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# Giant Quantum Freezing of Nanojunctions mediated by Environments

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We investigate the quantum heat exchange between a nanojunction and a many-body or electromagnetic environment far from equilibrium. It is shown that the two-temperature energy emission-absorption mechanism gives rise to a giant heat flow between the junction and the environment. We obtain analytical results for the heat flow in an idealized high impedance environment and perform numerical calculations for the general case of interacting electrons and discuss giant freezing and heating effects in the junction under typical experimental conditions.

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## I. INTRODUCTION

Electron transport in the presence of an electric field is always accompanied by heating of the charge carriers. This effect is especially pronounced in electronic devices where overheating leads to instabilities in the current-voltage characteristics.<sup>1,2</sup> This defines an urgent task to study far-from-equilibrium heating of charge carriers.

In this paper we study the heat flow between tunneling electrons and an environment in nanojunctions using a non-perturbative technique based on a quantum kinetic equation. We show that a regime exists in which the interaction with the environment leads to an effective (“giant”) environment cooling of the junction.

At high electron temperatures phonons play the role of the cooling agent. At low temperatures the direct energy transfer to the phonon bath becomes inefficient and the relaxation is dominated by energy exchange between tunneling electrons and an electromagnetic environment and/or environment of many-body excitations in the electrodes.<sup>3</sup> In both cases cooling follows the two-temperature emission-absorption mechanism:<sup>1,3</sup> the emission of environment modes with temperature equal to that of the tunneling charge carriers,  $T_e$ , and the absorption of environment excitations having the temperature of the environment,  $T_{\text{env}}$ . Moreover, not only temperatures, but also the distributions of emitted and absorbed environment modes may be different in the far from equilibrium regime.

The coupling between the tunneling electrons and the environment has a dispersion characterized by the “cut-off” frequency  $\omega_{\text{max}}$ . For example,  $\omega_{\text{max}} = E_c$  for a high impedance environment, with  $E_c$  being the charging energy of the tunnel junction;  $\omega_{\text{max}} = 1/\sqrt{LC}$  for an environment represented by an  $L - C$  circuit; and  $\omega_{\text{max}} = 1/R_T C$  for an Ohmic environment with  $R_T$  and  $C$  being the ohmic resistance<sup>4,5</sup> and capacitance of the tunnel junction, respectively.<sup>6</sup> We are interested in the regime where  $T_e, T_{\text{env}} > \omega_{\text{max}}$ . In this case the large number of environment modes,  $\mathcal{N} \sim \ln[\omega_{\text{max}}\tau_e(T_e)] \max\{T_e, T_{\text{env}}\}/\omega_{\text{max}} \gg 1$  ( $\tau_e$  is the en-

ergy relaxation rate), participate in the heat exchange between the environment and tunneling electrons in the nanojunction. At low electron temperatures when the environment has electromagnetic or many body origin this regime is easy to reach.<sup>6,7</sup> We show that in general the heat flux acquires the large factor  $\mathcal{N} \gg 1$  in all orders in electron-environment interaction increasing the efficiency of the heat exchange. Using the Landauer scattering theory<sup>8</sup> we express the density matrix as the direct product of the density matrices for emitted and absorbed environment excitations; this is a typical case for the two-temperature emission-absorption mechanism far from equilibrium. The validity of the Landauer approach implies that the effective energy relaxation length of the environment modes is larger than the size of the nanostructure. In our consideration a bath (phonons) is absent. Therefore our approach is valid when the inter-

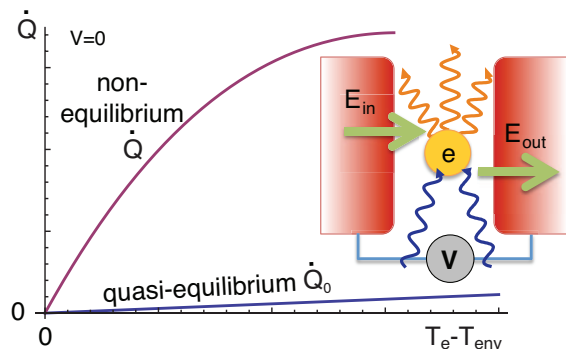


FIG. 1: (color online) Illustration of the non-equilibrium heating effects in a nanojunction. The electrons traversing the junction absorb external photons (incident wavy lines) and emit them leading to heating of the contact. The plots show the *giant* heating effect,  $\dot{Q}$ , as a function of the difference of electron and environment temperatures ( $V = 0$ ) compared to “quasiequilibrium” approximation where the radiation density matrix is equilibrium,  $\dot{Q}_0$ . The full non-equilibrium analysis gives an at least one order of magnitude more pronounced heating effect than for the latter case:  $\max(\dot{Q}/\dot{Q}_0) \sim \mathcal{N} > 10$ .

action time between electrons and environment is much smaller than the one between environment and bath, which is the case at not very high temperatures where the number of phonons is small.

