sigma}^{-}(\underline{k})} \end{aligned} \quad (22)$$

which the dispersion relations of the hybridized Hartree-Fock quasi-particle bands $E_{\sigma}^{\pm}(\underline{k})$ are given by

$$E_{\sigma}^{\pm}(\underline{k}) = \left(\frac{E_{f\sigma}^{\alpha}(\underline{k}) + \epsilon(\underline{k})}{2} \right) \pm \sqrt{\left(\frac{E_{f\sigma}^{\alpha}(\underline{k}) - \epsilon(\underline{k})}{2} \right)^2 + |V_{\alpha}(\underline{k})|^2} \quad (23)$$

The f and d characters of the bands have weights given by $|A_{\sigma}^{\pm}(\underline{k})|^2$ and $|B_{\sigma}^{\pm}(\underline{k})|^2$, respectively, where

$$\begin{aligned} |A_{\sigma}^{\pm}(\underline{k})|^2 &= \frac{1}{2} \left[1 \pm \frac{(E_{f\sigma}^{\alpha}(\underline{k}) - \epsilon(\underline{k}))}{\sqrt{(E_{f\sigma}^{\alpha}(\underline{k}) - \epsilon(\underline{k}))^2 + 4|V_{\alpha}(\underline{k})|^2}} \right] \\ |B_{\sigma}^{\pm}(\underline{k})|^2 &= \frac{1}{2} \left[1 \mp \frac{(E_{f\sigma}^{\alpha}(\underline{k}) - \epsilon(\underline{k}))}{\sqrt{(E_{f\sigma}^{\alpha}(\underline{k}) - \epsilon(\underline{k}))^2 + 4|V_{\alpha}(\underline{k})|^2}} \right] \end{aligned} \quad (24)$$

The k -dependence of the quasi-particle dispersion relations and the form factors are sketched in fig.(1). In the figure, the energies are given in units of half the conduction band width. The 5f density of states are shown in fig.(2). Due to the relatively small value of the hybridization ($V=1/10$) and the large value of the 5f band width ($W_f = 6/10$) caused by f-f hopping, the α and β 5f density of states have quite similar shapes

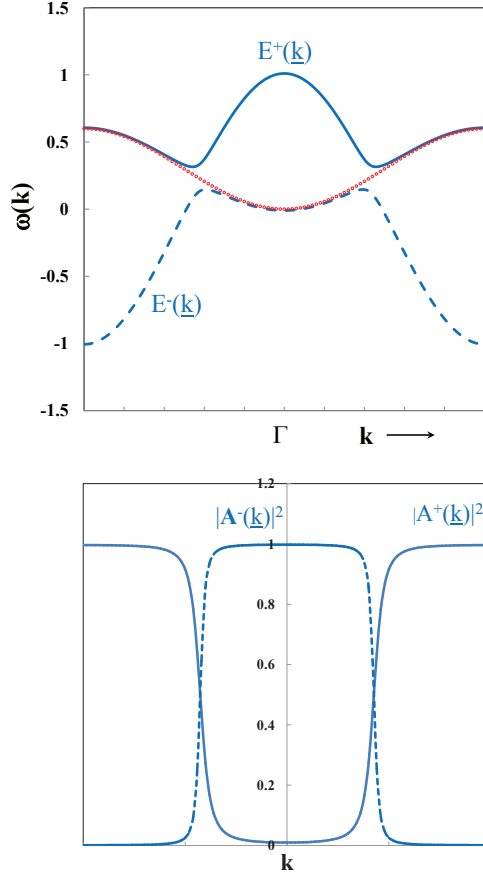


Figure 1: (Color online) [Upper Panel] The three dispersion relations for the bands of the under-screened Anderson Lattice Model in the normal state (Schematic). The momentum \underline{k} is taken to be along the (1, 1, 1) direction. The upper ($E^+(\underline{k})$) and lower ($E^-(\underline{k})$) bands are comprised of α 5f states hybridized with the conduction band. The band in the center marked by the red curve (open circles) is the unhybridized β -band. [Lower Panel] The 5f weights of the upper and lower hybridized bands ($|A^\pm(\underline{k})|^2$) are plotted as functions of \underline{k} . The β band has pure 5f character.

at energies removed from the hybridization gap. The hybridization gap is centered on $E_f/(1+W_f/2)$. It should be noted that near the band edges, the shape of the band is similar to that of the unhybridized bands except for small energy shifts of the order of $|V|^2$.

