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Phys. Rev. B **85**, 140506 — Published 10 April 2012

DOI: 10.1103/PhysRevB.85.140506

Kondo Resonance of a Microwave Photon

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We emulate renormalization group models, such as the Spin-Boson Hamiltonian or the anisotropic Kondo model, from a quantum optics perspective by considering a superconducting device. The infra-red confinement involves photon excitations of two tunable transmission lines entangled to an artificial spin-1/2 particle or double-island charge qubit. Focusing on the propagation of microwave light, in the underdamped regime of the Spin-Boson model, we identify a many-body resonance where a photon is absorbed at the renormalized qubit frequency and reemitted forward in an elastic manner. We also show that asymptotic freedom of microwave light is reached by increasing the input signal amplitude at low temperatures which allows the disappearance of the transmission peak.

PACS numbers: 03.65.Yz, 03.75.Lm, 42.50.-p, 85.25.-j

The asymptotic confinement phenomenon in the infra-red limit is omnipresent in condensed-matter systems and it plays a crucial role in quantum impurity systems, such as the Kondo model describing a single spin-1/2 particle interacting with a bath of conduction electrons [1]. The Kondo effect can also be considered as an example of asymptotic freedom, *i.e.*, the coupling of electrons and spin only becomes non-perturbatively strong at low temperatures and low energies. This model introduced to describe resistance anomalies in metals with magnetic impurities embodies the "hydrogen atom" of many-body physics [2, 3]. Distinct aspects of this infra-red confinement phenomenon can also be addressed through a one-dimensional boson bath (transmission line) entangling a spin-1/2 particle or two-level system resulting in the Spin-Boson model which can be mapped onto the anisotropic Kondo model and exhibits a plethora of interesting phenomena such as an underdamped-overdamped crossover in the spin dynamics and a quantum phase transition [4–6]. In this Letter, we consider the superconducting Josephson circuit of Fig. 1, which allows to investigate the quantum entanglement in the Spin-Boson model and therefore properties of the anisotropic Kondo model through transport of photons. In the underdamped limit, we prospect to reveal a related many-body Kondo resonance in the elastic power of a transmitted microwave photon. This circuit offers the opportunity to export many-body physics in quantum optics.

The superconducting system comprises an artificial spin or double-island charge qubit [7–9] interacting with the zero-point fluctuations of two long one-dimensional transmission lines envisioned from tunable one-dimensional Josephson junction arrays [10–12]. In order to maximize the elastic transmission of a microwave photon, the spin-1/2 object is built from a superconducting double Cooper-pair box where spin up and spin down states refer to the two degenerate charge states $(0, 1)$ and $(1, 0)$, respectively corresponding to one additional Cooper pair on either island [13]. Recently, geometries involving artificial atoms and transmission lines or cavities have already been realized experimentally [14, 15].

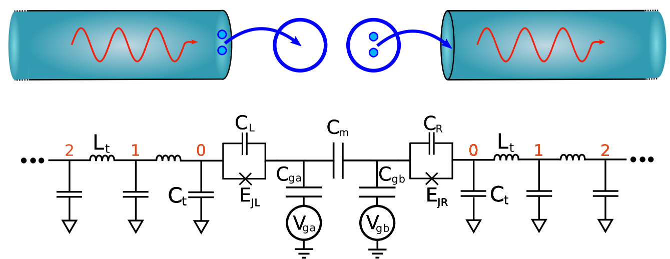

FIG. 1: (Color online) Superconducting circuit envisioned from a double-island charge qubit coupled to two one-dimensional Josephson junction arrays allowing to produce a Kondo resonance in the elastic power of microwave light.

Below, we will assume that the charging energy corresponds to the most dominant term in the Hamiltonian. In fact, close to a charge degeneracy line [7], we can apply the pseudospin representation for the charge states $(0, 1)$ and $(1, 0)$ reinterpreting them as spin-up and spin-down eigenstates of the operator σ_z [13]. The effective detuning $\epsilon = (E_{10} - E_{01}) \rightarrow 0$, where E_{10} (E_{01}) corresponds to the energy of the spin-down (spin-up) eigenstate, can be adjusted through the gate voltages V_{ga} and V_{gb}.

