Quantum physics inspired optical effects in tight-binding lattices: Phase-controlled photonic transport

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Quantum physics inspired optical effects in tight binding lattices – phase controlled photonic transport

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A theoretical proposal for photonic transport in an array of waveguides is presented. By coupling phase-displaced inputs into adjacent waveguides, and calculating the expectation value of the position space operator, we demonstrate directed motion in this system. The one way photonic transport arises from interferences due to the phase differences in the inputs to the waveguides.

I. INTRODUCTION

Integrated optical structures in the form of an array of coupled waveguides have, in recent years, become a paradigm for demonstrating a variety of phenomena which were hitherto difficult to realize. Some of these effects, such as Anderson localization[1], and Bloch oscillations[2,3], were ideas that were originally conceived in the context of condensed matter systems. However, coupled arrays of optical waveguides have been shown to be ideal systems to observe these effects. In addition, some distinctly quantum mechanical effects have also been realized in such waveguides. Two examples of such effects are random walk of photons[4], and the observation of Dirac zitterbewegung[5].

Historically, many ideas in quantum physics had their pre-cursors in classical wave optics. With the availability of integrated optical structures, it is now possible to realize quantum effects in seemingly classical systems. The wave equation in the paraxial approximation, which is normally used to describe the propagation of light in discrete structures, is nearly identical to the Schrödinger equation. Since the wave equation and the Schrödinger equation are mathematically similar, light propagation in integrated structures, such as an array of coupled waveguides, allows the possibility of studying quantum physics inspired effects over lengths scales ranging from micrometer to nanometer, hence making it easier to observe and measure these effects.

It has been shown previously that the Hamiltonian for the evolution of a wavepacket in an array of coupled waveguides is well described by the tight binding Hamiltonian which was originally used to describe the time evolution of an electron in a metallic medium. This in fact was the reason for sucess in realizing, using waveguide arrays, many of the condensed matter effects which, in addition to those mentioned in the first paragraph, include studies of defects in lattices[6], and rectification of light[7]. Further, many quantum optical effects can be studied using waveguide arrays. These include the demonstration of a classical analog of the quantum zeno effect[8], an investigation of the Hanbury-Brown-Twiss correlations in waveguide lattices[9], optical bloch oscillations with single photons[10] as well as with NOON states[11], Hong Ou Mandel interference[12], and Anderson localization with quantized fields[1,13].

In previous work, Peschel, et. al.[2] have shown theoretically that if a single waveguide in an array is excited, then under suitable conditions one can observe the breathing mode oscillations. The requirement for observing these oscillations was that the potential term in the tight binding model which describes this system be modified so that the refractive index increases linearly with the waveguide index. Subsequently, the predictions of this work were observed experimentally by Pertsch and co-workers,[3] and by Morandotti and co-workers[3]. This paper also showed that if a spatially broad wavepacket was launched into the array such that the initial wavepacket spanned a few waveguides, the light showed Bloch oscillation like behavior but confined itself to the high refractive index side of the array.

In this paper, we utilize the same system, viz.. one where the refractive index of the waveguides is linearly varying across the array, but instead of exciting the array with a delta-function input, i.e. single waveguide excitation, or a spatially broad input, we excite two neighboring waveguides with delta-function inputs that are phase displaced from each other. This type of input gives rise to novel effects that are not seen with spatially broad inputs that excite a few waveguides simultaneously, or with the excitation of a single waveguide. In particular, we demonstrate that by altering the relative phase between the two inputs, one can get an all-optical ratchet effect, where the light inside the waveguide array can be deflected towards the increasing refractive index side of the array or to the opposite direction. Such directed motion, or transport of light, is shown to be a consequence of the relative phase between the two inputs.

The results are reminiscent of the quantum ratchet that has been studied theoretically and experimentally in Bose-Einstein condensates[14]. The distinctions between the ratchet like behavior proposed here, versus the classical ratchets that have been studied in statistical physics, are: classical ratchets normally require an asymmetric potential, external noise and diffusion, whereas our proposed model does not require any external noise, and there is no quantum mechanical analog for diffusion. Instead, the directed motion of light in a preferred direction arises from interference due to the phase displaced inputs. Therefore, interference in our system plays a role like that of diffusion in classical ratchets. Therefore, while classical ratchets usually rely on an asymmetric potential with a symmetric input, here we essentially have a symmetric potential and an asymmetric input. In the usual systems dealing with massive particles, classical or quantum, one has a kinetic energy term and potential term. As we show later, in our case of waveguides, the analog of the kinetic energy term is the refractive index ramp; the analog of the potential term is the coupling between different waveguides. The last part is symmetric, whereas the ramp part is asymmetric. Thus, the total H is asymmetric, though the intra-waveguide interaction term is symmetric.