## II. MODEL

The rate of the heat flow between the tunnel junction and the environment is given by: (see Appendix A and Ref. [7])

$$\dot{Q} = \int_0^\infty \varepsilon \{n_\varepsilon P(\varepsilon) - [1 + n_\varepsilon] P(-\varepsilon)\} p(\varepsilon) d\varepsilon, \quad (1)$$

where  $P(\pm\varepsilon)$  is the probability density for the tunneling charge-carrier to lose [gain] the energy  $\varepsilon$  to [from] the environment. The distribution function  $n_\varepsilon$  in Eq. (1) can

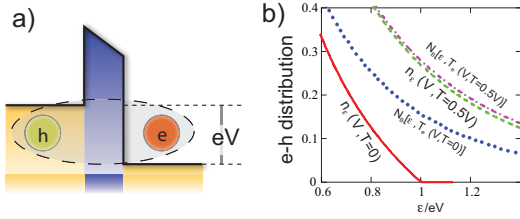


FIG. 2: (color online) a) Illustration of electron-hole pair generation in the tunnel junction, resulting in the distribution function  $n_\varepsilon$  [Eq. (1)] of these pairs (environment). b) Comparison of the distribution functions for  $T = 0$  in the leads and  $T = V/2$ .

be interpreted as the distribution function of electron-hole pairs that appear at the junction interface just after the tunneling process: the hole in the source lead and the electron in the drain, Fig. 2a). The effective temperature of “tunneling electrons”  $T_e$  should be identified with  $\lim_{\varepsilon \rightarrow 0} n_\varepsilon$ . If the distribution functions at the electrodes are Fermi functions with equal temperatures  $T$ , then  $n_\varepsilon = \{(\varepsilon - V)N_B(\varepsilon - V, T) + (\varepsilon + V)N_B(\varepsilon + V, T)\}/2\varepsilon$ , with  $N_B(\varepsilon \pm V, T)$  being the equilibrium Bose distribution function. For this case,  $T_e = \frac{V}{2} \coth \frac{V}{2T}$ . At low applied voltages,  $V \ll T$ , and  $T_e \approx T$ . In the opposite case,  $V \gg T_e$ , we obtain  $T_e \approx V/2$ . The function  $p(\varepsilon)$  in Eq. (1) is the weight function for a junction between two normal metals, Fig. 1, and can be calculated for any choice of the electron distribution function in the leads, resulting in  $p(\varepsilon) = 4\varepsilon/R_T$ .

## III. HEAT FLOW

To calculate  $\dot{Q}$  one has to specify the probability density, which can be written in the form  $P(\varepsilon) = \int_{-\infty}^\infty dt \exp[J(t) + i\varepsilon t]$ , where the function  $\exp[J(t)]$  reflects the fact that tunneling electrons acquire random phases due to interaction with the Bosonic environment.

The equilibrium situation where the distribution function of the environment modes is the Bose distribution was discussed in Ref. [6]. In the general far-from-equilibrium situation, the function  $J(t)$  can be written as<sup>3</sup>

$$J(t)/2 = \int_{[\tau_e(T_e)]^{-1}}^\infty \frac{d\omega}{\omega} \rho(\omega) \left[ N_\omega^{(\text{in})} e^{i\omega t} + (1 + N_\omega^{(\text{out})}) e^{-i\omega t} - B_\omega \right], \quad (2)$$

where the terms proportional to  $N_\omega^{(\text{in})}$  and  $1 + N_\omega^{(\text{out})}$  correspond to the absorbed and emitted environment excitations, respectively, and  $B_\omega = 1 + N_\omega^{(\text{out})} + N_\omega^{(\text{in})}$ . In equilibrium  $N_\omega$  reduces to the Bose-function and the functional  $P(\omega)$  recovers the result of Ref. [6]. The energy relaxation time  $\tau_e$  in expression for  $J(t)$  determines the low energy cut-off, since the electrons start to equilibrate on larger time scales, i.e. the non-equilibrium description does not hold any more. The spectral function  $\rho(\omega)$  is the probability of the electron–environment interaction and characterizes the particular system under consideration.