Transfer of Cooper pairs between superconducting islands and leads is described through the Josephson terms E_{JL} and E_{JR} in Fig. 1. In the weak tunneling limit $(E_{JL}, E_{JR}) \ll \min(E_{11} - E_{10}, E_{00} - E_{10})$ one can perform a standard perturbation theory and cotunneling of Cooper pairs then costs an effective energy [13]:

$$E_J = \frac{E_{JL}E_{JR}}{4} \sum_{j=0,1} \left[\frac{1}{E_{jj} - E_{01}} + \frac{1}{E_{jj} - E_{10}} \right], \quad (1)$$

where E_{11} (E_{00}) corresponds to the energy to add (remove) one extra Cooper on the double-island. The Josephson Hamiltonian then takes the form $-(E_J/2)\sigma^+ \exp[i(\phi_l - \phi_r)(x = 0)] + h.c.$ [16] where the Josephson phases $\phi_l(x = 0)$ and $\phi_r(x = 0)$ of the left and right one-dimensional transmission lines read ($j = l, r$)

$$\phi_j(x=0) = i \sum_{k>0} \frac{2e}{\sqrt{\mathcal{L}c}} \frac{1}{\sqrt{\hbar\omega_k}} (b_{jk} - b_{jk}^\dagger). \qquad (2)$$

This Josephson term captures the transmission of a given Cooper pair across the system in Fig. 1. The superconducting reservoirs are explicitly modeled by one-dimensional transmission lines revealing low-energy photon excitations. The left and right transmission lines are described by two distinct sets of harmonic oscillator (photon) operators b_{lk} and b_{rk}. Below, we consider the limit where $(C_L, C_R) \ll C_t$ and $(E_{JL}, E_{JR}) \ll \hbar^2/L_T$ in Fig. 1. The spatial solution of the modes can be expressed in terms of the wavevectors $k \approx m\pi/(2\mathcal{L})$, where m is odd for symmetric modes and even for antisymmetric modes, and a transmission line is diagonalized introducing bosonic creation and annihilation operators. Here, $\mathcal{L}$ corresponds to the length of each transmission line and $c = C_t/a$ to the capacitance per unit length; a is the size of a unit cell in each transmission line and we consider the thermodynamic limit $a/\mathcal{L} \to 0$. The photon waves propagate at the speed $v = \omega_c a$ where $\omega_c = 1/\sqrt{L_t C_t}$, the inductances L_t are defined in Fig. 1, and $\omega_k = v|k|$.

To build an explicit analogy with the spin-boson Hamiltonian, we rewrite the Josephson term as a transverse field $H_J = -(E_J/2)\sigma_x$ performing a unitary transformation or spin rotation (see footnote in [29]). Such a procedure, also referred to as a polaron transformation [4, 5], has been applied in the case of a spin-1/2 interacting with the sound modes of a Bose-Einstein condensate [17]. Since the Hamiltonian of the transmission lines does not commute with the spin rotation this produces an effective interaction between the two-level system and the photon excitations. This term can be combined with the capacitive couplings C_L and C_R of Fig. 1. More precisely, since the electrical potential (operator) at the end of a transmission line, *i.e.*, at $x = 0$, takes the form

$$V_j(x=0) = \frac{1}{\sqrt{c\mathcal{L}}} \sum_{k>0} \sqrt{\hbar\omega_k} (b_{jk} + b_{jk}^\dagger), \qquad (3)$$

this results in the Spin-Boson Hamiltonian:

$$\begin{aligned} H &= \sum_{j=l,r} \sum_{k>0} \hbar v|k| \left[b_{jk}^\dagger b_{jk} + \frac{1}{2} \right] - \frac{\epsilon}{2}\sigma_z - \frac{E_J}{2}\sigma_x \quad (4) \\ &+ \sum_{k>0} \alpha_k \left(-\gamma_l (b_{lk} + b_{lk}^\dagger) + \gamma_r (b_{rk} + b_{rk}^\dagger) \right) \frac{\sigma_z}{2}. \end{aligned}$$

