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Spin current generation from Coulomb-Rashba interaction in semiconductor bilayers

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Electrons in double-layer semiconductor heterostructures experience a special type of spin-orbit interaction which arises in each layer from the perpendicular component of the Coulomb electric field created by electron density fluctuations in the *other* layer. We show that this interaction, acting in combination with the usual spin-orbit interaction, can generate a spin current in one layer when a charge current is driven in the other. This effect is symmetry-wise distinct from the spin Hall drag. The spin current is not, in general, perpendicular to the drive current.

Spin-orbit interactions in semiconductors are traditionally studied within a one-electron picture, but there are instances in which electron-electron interactions latch onto spin-orbit interactions to produce intriguing effects, which may lead to the creation of radically new spin-based electronic devices^{1–3}. Of particular interest is the generation of spin currents and spin accumulation by an electric current, through the so-called *spin Hall effect*⁴. Recently, a special type of spin Hall effect was predicted to occur in double-layer heterostructures, i.e., two parallel quantum wells separated by an essentially impenetrable potential barrier, with a quasi-two dimensional electron gas in each layer. This effect, called “spin Hall drag”⁶, consists of the generation of transverse spin accumulation in one layer by an electric current flowing along the other layer and is caused by the component of the Coulomb electric field parallel to the two layers. In this paper we study a different and novel effect, which is driven by the component of the Coulomb electric field *perpendicular* to the layers. This perpendicular field creates an inhomogeneous Rashba spin-orbit interaction, with spatial variation in the plane of the layer⁷. We refer to this interaction as the “Coulomb-Rashba interaction”.

The system under study is shown in Fig. 1. A steady electric current is driven in the active layer 2. We show that the interplay of the Coulomb-Rashba interaction with the ordinary cubic Dresselhaus spin-orbit interaction (characteristic of semiconductors of the zincblende structure), provides a new mechanism for the generation of a spin current in the passive layer 1. This is particularly remarkable in view of the fact that ordinary spin Hall effect and spin Hall drag are *suppressed* by spin precession. But, in this case the presence of spin precession is absolutely essential to the effect.

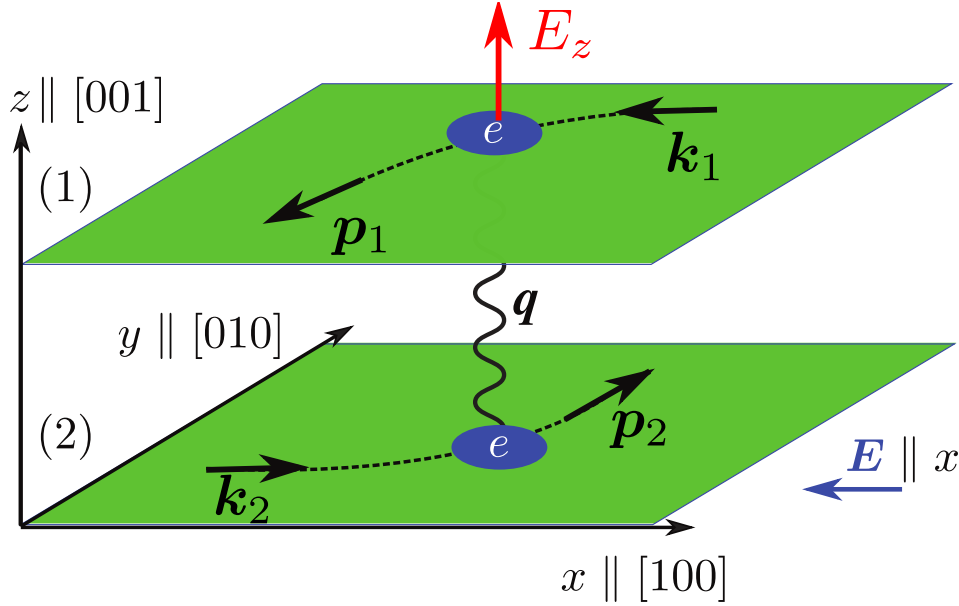


Figure 1. Scheme of the device under study: an electric field, \mathbf{E} , is applied to the active layer (2), a spin current is generated in the passive layer (1). Blue circles depict electrons, and the wavy line shows the inter-layer Coulomb interaction. Arrows marked \mathbf{k}_1 , \mathbf{k}_2 and \mathbf{p}_1 , \mathbf{p}_2 are the wave vectors in the initial and final states. The vertical arrow emphasizes the relevant component of the interlayer Coulomb field.