According to scattering theory, modes coming from one “reservoir” into the other have the temperature of the “reservoir” of their origin.<sup>8</sup> Then the second quantization operators  $c_\alpha^{(\text{in/out})}$  of the emitted (absorbed) environment mode  $\alpha$  enter the following density matrix:

$$\varrho = e^{-\sum_\alpha (c_\alpha^{(\text{in})})^\dagger c_\alpha^{(\text{in})}/T_{\text{in}}} \times e^{-\sum_\alpha (c_\alpha^{(\text{out})})^\dagger c_\alpha^{(\text{out})}/T_{\text{out}}}, \quad (3)$$

where  $T_{\text{in}} = T_{\text{env}}$  and  $T_{\text{out}} = T_e$ . Thus  $N_\omega^{(\text{in})} = \langle (c_\omega^{(\text{in})})^\dagger c_\omega^{(\text{in})} \rangle = N_B(\omega, T_{\text{env}})$  and  $N_\omega^{(\text{out})} = N_B(\omega, T_e)$ . The quasi-equilibrium approximation mentioned above corresponds to Gibbs distribution of the environment modes:  $\varrho_0 = \exp\{-\sum_\alpha c_\alpha^\dagger c_\alpha/T_{\text{env}}\}$ .

To estimate the magnitude of the heat flow  $\dot{Q}$  we first expand the distribution function  $P(\varepsilon)$  in Eq. (1), in the first order in  $\rho(\varepsilon)$ :

$$\dot{Q}^{(1)} = \frac{8}{R_T} \int_{\tau_e^{-1}}^\infty d\varepsilon \varepsilon \rho(\varepsilon) \left\{ n_\varepsilon (1 + N_\varepsilon^{(\text{out})}) - (1 + n_\varepsilon) N_\varepsilon^{(\text{in})} \right\}. \quad (4)$$

This expression becomes zero if  $n_\varepsilon = N_\varepsilon^{(\text{in})} = N_\varepsilon^{(\text{out})}$ . If the distribution functions are not equal to each other, we can expand  $\dot{Q}^{(1)}$  with respect to their difference. We consider the case where the voltage bias at the nano-junction is zero but the temperatures of electrons at the leads and those that comprise the environment are slightly different,  $T_e = T + \delta T/2$  and  $T_{\text{env}} = T - \delta T/2$ . Thus,  $n_\varepsilon = n_\varepsilon(T + \delta T/2)$ ,  $N_\varepsilon^{(\text{in})} = n_\varepsilon(T - \delta T/2)$ ,  $N_\varepsilon^{(\text{out})} = n_\varepsilon(T + \delta T/2)$ , where  $n_\varepsilon$  is the Bose distribution function. Expanding  $\dot{Q}^{(1)}$  in the first order in small parameter  $\delta T/T \ll 1$  we find

$$\dot{Q}_\theta^{(1)} \approx \delta T \frac{8}{R_T} \int_{\tau_e^{-1}}^\infty d\varepsilon \varepsilon \rho(\varepsilon) n'_\varepsilon(T) (1 + \theta n_\varepsilon(T)) \quad (5)$$

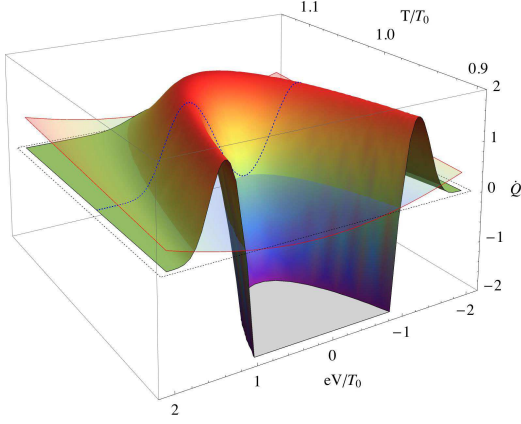


FIG. 3: (color online) Typical heat exchange  $\dot{Q}$  in Eq. (1) of the Ohmic environment with the tunnel junction between two normal leads.  $\dot{Q}(T_{\text{eff}}, T, V)$  vs  $T/T_0$  and voltages  $eV/T_0$  (scaling factor  $T_0 = 30\omega_{\text{max}}$ ). We used  $\omega_{\text{max}}/\omega_{\text{min}} = 100$ ,  $T_{\text{env}}/T_0 = 1$ ,  $\rho(0) = 10$ .  $\dot{Q}$  is measured in units of  $10^3 \omega_{\text{max}}^2 / (e^2 R_T)$ .