The charge operators on the two islands take the forms $Q_b = \frac{2e}{2}(1 + \sigma_z)$ and $Q_a = \frac{2e}{2}(1 - \sigma_z)$. Hereafter the detuning will be fixed to $\epsilon \to 0$ and $\alpha_k = (2e/\sqrt{c\mathcal{L}})\sqrt{\hbar\omega_k}$. The couplings γ_r and γ_l are given by:

$$\begin{aligned} \gamma_r &= -1 + \frac{C_R}{2} \left(\frac{C_{\Sigma a}}{C_{\Sigma a} C_{\Sigma b} - C_m^2} - \frac{C_m}{C_{\Sigma a} C_{\Sigma b} - C_m^2} \right) \quad (5) \\ \gamma_l &= -1 + \frac{C_L}{2} \left(\frac{C_{\Sigma b}}{C_{\Sigma a} C_{\Sigma b} - C_m^2} - \frac{C_m}{C_{\Sigma a} C_{\Sigma b} - C_m^2} \right). \end{aligned}$$

Following Ref. 7 and the notations of Fig. 1, we have defined the total capacitances seen by each superconducting island: $C_{\Sigma a} = C_L + C_{ga} + C_m$ and $C_{\Sigma_b} = C_R + C_{gb} + C_m$.

The analogy with the Spin-Boson model [4, 5] becomes complete when rewriting the Hamiltonian in terms of the symmetric and antisymmetric bosonic combinations:

$$\begin{aligned} b_{sk} &= \cos\theta b_{lk} + \sin\theta b_{rk} \quad (6) \\ b_{ak} &= \sin\theta b_{lk} - \cos\theta b_{rk}. \end{aligned}$$

Choosing $\cos\theta = \gamma_r/\sqrt{\gamma_l^2 + \gamma_r^2}$ and $\sin\theta = \gamma_l/\sqrt{\gamma_r^2 + \gamma_l^2}$, we note that the boson operator b_{ak} only couples to the two-level system through the coupling $\lambda_k = \alpha_k \sqrt{\gamma_l^2 + \gamma_r^2}$. Each transmission line mimics a physical resistor then producing dissipation in the system. In the present circuit, the spectral function of the environment is defined as $J(\omega) = (\pi/\hbar) \sum_{k>0} \lambda_k^2 \delta(\omega - \omega_k) = 2\pi\hbar\alpha\omega e^{-\omega/\omega_c}$ where $\omega_c \gg E_J/\hbar$ represents the high-frequency cutoff of this Ohmic environment [30] and α is given by

$$\alpha = \frac{2R}{R_Q}(\gamma_l^2 + \gamma_r^2). \qquad (7)$$

Here, $R_Q = h/(2e)^2$ denotes the quantum of resistance and $R = \sqrt{L_t/C_t}$ is the resistance of each transmission line. It is instructive to observe that in the limit of negligible capacitances C_L and C_R the system naturally converges towards the symmetric condition $\gamma_l = \gamma_r = -1$.

The Spin-Boson Hamiltonian with Ohmic dissipation is intimately related to the Kondo model in the anisotropic regime via bosonization [18]. Other Spin-Boson Hamiltonians such as the Jaynes-Cummings model, in contrast, involve a two-level system interacting with a single mode of a cavity [19]. Other impurity models with photons have also been considered [20, 21]. We are interested in the underdamped regime ($0.1 \preceq \alpha \preceq 0.2$) of the Spin-Boson model where the two-level system displays visible Rabi oscillations but dissipation modifies the qubit frequency which is related to the Kondo energy [4, 5]

$$E_R(\alpha) = \hbar\omega_R = E_J \left(E_J/\hbar\omega_c \right)^{\alpha/1-\alpha}. \qquad (8)$$