We start with the quantum mechanical Hamiltonian for our system, and deduce the Heisenberg equations for the time evolution of the field operator in each waveguide. Subsequently, we consider a classical description where the field operators are regarded as numbers. Note that up to this point, the quantum and the classical descriptions are mathematically identical, a direct consequence of the wave equation and the Schrödinger equation being similar. The ratchet-like effect, i.e. the preferred direction of photonic transport, that we propose here can then therefore be regarded as being a classical analog of a quantum ratchet.

II. THEORETICAL MODEL

In this section we describe the theoretical basis for photonic transport that is controlled by interference. We consider an array of waveguides, as shown in Fig. 1, in which the nearest neighboring waveguides are evanescently coupled to each other. The Hamiltonian for this system is known to be isomorphic to the Hamiltonian in the tight binding model, and is given by

$$H = \hbar \sum_j \beta(j) a_j^\dagger a_j + \hbar C \sum_j (a_{j+1}^\dagger a_j + a_{j-1}^\dagger a_j) \tag{1}$$

where the individual waveguides in the array are labeled by the index j and $\beta(j)$ is related to the refractive index of the j^{th} waveguide.

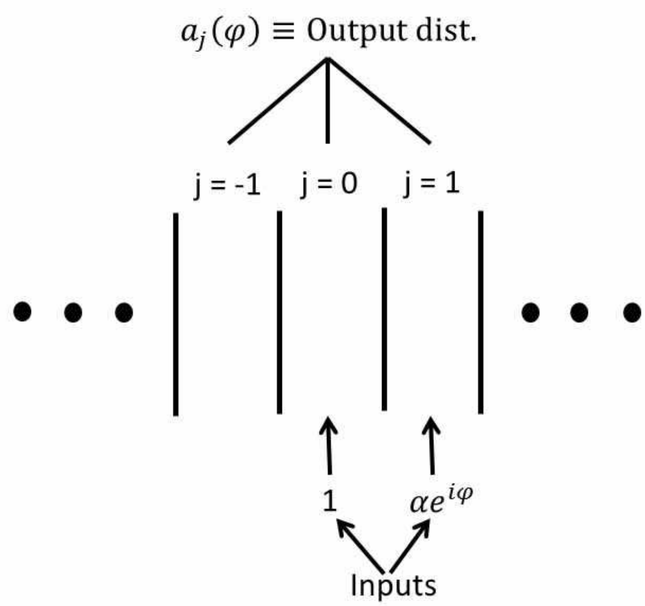

FIG. 1. A large waveguide array with the input fields shown.

In Eq. (1), C is the coupling between adjacent waveguides, and $a_j^\dagger(a_j)$ is the creation (annihilation) operator for the jth waveguide. These operators obey the Heisenberg equations, given by

$$\dot{a}_j = -i\beta(j)a_j - iC(a_{j+1} + a_{j-1}) \tag{2}$$

We now turn to a classical description (for a fully quantum mechanical description, see Ref. 23) and so a_j's are regarded as numbers. We now write,

$$a_j = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \tilde{a}(k) e^{ikj} dk \tag{3}$$

$$\tilde{a}(k) = \frac{1}{\sqrt{2\pi}} \sum_j a_j e^{-ikj} \tag{4}$$

Eq. (2) can now be written in the Fourier space, by using Eq. (2) and Eq. (3), as

$$\dot{\tilde{a}}(k) = -2iC\tilde{a}(k)\cos k - i\beta\left(\frac{\partial}{\partial(-ik)}\right)\tilde{a}(k) \tag{5}$$

Clearly, Eq. (5) now has a term describing a periodic potential, i.e. the term containing C. The β term is in a sense the analog of the kinetic energy term as it involves the derivatives, though its explicit form depends on the functional form of β. We now choose

$$\beta(j) = j\beta \tag{6}$$

and then Eq. (5) can be written as

$$\dot{\tilde{a}}(k) = -2iC\tilde{a}(k)\cos k - ij\beta\frac{\partial}{\partial(-ik)}\tilde{a}(k) \tag{7}$$

The choice of the waveguide index, as given by Eq. (6), has been experimentally realized in studies on Bloch oscillations in waveguide arrays. It is important to emphasize here that for the waveguide structure under investigations, the periodic potential is in the Fourier space, k. Therefore, to study the photonic transport properties of this system, we study the directed motion in the site-space j.

The input conditions on the field amplitude are chosen as

$$a_j(t=0) = \delta_{j,0} + \alpha\delta_{j,1}e^{i\varphi} \tag{8}$$

We assume that the array index runs from $j = -\infty$ to ∞ (see Fig. 1), where $j = 0$ is the middle waveguide. The $|a_j|^2$ play the role of density distributions. The directed transport that we are interested in is obtained from a calculation of $|a_j|^2$ for non-zero α and φ. The possibility of additional interference effects due to a no-zero α leads to photonic transport, and can be viewed as the classical analog of a quantum ratchet.