where  $n'_\varepsilon(T) = dn_\varepsilon(T)/dT$ . The index  $\theta$  is 0 for the quasi-equilibrium situation when the temperatures of emitted and absorbed environment excitations are equal and 1 for the non-equilibrium case (the index 1 is skipped throughout this paper). Since  $n_\varepsilon(T)$  in Eq. (5) is always positive, the following inequality is valid  $|\dot{Q}_0^{(1)}| < |\dot{Q}^{(1)}|$ , where  $\dot{Q}_0^{(1)}$  and  $\dot{Q}^{(1)}$  refer to the heat flux in quasi-equilibrium and in non-equilibrium cases, respectively. The interaction function  $\rho(\varepsilon)$  in Eq. (5) quickly decays at energies larger than some characteristic frequency  $\omega_{\text{max}}$ . For temperatures  $T > \omega_{\text{max}}$  we can approximate  $n_\varepsilon(T) \approx T/\varepsilon \gg 1$  and find

$$\frac{|\dot{Q}^{(1)}|}{|\dot{Q}_0^{(1)}|} \approx \frac{\int_{\tau_e^{-1}}^{\infty} \frac{T\rho(\varepsilon)d\varepsilon}{\varepsilon}}{\int_{\tau_e^{-1}}^{\infty} \rho(\varepsilon)d\varepsilon} \approx \frac{T}{\omega_{\text{max}}} \ln(\omega_{\text{max}}\tau_e) \equiv \mathcal{N} \gg 1. \quad (6)$$

Remarkably, in higher orders with respect to  $\rho(\varepsilon)$  the non-equilibrium heat flow  $\dot{Q}$  differs from the equilibrium flow  $\dot{Q}_0$  by the same factor. This result holds even for a finite electric current flowing through the junction. Thus, the heat flow between the junction and the environment appears much larger than what the quasi-equilibrium estimates predict.

#### IV. OHMIC APPROXIMATION

We now turn to the simplest case, an environment with a very high impedance as compared to the quantum resistance,  $R_Q$ . In this limit tunnelling electrons easily excite the environment modes. The spectral density  $\rho(\omega)$  of these modes is sharply peaked at the zero frequency,  $\omega = 0$ . For the correlation function  $J(t)$  the concentration of the environment modes at low frequencies implies that the expansion of  $J(t)$  over  $t$  up to the second order

yields  $J(t) \approx -iat - (b/2)t^2$ , where the coefficients  $a$  and  $b$  are defined as  $a = \int_{\tau_e^{-1}}^{\infty} (1 + N_\omega^{(\text{out})} - N_\omega^{(\text{in})}) \rho(\omega) d\omega$  and  $b = \int_{\tau_e^{-1}}^{\infty} \omega \rho(\omega) B_\omega d\omega$ . Using this expansion for  $J(t)$  we obtain the following result for the density function  $P(\omega)$

$$P(\varepsilon) = (1/\sqrt{2\pi b}) \exp[-(\varepsilon - a)^2/2b]. \quad (7)$$

Here the expansion parameter  $a$  can be estimated as follows  $a = a_0 \left(1 + \frac{(T_e - T_{\text{env}}) \ln(\omega_{\text{max}} \tau_e)}{\pi \omega_{\text{max}}}\right)$ , where  $a_0 = 2 \int \rho d\omega \approx 2\rho(0)\omega_{\text{max}} \approx 2E_c$  with  $E_c$  being the charging energy of the tunnel junction,  $T_e$  is the electron temperature in the junction,  $T_{\text{env}}$  is the temperature of environmental modes,  $\omega_{\text{max}} \approx 1/(R_T C)$ . Similar for coefficient  $b$  in Eq. (7) we obtain  $b \approx a_0(T_e + T_{\text{env}})$ .

Substituting the density  $P(\omega)$ , Eq. (7), into the heat flux  $\dot{Q}$ , Eq. (1), we obtain our first main result for the typical heat exchange of the Ohmic environment with the tunnel junction between two normal leads. The full temperature and voltage dependence is shown in Fig. 3.