To understand the physical content of the energy E_R, it is relevant to apply the unitary transformation $U = \exp(A_l - A_r)$ where $A_j = \sum_{k>0} \frac{\alpha_k \gamma_j}{\hbar\omega_k}(b_{jk}^\dagger - b_{jk})\sigma_z/2$ such that the Hamiltonian can be rewritten as ($\tilde{H} = U^\dagger H U$):

$$\tilde{H} = -\frac{E_J}{2}\sigma^+ e^{i(\Phi_l - \Phi_r)} + h.c. + \sum_{j=l,r} \sum_{k>0} \hbar v|k| \left[b_{jk}^\dagger b_{jk} + \frac{1}{2} \right], \qquad (9)$$

where the phases $\Phi_l = -\gamma_l \phi_l(x = 0)$ and $\Phi_r = -\gamma_r \phi_r(x = 0)$ contain Josephson physics as well as (weak) charging effects. Then, we can define an effective transverse field acting on the dissipative two-level system as $\Delta = E_J \langle \cos(\Phi_l - \Phi_r) \rangle$ such that the artificial atom is described by the effective Hamiltonian

$\tilde{H}_{eff} = -(\epsilon/2)\sigma_z - (\Delta/2)\sigma_x$. Bethe ansatz calculations [22, 23] and the adiabatic renormalization [4] in the underdamped limit where $0.1 \preceq \alpha \preceq 0.2$ indeed confirm that $\Delta = E_R$. *The bare qubit frequency $E_J/\hbar$ of the two-level system is modified due to the strong renormalization effects associated with the photon bath.* One way to experimentally measure the Kondo energy E_R would be through charge measurements since the Fermi liquid ground state imposes that $\langle\sigma_z\rangle \propto \epsilon/E_R$ at small detuning and low temperatures $k_B T \ll E_R$, and the prefactor is accessible from Bethe Ansatz calculations [23].

Below, we show that in the underdamped regime and for temperatures $k_B T \ll E_R$, the Kondo energy E_R can be directly measured based on the (elastic) resonant propagation of a photon. When the system is driven by an external coherent source, the drive, the circuit and the outgoing waves can be treated through the input-output theory [24]. Previous works have studied the limit $\alpha \to 0$ where many-body effects can be fully ignored (the elastic resonance is centered at the bare frequency of the two-level system and converges to a δ-function) [14]. We assume perfect transmission of the microwave signal in the transmission lines such that the input signal reads

$$V_l^{in}(t) = \sum_{k>0} \frac{\alpha_k}{2e}\left(e^{-i\omega_k(t-t_0)} b_{lk}(t_0) + e^{i\omega_k(t-t_0)} b_{lk}^\dagger(t_0)\right). \tag{10}$$

Here, $t_0 < t$ denotes a time in the distance past before any wave packet has reached the two-level system. Similarly, an output field in the left transmission line at time $t_1 > t$ being a time in the distant future after the input field has reached the double-island Cooper box system reads

$$V_l^{out}(t) = \sum_{k<0} \frac{\alpha_k}{2e}\left(e^{-i\omega_k(t-t_1)} b_{lk}(t_1) + e^{i\omega_k(t-t_1)} b_{lk}^\dagger(t_1)\right). \tag{11}$$

Through the Heisenberg relation $\dot{b}_{lk} = (i/\hbar)[H, b_{lk}] = -i\omega_k b_{lk} + (i/2\hbar)\gamma_l \alpha_k \sigma_z$ we relate the properties of the input signal to those of the two-level system.

Below, since we focus on the underdamped limit of the Spin-Boson model which is characterized by a (Rabi) resonance at $\omega = \omega_R$, we establish $\langle\sigma_z(\omega)\rangle \approx \gamma_l \chi(\omega, P_{in})\langle V_l^{in}(\omega, P_{in})\rangle$ for frequencies in the vicinity of ω_R where $P_{in} = \langle (V_l^{in})^2\rangle/R$ is the average input power [31]; see EPAPS [25]. Then, we can introduce the reflection coefficient $r(\omega, P_{in}) = \langle V_l^{out}(\omega, P_{in})\rangle / \langle V_l^{in}(\omega, P_{in})\rangle$. Defining the output signal in the right transmission line as

$$V_r^{out} = \sum_{k>0} \frac{\alpha_k}{2e}\left(e^{-i\omega_k(t-t_1)} b_{rk}(t_1) + e^{i\omega_k(t-t_1)} b_{rk}^\dagger(t_1)\right). \tag{12}$$

the transmission coefficient is $t(\omega, P_{in}) = \langle V_r^{out}(\omega, P_{in})\rangle / \langle V_l^{in}(\omega, P_{in})\rangle$. Using the Heisenberg relation of $\dot{b}_{lk}$ with the definitions above we obtain (see EPAPS [25]):

$$\begin{aligned} r(\omega, P_{in}) &= \left(1 + \frac{2i\gamma_l^2}{\gamma_l^2 + \gamma_r^2} J(\omega)\chi(\omega, P_{in})\right), \\ t(\omega, P_{in}) &= -\frac{2i\gamma_r\gamma_l}{\gamma_l^2 + \gamma_r^2} J(\omega)\chi(\omega, P_{in}). \end{aligned} \tag{13}$$

