Propagation of disturbances in degenerate quantum systems

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Propagation of Disturbances in Degenerate Quantum Systems

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Abstract

Disturbances in gapless quantum many-body models are known to travel an unlimited distance throughout the system. Here, we explore this phenomenon in finite clusters with degenerate ground states. The specific model studied here is the one-dimensional J1-J2 Heisenberg Hamiltonian at and close to the Majumdar-Ghosh point. Both open and periodic boundary conditions are considered. Quenches are performed using a local magnetic field. The degenerate Majumdar-Ghosh ground state allows disturbances which carry quantum entanglement to propagate throughout the system, and thus dephase the entire system within the degenerate subspace. These disturbances can also carry polarization, but not energy, as all energy is stored locally. The local evolution of the part of the system where energy is stored drives the rest of the system through long-range entanglement. We also examine approximations for the ground state of this Hamiltonian in the strong field limit, and study how couplings away from the Majumdar-Ghosh point affect the propagation of disturbances. We find that even in the case of approximate degeneracy, a disturbance can be propagated throughout a finite system.

1 Introduction

This paper uses quantum information measures, such as entanglement, and trace distance to study quantum many body systems. Unlike physical observables, such quantities usually cannot be directly measured [1], but can give an important insite into the properties of the system. Abstract concepts such as quantum entanglement have been important for almost as long as quantum mechanics has existed [2]. The power of these information theoretical quantities is that they represent general ideas that can be applied to any system which can be considered quantum. By studying such abstract quantities one can more easily generalize a result for a specific system to more universal behavior. Examples of successful application of quantum information measures to the study of quantum many body systems are many, a few examples are [3, 4, 5, 6, 7, 8]. The specific uses of these quantities can be diverse, for example in [4] the authors use the concept of trace distance from an averaged density matrix to define a type of quantum equilibration which would be analogous to equilibration in classical thermodynamics. Similar questions are examined, but with different methods, in [5, 6], where the concept of equilibration is used to detect criticality in a system. In [3] a quantity related to fidelity is used to detect quantum chaos. This paper will make broad use of such quantum informational quantities, but will deal with relatively few direct observables. This is because our intention is to provide a study which can be easily related to other quantum systems, and to quantum many body theory in general.

The central result of this paper involves a type of local quench which can propagate disturbances an unlimited distance in a J1-J2 Heisenberg spin chain. The unitary dynamics of spin chains which can be studied through quenches can be realized experimentally with trapped cold atoms [9, 10]. Certain superconducting qubit arrays can also provide promising physical realizations of spin chain Hamiltonians [11, 12]. Quenches are also important from a theoretical perspective. For example, quantum equilibration can be induced and studied in spin chains using various quenches [4, 5, 6, 13]. Certain local quenches have also been proposed as a way to physically measure entanglement entropy [1]. Furthermore local magnetic field quenches similar to those studied in this paper have been used to study entanglement specifically in Heisenberg spin chains[7], as well as other quantum systems [8]. A generalization of the specific system which is studied in this paper has also been proposed as being possibly useful in quantum computation [14].

The frustrated spin-1/2 anti-ferromagnetic Heisenberg chain has one of the most prototypical matrix product ground states, featuring a two-fold degeneracy at the so-called Majumdar-Ghosh point [15], when the nearest-neighbor and next-nearest-neighbor exchange integrals are the same. The Hamiltonian of this system is given by

$$H_{MG} = \sum_{j=1}^{N} \left(\vec{S}_j \cdot \vec{S}_{j+1} + \frac{1}{2} \vec{S}_j \cdot \vec{S}_{j+2} \right), \tag{1}$$

where the sum extends over N lattice sites, and the two terms represent anti-ferromagnetic nearest-neighbor and next-nearest neighbor Heisenberg interactions respectively. The ground state of this model is exactly known [15],

$$|\psi_{1,MG}\rangle = \bigotimes_{l=1}^{\frac{N}{2}} \frac{(|\uparrow_{2l-1}\downarrow_{2l}\rangle - |\downarrow_{2l-1}\uparrow_{2l}\rangle)}{\sqrt{2}}, \tag{2}$$

i.e. the product of nearest-neighbor spin singlets, assuming an even number of lattice sites. For the case of open boundary conditions, this state is unique, whereas for periodic boundary conditions it is two-fold degenerate, as the underlying lattice can be decorated by the singlet product state in another unique way,

$$|\psi_{2,MG}\rangle \quad = \quad \bigotimes_{l=1}^{\frac{N}{2}} \frac{1}{\sqrt{2}}(|\uparrow_{mod_N 2l}\downarrow_{mod_N(2l+1)}\rangle - |\downarrow_{mod_{\frac{N}{2}} 2l}\uparrow_{mod_{\frac{N}{2}}(2l+1)}\rangle).$$

The resulting ground state for the periodic system is a superposition,

$$|\psi_{PB,MG}\rangle = a|\psi_{1,MG}\rangle + b|\psi_{2,MG}\rangle, \tag{3}$$

where the two terms are not automatically orthogonal. [16] Hence, changing the boundary conditions of the Hamiltonian from open to periodic one goes from a unique to a two-fold degenerate ground state, thus allowing us to study the effects of a ground state degeneracy.

Local disturbances of this ground state can be introduced by applying a local magnetic field h to a subset of N' adjacent spins,

$$H(h, N') = H_{MG} - h\sum_{j=1}^{N'} S_j^z, \tag{4}$$

where without loss of generality we consider the direction of the applied field to be along the z-direction.

One can take advantage of the fact that spin polarization is conserved in this system, allowing one to reduce the complexity of the problem by dividing the Hamiltonian into independent spin sectors, which may each be diagonalized independently. These sectors correspond to the total polarization of the system in the z direction, and may be diagonalized independently. The polarization sector which contains the global ground state of the system changes with field strength, therefore figures 3, 3,5,9, 10, and 11 all show curves for three different polarization sectors. Each sector is labeled with the total z polarization of the entire spin chain in that sector, which is conserved under the action of all Hamiltonians considered in this paper. For example in the basis where S_j^z is diagonal, all of the basis states in the L=0 sector will have the same number of spins pointing in +z as -z, in the L=-1 state, 2 more spins will be facing in -z than +z, etc.

In this study, we identify several effects induced by the application of a local magnetic field, as depicted in Fig. 1. Here we briefly summarize our findings.

Firstly, for sufficiently small field amplitudes polarization induced by the local magnetic field is stored in the vicinity of the region to which the field is applied, instead of spreading throughout the entire system. Only beyond a certain threshold field, i.e. once some of the polarization in this boundary region has saturated, can it spread throughout the entire system. We argue that this is to be expected because at the Majumdar-Ghosh point the energy spectrum of the J1-J2 Heisenberg Hamiltonian is gapped. Provided that the energy gained from the locally applied magnetic field is small compared to the coupling energy

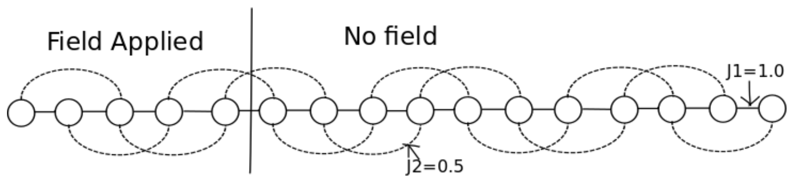

Figure 1: Example of a local field applied to the Majumdar-Ghosh Hamiltonian.

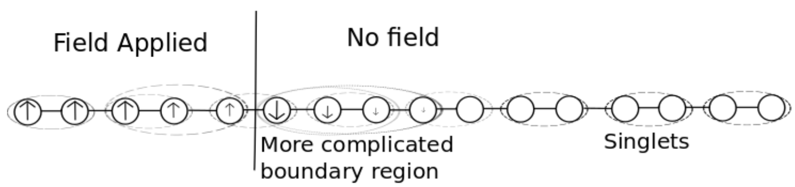

Figure 2: Sketch of a typical state of the spins when exposed to a field. For all field strengths studied here, the ground state of at least one total spin sector behaves like this, and one of these states always is the global ground state of the system. Ovals represent entanglement, arrows indicate spin polarization.

of the spins, any state which keeps the majority of spins in a matrix product configuration similar to the zero field ground state will have a lower energy. For an even number of spins in the non-field region, the system can only accomplish this if the total polarization of a given subsystem far from the field region is zero. The spins in the field region align in the direction of the applied field, thus in turn leading to an excess opposite polarization of the spins not directly subjected to the field. This induced polarization is typically localized near the edge of the field region. We will show, however, that this effect does not occur if the two degenerate ground states lie in different polarization sectors, because in this case the polarization can spread through the degenerate subspace at no energy penalty.