Magnetic Instabilities

In the Hartree-Fock approximation, the paramagnetic phase is gener-

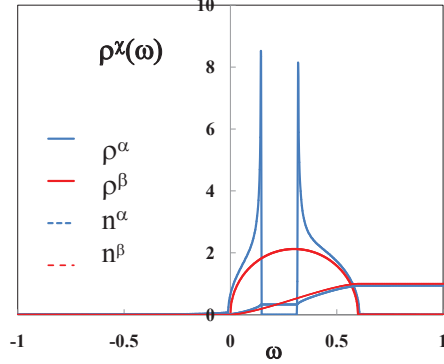


Figure 2: (Color online) The α and β components of the 5f density of states per spin versus ω (solid lines), and n_f^α and n_f^β electron per spin (dashed lines) as functions of μ , in which the Hartree-Fock energies of the 5f orbitals have been kept constant. The simple cubic tight-binding density of states has been approximated by a semi-elliptical form.

ally expected to retain its stability relative to magnetic states for small values of the Coulomb U and the exchange J interactions. The dynamic spin susceptibility for the Hartree-Fock state can be found from the equations of motion in the Random-Phase Approximation[32]. The static susceptibility found by setting $\omega = 0$, shows a pole when

$$(1 - U \chi_f^{\alpha, \alpha(0)}(\underline{q}, 0)) (1 - U \chi_f^{\beta, \beta(0)}(\underline{q}, 0)) - J^2 \chi_f^{\alpha, \alpha(0)}(\underline{q}, 0) \chi_f^{\beta, \beta(0)}(\underline{q}, 0) = 0 \quad (25)$$

which signals the instability to a spin density wave with wave vector q . In the above equation, the quantity $\chi_f^{\beta, \beta(0)}(\underline{q}, 0)$ represents the response of the β -band to a Weiss field and is given by

$$\chi_f^{\beta, \beta(0)}(\underline{q}, 0) = \frac{1}{N} \sum_{\underline{k}} \left(\frac{f(E_f^\beta(\underline{k} + \underline{q})) - f(E_f^\beta(\underline{k}))}{E_f^\beta(\underline{k}) - E_f^\beta(\underline{k} + \underline{q})} \right) \quad (26)$$

and $\chi_f^{\alpha, \alpha(0)}(\underline{q}, 0)$ is the reduced Hartree-Fock 5f spin susceptibility for the α -band which is given by

$$\chi_f^{\alpha, \alpha(0)}(\underline{q}, 0) = \frac{1}{N} \sum_{\underline{k}; \pm, \mp} |A^\pm(\underline{k} + \underline{q})|^2 |A^\mp(\underline{k})|^2 \left(\frac{f(E^\pm(\underline{k} + \underline{q})) - f(E^\pm(\underline{k}))}{E^\mp(\underline{k}) - E^\pm(\underline{k} + \underline{q})} \right) \quad (27)$$

The quantity $\chi_f^{\alpha, \alpha(0)}(\underline{q}, 0)$ has the same form as the reduced 5f susceptibility found from an R.P.A. study of the Anderson Lattice[21]. Due to the E_f dependence of the form factors in the α -band 5f electron density of states, the susceptibility $\chi_f^{\alpha, \alpha(0)}(\underline{q}, 0)$ has a band Van-Vleck component in addition to a modified Pauli-paramagnetic term. Both Hartree-Fock susceptibilities are always positive. It is expected that if the β -band is described in a tight-binding approximation, the reduced susceptibility $\chi_f^{\beta, \beta(0)}(\underline{q}, 0)$

may show a divergence at the β -band nesting vector $\underline{Q} = (1, 1, 1)$ when μ approaches the center of the band. In any case, it should be noted that the magnetic instability is promoted by nesting the Fermi-surface sheets that have the same 5f orbital characters.

3.2 The Instability to a Novel State

We will now introduce a Novel state which is based on a non zero value of $z_{\underline{Q},\sigma}^*$ given by the equation (18). This new parameter links the two f-electrons of different orbitals α and β and exists in the situation of the underscreened Anderson Lattice model, but not in the classical Anderson Lattice model. This model is used here since we are now discussing the case of the uranium compound URu₂Si₂ [23].