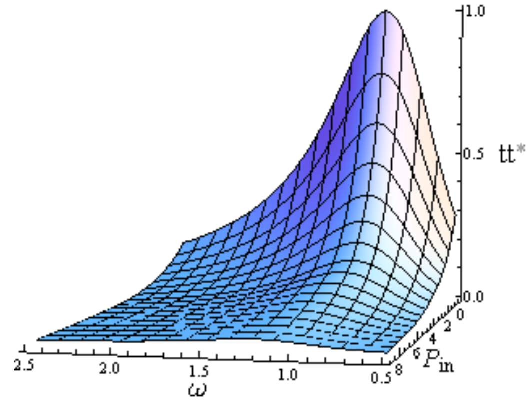

FIG. 2: (Color online) Normalized elastic transmitted power $tt^*(\omega)$ as a function of frequency and driving power for $\gamma_l = \gamma_r$. The parameters are chosen as $\alpha = 0.15$, $\omega_R = 1 = P_R$, $E_J \approx 1.9$, $\omega_c = 50$ and $\hbar = 1$ (we set the ratio $E_J/\hbar\omega_c$ to be moderate, but the "many-body" resonance frequency or renormalized qubit frequency ω_R is distinct from $E_J/\hbar$).

First, we consider the linear regime where the amplitude of the input signal is very small, $P_{in} \to 0$ (see footnote in [29]). For frequencies close to the confinement frequency, again assuming the underdamped limit ($0.1 \preceq \alpha \preceq 0.2$), we derive an expression of the spin susceptibility which agrees with Numerical Renormalization Group results [26]; see EPAPS [25]. This leads to

$$\chi(\omega) = \frac{\omega_R/\hbar}{\omega_R^2 - \omega^2 - i\gamma(\omega)}, \tag{14}$$

where the dissipation factor takes the form $\gamma(\omega) = \omega_R J(\omega)/\hbar$ and is in agreement with the (many-body) Fermi-liquid type ground state [5]. In the linear regime of small input power, we check that the scattering matrix is unitary, $|r|^2 + |t|^2 = 1$, since $J(\omega_R)\Im m\chi(\omega_R) = 1$ showing that the photon propagation is purely elastic close to the resonance (see Fig. 2). We corroborate that the normalized (elastic) power $tt^*(\omega_R)$ flowing to the right transmission line reaches unity since here $\gamma_l \approx \gamma_r$.

In the underdamped regime, the photon propagation across the system is characterized by a many-body resonance at the frequency ω_R [4–6]: a photon is absorbed at the frequency ω_R and reemitted forward in a purely elastic manner. In the underdamped regime of the circuit, the qubit is described by a resonance which turns the "photon+Cooper pair" system into an ideal conductor. The phase associated with the reflection coefficient satisfies the following properties. For small γ_l, the phase vanishes since $V_l^{out} = V_l^{in}$ for an open termination and for $\gamma_r = 0$ the phase is consistent with the Kondo-type $2 \times \pi/2$ phase shift of a right-moving wave.