#### V. DYNAMIC COULOMB INTERACTION

Next we discuss the more realistic situation where the tunneling junction is connected to two disordered conductors (leads). Following Ref. 9, one can find the spectral probability function  $\rho(\omega)$  corresponding to the electron–environment interaction

$$\rho_{ij}(\omega) = \frac{\omega}{2\pi} \text{Im} \sum_{\mathbf{q}} \frac{\left(\frac{2\pi}{L}\right)^2 (2\delta_{ij} - 1) \tilde{U}_{ij}(\mathbf{q}, \omega)}{(\mathcal{D}_i q^2 - i\omega)(\mathcal{D}_j q^2 - i\omega)}, \quad (8)$$

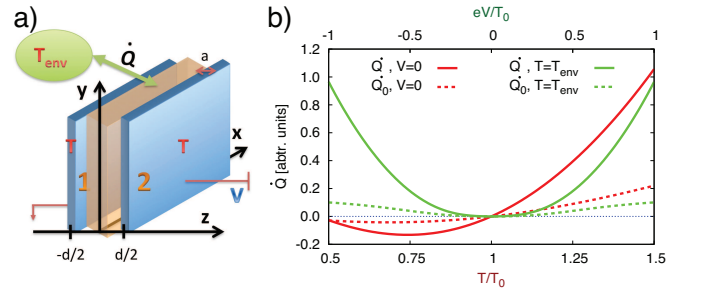


FIG. 4: (color online) **a)**: Schematic presentation of the system: single contact junction, with contacts consisting of two thin plates, which are distance  $d$  apart. Their thickness  $a$  is much less than the extension in  $x$  and  $y$  directions, such that they can be treated as 2D contacts. The temperature of the contacts  $T$  is kept constant, while the environment temperature  $T_{\text{env}}$  can be different, which results in heat production or removal in the junction. **b)**: Heating of a tunnel junction taking into account dynamic Coulomb interactions for the zero bias case ( $V = 0$ ) [red lines, lower x-axis] and the voltage dependence for  $T = T_e$  [green lines, upper x-axis]. The solid curves represent the quasi-equilibrium curves and the dashed assuming an equilibrium distribution for  $N_\omega$ . (temperature in units of  $T_0 = 0.1E_{\text{th}}$ )

where  $i, j = 1, 2$  are the lead indices,  $\mathcal{D}_{1(2)}$  are diffusion coefficients within respective electrodes, and  $\tilde{U}_{ij}(\mathbf{q}, \omega)$  are the dynamically screened Coulomb interactions within (across) the electrodes. The form of spectral probability  $\rho(\omega)$  [ $\rho(\omega) = 2\rho_{12} + \rho_{11} + \rho_{22}$ ] depends on the structure of the environmental excitations spectrum and, thus, on the external bias.

The system under consideration is shown in Fig. 4a): two contacts are separated by distance  $d$  and their thickness is  $a$ . The external bias is  $V$  and the contacts are kept at temperature  $T$  and the environment at temperature  $T_{\text{env}}$ .

The screened Coulomb interaction in Eq. (8) in Fourier space has the form  $\tilde{U}(\mathbf{q}, \omega) = \{[\underline{U}^{(0)}(\mathbf{q}, \omega)]^{-1} + \underline{\mathcal{P}}(\mathbf{q}, \omega)\}^{-1}$ , where  $\underline{U}^{(0)}(\mathbf{q}, \omega) = u(q)\underline{I} + v(q)\underline{\sigma}_x$  is the bare Coulomb interaction and  $\underline{\mathcal{P}}(\mathbf{q}, \omega)$  the polarization matrix respectively with  $\mathcal{P}_{ij} = \nu_i \mathcal{D}_i q^2 (\mathcal{D}_i q^2 - i\omega)^{-1} \delta_{ij}$ .  $\nu_i$  is the electron density of states at the Fermi surface in lead  $i$ .

Below we concentrate on quasi 2D infinite leads. For this geometry with  $a \ll L$ , where  $L$  is the characteristic lead size in the  $x$  and  $y$  directions, the bare Coulomb interaction has the form

$$U_{ij}^{(0)}(\mathbf{r}_i - \mathbf{r}_j) = e^2 \int dz_i dz_j \frac{\delta(z_i - z_i^{(0)}) \delta(z_j - z_j^{(0)})}{|\mathbf{r}_i - \mathbf{r}_j|}, \quad (9)$$

with  $z_i^{(0)} = (1/2 - \delta_{i1})d$ , leading to  $u(q) = 2\pi e^2/q$  and  $v(q) = 2\pi e^2 e^{-qd}/q$ .