The spin current generation can be understood as a two-stage process, which is schematically described in Fig. 2¹⁰. In the first stage, the steady current of electrons in layer 2 induces a quadrupolar spin distribution in layer 1. To understand qualitatively how this comes about, observe that Coulomb collisions between electrons in the two layers take place on average with a positive momentum transfer from layer 2 to layer 1 along the x axis. Moreover, due to the spin-dependent terms in the scattering rate (see Eq. 1 below), the scattering efficiency depends on the relative orientations of electron spin and its initial and final wavevectors, denoted by \mathbf{k} and \mathbf{p} respectively. Consider electrons with spins parallel or antiparallel to the y axis: $s_y > 0$ and $s_y < 0$ respectively. For $s_y > 0$ the scattering rate is maximal for $k_x + p_x > 0$ and minimal for $k_x + p_x < 0$. For $s_y < 0$ the situation is reversed: the scattering rate is maximal for $k_x + p_x < 0$ and minimal for $k_x + p_x > 0$. The strongest transitions are marked by solid arrows in Fig. 2(a). As a result, states with large values of $|k_y|$ ($|k_y| \sim k_F$) become depleted of $s_y > 0$ electrons and filled with $s_y < 0$ electrons. At the same time, states with large values of $|k_x|$ ($|k_x| \sim k_F$) are depleted of $s_y < 0$ electrons and filled with $s_y > 0$ electrons. Hence, a quadrupolar spin distribution is formed, as shown in Fig. 2(b).

The second stage of the spin current generation is related to the spin precession in the cubic Dresselhaus field. This is also illustrated in Fig. 2(b), where the green arrows show the direction of the spin precession induced by the cubic Dresselhaus field, $\mathbf{\Omega}_{\mathbf{k}}$, (see Eq. 4 below). The field tilts the spins out of the $x - y$ plane, thus creating a

