

This is the accepted manuscript made available via CHORUS. The article has been published as:

# Measurement of the phonon density of states of $\text{PuO}_2$ (+2% Ga): A critical test of theory

M. E. Manley, J. R. Jeffries, A. H. Said, C. A. Marianetti, H. Cynn, B. M. Leu, and M. A. Wall

Phys. Rev. B **85**, 132301 — Published 13 April 2012

DOI: [10.1103/PhysRevB.85.132301](https://doi.org/10.1103/PhysRevB.85.132301)

# Measurement of the phonon density of states of $\text{PuO}_2(+2\%\text{Ga})$ : A critical test of theory

M. E. Manley<sup>1</sup>, J. R. Jeffries<sup>1</sup>, A. H. Said<sup>2</sup>, C. A. Marianetti<sup>3</sup>, H. Cynn<sup>1</sup>, B. M. Leu<sup>2</sup>, M. A. Wall<sup>1</sup>

<sup>1</sup>*Lawrence Livermore National Laboratory, Livermore, California 94550, USA*

<sup>2</sup>*Argonne National Laboratory, Argonne, Illinois 60439, USA*

<sup>3</sup>*Department of Applied Physics, Columbia University, New York, New York 10027, USA*

Using inelastic x-ray scattering, we determine the phonon density of states (DOS) of  $\text{PuO}_2(+2\%\text{Ga})$  and compare results with recent predictions made using Density Functional Theory (DFT), DFT plus the Hubbard U (DFT+U), and Dynamical Mean-Field Theory (DMFT). The DFT prediction underestimates the measured energies of most features. The DFT+U prediction accurately reflects the low-energy features but incorrectly splits off an isolated high-energy oxygen mode. Ramifications for predictions of thermodynamic and transport properties of this nuclear fuel material are discussed.

PACS numbers: 63.20.—e, 78.70.Nx

Plutonium dioxide ( $\text{PuO}_2$ ) has been a subject of intense interest in recent years owing to its use in mixed-oxide nuclear fuels, and for the theoretical challenges posed by its  $5f$  electrons, which inhibit development of predictive capabilities for its use and storage. Because of difficulties in preparing and handling  $\text{PuO}_2$ , much of the theoretical work has proceeded without rigorous experimental testing. In particular, the lattice vibrations, which are critical to understanding phase stability [1] and thermal transport [2], are known only through predictions made using various theoretical approaches including Density Functional Theory (DFT) [3], DFT plus the Hubbard  $U$  (DFT+ $U$ ) [4], and Dynamical Mean Field Theory (DMFT) [5]. However, DFT is known to incorrectly describe  $\text{PuO}_2$  as a ferromagnetic conductor [6], when it is actually an insulator [7], and DFT+ $U$  obtains an insulator but incorrectly obtains an antiferromagnetic ground state [8, 9-11]. Furthermore, neither the DFT nor DFT+ $U$  calculations include spin-orbit coupling, which has been shown to be a strong perturbation on the electronic structure [12], though it is unclear how relevant this is to the phonons. The DMFT calculation by Yin et al. [5] includes spin-orbit coupling but is done with the Hubbard-I approach, which should be well suited to  $\text{PuO}_2$  but is still an approximation. Here, we use high-resolution inelastic x-ray scattering (IXS) at a synchrotron source to measure the phonon density of states (DOS) of  $\text{PuO}_2(+2\%\text{Ga})$ . We find that the limitations of these predictions also manifest as differences between theoretical and experimental phonons. Ramifications of these errors on predictions made of the thermal conductivity and phase stability are also discussed.