In the following, we consider the case of identical leads with same diffusion coefficients  $\mathcal{D}_1 = \mathcal{D}_2 \equiv \mathcal{D}$  and densities of states,  $\nu_1 = \nu_2 \equiv \nu$ . The dimensionless matrix elements  $\tilde{U}_{ij}$  of the dynamically screened Coulomb interaction (in units of  $e^2 d$ ) are then given by

$$\tilde{U}_{ii} = \frac{4\pi}{\tilde{q}} \frac{\chi(\tilde{q})}{\chi^2(\tilde{q}) - \coth^2(\tilde{q})}, \quad \tilde{U}_{i \neq j} = \frac{\tilde{U}_{ii}}{\chi(\tilde{q}) \coth(\tilde{q})} \quad (10)$$

where  $\tilde{q} = dq$  and  $\tilde{\omega} \equiv \omega(d^2/\mathcal{D})$  with the dimensionless function  $\chi(\tilde{q}) \equiv 1 + \coth(\tilde{q}) + \frac{4\pi e^2 d \nu x}{q^2 - i\tilde{\omega}}$ . Using these expressions, we can write Eq. (8) as

$$\rho(\tilde{\omega}) = \frac{2e^2 d}{\mathcal{D}} \tilde{\omega} \text{Im} \int_0^\infty \tilde{q} d\tilde{q} \frac{\tilde{U}_{11} [1 - (\chi(\tilde{q}) \coth(\tilde{q}))^{-1}]}{(\tilde{q}^2 - i\tilde{\omega})^2}.$$

Using this expression we can calculate the heat flux  $\dot{Q}$  in Eq. (1) between environment and nanojunction with dynamic Coulomb interaction. The typical energy scale is given by the Thouless energy for the junction of distance  $d$ ,  $E_{\text{th}} = \mathcal{D}/d^2$  which we use to rewrite all expressions in dimensionless units. For a typical temperature  $E_{\text{th}} \approx 100K$ , the temperature and voltage dependence is numerically calculated and shown in Fig. 4b). Again, the non-equilibrium heat flow  $\dot{Q}$  is up to an order of magnitude larger and the quasi-equilibrium approximation  $\dot{Q}_0$ . We remark, that in this case the function  $\rho(\omega)$  introduces a natural cut-off for  $J(t)$  which behaves as  $\sim -|t|$  for large  $t$ .

## VI. DISCUSSION

Above we assumed that the density of hot electrons is high enough so that the electron-electron scattering time is smaller than the time of energy relaxation (this time is large because of quasi-elastic nature of interaction between the electrons and environment). In this case the electron distribution function is close to an equilibrium one with an electron temperature  $T_e$ , which is in high voltage limit is higher than the environment temperature  $T_{\text{env}}$ .

In summary, we discussed the influence of far from equilibrium heating effects on properties of nanojunctions. Based on a quantum-kinetic approach we calculated the non-linear heat flux between environment and junction. We showed that the resulting freezing or heating effect far from equilibrium are by orders of magnitude larger than estimates based on quasi-equilibrium environment theory. We obtained analytical results for the heat flow in an idealized high-impedance environment and demonstrated, numerically, that these results hold for the more general case of an environment with Coulomb interaction. We showed that the environment can be a very effective freezing agent if the effective temperature well exceeds the high frequency cut-off  $\hbar\omega_{\text{max}}$ . From the experimental point of view the temperature regime in which the effect is present is readily accessible. However, one needs to measure the time dependence of the junction temperature in order to extract  $\dot{Q}$  which could be technically challenging for a nanojunction and the presence of a substrate might need consideration.

One can expect that our results, in particular the giant freezing effect, will be important for the electronic transport in junction arrays,<sup>10</sup> which will be subject of a forthcoming work.

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## Appendix A: Heat Flow rate

In this Appendix we present a derivation of Eq. (1) for the rate of the heat flow. A general formula for the heat current going from the left electrode (1) towards the right electrode reads:

$$I_q^{1 \rightarrow} = -(\Gamma_q^{1, \text{in}} - \Gamma_q^{1, \text{out}}), \quad (\text{A1})$$



where  $\Gamma_q^{1\rightarrow} (\Gamma_q^{1\leftarrow})$  is the heat transfer tunneling rate calculated in the left electrode:

$$\begin{aligned}\Gamma_q^{1,\text{out}} &= \frac{1}{R_T} \int_{\epsilon\epsilon'} (\epsilon - \phi_1) f_\epsilon^{(1)} (1 - f_{\epsilon'}^{(2)}) P(\epsilon - \epsilon'), \\ \Gamma_q^{1,\text{in}} &= \frac{1}{R_T} \int_{\epsilon\epsilon'} (\epsilon' - \phi_1) (1 - f_{\epsilon'}^{(1)}) f_\epsilon^{(2)} P(\epsilon - \epsilon').\end{aligned}$$

Under the gauge transformation  $\phi \rightarrow \phi - \partial_t k$ , the distribution functions transform like  $f^{(i)}(\epsilon) \rightarrow f^{(i)}(\epsilon + \partial_t k)$ . Therefore the rates and the heat current defined above are gauge invariant.