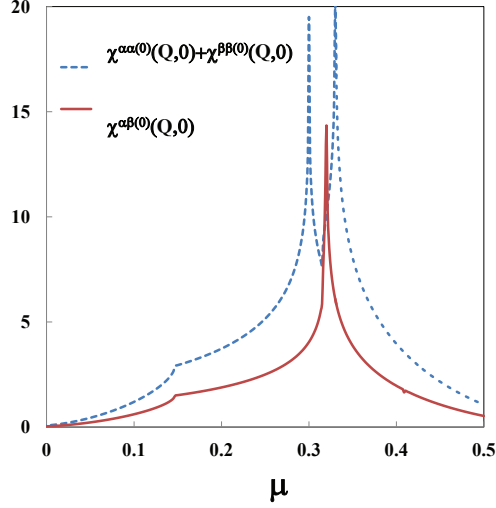


Figure 3: (Color online) The interband susceptibility $\chi_f^{\alpha,\beta(0)}(\underline{Q})$ (solid line) and the sum of the intraband susceptibilities $\chi_f^{\alpha,\alpha(0)}(\underline{Q}) + \chi_f^{\beta,\beta(0)}(\underline{Q})$ (dashed line) as functions of μ , with fixed Hartree-Fock f electron bands $E_{f,\sigma}^x(\underline{k})$. The unhybridized f and conduction band dispersion relations have been described within a simple cubic tight-binding model.

The self-consistency condition for $z_{\underline{Q},\sigma}^*$ can be expressed as

$$z_{\underline{Q},\sigma}^* = -\kappa_{\underline{Q},\sigma}^* \frac{1}{N} \sum_{\underline{k}} \int_C \frac{d\omega}{2\pi i} \frac{(\omega - \epsilon(\underline{k}))f(\omega)}{D_\sigma(\underline{k}, \omega)} \quad (28)$$

If the novel transition is second-order, then at the transition we expect that $\bar{F}_\sigma \sim 0$, so the above set of equations can be linearized to yield

$$\left[1 - (U - J)\chi_{f,\sigma}^{\alpha,\beta,(0)}(\underline{Q}, 0) \right] z_{\underline{Q},\sigma}^* = -z_{\underline{Q},-\sigma}^* J\chi_{f,\sigma}^{\alpha,\beta}(\underline{Q}, 0)$$

$$\left[1 - (U - J)\chi_{f,-\sigma}^{\alpha,\beta,(0)}(\underline{Q}, 0)\right] z_{\underline{Q},-\sigma}^* = -z_{\underline{Q},\sigma}^* J \chi_{f,-\sigma}^{\alpha,\beta}(\underline{Q}, 0) \quad (29)$$

where

$$\chi_{f,\sigma}^{\alpha,\beta,(0)}(\underline{Q}, 0) = \frac{1}{N} \sum_{\mathbf{k}, \pm} |A_{\sigma}^{\pm}(\mathbf{k})|^2 \left(\frac{f(E_{\sigma}^{\pm}(\mathbf{k})) - f(E_{f,\sigma}^{\beta}(\mathbf{k} + \underline{Q}))}{E_{f,\sigma}^{\beta}(\mathbf{k} + \underline{Q}) - E_{\sigma}^{\pm}(\mathbf{k})} \right) \quad (30)$$

The summand in $\chi_{f,\sigma}^{\alpha,\beta,(0)}$ is manifestly positive. Hence, for positive J we may have a state for which the z -component of the spin magnetization for each 5f band vanishes (i.e. $\bar{n}_{f,\sigma}^{\chi} = \bar{n}_{f,-\sigma}^{\chi}$) and the spin-up with the spin-down dispersion relations are identical, but for which $z_{\underline{Q},\sigma} = -z_{\underline{Q},-\sigma}$. For negative values of J , one may have a paramagnetic solution for which $z_{\underline{Q},\sigma} = z_{\underline{Q},-\sigma}$. If $|z_{\underline{Q},\sigma}| \neq |z_{\underline{Q},-\sigma}|$, the system may have a non-zero value of the z -component of the magnetization. We shall, henceforth, restrict our attention to positive J . The critical value of J , J_c , at which this new phase may occur is given by the expression

$$\left[1 - (U - J_c)\chi_{f,\sigma}^{\alpha,\beta,(0)}(\underline{Q}, 0)\right] \left[1 - (U - J_c)\chi_{f,-\sigma}^{\alpha,\beta,(0)}(\underline{Q}, 0)\right] = J_c^2 \chi_{f,-\sigma}^{\alpha,\beta,(0)}(\underline{Q}, 0) \chi_{f,\sigma}^{\alpha,\beta,(0)}(\underline{Q}, 0) \quad (31)$$