We will also show that, for a sufficiently small fraction of the spins subjected to the field, there exists at least one state in one of the polarization sectors which looks locally like the zero field (MPS) ground state far from the field (Fig. 2). For the systems studied in this paper one of these states is always the ground state. [17]

For the case of periodic boundary conditions, any state which lies locally in the degenerate subspace far from the local magnetic field region will have the minimum local contribution to the energy. This means that even for a system with many more spins outside of the field than within it, a disturbance can easily propagate throughout the entire zero field region.

This paper is organized as follows. In the following section 2, we introduce the observables on which we focus to understand the effects of a local applied magnetic field on this many-body system. The cases of open and periodic boundary conditions need to be treated separately. In section 3, we then discuss the physics of open chains, and in section 4 the phenomena observed in periodic systems. In section 5, we consider how these results are affected when one departs from the Majumdar-Ghosh point in the underlying Hamiltonian. This is followed by conclusions in section 6.

2 Physical observables

2.1 Open boundary conditions

For open boundary conditions the field is applied to N' spins on one end of the chain. Unless otherwise stated, we consider finite chains with a total number of spins, N, performing full numerical diagonalizations of the frustrated Majumdar-Ghosh Heisenberg Hamiltonian.[18] Several observables are studied. The first is the total polarization outside of the region subjected to the applied field. While the total spin polarization of the chain is conserved, local polarization is not. This quantity is defined as

$$L_{\neg N'} = \left\langle \psi \mid \textstyle\sum_{j=N'+1}^{N} S_j^z \mid \psi \right\rangle \tag{5}$$

Furthermore, we study the trace distance from a singlet state of the two spins at the end of the chain opposite to the region of the applied magnetic field, i.e. the spins located at sites $N-1$ and N. This observable is defined as

$$d_s = \frac{1}{2}\|\rho_s - \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)(\langle\uparrow\downarrow| + \langle\downarrow\uparrow|)\|_1, \tag{6}$$

where

$$\begin{aligned} \rho_s &= \mathrm{Tr}_{\neg s}(|\psi\rangle\langle\psi|), & (7) \\ \|O\|_1 &\equiv \mathrm{Tr}\sqrt{O^\dagger O}. & (8) \end{aligned}$$

Finally, we focus on the polarization of the spins at sites $N-1$ and N, defined the same as in Eq. 5, but with the sum running from N-1 to N. In this paper the subsystem of the 2 furthest spins will be labeled f. This observable tells about whether the polarization has been allowed to spread to the furthest 2 spins from the field.

Two different sizes of field regions are considered, N'=5 and N'=4. The reason that both are considered separately is that there are significant even-odd effects.

In this paper, no actual quenches are performed in the system with open boundary conditions, and all observables are given for the ground state of a given sector.

2.2 Periodic boundary conditions

For periodic boundary conditions, the field is applied to a region of N' adjacent spins. In this case, we are considering chains with an even number of sites. While the observables studied in the periodic case are defined in analogy to those studied in the open case, some extra care is necessary. In particular, a complication arises for the trace distance from a singlet for the two spins furthest from the field region. For periodic boundary conditions, there is no unique choice of singlet covering for the system. Two different approaches to this problem are examined. Firstly, one can consider the distance from the closest of the two singlet coverings for a subsystem,

$$d_{s,cover} = min(\|\rho_s - \mathrm{Tr}_{\neg s}(|\psi_{1,NF}\rangle\langle\psi_{1,NF}|)\|_1, \|\rho_s - \mathrm{Tr}_{\neg s}(|\psi_{2,NF}\rangle\langle\psi_{2,NF}|)\|_1). \tag{9}$$

However, this quantity has a drawback, i.e. all but a zero measure set of states in the degenerate subspace will have a finite distance to either of these coverings. An alternative approach is to look at the distance from the closest point in the subspace to the reduced density matrix,

$$d_{s,subspace} = min_{a,b}(\|\rho_s - (\|a|\psi_{1,NF}\rangle + b|\psi_{2,NF}\rangle\|_2)^{-2} \times (\mathrm{Tr}[(a|\psi_{1,NF}\rangle + b|\psi_{2,NF}\rangle)(a^\dagger\langle\psi_{1,NF}| + b^\dagger\langle\psi_{2,NF}|)]\|_1). \tag{10}$$

This equation appears as though it can be further simplified in an obvious way, but remember that the two wave functions are not orthogonal. The norm in the denominator is the usual L2 norm for a vector. Also in this case the minimization is actually simpler than it looks, by realizing that it can be reduced to: $d_{sing,subspace} = min_{0\leq\alpha\leq 1}\|\rho_s - ((1-\alpha)\times\frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)(\langle\uparrow\downarrow| + \langle\downarrow\uparrow|) + \alpha\times 1_4)\|_1$ where $1_4$is the 4-dimensional identity operator.

While the observables presented in this section could be considered as time dependent variables, in this paper they are always studied for the ground state of a given polarization sector.

2.3 Small magnetic field quenches

Because of the degeneracy caused by the periodic boundaries there is another quantity which is interesting to look at, relating to a field quench performed by changing the magnetic field instantaneously and subsequently monitoring the time evolution of the system, especially in regions far from where the local field is applied. Unitary evolution gives the time evolution of a system following a quench at time $t = 0$, in terms of energies E_n,

$$\rho_{m,n}(t) = c_m^* c_n \exp[-\imath(E_n - E_m)t] \quad , \tag{11}$$

where $c_m = \langle m \mid \psi\rangle$, where $|\psi\rangle$is the pre-quench ground state of the system. This leads to a definition of the time averaged state,

$$\bar{\rho}_{m,n} = c_m^* c_n \delta(E_n - E_m) \quad . \tag{12}$$

The field quench is performed by taking $|\psi(t=0)\rangle = |\psi_0\rangle$ to be the ground state of a Hamiltonian with a slightly stronger field, $H_0 = H - \epsilon \sum_{j=1}^{N'} S_j^z$. At $t = 0$, ϵ is instantaneously turned off. In our analysis of the time evolution, we will focus on the trace distance from the time averaged (or dephased) state of the density matrix of the two spins furthest away from the field region

$$d_{av}(t) = \|\mathrm{Tr}_{\neg s}(|\psi(t)\rangle\langle\psi(t)|) - \bar{\rho}_s\|_1. \tag{13}$$

2.4 Large magnetic field quenches

We also examine the time evolution due to large local field quenches. Several statistical distributions are studied to understand the ensuing equilibration behavior. These quenches are performed in a regime where quenches are shown to disturb the entire system, even regions far away from the field region. A global quantity which is studied is the Loschmidt echo, a measure of the overlap of the time evolved system with the initial state,

$$LE(t) = |\,\langle\psi \mid exp(-\imath\, H\, t) \mid \psi\rangle\,|^2. \tag{14}$$

Two local linear quantities are examined as well. In the region subjected to the local external field, the local polarization is studied. This is simply the expectation value of the magnetization operator with respect to the local density matrix,

$$L_{N'}(t) = \mathrm{Tr}(\rho_{N'}(t)M). \tag{15}$$

In the region far from the spins where the local magnetic field is applied, all of the states are expected to be locally within the degenerate ground state subspace and therefore have zero magnetization. Therefore, a more appropriate observable to use is the overlap with a singlet state,

$$O_s(t) = \mathrm{Tr}(\rho_s(t)\mathrm{Tr}_{\neg s}(|\psi_{1,NF}\rangle\langle\psi_{1,NF}|)). \tag{16}$$

Finally an important non-linear local quantity is studied far from the local magnetic field, the time evolving distance to the average state, defined by

$$d_s(t) = \|\rho_s(t) - \bar{\rho}_s\|_1. \tag{17}$$

This quantity is important, as it provides a direct measure of equilibration locally, and can thus be used to show that the quench not only disturbs the system far from the field, but also that these disturbances can cause equilibration.