In order to study the photonic transport, we calculate the expectation values for $< j >$ and $< j^2 >$ which are analogous to the expectation values of momentum and energy respectively. These expectation values are calculated as $< j >= \sum_{j=-\infty}^{\infty} jI_j$ and $< j^2 >= \sum_{j=-\infty}^{\infty} j^2 I_j$ where I_j is the output intensity of the j^{th} waveguide. We write the solution to the coupled equations, Eq. (2), in the form

$$a_j = \sum_{j'} G_{j,j'}a_{j'}(0) \tag{9}$$

where $a_j(0)$ is given by Eq. (8) and the Green's function is given by Eq. (21) from Ref 2

$$G_{j,j'} = \exp\left[i\beta z + \frac{i(j-j')(\beta z - \pi)}{2}\right]$$

$$\times J_{j'-j}\left[\frac{4C}{\beta}\sin\left(\frac{\beta z}{2}\right)\right] \tag{10}$$

Using the initial condition of Eq. (8), the output intensity from the waveguide array can be written as

$$I_j = |G_{j,0}|^2 + |\alpha G_{j,1}|^2 + \alpha G_{j,0}G^*_{j,1}e^{-i\varphi} + \alpha G^*_{j,0}G_{j,1}e^{i\varphi}$$

$$= |J_{-j}\left[\frac{4C}{\beta}\sin\left(\frac{\beta z}{2}\right)\right]|^2 + |\alpha J_{1-j}\left[\frac{4C}{\beta}\sin\left(\frac{\beta z}{2}\right)\right]|^2$$

$$-2\alpha J_{-j}\left[\frac{4C}{\beta}\sin\left(\frac{\beta z}{2}\right)\right]J_{1-j}\left[\frac{4C}{\beta}\sin\left(\frac{\beta z}{2}\right)\right]$$

$$\times \sin\left(\frac{\beta z}{2} - \varphi\right) \tag{11}$$

Using Eq. (11), the expectation value of the site position can be written as (after using properties of Bessel functions)

$$\sum_{j=-\infty}^{\infty} jI_j = |\alpha|^2 + \frac{4\alpha C}{\beta} \sin\left(\frac{\beta z}{2}\right) \sin\left(\frac{\beta z}{2} - \varphi\right) \tag{12}$$

Similarly, the expectation value for the analog of energy is given by

$$\sum_{j=-\infty}^{\infty} j^2 I_j = |\alpha|^2 + \frac{4\alpha C}{\beta} \sin\left(\frac{\beta z}{2}\right) \sin\left(\frac{\beta z}{2} - \varphi\right)$$

$$+ \frac{1+|\alpha|^2}{2} \left(\frac{4C}{\beta} \sin\left(\frac{\beta z}{2}\right)\right)^2 \tag{13}$$

For small values of z, the average position, Eq. (12), is proportional to $-\alpha \sin(\varphi) z$, which shows that the direction of transport for the photons is dependent on the relative phase of the inputs.

III. RESULTS

We now describe the results on photonic transport that can arise in a waveguide array when phase-displaced inputs are utilized. For all results shown, the inputs are at the $j = 0$ and $j = 1$ waveguides. Fig. 2(a) shows the output intensity from the array, given by Eq. (11), when α is equal to zero. This results in profiles which are symmetric about the center of the array, and one sees the breathing modes. However, when α is non-zero, the transport effect is immediately evident. For example, in Fig. 2(b), we display the intensity profiles when $\varphi = 37°$. These profiles are asymmetric about the j-axis, indicating transport to the lower index side of the array. However, when $\varphi = 217°$, as in Fig. 2(c), the direction of the asymmetry is reversed, indicating that the transport is now to the high index side of the array. These results suggest that the photonic transport is a consequence of interference, and that one can control the direction of the transport through the relative phase of the inputs. In particular, when φ lies between $0°$ and $180°$, one gets a transport to the low index side, and for phi between $180°$ and $360°$, to the high index side.

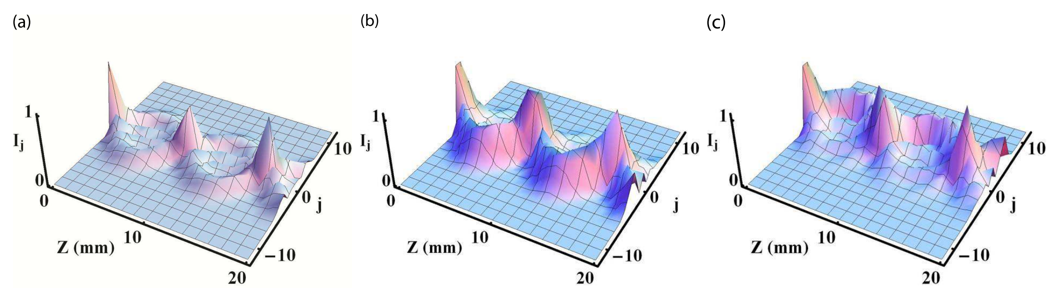

FIG. 2. (Color Online) Output intensity distribution, I_j, as a function of z for $j = -12, ..., 12$ The values of the parameters are $\beta/C = 073$ with (a) $\alpha = 0$ (b) $\alpha = 1$, $\varphi = 37°$, and (c) $\alpha = 1$, $\varphi = 217°$.