It is seen that the novel transition is promoted by the Fermi-surface nesting between the two bands with different 5f orbital characters. In this aspect our theory bares a resemblance to the that of Dubi and Balatsky[17]. However, in their case the hybridization wave is driven by a spin-independent interaction, whereas in our theory the transition is driven by the spin-flip part of the Hund's rule exchange and, thus, breaks spin-rotational invariance. The interband susceptibility $\chi_f^{\alpha,\beta,(0)}(\underline{Q}, 0)$ is shown as a function of μ in fig.(3). The graph determines the μ -dependence of the critical value of J_c^{-1} at which the novel state first becomes stable, when $U = J$. The figure also shows $\chi_f^{\alpha,\alpha(0)}(\underline{Q}, 0) + \chi_f^{\beta,\beta(0)}(\underline{Q}, 0)$ which, when $U = J$, determines the critical value of J^{-1} required for the antiferromagnetic instability. The magnetic susceptibility of the β -band diverges logarithmically when the Fermi-surface approaches the perfect nesting condition $\mu = E_{f,\sigma}^{\beta}(\mathbf{k}) = E_{f,\sigma}^{\beta}(\mathbf{k} + \underline{Q})$ at $\mu = 0.3$. The second peak originates from the α -band susceptibility. This is expected since the hybridized α -band follows a similar dispersion relation to the β -band, except it is split and shifted by an energy of the order of $|V|^2$ by the hybridization and, therefore, should exhibit a similar structure at a shifted value of the chemical potential. The interband susceptibility shows a similar nesting peak at an intermediate value of μ . The graph indicates that the commensurate novel state may have a narrow region of phase space where it is stable against paramagnetism and Neel antiferromagnetism. The instability criterion can be extended to incommensurate wave vectors \underline{q} by using the Random Phase Approximation. For an incommensurate transition, where the band folding is not appropriate, one should see clear signs of gapping in angle resolved photoemission, at wave vectors corresponding to the nesting of sheets of the Fermi-surface with different orbital characters. However, the gapping is expected to produce a new branch that only has appreciable

intensity in the nested region of the Fermi-surface. Since the Neel antiferromagnetic state is expected to be produced by intra-band nesting, and since this model is characterized by a close proximity of the intra and inter band nesting vectors, it is expected that these two phases are competing for the same regions of the Fermi-surface. It should be noted the role of nesting in the electronic structure of URu₂Si₂ has been investigated within the LDA[27, 28]. The LDA studies[27, 28] have identified a commensurate nesting vector associated with the antiferromagnetic phase and have also discovered the pure character of the 5f bands in the normal state. The underscreened Anderson Lattice Model, although generic, captures these features. Our description is also consistent with the interpretation of inelastic neutron scattering measurements[11, 12]. Hence, it seems highly unlikely that this novel state will coexist homogeneously with antiferromagnetism. This is consistent with the experimental observations[9] on URu₂Si₂. For $U = J$ and a general value of J away from the quantum critical value J_c , the temperature dependence of the gap $|\kappa_{\underline{Q},\sigma}(T)|$ has the usual mean-field variation shown in fig.(4). The calculated ratio of

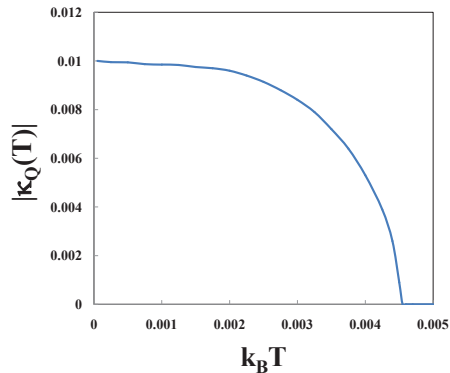


Figure 4: The temperature dependence of the gap parameter $|\kappa_{\underline{Q},\sigma}(T)|$ for a value of $J=0.128$ and $U = J$. This value of J is comparable to the assumed value of 0.6 for the 5f band-width due to direct f-f hopping.

the gap parameter to T_c of ~ 2.25 should be compared with the experimentally determined value[5] of 2.9 ± 0.15 . The result shown on fig.(4) is important for the comparison with the compound URu₂Si₂. It supports our hypothesis that the change occurring at 17.5 K is due to an opening of a gap resulting from the underscreened band structure with a two-fold degenerate 5f-level, as expected in uranium compounds.