In fact, the appearance of resonances in such a circuit is not so surprising. For example, let us ignore the Coulomb blockade physics in the two islands completely and replace the Josephson junctions E_{JL} and E_{JR} by purely linear inductances L_L and L_R. When $C_L = C_R = C$ and $L_L = L_R = L$, then we corroborate a resonance with $r = 0$ at the frequency $\omega^* = 1/\sqrt{CL + 2C_m L}$; however, we emphasize that ω^* is distinct from ω_R which has a many-body origin. In addition, nonlinear effects unavoidably appear in the Josephson circuit of Fig. 1 when increasing the amplitude of the input signal. Under a strong drive, this produces the accumulation of a macroscopic number of photons in the left transmission line which will cause the saturation of the two-level system excitation and the destruction of the resonance peak. More precisely, evaluating the Franck-Condon factor $\langle \cos(\Phi_l - \Phi_r)\rangle = \exp -[\langle(\Phi_l - \Phi_r)^2\rangle/2]$ when increasing the input signal amplitude, we observe that $\langle \Phi_l^2 \rangle$ yields an extra contribution (note, $P_{in} = \langle V_l(x=0)^2\rangle/R$)

$$\frac{1}{(\hbar\omega_R)^2} \sum_{q\in'} \alpha_q^2 \gamma_l^2 \langle b_{lq}^\dagger b_{lq}\rangle = \frac{P_{in} R (2e)^2 \gamma_l^2}{(\hbar\omega_R)^2}, \tag{15}$$

and the symbol $'$ refers to momenta such that $\omega_q \sim \omega_R$ in the case of a monochromatic signal with a frequency close to ω_R. The Josephson process in Eq. (9) becomes exponentially diminished and this results in an exponential suppression of $\Im m\chi(\omega = \omega_R)$ (see EPAPS [25])

$$\Im m\chi(\omega_R, P_{in}) J(\omega_R) = \exp - \left(\frac{P_{in}}{P_R} \frac{R}{R_Q} \pi \gamma_l^2 \right), \tag{16}$$

where $P_R = \hbar\omega_R^2$. The nonlinearity of the two-level system produces an exponential decrease of the spin susceptibility at $\omega = \omega_R$. Then, this causes the disappearance of the Rayleigh transmission resonance; see Fig. 2. When $P_{in} \gg P_R$ one reaches the asymptotic freedom of microwave light where $r(\omega_R) \sim 1$; see Eqs. (13) and (16). These nonlinear effects are driven by the Josephson-type Hamiltonian in Eq. (9). Increasing the driving power P_{in}, the scattering matrix becomes non-unitary since $J(\omega_R)\Im m\chi(\omega_R, P_{in}) < 1$ (which hides the presence of additional inelastic Raman corrections [14]). An open question concerns the inelastic Raman spectrum, which should become prominent in the overdamped limit of the Spin-Boson model. The asymptotic freedom of light, resulting in $r(\omega_R) \sim 1$ can also be reached when increasing the temperature producing a prominent decoherence of the two-level system and a strong decrease of $\langle \sigma_x \rangle$ and of the Josephson coupling in Eq. (9); see EPAPS [25].

To summarize, in the underdamped regime of the Spin-Boson model, sending a microwave photon produces a many-body Kondo resonance. For moderate and accessible values of the dissipation strength $0.1 \preceq \alpha \preceq 0.2$, the confinement frequency ω_R is clearly distinguishable from the bare Josephson frequency $E_J/\hbar$ of the two-level atom as a result of the continuum of photon modes in the (very long) transmission lines. The width of the resonance peak reflects the Fermi-liquid type ground state. We assumed that the detuning ϵ and thermal effects through $k_B T$ are smaller than the Kondo energy E_R, and in the underdamped regime, E_R is not too small compared to the Josephson energy E_J. For Josephson junction arrays with large resistances, this circuit would offer the opportunity to study overdamped spin dynamics and quantum phase transitions using nonlinear optics. Note that the development of techniques such as the numerical renormalization group [27] or a stochastic-type approach [28] would be necessary to combine the traditional input-output theory with many-body physics of quantum impurity models. Finally, such superconducting quantum devices can be used for controllable (single-)photon sources in which a plethora of novel effects related to many-body physics and nonlinear quantum optics can be realized.

This work was supported by DOE, under the grant DE-FG02-08ER46541. We thank M. Devoret for discussions.

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[29] This is analogous to U^{-1} below Eq. (8) with $\gamma_j = -1$.
[30] The plasma frequencies related to the capacitances C_L and C_R are assumed to be comparable or larger than ω_c.
[31] Assuming $V_l^{in}(t) = V_0 \cos(\omega t)$ then $P_{in} = V_0^2/2R$.