Measuring the phonons of plutonium-bearing materials is challenging, because neutron scattering, the standard approach, requires large quantities of special isotopes with low neutron-absorption cross sections. Here, we avoid this constraint by using high-resolution IXS with the HERIX instrument at the Advanced Photon Source at Argonne National Laboratory [13-15]; this IXS technique has proven effective for measuring phonon dispersions in actinides [16, 17] and the phonon density of states (DOS) of plutonium [18]. To obtain an accurate representation of the phonon DOS from IXS on a powder sample, it is necessary to measure over a large range of momentum transfers ( $Q$ ) so as to cover several Brillouin zones. Partial averages using a limited number of  $Q$  points [19] have been shown with neutron scattering to lead to errors in the evaluation of thermodynamic quantities [20]. Using a 23.8 keV x-ray beam with nine equally spaced analyzers positioned at a series of overlapping angles, a complete  $Q$  range from 2–7  $\text{\AA}^{-1}$  was measured. Since for  $\text{PuO}_2(+2\%\text{Ga})$  a reciprocal unit length is  $a^* = 1.16 \text{\AA}^{-1}$ , the measured range provides a good reciprocal space average. The detector signals were each normalized by the detector efficiency, corrected for the x-ray form factors,  $f^2(Q)$ , (for this part the Pu and O modes were treated as separable in the spectrum, which is justified according to partial phonon DOS calculations [4]) and the  $(\cos 2\theta)^2$  term, and then the spectra were summed over  $Q$ . A phonon DOS was extracted from the reduced spectra by subtracting the elastic peak and the incoherent multiphonon scattering, which was determined iteratively [21-23], and dividing out the thermal occupation and Debye-Waller factors. X-ray diffraction patterns collected in situ revealed that 10% unreacted metallic  $\delta$ -Pu was contained

within the scattering volume. Since the phonon DOS of  $\delta$ -Pu resides at lower energies than  $\text{PuO}_2$  the presence was also evident in the inelastic spectra, which was corrected using spectra measured on the metal with the same Ga concentration ( $\delta$ -Pu+2%Ga) using the same instrument [18]. The size of the correction needed in the phonon DOS independently confirmed that the unreacted material in the beam occupied a 10% volume fraction. However, the need for the correction introduces uncertainty in the low-energy sloping part of the oxide phonon DOS, which retained artifacts (slight wiggles) not expected at these low energies (see Fig. 1a at energies below  $\sim 10$  meV).

Figure 1 compares the measured phonon DOS, Fig. 1a, with several theoretical phonon DOS, Fig. 1b, and theoretical phonon dispersion curves, Fig. 1c. At low energies the transverse acoustic (TA) peak in the DOS measured at 13.8 meV is fairly well matched by all three predictions. However, the longitudinal acoustic (LA) peak measured at 22.8 meV is significantly higher in energy than the 19.5 meV prediction by Minamoto et al. using DFT [3]. The inclusion of the Hubbard U (DFT+U) by Zhang et al. [4] appears to correct for this shortcoming, increasing the energy of the LA mode putting it in good agreement with measurement, Fig. 1a&b. The DMFT prediction by Yin et al. [5] also correctly stiffens the LA modes to about the same level as the Zhang calculation, as shown in the dispersion curves in Fig. 1c. On the other hand, the DFT+U calculation misplaces the small feature between the TA and LA peaks, placing it as closer to the TA while the measurement shows this feature about half way in between these peaks, as does the DFT calculation. Judging from the overlap in the DMFT and DFT+U calculated dispersions in the acoustic