4 Results and Discussion

The novel state is described by a non-zero value of the complex order parameter $Z_{\underline{Q}}$, defined as the trace over spins of a spin-dependent expect-

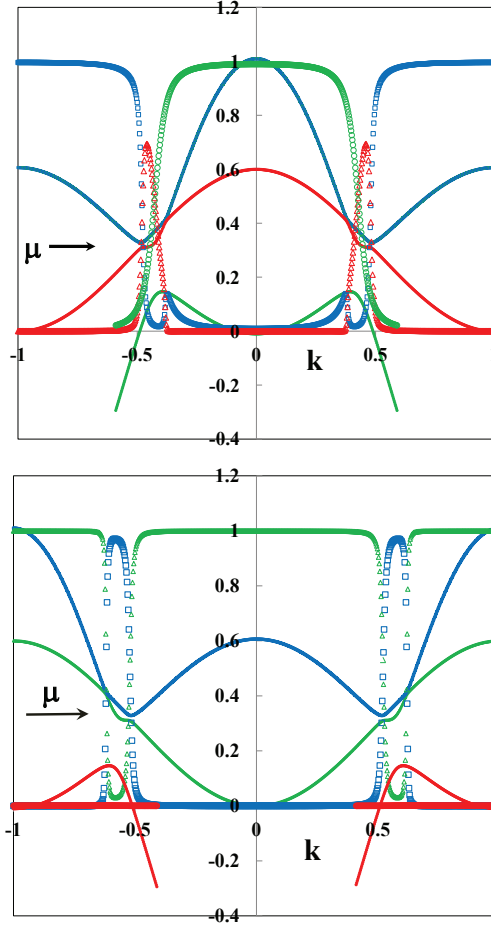


Figure 5: (Color online) [Upper Panel] A (schematic) close up view of the dispersion relations (filled markers) for the bands with α character in the novel state and their 5f- α weights (unfilled markers). The wave vector \underline{k} is directed along the (1,1,1) direction. The position of the Fermi-energy μ is marked by the arrow. The α intensities for the various branches of dispersion relations are indicated by symbols of the same type. [Lower Panel] The dispersion relations for the bands with β character and their 5f- β weights are plotted as functions of \underline{k} .

tation value

$$Z_{\underline{Q}} = \frac{1}{2N} \sum_{\underline{k}, \sigma} \sigma \langle f_{\underline{k}+\underline{Q}, \sigma}^{\dagger, \beta} f_{\underline{k}, \sigma}^{\alpha} \rangle \quad (32)$$

which characterizes a type of spin inter-5f orbital density wave. Since this appears to be a second-order instability which breaks a continuous

gauge symmetry through short-ranged interactions[33], there should be a branch of collective Goldstone modes associated with it[34]. However, unlike the case of ordinary magnetic instabilities[32], the Goldstone modes are not expected to be easily accessed via inelastic neutron scattering spectroscopy. Therefore, we shall outline the signatures of the novel state that may be accessed in orbitally-sensitive angle resolved photoemission measurements.

In the novel state, the 5f quasi-particle dispersion relations are modified and the orbital characters of the bands are mixed. The dispersion relations and the 5f-orbital characters of the bands are shown in fig.(5), for wave vectors along the nesting direction. The weights for each character when summed over the bands yields unity. It can be seen that the set of bands containing α -character contains a pair of adjacent band segments with disjoint dispersion relations which are quite similar to those of the upper and lower hybridized bands of the normal state. The 5f- α intensities of this pair of band segments grossly follow the same pattern as the intensities of the pair of hybridized bands in the normal state. However, the individual α -weights of the segments do show sharp jumps in the regions where gaps, either at the Fermi-surface gaps or above, are found. It is the gapping of the Fermi-surface that stabilizes the novel state. A Fermi-surface gap occurs for $|k| \sim 0.48$, and a smaller above-the-Fermi-surface-gap can be seen for $|k| \sim 0.37$. It should be noted that Fermi-surface gaps at $k \sim \pm 0.56$ have been inferred from the experimental measurements of Dakovski *et al.* [36]. In the momentum intervals enclosed by these gaps, the dispersion relation marked by solid red triangles has a gradual variation in intensity caused by the hybridization of the 5f α -band with the conduction band. The set of bands with β -character contain two adjacent segments that, when combined, resembles the pure β -band of the normal state. The β -weights of these two adjacent segments form a (disjoint) curve that remains almost constant over almost the entire Brillouin zone, similar to the normal state. However, there are rapid changes in the intensity for the k values where the energy gaps occur between the consecutive segments. It is seen that the Fermi-surface gaps for the β -character bands occur at $k \sim \pm 0.52$, which slightly differ from the k values for which the Fermi-surface gaps occur in the bands with α character. This leads to a double gap structure seen in fig.(6). It should be noted, that despite the gapping of the Fermi-surface for the normal state derived bands, the occurrence of new bands that cross the Fermi-energy, albeit with reduced intensities, allows the novel (mean-field) phase to be classified as metallic. This can be seen in the density of states shown in fig.(7). For the commensurate case considered here, the picture greatly simplifies if one folds the Brillouin zone. Indeed, evidence of a modified periodicity (such as from simple tetragonal to body-centered tetragonal) in the “Hidden Order” state of URu₂Si₂ has been inferred from angle resolved photo-emission experiments[35].