Similarly we can find,

$$I_q^{2\rightarrow} = -(\Gamma_q^{2,\text{out}} - \Gamma_q^{2,\text{in}}), \quad (\text{A2})$$

where  $\Gamma_q^{2\rightarrow} (\Gamma_q^{2\leftarrow})$  is the heat transfer tunneling rate calculated in the right electrode:

$$\begin{aligned}\Gamma_q^{2,\text{out}} &= \frac{1}{R_T} \int_{\epsilon\epsilon'} (\epsilon - \phi_2) f_\epsilon^{(2)} (1 - f_{\epsilon'}^{(1)}) P(\epsilon - \epsilon'), \\ \Gamma_q^{2,\text{in}} &= \frac{1}{R_T} \int_{\epsilon\epsilon'} (\epsilon' - \phi_2) (1 - f_{\epsilon'}^{(2)}) f_\epsilon^{(1)} P(\epsilon - \epsilon').\end{aligned}$$

The gradient of the heat current,  $\nabla I_Q$  at the contact is

$$\nabla I_Q = I_q^{2\rightarrow} - I_q^{1\rightarrow} = \{\Gamma_q^{1,\text{out}} - \Gamma_q^{2,\text{in}}\} + \{\Gamma_q^{2,\text{out}} - \Gamma_q^{1,\text{in}}\}.$$

Finally we find

$$\begin{aligned}\nabla I_Q &= (\phi_2 - \phi_1)I + \\ &\frac{1}{R_T} \int_{\epsilon\epsilon'} (\epsilon - \epsilon') P(\epsilon - \epsilon') \left\{ f_\epsilon^{(1)} (1 - f_{\epsilon'}^{(2)}) + f_\epsilon^{(2)} (1 - f_{\epsilon'}^{(1)}) \right\}.\end{aligned}$$

On the other hand the conservation law demands:

$$\dot{Q} + \nabla I_Q = \mathcal{E}I, \quad (\text{A3})$$

where the right-hand side is the Joule heat which is related to the work of the electric field. The Joule heat is dissipated in the bulk of the electrodes at the distance

$l_E$  from the junction, where  $l_E$  is the energy relaxation length. The heat  $\dot{Q}$  is the heat dissipated into the environment:

$$\dot{Q} = \frac{1}{R_T} \int_{\epsilon\epsilon'} (\epsilon - \epsilon') \sum_{i,j} f_\epsilon^{(i)} \sigma_{ij}^x (1 - f_{\epsilon'}^{(j)}) P(\epsilon - \epsilon'), \quad (\text{A4})$$

where  $\sigma_x$  is the Pauli matrix. Equation (A4) can be rewritten in terms of "Bose" distribution functions as follows

$$\dot{Q} = \int_0^\infty d\varepsilon \varepsilon p(\varepsilon) \{n_\varepsilon P^<(\varepsilon) - [1 + n_\varepsilon] P(-\varepsilon)\}, \quad (\text{A5})$$

with  $p(\varepsilon) = 4\varepsilon/R_T$  and  $n_\varepsilon = \{(\varepsilon - V)N_B(\varepsilon - V, T) + (\varepsilon + V)N_B(\varepsilon + V, T)\}/2\varepsilon$ . Equation (A5) coincides with Eq. (1) in the body of the paper. In the zero-voltage limit Eq. (A5) agrees with the corresponding expression in Ref. [5].

#### Appendix B: Heat flow in the second order in $\rho(\varepsilon)$

Below Eq. (3) we derived the heat flow  $\dot{Q}$  in the leading (first) order in spectral function  $\rho(\varepsilon)$ . In this appendix we show that the heat flow  $\dot{Q}$  in the second order in electron-environment interaction and in the first (leading) order in temperature difference  $\delta T = T_e - T_{\text{env}}$  leads to the same enhancement as the first order term.

The heat flow can be written as a sum of two terms,  $\dot{Q} = (W_1 + W_2)\tau$ , where

$$W_1 = \frac{1}{2} \int_{-\infty}^\infty d\varepsilon \varepsilon p(\varepsilon) [\partial_{\tilde{T}} n_\varepsilon] P(\varepsilon), \quad (\text{B1})$$

$$W_2 = \int_0^\infty d\varepsilon \varepsilon p(\varepsilon) n_\varepsilon \partial_\tau \{P(\varepsilon) - P(-\varepsilon)\}. \quad (\text{B2})$$

Here  $\tilde{T} = (T_e + T_{\text{env}})/2$ . Typically  $W_1 \lesssim W_2$ , therefore we concentrate on contribution  $W_2$  below.