region, Fig. 1c, it appears that DMFT also probably misplaces the feature between the main TA and LA peaks. It is difficult, however, to uniquely identify the origin of this feature since in the DFT+U prediction an optic mode drops down to this energy at the X high symmetry point, yet for the DFT and DMFT prediction only the split in TA branch accounts for this feature. At mid-range energies the DFT calculation continues to under estimate the energies of the features by about 6 meV, while the DFT+U manages to accurately capture the two most significant optic features, labeled O1 and O2 in Fig. 1. The dispersion data in the DMFT prediction by Yin et al. [5] is too limited to pinpoint optic features of the phonon DOS in this range, although it is clearly different than the DFT+U result. At the highest energies ( $>50$  meV), there is little correspondence that can be found between any of the measured and predicted features. The measured phonon DOS shows two significant peaks separated by 4.2 meV located at 58 meV and 62.2 meV. The DFT and the DFT+U appear dramatically different, with the DFT showing essentially no split while the DFT+U shows a very large  $\sim 9$  meV split with a gap actually opening up. The gap in the Zhang et al. DFT+U [4] calculation is attributed to a lifting of mode degeneracy in the dispersion curves by spin and orbital ordering. A similar, but smaller splitting is also apparent in the DMFT calculations [5]; in this case there is only orbital order splitting. These splits/gaps can be seen in the calculated dispersion curves shown in Fig. 1c. Comparing all results, it appears that measurement is revealing some splitting as might be expected for orbital ordering in the insulating state, but that the calculations are overestimating the amount of splitting.

In the analysis of thermal conductivity using the DMFT calculated phonons, Yin et al. [5] find that the LA mode dominates the thermal conductivity owing to its large group velocity and small anharmonicity implied by its Grüneisen constant [5]. Unlike the TA mode, however, which all three calculations and the experiment agree on (Fig. 1), the LA mode is not handled consistently by the calculations. The DFT calculation underestimates the experimental upper bound of the LA mode (Fig. 1a,b), and the both DMFT and DFT predict a maximum slope (group velocity) along X that is about 1.6 times less than the DFT+U calculation (Fig. 1c). A lower LA mode group velocity in the DMFT calculations might partially explain why Yin et al. [5] predict a thermal conductivity at 1000 K ( $1.74 \text{ W m}^{-1} \text{ K}^{-1}$  [5]) that is about a factor of two lower than the experimental value ( $3.8 \text{ W m}^{-1} \text{ K}^{-1}$  [24]). However, they also underestimate the thermal conductivity in  $\text{UO}_2$  by a factor of two [5] even though that calculation matches the previously measured  $\text{UO}_2$  phonon dispersion curves [25]. Furthermore, uncertainty in the phonon energies prediction suggests uncertainty in the predicted mode Grüneisen constant and its use to estimate the phonon mean free path (phonon-phonon scattering rate) contribution to the thermal conductivity [2]. Finally, although Minamoto et al. [3] and Zhang et al. [4] obtain reasonably accurate predictions of the experimental heat capacity curve using quite different phonon DOS curves, such success does not translate to phase stability predictions where even small shifts in the phonon DOS produce vibrational entropy changes that can dramatically reshape phase diagram predictions [1]. Hence, there is a practical need to improve calculations of the lattice vibrations to

enhance the predictive capabilities regarding phase stability and thermal transport in extreme environments not easily accessible by experiment.

High resolution inelastic x-ray scattering measurements of the phonon DOS of PuO<sub>2</sub>(+2%Ga) provides critical guidance for improving theoretical predictions. Comparing our results with a series of theoretical predictions shows how adding new physics to the calculation (e. g. DFT to DFT+U or DMFT) improves on the predictive capability of electronic structure theory when applied to the lattice vibrations, but that further work is needed. Future calculations incorporating these advances will surely provide more reliable predictions of thermodynamic and transport properties of this important nuclear fuel material.

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract No. DE-AC52-07NA27344. Use of the Advanced Photon Source at Argonne National Laboratory was supported by the US Department of Energy, Office of Science, Office of Basic Energy Sciences, under Contract No. DE-AC02-06CH11357. The construction of HERIX was partially supported by the NSF under Grant No. DMR-0115852. C.A.M. acknowledges funding from the Columbia RISE program.

[1] B. Fultz, Prog. Mater. Sci. **55**, 247-352 (2010).

[2] *Thermal Conductivity: Theory, Properties, and Applications*, edited by Terry M. Tritt (Kluwer Academic, New York, 2004).

[3] S. Minamoto, M. Kato, K. Konashi, Y. Kawazoe, J. Nucl. Mater. **385**, 18-20 (2009).