2.5 Entanglement maps

A tool which is used in this paper for visualizing quantum states is a map of two point entanglement. In these graphics, colors are used to indicate entanglement strength between single spins using von Neumann entropy,

$$S_{VN}(\rho) \equiv \mathrm{Tr}(\rho \log(\rho)), \tag{18}$$

as a measure of entanglement.

These graphics consist of arrays of colored squares where, for off-diagonal elements, the color corresponds to the entanglement between the two spins. The diagonal elements correspond to the difference between the maximum possible entropy on a spin and the actual entropy. This represents the the amount of information left about a spin after the rest of the system is measured. These maps are created using

$$\begin{aligned} entMap(i,j) &= (1-\delta_{ij})*((S_{VN}(\rho_i)+(S_{VN}(\rho_j)-S_{VN}(\rho_{ij}))+ \\ & \quad \delta_{ij}*(S_{VN}(\frac{1}{2}*1_2)-S_{VN}(\rho_i)) \end{aligned} \tag{19}$$

The color scale with the maximum entanglement normalized to 1 appears in Fig. 4. It is important to note that while these figures can give a good general impression of entanglement behavior of the system, they do not tell the whole story, i.e. they only give information about two-point entanglement. Just because one of these figures shows no two point entanglement for a pair of spins, this does not mean that they are not entangled in a more complicated way.[19]

Although in principle there is nothing preventing one from obtaining entanglement maps for time averaged states, in this paper we only use this technique to study eigenstates.

3 Local magnetic field applied to Majumdar-Ghosh chains with open boundaries

Subjecting a local region of a Majumdar-Ghosh spin chain to an external magnetic field forces the exposed spins to align with the field. Because of polarization conservation, excess polarization opposite to the direction of the field is generated in the field-free region of the system. In the sector of zero total spin polarization ($L = 0$), and for sufficiently large magnetic field strengths, this can cause spins far from the field region to switch to non-trivial polarized configurations, whereas for smaller applied fields they remain in a spin singlet product state. In contrast, in polarization sectors with $L \neq 0$ excess polarization is trapped close to the region where the field is applied, and singlets are pushed far away from this field region. This is demonstrated graphically in Fig. 3 where parts (a) and (b) show the trace distance of two spins far from the locally applied magnetic field from a singlet for fields on an even and odd number of spins respectively. Parts (c) and (d) show the polarization stored in the region with no applied magnetic field versus field strength, again for fields on an even and odd number of spins respectively. As Figs. 3(a) and (b) show, for local fields applied to regions with both an even and odd number of spins, there is always at least one polarization sector for which singlets are located far away from the field region. Even for relatively small finite systems, such as the ones studied here, the ground state always lies in one of these sectors.

It is also interesting to note from Figs. 3(c) and (d) that for a small field in the L=0 sector, the spins in the field-free area behaves differently, depending

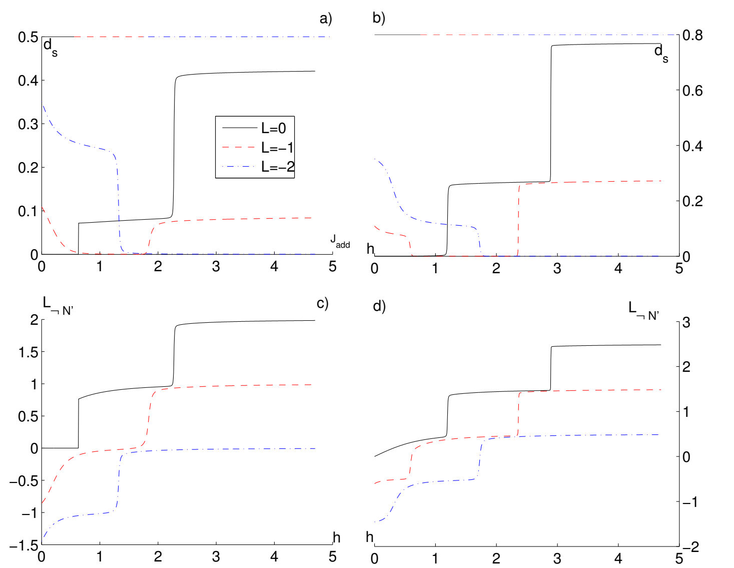

Figure 3: (color online) (a) and (b): Trace distance from a singlet state of the two spins at the end of the chain opposite to the region subjected to the local magnetic field. (c) and (d): Total spin polarization outside of the region subjected to the local field. (a) and (c) are for local fields applied to 4 spins, and (b) and (d) are for local fields applied to 3 spins. On all figures, the solid line is the L=0 sector, dashed lines indicate the L=-1 sector and the dot dashed lines indicate the L=-2 sector. Note that for sufficiently small local fields, the global ground state lies in the L=0 sector, whereas for larger local field strengths it lies in higher polarization sectors. In both cases the global ground state is locally close to the singlet state on spins far from the field. These plots are all properties realting to the ground states of given sectors.

on whether the local field is applied to an odd or to an even number of spins. This can be explained by the fact that for a field on an odd number of spins, the boundary between the field and the region with no field cuts through a singlet, in the original ground state. E.g. the field gradient makes one component of the singlet more energetically favorable than the other. By rotating these two spins between the singlet and the classical $|\downarrow\uparrow\rangle$ state, the ground state can be adjusted locally. When, however the field boundary is between two singlets, a critical local field strength must be reached for any polarization to be transferred from the field region to the field-free region as Fig. 3(c) demonstrates. This is because the matrix product state of singlets is still an eigenstate of the Hamiltonian for any field strength in this case, and a level crossing must occur before the ground state can change. [20]

Fig. 4 shows the entanglement map of a system in the L=-1 global spin

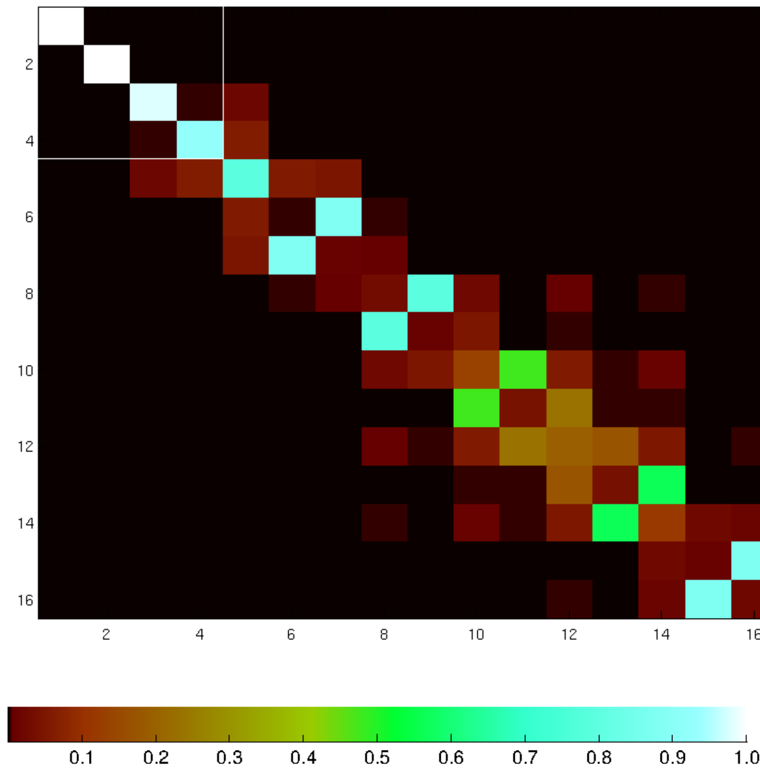

Figure 4: (color online) Entanglement map for the L=-1 sector ground state (note that this is not the global ground state) for a Majumdar-Ghosh chain of 16 spins with 4 adjacent spins, whose position is indicated by the white square, subjected to a local magnetic field of strength h=5J . The color scale is normalized to 1 as shown.

sector, with a magnetic field of h=5J applied on 4 of 16 spins. Fig. 4 suggests that for a range of field values, the distance from a singlet is caused by frustration from having an effectively odd number of spins available in the Majumdar-Gosh Hamiltonian. In this case, however, the frustration is alleviated by an intermediate transition region between the field behavior and far from field behavior changing its length (at the cost of some energy). [21]

3.1 Polarization effects

The way the system distributes polarization depends strongly on even-odd effects. To study the effects of polarization we examine Fig. 5 which shows the dependence of trace distance from a singlet for spins far from the locally ap-