In the second order in electron-environment interaction (function  $\rho(\varepsilon)$ ) we obtain for the heat flow the following result

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$$\begin{aligned}\dot{Q}^{(2)} &\propto \int_0^\infty d\varepsilon d\varepsilon_1 d\varepsilon_2 \varepsilon p(\varepsilon) \frac{\rho(\varepsilon_1)}{\varepsilon_1} \frac{\rho(\varepsilon_2)}{\varepsilon_2} \times \\ &\left\{ n_\varepsilon^{(12)} (1 + N_{\varepsilon_1}^{(\text{out})}) (1 + N_{\varepsilon_2}^{(\text{out})}) \delta_{\varepsilon - \varepsilon_1 - \varepsilon_2} + n_\varepsilon^{(12)} N_{\varepsilon_1}^{(\text{in})} (1 + N_{\varepsilon_2}^{(\text{out})}) \delta_{\varepsilon + \varepsilon_1 - \varepsilon_2} + n_\varepsilon^{(12)} (1 + N_{\varepsilon_1}^{(\text{out})}) N_{\varepsilon_2}^{(\text{in})} \delta_{\varepsilon - \varepsilon_1 + \varepsilon_2} - \right. \\ &\left. (1 + n_\varepsilon^{(12)}) N_{\varepsilon_1}^{(\text{in})} N_{\varepsilon_2}^{(\text{in})} \delta_{\varepsilon - \varepsilon_1 - \varepsilon_2} - (1 + n_\varepsilon^{(12)}) (1 + N_{\varepsilon_1}^{(\text{out})}) N_{\varepsilon_2}^{(\text{in})} \delta_{\varepsilon + \varepsilon_1 - \varepsilon_2} - (1 + n_\varepsilon^{(12)}) N_{\varepsilon_1}^{(\text{in})} (1 + N_{\varepsilon_2}^{(\text{out})}) \delta_{\varepsilon - \varepsilon_1 + \varepsilon_2} \right\}. \quad (\text{B3})\end{aligned}$$

At low frequencies ( $T \gg \varepsilon$ ) we find the  $W_2$  contribution to  $\dot{Q}^{(2)}$  as follows

$$\begin{aligned}n_\varepsilon^{(12)} (1 + N_{\varepsilon_1}^{(\text{out})}) (1 + N_{\varepsilon_2}^{(\text{out})}) - (1 + n_\varepsilon^{(12)}) N_{\varepsilon_1}^{(\text{in})} N_{\varepsilon_2}^{(\text{in})} &\approx \\ n_\varepsilon^{(12)} \frac{(T_e)^2}{\varepsilon_1 \varepsilon_2} - (1 + n_\varepsilon^{(12)}) \frac{(T_{\text{env}})^2}{\varepsilon_1 \varepsilon_2} &\approx \dots + (T_e - T_{\text{env}}) (1 + 2n_\varepsilon^{(12)}) \frac{T_e + T_{\text{env}}}{2\varepsilon_1 \varepsilon_2} + \dots \quad (\text{B4})\end{aligned}$$

Here dots represent the terms that finally cancel in Eq. (B3). We mention the presence of a large enhancement factor  $(1 + 2n_\varepsilon^{(12)}) \approx (T_e + T_{\text{env}})/\varepsilon \gg 1$  in Eq. (B4).

In quasi-equilibrium case we do not have this large factor. Indeed, in this case we have

$$n_\varepsilon^{(12)}(1 + N_{\varepsilon_1}^{(\text{out})})(1 + N_{\varepsilon_2}^{(\text{out})}) - (1 + n_\varepsilon^{(12)})N_{\varepsilon_1}^{(\text{in})}N_{\varepsilon_2}^{(\text{in})} \approx n_\varepsilon^{(12)}\frac{(T_e)^2}{\varepsilon_1\varepsilon_2} - (1 + n_\varepsilon^{(12)})\frac{(T_e)^2}{\varepsilon_1\varepsilon_2} \approx \dots + (T_e - T_{\text{env}})\frac{T_e + T_{\text{env}}}{2\varepsilon_1\varepsilon_2} + \dots \quad (\text{B5})$$

To conclude, in the second order in function  $\rho(\varepsilon)$ , the heat flow  $\dot{Q}$  is enhanced by the same factor  $(T_e + T_{\text{env}})/\varepsilon \gg 1$  as in the first order. Similar result can be proofed in higher orders in  $\rho(\varepsilon)$ .

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