[4] P. Zhang, B.-T. Wang, X.-G. Zhao, Phys. Rev. B **82**, 144110 (2010).



- [5] Q. Yin, S. Y. Savrasov, Phys. Rev. Lett. **100**, 225504 (2008).
- [6] C. E. Boettger, A. K. Ray, Int. J. Quantum Chem. **90**, 1470 (2002).
- [7] C. McNeily, J. Nucl. Mater. **11**, 53 (1964).
- [8] G. Raphael, R. Lallement, Solid State. Comm. **6**, 383-385 (1968).
- [9] F. Jollet, G. Jomard, B. Amadon, J. P. Crocombette, D. Torumba, Phys. Rev. B **80**, 235109 (2009).
- [10] G. Jomard, B. Amadon, F. Bottin, and M. Torrent, Phys. Rev. B **78**, 075125 (2008).
- [11] I. D. Prodan, G. E. Scuseria, and R. L. Martin, Phys. Rev. B **73**, 045104 (2006).
- [12] H. Nakamura, M. Machida, and M. Kato, Phys. Rev. B **82**, 155131 (2010).
- [13] H. Sinn, E. E. Alp, A. Alatas, J. Barraza, G. Bortel, E. Burkel, D. Shu, W. Sturhahn, J. P. Sutter, T. S. Toellner, and J. Zhao, Nucl. Instrum. Methods Phys. Res. A **467-468**, 1545 (2001).
- [14] T. S. Toellner, A. Altas and A. H. Said, J. Synchrotron Rad. **18**, 605 (2011).
- [15] A. H. Said, H. Sinn, and R. Divan, J. Synchrotron Rad. **18**, 492 (2011).
- [16] M. E. Manley, G. H. Lander, H. Sinn, A. Alatas, W. L. Hults, R. J. McQueeney, J. L. Smith, J. Willit, Phys. Rev. B **67**, 052302 (2003).
- [17] J. Wong, M. Krisch, D. L. Farber, F. Occelli, A. J. Schwartz, T. C. Chiang, M. Wall, C. Boro, and R. Q. Xu, Science **301**, 1078 (2003).
- [18] M. E. Manley, A. H. Said, M. J. Fluss, M. Wall, J. C. Lashley, A. Alatas, K. T. Moore, and Yu. Shvyd'ko, Phys. Rev. B **79**, 052301 (2009).
- [19] J. L. Robertson, H. N. Frase, P. D. Bogdanoff, M. E. Manley, B. Fultz, R. J. McQueeney, Philos. Mag. Lett. **79**, 297-304 (1999).

- [20] M. E. Manley, R. J. McQueeney, J. L. Robertson, B. Fultz, D. A. Neumann, *Philos. Mag. Lett.* **80**, 591-596 (2000).
- [21] G. L. Squires, *Introductions to the Theory of Neutron Scattering* (Cambridge University Press, Cambridge, 1978).
- [22] M. E. Manley, R. J. McQueeney, B. Fultz, R. Osborn, G. H. Kwei, and P. D. Bogdanoff, *Phys. Rev. B* **65**, 144111 (2002).
- [23] A. Bosak and M. Krisch, *Phys. Rev. B* **72**, 224305 (2005).
- [24] R. L. Gibby, *J. Nucl. Mater.* **38**, 163-177 (1972).
- [25] G. Dolling, R. A. Cowley, A. D. B. Woods, *Can. J. Phys.* **43**, 1397 (1965).

**Figure Caption:**

FIG. 1 (color online). Comparison of experimental and theoretical phonons: (a) experimental phonon density of states (DOS) determined by inelastic x-ray scattering in this study; (b) theoretical phonon DOS determined using DFT by Minamoto *et al.* [3] (blue dashed line) and DFT+U by Zhang *et al.* [4] (red dotted line); and (c) theoretical phonon dispersion curves associated with the DFT (blue) and DFT+U (red) phonon DOS above, plus partial phonon dispersion curves calculated using DMFT by Yin *et al.* [5] (green symbols).