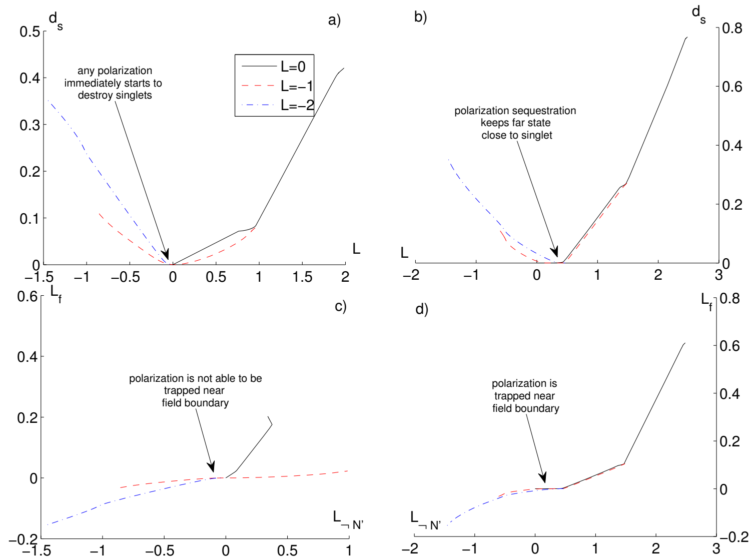

Figure 5: (color online) (a) and (b): Trace distance from a singlet state of the two spins at the end of the chain opposite to the region subjected to the local magnetic field versus polarization of the entire field free region. (c) and (d): Spin polarization of the 2 furthest spins versus polarization of the entire field free region. Local field on 4 of 16 spins with periodic boundary conditions (left column). Local field on 5 of 16 spins with periodic boundary conditions (right column). In all parts, the solid line is the L=0 sector, dashed lines indicate the L=-1 sector and the dot dashed lines indicate the L=-2 sector. These plots are all properties realting to the ground states of given sectors.

plied magnetic field on the polarization in the non-field region in parts (a) and (b) for fields on an even and odd number of spins respectively. Parts (c) and (d) show the polarization of the last 2 spins rather than trace distance from a singlet. From Figs. 5(c) and (d) one can tell that if the field is placed on an even number of spins, any polarization that is in the non-field region will be immediately spread, even to the furthest spins. In the case where the field is placed on an odd number of spins, however, a finite amount of polarization can be sequestered near the boundary. Figs. 5(a) and (b) show that this trend is mirrored in distance from a singlet for far-away spins.

The differences between the even-spin and odd-spin ground state for the zero-field spin chain can be used to explain why polarization sequestration can occur in one case and not the other. Any spin $\frac{1}{2}$ spin chain with an odd number of spins and no applied local field must have a degenerate ground state because the particle-hole duality. The degenerate ground states also have different polarization and, therefore, 2 degenerate ground states with a continuum

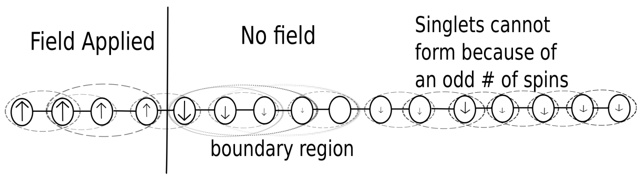

Figure 6: Cartoon representation of the effect which prevents polarization sequestration for a field on an even number of spins.

of polarization between $L = -\frac{1}{2}$ and $L = \frac{1}{2}$ are possible. This means that for a chain which is effectively "odd", there is no energy penalty for being anywhere in this range. This effect allows polarization to be spread throughout the no-field region without increasing the energy in that region. Polarization can effectively be moved through this locally degenerate subspace, therefore polarization sequestration does not occur. Conversely, for a spin chain which is effectively "even", the ground state is unique, and polarization will tend to be localized in the ground state to avoid raising the energy of all of the no-field spins. As Fig. 4 suggests, for certain field ranges in a given sector, the length of the non-field region of the chain is effectively "odd". When this happens polarization can be spread freely throughout the non-field region, and sequestration does not occur, see Fig. 6.

3.2 Field Induced Effects

For very strong fields, the spins within the field should have no entanglement with the rest of the system, in low energy states. This is because the spins subjected to the field will align with the field. Therefore an effective Hamiltonian which acts only on the spins outside of the field should be able to describe the system in low energy states. A simple model for this Hamiltonian would be to alter the coupling between the two spins closest to the field, with the supposition that the coupling with the field spins acts to mediate the interaction between the two spins coupled to them (see Fig. 7). The overlap between the known ground state, and the ground state calculated using the approximation shown in Fig. 7 for different added coupling strengths and different spins in the field region appear in Fig. 3.2. Fig. 3.2 supports the claim that this approximation works fairly well in the ground state for a field on an odd number of spins. For numerical results see table 1.

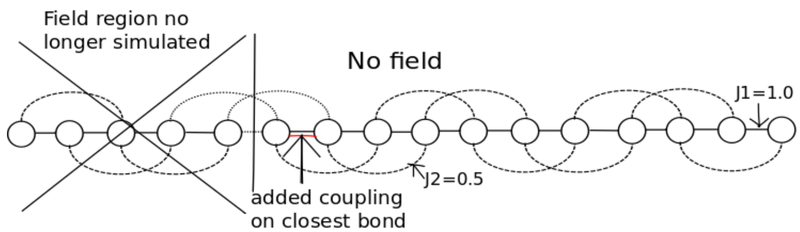

Figure 7: (color online) The approximation used to simulate behavior with a strong field.

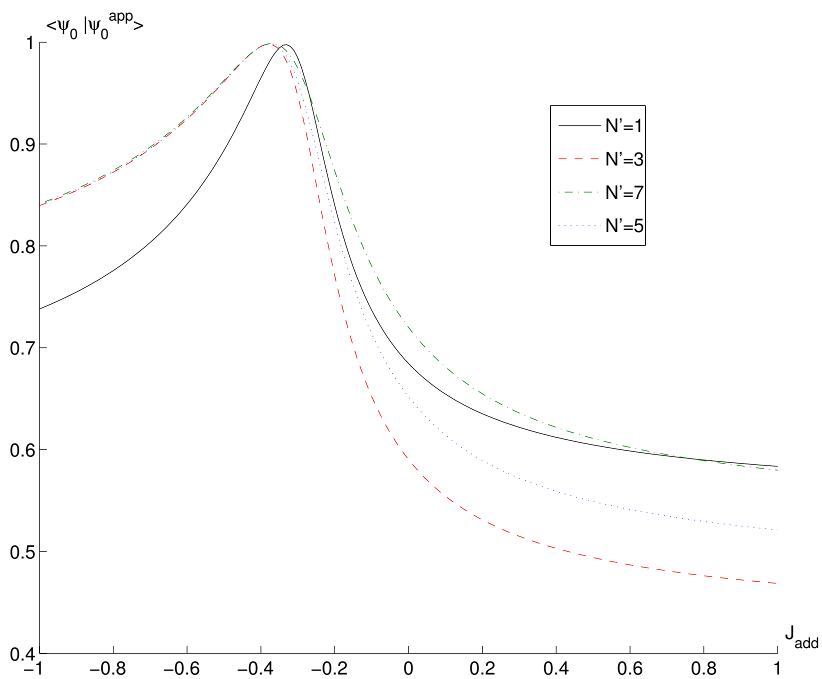

Figure 8: (color online) Overlap between actual ground state, and ground state of a Hamiltonian which is applied only to the non-field spins (tensored with spins opposing the field in the field region), but with a modified coupling on the two spins closest to the field. X axis is the additional coupling added to the Majumdar-Ghosh Hamiltonian. Data was taken with h=100, N=16 (open boundaries). Different lines are as follows solid-N'=1, dashed-N'=3, dotted-N'=5, dot dashed-N'=7. Even N' (not shown) are not accurately represented by this model.