In summary, we propose that a novel concept of a spin inter-orbital density wave may describe the “Hidden Order” state of URu₂Si₂. The order parameter is complex, indicating that the transition breaks a con-

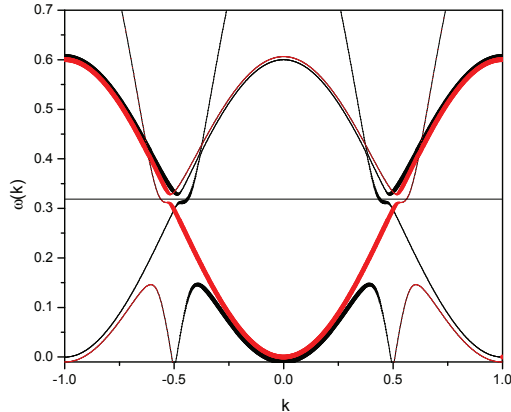


Figure 6: (Color online) The dispersion relations for the 5f α (black line) and 5f β (red line) electrons along the (k, k, k) direction. The widths of the lines are proportional to their intensities. The Fermi-energy μ is set at about 0.32 and is indicated by the horizontal line. The α -dispersion relations shows the existence of a direct gap of the order of $2|V| \sim 0.2$ between the two branches. For other wave vectors, the α and β branches follow similar dispersion relations, however, their degeneracy is lifted by a small energy of the order of $|V|^2$. Gaps at the Fermi-energy are seen to occur at points which are connected by the commensurate nesting vector $\underline{Q} = (1, 1, 1)$. [Figure courtesy of T. Durakiewicz]

tinuous gauge symmetry of the Hamiltonian. The correlations in this state are not readily accessible by purely spin or purely orbital measurements. However, the novel state is produced by the nesting between sheets of the Fermi-surface with different (unmixed) orbital characters. Furthermore, below the transition temperature, the Fermi-surface will gap at these points, leading to the formation of small patches of dispersion relations that describe electrons with mixed orbital characters. The existence of two distinct gaps with mixed characters of 5f states may be identifiable through orbitally sensitive angle resolved photoemission measurements. Indeed, such gaps shifting states away from the Fermi-level at specific locations in the Brillouin zone have been seen[36] to evolve in the “Hidden Order” phase of URu₂Si₂. Since antiferromagnetism is favored by nesting between sheets with the same (unmixed) orbital characters, and since the inter-band and intra-band nesting points are close, the novel ordering is expected to compete with antiferromagnetism, as has been found to be the case for URu₂Si₂. Thus, we have presented a new model starting from two 5f-localized bands hybridized with a conduction band within the underscreened Anderson Lattice model. We have obtained an electronic structure with gaps which increase with decreasing temperature. Our model can account for the opening of a gap observed in a number of

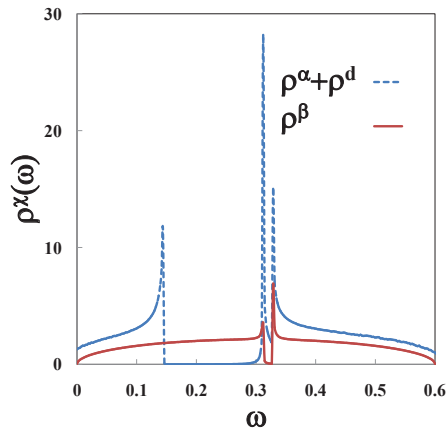


Figure 7: (Color online) The combined α 5f $\rho^\alpha(\omega)$ and conduction band density of states for the ordered state, as a function of ω . The β 5f density of states $\rho^\beta(\omega)$ is also shown as a function of ω . It is seen that the gap structure associated with the novel ordering is highly asymmetric.

different experiments (including recent photoemission measurements) in the “Hidden Order” phase of URu_2Si_2 .

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