The results described here exploit asymmetric inputs in a system with a symmetric, periodic potential, to obtain photonic transport. Thus, this work is quite different from other work wherein either a single waveguide was excited, or several waveguides were excited simultaneously. The novel feature here is the phase displaced inputs, which gives us an additional parameter, viz. the relative phase of the inputs, to control the direction of transport.

Next, we discuss the consequences of Eq. (12) and Eq. (13), viz. the analogs of momentum and energy respectively. Fig. 3(a) shows the behavior of $< j >$ as a function of the propagation distance within a waveguide for $\varphi = 37°$. Note that the profiles are oscillatory, indicating that the direction of the transport within a waveguide is periodically alternating. However, for very small values of z, the slope of the curve is negative, indicating a deflection to the low index side of the array. Fig. 3(b) shows the behavior of energy as a function of propagation distance.

Once again the profiles are oscillatory. When the phase is taken to $217°$, as in Fig. 4, we find that both the momentum and energy show periodic oscillations, but that the deflection of photons is now to the high index side.

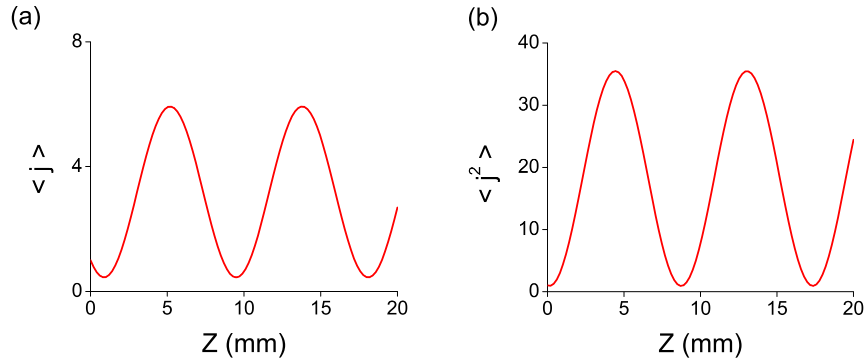

FIG. 3. (Color Online) Plots showing ratchet like behavior for (a) the momentum and (b) the energy. The values of the parameters are $\alpha = 1$, $\beta/C = 0.73$, and $\varphi = 37°$.

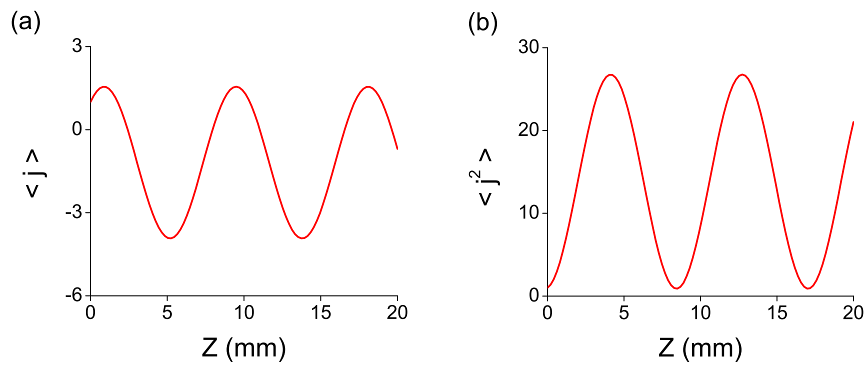

FIG. 4. (Color Online) Plots showing ratchet like behavior for (a) the momentum and (b) the energy. The values of the parameters are $\alpha = 1$, $\beta/C = 0.73$, and $\varphi = 217°$.

IV. SUMMARY

In summary, we have described all-optical, quantum physics inspired photonic transport in a system of evanescently coupled waveguide array. Starting from the tight binding Hamiltonian that is used to describe such systems, we have derived an equation of motion that is reminiscent of the kicked rotor model. However, unlike that model where the periodic potential is usually in coordinate space, here the periodic potential is in Fourier space. This requires one to study the photonic transport in the site-space, which is fortunate because all waveguide structures are usually studied in site-space. The key element in our model for obtaining photonic transport is the use of coherent amplitudes which are displaced in phase.

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