	$\langle \psi_0 \mid \psi_0^{app} \rangle$	J_{add}
N'=1	0.9976	-0.3323
N'=3	0.9980	-0.3786
N'=5	0.9983	-0.3756
N'=7	0.9987	-0.3706

Table 1: Statistics considering a field of h=100 placed on N' spins, comparing the approximate to the actual Hamiltonian. The coupling listed here is the additional coupling added to the 2 closest spins to the field

4 Local magnetic field applied to Majumdar-Ghosh chains with periodic boundary conditions

Unlike open boundary conditions, periodic boundaries present a case where the unperturbed Hamiltonian has a degenerate ground state. Therefore, the local Hamiltonian for the spins far away from the region subjected to the local field will also always have a degenerate ground state. The complications from this degeneracy add a new series of effects which are not observed in the open-boundary case. These effects are illustrated by Fig. 9 which shows in parts (a) and (b) the closest distance from the singlet subspace for the two furthest spins from the region of the locally applied magnetic field versus polarization on all non-field spins for 3 of 20 and 4 of 20 spins in the field respectively. Parts (c) and (d) show polarization on the two furthest spins from the locally applied field versus total polarization in the non-field region, again for field on 3 of 20 and 4 of 20 spins respectively.

The most immediately obvious difference is that if the local magnetic field is placed on an odd number of spins, neither spin sequestration nor closeness in trace distance to the singlet subspace for any spins are observed, except for the L=0 subspace in weak local fields. Figs. 9(a) and (c) show the trace distance from a singlet in spins far from the applied magnetic field and local angular momentum for spins far from the local magnetic field respectively, both versus total angular momentum in the field free region. For larger fields, the spins far from the region where the external magnetic field is applied do not approach the singlet subspace because of frustration caused by having an odd number of spins in the non-field region. In the case of periodic boundary conditions, the effects of the frustration are stronger than in the case of open boundaries. This is because here a change in the length of the field-to-far-from-field transition region will do nothing to relieve the frustration because of the symmetry between the two field boundaries. Regardless of whether the length of one of these regions is odd or even, the total length of transition regions is always even because it is the length of a single transition region multiplied by two.

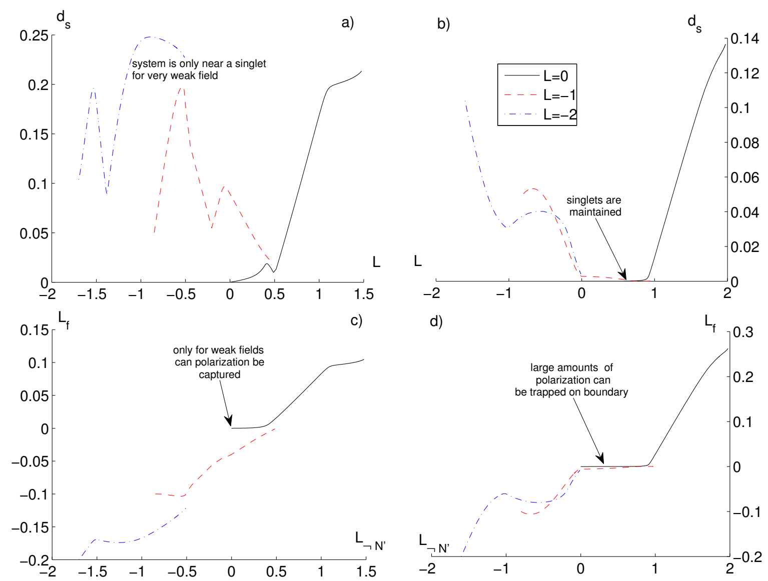

Figure 9: (color online) Closest local distance from singlet subspace of 2 furthest spins from the locally applied magnetic field versus polarization on all non-field spins (top row). Polarization on 2 furthest spins versus polarization on all non field spins (bottom row). Field on 3 of 20 spins with periodic boundary conditions (left column). Field on 4 of 20 spins with periodic boundary conditions (right column). On all figures, the solid line is the L=0 sector, dashed lines indicate the L=-1 sector and the dot dashed lines indicate the L=-2 sector, where I call negative L to be in the direction of the field. These plots are all properties realting to the ground states of given sectors.

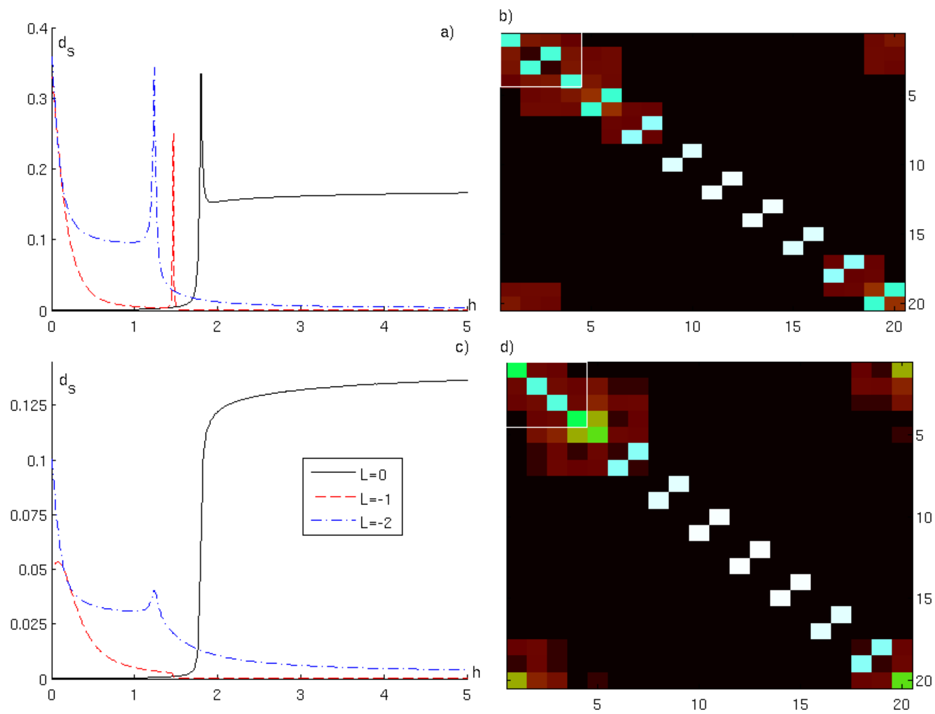

Figure 10: (color online) a) Trace distance between the two spins furthest from the field and the nearest singlet covering (see Eq. 9) versus field for 20 spins with periodic boundary conditions and a field placed on 4 of the spins. b) Entanglement map for 16 spins with a field of h=1.3J placed on spins 1-4 (indicated by the white rectangle) with periodic boundary conditions c) Same as (a), but now with distance to the closest state in the degenerate subspace (see Eq. 10) d) Same as (b) but with a field of h=1.6J. On all figures, the solid line is the L=0 sector, dashed lines indicate the L=-1 sector and the dot dashed lines indicate the L=-2 sector, where negative L is in the direction of the field. All plots in this figure are for eignestates of the Hamitlonian.

4.1 Effect of local degeneracy on small quenches

Shifting the focus to the case where the external field is placed on an even number of spins, one can consider the effects of now having a locally degenerate ground state, i.e. having a Hamiltonian which has a ground state degeneracy when no field is applied, and therefore is degenerate in a local sense far from the spins with an applied magnetic field. Fist the ground state can be studied by observing Fig. 10, this figure shows in parts (a) and (c) the trace distance from the closest singlet covering and minimum distance from the manifold of singlet coverings respectively for the two furthest spins from the locally applied magnetic field versus field strength, for a field applied to 4 of 20 spins with periodic boundary conditions. Parts (b) and (d) show entanglement maps for a local magnetic field strength of h=1.3J and h=1.6J respectively, again for a field on 4 of 20 spins with periodic boundaries. Fig. 10(c) indicates that the global ground state of the system is always close to the singlet subspace far from the field, however 10(a) suggests that around a field strength of 1.5 the system may undergo a switch between singlet coverings far from the spins to which the field is applied. Figures 10(b,d) confirm this suspicion by showing that indeed before the peak in 10(a) there are an even number of dimers outside of the field region, while after there are an odd number of dimers. This indicates that disturbances from the local field can be felt far from the spins with an applied field, but only for a narrow range of field values.