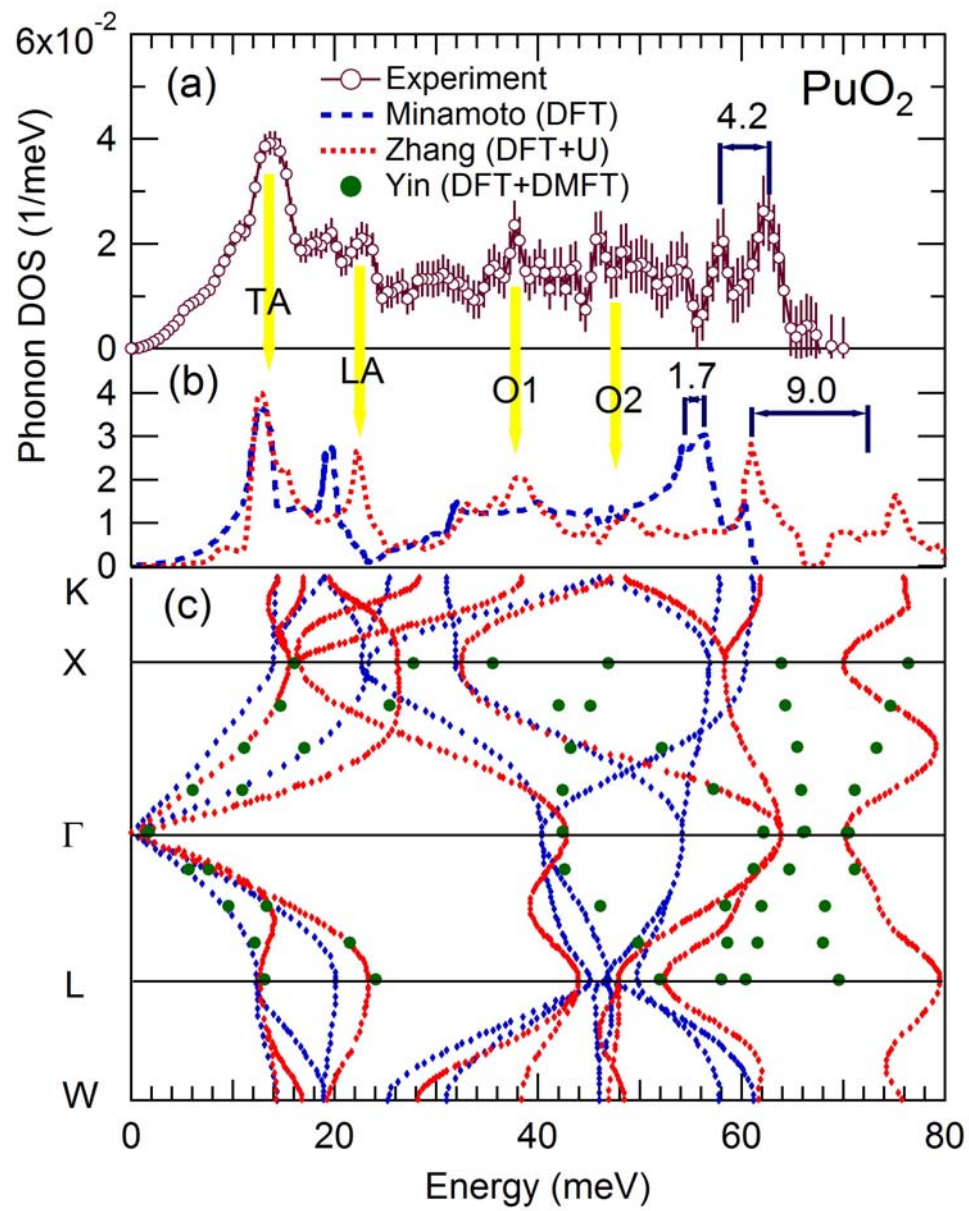


FIG. 1