One can now consider the effect of small quenches at various applied field strengths on spins far from the field spins. The results of such quenches are shown in Fig. 11, parts (a) and (b) show the trace distance to average for the two furthest spins from the locally applied magnetic field, after a quench which involves a small change in field strength versus the strength of that field for a field on 3 of 20 and 4 of 20 spins respectively with periodic boundary conditions. Parts (c) and (d) show the polarization of the two furthest spins from the locally applied magnetic field versus field, and are included to emphasize the important role played by polarization in this system. One would expect that these disturbances can only be propagated through the still locally degenerate ground state subspace of the no-field Hamiltonian and therefore would only have an effect when the coverings shift. Fig. 11(b) shows that in fact a small quench does disturb the system strongly at the point where the coverings switch. The other two peaks in Fig. 11(b) are less relevant because they occur in the ground state of a spin sector, but not in the global ground state of the system. Also none of the quench disturbances which occur far from the spins with an applied field occur in the global ground state in Fig. 11(a), demonstrating another difference caused by even-odd effects. This is to be expected, because the the two degenerate ground states of an odd length Majumdar-Gosh chain lie in different polarization sectors and therefore cannot exhibit level repulsion, at least locally, in the region far from the applied magnetic field.

Note also that Fig. 11 suggests that there is a strong correlation between polarization outside of the subsystem where a magnetic field is applied and quench disturbance to the far spins, in the sense that when the quench has

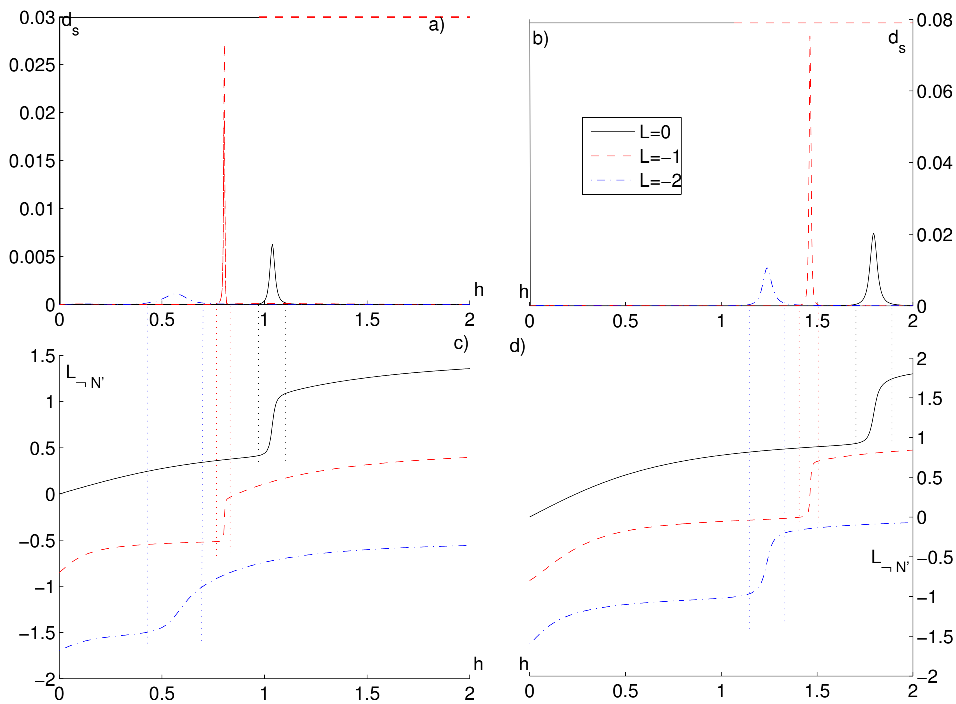

Figure 11: (color online) Initial trace distance from average (see Eq. 13) for a subsystem far from the fields after a small field quench $\epsilon = 0.001$ (top row). Polarization on all non-field spins versus field strength (bottom row). 20 spins with field placed on 3 of them and periodic boundary conditions (left column). same with field placed on 4 spins (right column). Dotted vertical lines have been added to emphasize correlation between the two graphs. On all figures, the solid line is the L=0 sector, dashed lines indicate the L=-1 sector and the dot dashed lines indicate the L=-2 sector. Lines at the top are added to show which spin sector the global ground state is in. The top two plots are time averaged quantities from a quench, while the bottom two figures are properties of the ground state of each sector.

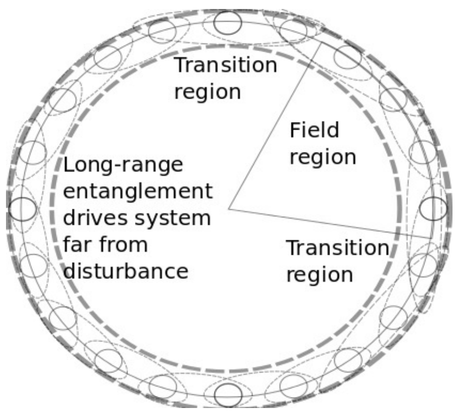

Figure 12: Cartoon of field quench for periodic boundaries.

a strong effect, there is a rapid change in polarization in the non-field region. The converse however is not supported by this figure. This demonstrates than polarization plays a strong role in the global behavior of this system.

The same energy arguments used in the static case for behavior of spins far from the spins with an externally-applied magnetic field should be usable as a dynamical argument. A finite local field can only introduce a finite amount of energy into the system. Therefore only states which lie sufficiently close in energy to the ground state can be accessed in any significant way. For a large enough system, all of the low energy states will have to be locally close to the ground state for most of the spins far from the locally-applied magnetic field, therefore locally, far from the field, spins can only be disturbed within the degenerate subspace. Put another way, in gapped systems the effects of a local quench have to be localized, unless there exists a locally degenerate subspace far from the region where the quench is applied. When such a subspace exists it may be able to transport conserved charges, quantum entanglement and dynamical disturbances an unlimited distance away from the disturbance site. A locally degenerate ground state can be thought of as a special symmetry which allows transport of information and charges (but not energy) with no losses throughout the part of a system far from the quench. [22]

Long-range entanglement allows a part of the system which lies entirely in a degenerate subspace to have its evolution driven by local evolution far away, see Fig. 12.

4.2 Large quenches

Now that it is established that a disturbance will be able to be propagated throughout the entire system from a small quench, one can perform a large quench from h=1.6 to h=1.3 for a local field applied to 4 adjacent spins of 20 total spins with periodic boundary conditions. One can then examine the time statistics of various properties of the system. These statistics are shown in Fig.

13, in this figure part (a) is the trace distance of 4 spins far from the locally applied magnetic field from a singlet state, part (b) is the time statistics of the Loschmidt echo of the entire system, part (c) is the time statistics of the distance from the time averaged state for 4 spins far from the locally applied field, and part (d) is the time statistics of the magnetization of the spins subjected to the field. These statistics will show the ability of the system to equilibrate, even locally for spins far from the spins where the local magnetic field is applied. In the case studied here, the system only equilibrates poorly, even in the global sense, not surprisingly, poor equilibration is also shown in local observables both close to and far from the spins with an applied magnetic field.

The double-peaked pattern of equilibration seen here is typical of small systems, see [4], and is thus consistent with the theory that although the system itself is rather large [23], the actual evolution is only taking place on a few spins in or near the region of externally applied field, the rest of the system is simply being drug along by long range entanglement with these spins. As Fig. 13(c) demonstrates, even though the dynamics is driven by long range entanglement with far away spins, a subsystem of spins is still able to be pushed toward equilibration in the trace distance sense. The fact that there is no local energy difference does not seem to interfere at all with equilibration of these spins. The trace distance from the average is observed quite close to zero at some times, unlike in similar quenches performed at the Majumdar-Ghosh point in [4]. This is because an undisturbed singlet somewhere in the region being observed would yield a large distance from the average at all times as shown in [4] where the quench did not cause a change in singlet coverings. In the case we are observing, where the coverings switch, there are no undisturbed singlets in the region away from the field spins.

Although the equilibration is globally poor in this system, there are no signs that equilibration via long-range entanglement through a locally degenerate subspace is any less effective than direct equilibration of the spins to which the field is applied. The data from this quench therefore indicate that the entire system can be equilibrated (at least somewhat) by a quench which only affects a very small region. In fact a system of any size should be able to be brought locally close to equilibrium in this way. Because all states of the far spins locally have the same energy, than they cannot affect the time evolution of the system, therefore the same behavior would be expected for a spin chain of any sufficiently long (even) length.

5 Other Coupling Strengths

One can now ask what would happen if the coupling were changed such that the system was no longer using the Majumdar-Ghosh Hamiltonian, but allowed the next nearest neighbor coupling to take on arbitrary values, see Eq. 20. This study is done with 20 spins and periodic boundary conditions, with a local magnetic field on 4 adjacent spins.

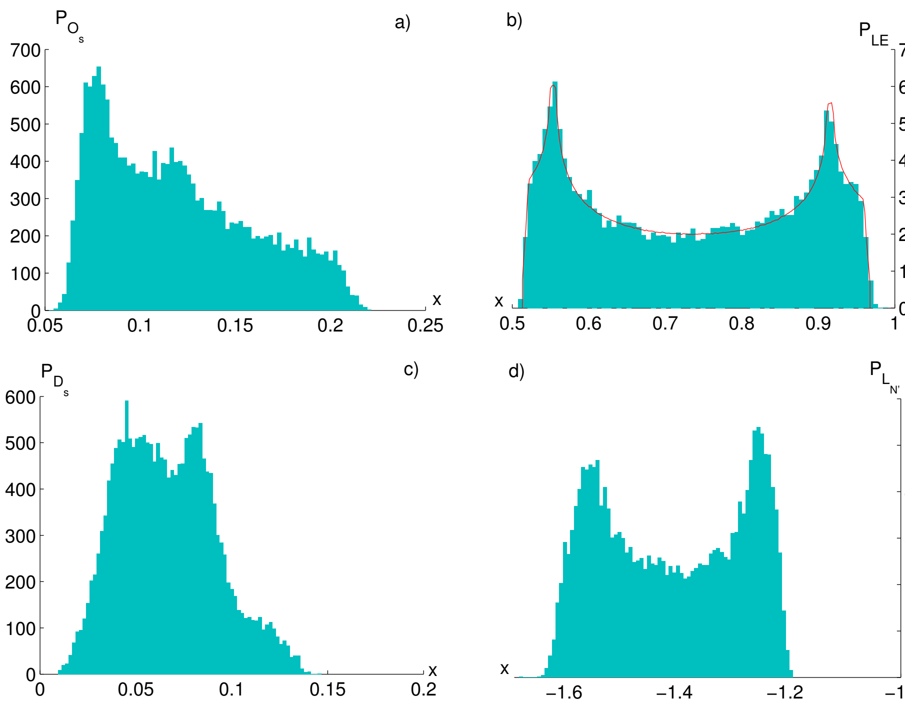

Figure 13: (color online) Equilibration statistics for N=20, N'=4 with a quench from $h_{(i)}$=1.6 to $h_{(f)} = 1.3$. a) Time statistics of the trace distance of 4 spins far from the locally applied magnetic field from a singlet state (Eq. 16). b) Time statistics of the Loschmidt echo (Eq. 14) with an approximation based on few frequencies. c) Time statistics of distance from time averaged state (Eq. 17) for 4 spins far from the locally-applied magnetic field. d) Time statistics of local magnetization (Eq. 15) of the field spins. These plots are all time statistics obtained from evolution.

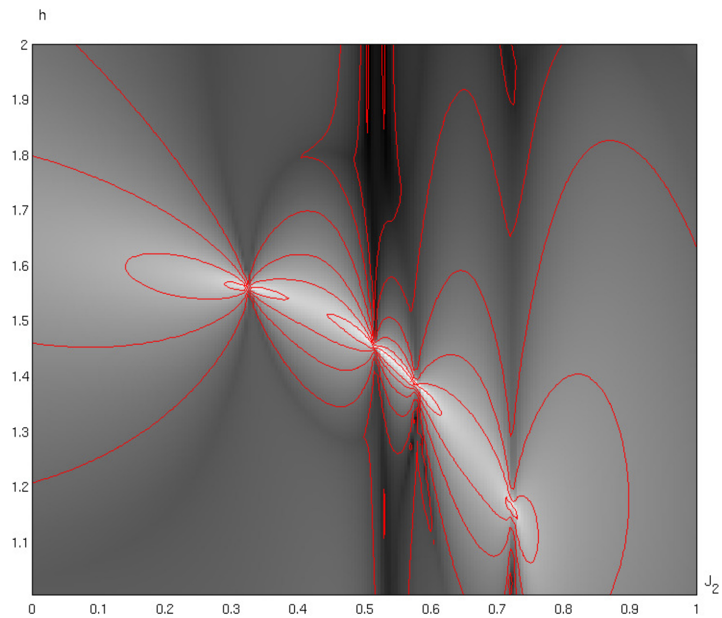

Figure 14: (color online) Initial trace distance to average for far spins after a small field quench, larger distances are lighter, smaller distances are darker. Trace distance is plotted on a logarithmic scale, contour lines (red on-line) are included for clarity. Data using 20 spins with periodic boundaries in the L=-1 sector.

$$H_{J2} = \sum_{j=1}^{N} \vec{S}_j \cdot \vec{S}_{j+1} + J_2 \sum_{j=1}^{N} \vec{S}_j \cdot \vec{S}_{j+2} \tag{20}$$

Small field quenches can be considered on this new Hamiltonian exactly in the same way they can be considered for the Majumdar-Ghosh Hamiltonian, the results appear in Fig. 14 which shows the initial trace distance from average for a small field quench versus coupling and field strength. This data shows that for a wide range of coupling near the Majumdar-Ghosh point, small field quenches can drastically affect spins very far from the spins with an applied magnetic field at specific field strengths. However, when the field strength is eventually different enough, these peaks broaden out and disappear (note logarithmic scale in Fig. 14). The basic behavior seen previously in this paper holds for a wide range of couplings, where the ground state is no longer degenerate.

For an infinite system one would expect that, far spins from the local magnetic field could not be disturbed by a local field quench unless either the system is gapless or there exists a degenerate ground state. For a finite system, this would only be necessarily true if the gap between the ground state and first excited state is sufficiently large compared to the energy introduced by the applied local magnetic field, in which case the field will be unable to introduce enough energy to affect the entire system. In this case the order of magnitude of the energy which the field introduces can be estimated by simply multiplying the field strength by the number of spins it is applied to. Because both of the quantities are of order 1, one would expect that the energy introduced would also be of order 1.

The energy gap in the system which will be used for this calculation can be determined by exact diagonalization. The energy of the gap between the ground state and first excited state of this system are shown in Fig. 15 part (c) which shows the gap energy versus coupling at zero applied field, part (a) shows the initial distance from the average for 4 spins far from the locally applied magnetic field after a large quench (within the $L = -\frac{1}{2}$ sector), part (b) shows the local trace distance from the nearest singlet covering for spins far from the local field in the ground state of the $L = -\frac{1}{2}$ sector versus field strength and coupling, and part (d) is the same as (b), but with trace distance from the nearest state in the ground state manifold. It can be seen from Fig. 15(c) that the gap energy is at most of order 0.1, therefore, one would expect that for the entire range of couplings, the far spins could be disturbed by the local field. The results seen in Fig. 14 are as expected, however if the system size were increased to infinity, one would expect that in the gapped region for $J_2 \gtrsim 0.25$, the peaks in the distance would have to disappear except for exactly at J_2=0.5, or any other point with a degenerate ground state. Twenty spins, however, is still too small a system for changes in the coupling to destroy the ability to dephase far spins with a local field, in other words the system can be considered to have an approximately degenerate (wrt. the energy scale associated with the field) ground state for all values of J_2, the next nearest neighbor coupling.

One can now ask whether the effects seen in Fig. 14 away from the Majumdar-Ghosh point are also caused by some kind of shift in singlet covering. To answer this question, one can compare Fig. 15(b) to Fig. 15(d) and notice that where the peaks in Fig. 14 are located, the trace distance from either covering tends to be relatively large, but the distance from the subspace tends to be relatively small. This indicates that movement within the singlet subspace is the cause of much of the disturbance in the far spins. Also interesting to note is that for a significant portion of the couplings, the spins far from the locally applied magnetic field are closest to the singlet subspace when the small field quenches have the most effect on far spins. It appears that even at many couplings away from the Majumdar-Ghosh point, the model of switching between coverings as a way to spread a disturbance throughout the system is accurate. In fact for many values of J_2, the system appears to move into the singlet subspace for a narrow range of fields only when the covering change occurs. For $J_2 \gtrsim 0.6$ this model seems to break down, but it is still at least relevant for a large range of J_2. Although not directly related to the quench, it is interesting to note that above a certain local magnetic field strength the spins far from the field seem to lie on the singlet superposition manifold for a fairly large range of coupling

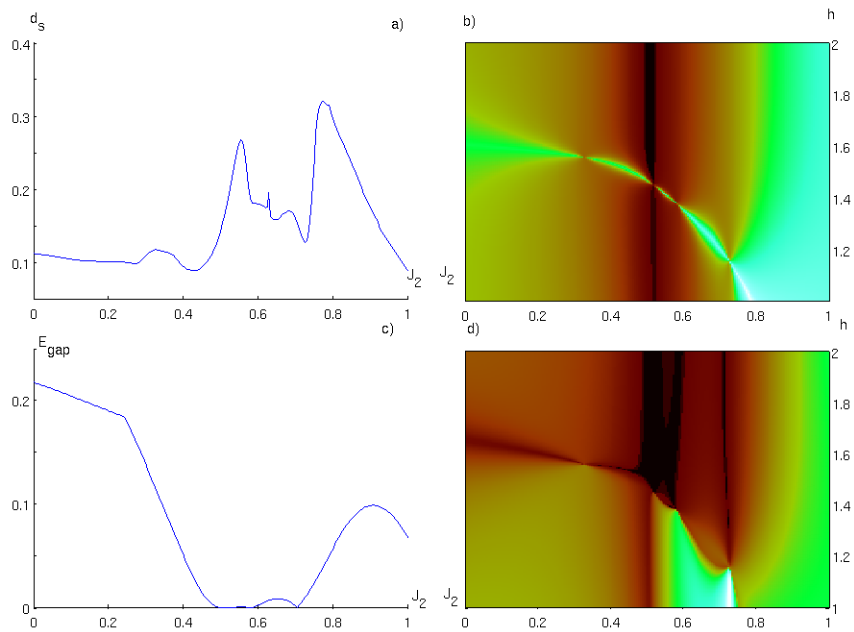

Figure 15: (color online) a) Initial distance from average for 4 spins far from the locally applied field for a large quench from h=2 to h=1 for the L=-1 sector. b) distance of far spins from nearest singlet covering (Eq. 9) versus h and J_2, L=-1 sector, color scale same as for entanglement maps, but normalized to largest value. c) gap between ground state energy and first excited state, for different $J_2$and h=0. d) distance from singlet subspace for far spins (Eq. 10) versus h and J_2, L=-1 sector, color scale same as for entanglement maps, but normalized to largest value. All plot except for (a) are static quantities relating to eigenstates.

strengths near Majumdar-Ghosh coupling, as well as a narrow strip between $J_2 = 0.6$ and $J_2 = 0.8$, the reason for this is not known.

The results of a large local magnetic field quench over varying J_2 as shown in Fig.15(a) simply helps to underscore what has already been noted about changing coupling not being an effective way of preventing disturbances from propagating throughout the system at this system size. Not only do the large quenches have a significant effect on far spins from the local magnetic field for all coupling strengths, but the expected trend of decreasing quench effect with increasing gap is not visible in any definitive way, indicating that, not only is the energy scale of the gap (Fig. 15(c)) too small to be the dominating factor in the quench effectiveness, it seems to not even play a very significant role. This result is consistent with the previous energy scale argument, the energy scale associated with the field is always at least an order of magnitude larger than the gap between the first 2 eigenstates.

6 Conclusions

In systems with degenerate ground states, quantum entanglement, disturbances, and charges can propagate freely, as long as the quench crosses between pre and post quench ground states which are locally different from each other far away from the region affected by the local quench. This effect is different and independent from gapless excitations, and has been demonstrated to occur in a gapped system. Unlike in gapless systems where excitations carry an arbitrarily small amount of energy far from the quench, these excitations store all energy locally near the quench, and evolution far away is driven solely by long-range entanglement. The local energy far from the region affected by a local quench Hamiltonian is exactly zero in these systems, not arbitrarily small.

To allow a charge to be propagated through a degenerate subspace, the two degenerate ground states must have different local expectation values for said charge far from the region affected by a local quench Hamiltonian. An effectively odd spin chain far from the field is allowed to propagate polarization throughout the far region for example. Again, in such a case, long range entanglement can propagate the charge, but does not propagate any energy far from the field. In cases where two degenerate ground states with different expectation values for a charge far from the region where the quench is applied do not exist, the charge can become locally trapped in part of the system. In the case of the Majumdar-Ghosh Hamiltonian, the polarization is trapped near the boundary of the local magnetic field region. A locally unique ground state (in a gapped system) means that charges, as well as disturbances, are confined after a local quench. Energy arguments prevent a disturbance from traveling throughout the system and also therefore forbid charges from moving outside of a small area.

In the system studied here, a large local magnetic field causes the spins within the field to become effectively 'fixed', facing in the direction of the field in the ground state. An approximation which does not include these spins directly but includes an effective modulation in coupling between the two spins neighboring the field can faithfully reproduce the ground state when an odd number of spins remain. For the case that an even number of spins are left out of the field region, this simple approximation fails. We believe that the effective transition region between the field and non-field region consists of an odd number of spins, and that this ground state cannot be faithfully reproduced in this way because of odd length frustration effects.

For Majumdar-Ghosh spin chains with periodic boundaries, with a local magnetic field on some even number of spins, there exists a range of fields where a small field quench can propagate a disturbance through the entire system using long range entanglement. This disturbance is propagated locally through the degenerate subspace of the local ground state. In this case, this range of fields is relatively narrow. A quench across this entire range does not only cause equilibration near the field, but also moves the far spins towards equilibration, within the locally degenerate subspace.

Study of systems with different next nearest neighbor coupling indicate that the basic effect which causes disturbances to be propagated to far spins can, at least for small enough systems, be extended away from the Majumdar-Ghosh point. For finite systems, if the gap between the ground state and the first excited state is small enough, the same effect which was described here for degenerate systems can also be applied to systems where the first 2 states are close in energy. In other words, under the right conditions, an approximate degeneracy will work in place of an exact degeneracy. For a system of 20 spins any value of J_2 between 0 and 1 still allows the far spins to be significantly affected by the field. We strongly suspect that for the values of $J_2$where the Hamiltonian is gapped and for which a degenerate ground state does not exist, spins far from a local magnetic field applied to a few spins cannot be affected in the large system limit.

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[16] However, one can orthogonalize them, still preserving the symmetry under $a \longleftrightarrow b$, $2l \rightarrow mod_N(2l+1)$, by adding a term $-(a \times \bigotimes_{k=1}^{\frac{N}{2}} \frac{|\uparrow_{2k-1}\downarrow_{2k}\rangle}{\sqrt{2}} + b \times \bigotimes_{k=1}^{\frac{N}{2}} \frac{|\downarrow_{2k-1}\uparrow_{2k}\rangle}{\sqrt{2}})$.

[17] The only case where this is not observed is for the case of periodic boundary conditions with a local field applied to an even number of spins. In this case these spins orient locally like in the frustrated ground state of a Majumdar-Ghosh chain with an odd number of spins.

[18] Note that due the inhomogeneity introduced by the applied local field, translational symmetry is broken.

[19] An example of this would be to consider a superposition of the two singlet coverings. Any non-adjacent spins will have exactly zero two-point entanglement (see Eq. 19). However, any set of two pairs of adjacent spins will have a finite entanglement between them. To see this, consider the case where one set of two spins is measured to be both in the up direction. If there is an odd number of spins between the two spin pairs, this forces the other pair to be a singlet, regardless of the distance between the pairs.

[20] Showing that this happens consists of demonstrating that the singlet covering is an eigenstate of any field which does not change within a singlet. This can be seen by realizing that the field operator on the first spin of the singlet will give the zero magnetization triplet state $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, while the operator on the second spin will give the same but with a negative sign. Thus the two will cancel making the singlet covering an eigenstate with a zero eigenvalue.

[21] The closeness to the boundary may also be a factor in the trace distance of the last 2 spins from a singlet, the fact that no entanglement can cross the open boundary may cause spins close to it to assume more localized states, however while this effect could make finite distances smaller, it should not be able to make the trace distance (virtually) zero as it is in many parts of Fig. 3(a).

[22] Note that the excitations which travel through a locally degenerate ground state are not the same as gapless excitations, which locally carry an arbitrarily small amount of energy. The parts of these excitations which exist far from the region affected by the local quench Hamiltonian carry exactly zero energy locally.

[23] One could argue that 20 spins is not such a large system, but it has been shown that the spins far from the evolution are always locally in the degenerate subspace. Adding more far spins and making the system into one which all of the readers would agree would be “large” (for example making the system size 100,000 spins) would not effect the dynamics, and the double peaked pattern would remain.