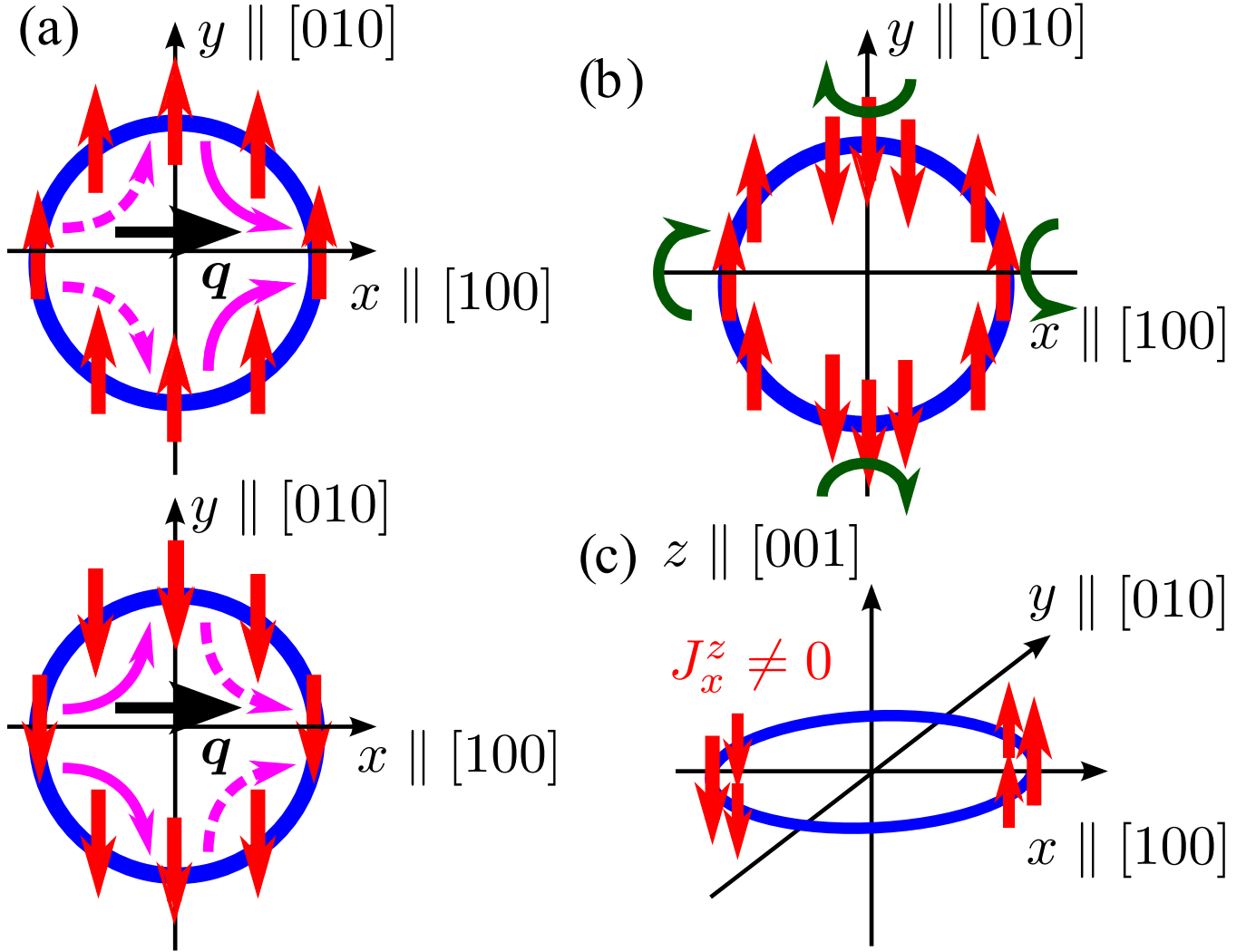


Figure 2. Schematic illustration of the spin current generation in the passive layer¹⁰. The circle is the Fermi surface, the arrows are the electron spins. Panel (a) shows the scattering stage of the process: top part shows electrons with spin parallel to the y axis, bottom part shows electrons with spin antiparallel to the y axis. The arrows show the spin-dependent scattering process: solid arrows indicate the stronger transitions, having $k_x + p_x > 0$ for $s_y > 0$ and $k_x + p_x < 0$ for $s_y < 0$, while dashed arrows indicate the weaker transitions. Scattering processes that increase the x component of the wave vector (i.e. with $q_x > 0$) dominate due to the current flowing in the active layer. (b) Precession stage of the process. The resulting spin distribution after the scattering contains second angular harmonics. Green arrows demonstrate the spin precession caused by the Dresselhaus field Ω_k , Eq. (4). Electron spins with opposite wave vectors precess in opposite directions. The resulting dipolar distribution of electron spins is presented in the panel (c).

dipolar distribution of the z -component of the spin as shown in Fig. 2(c). As a result, a z -spin current is formed. The characteristic C_{2v} symmetry of systems with both Rashba and Dresselhaus interactions causes the spin current to be parallel to the driving electric field \mathbf{E} , when \mathbf{E} is along one of the cubic $[100]$ or $[010]$ axes. Notice that this is completely different from the spin Hall drag current, which is always perpendicular to the electric field. However, the spin current can also be made perpendicular to \mathbf{E} , by orienting \mathbf{E} is along one of the principal axes $[110]$ or $[\bar{1}10]$.

In the rest of the paper we present some details of our theoretical analysis, and provide a numerical estimate for the size of the effect. The salient conclusions are as follows: (i) The spin transresistivity is proportional to $(T/E_F)^2$ which is characteristic of Coulomb drag phenomena^{6,12,13} (ii) It is inversely proportional to the fourth power of the interlayer separation and (iii) It is parametrically stronger than the spin Hall drag⁶ in the clean limit since the transresistivity is proportional to $\Omega\tau$, whereas the T^2 contribution to the spin-Hall drag is independent of τ .

Theory – The part of the “spin-orbit dressed” Coulomb interaction which is relevant to the effect described in this

paper has matrix elements^{8,14,15}

$$\begin{aligned}
M(\mathbf{k}_1 s_1, \mathbf{k}_2 s_2 \rightarrow \mathbf{p}_1 s'_1, \mathbf{p}_2 s'_2) &= \{V_q - \frac{\lambda^2}{2} V_q (q + q_s) \\
&\langle \chi_{s'_1} \chi_{s'_2} | [\hat{\boldsymbol{\sigma}}_1 \times (\mathbf{p}_1 + \mathbf{k}_1)]_z - [\hat{\boldsymbol{\sigma}}_2 \times (\mathbf{p}_2 + \mathbf{k}_2)]_z | \chi_{s_1} \chi_{s_2} \rangle \} \\
&\delta_{\mathbf{k}_1 + \mathbf{k}_2, \mathbf{p}_1 + \mathbf{p}_2},
\end{aligned} \tag{1}$$

where the subscripts 1 and 2 refer to electrons in layers 1 and 2, respectively, $\hat{\boldsymbol{\sigma}}_1$ and $\hat{\boldsymbol{\sigma}}_2$ are the spin operators of the first and second electron acting on spinors $|\chi_{s_1}\rangle, |\chi_{s'_1}\rangle$ and $|\chi_{s_2}\rangle, |\chi_{s'_2}\rangle$, respectively. The normalization area is set to unity. Here $\mathbf{q} = \mathbf{p} - \mathbf{k}$ is the transferred wave vector,

$$V_q = \frac{2\pi e^2}{\kappa} \frac{q e^{-qL}}{(q + q_s)^2 - q_s^2 e^{-2qL}}$$

is the Fourier transform of the screened Coulomb interaction between the layers, L is the distance between the layers, κ is a background dielectric constant and q_s is the screening wave vector. The strength of the spin-orbit interaction is controlled by the “effective Compton wavelength” λ for the semiconductor:

$$\lambda^2 = -\frac{P^2}{3} \frac{\Delta(2E_g + \Delta)}{E_g^2(E_g + \Delta)^2}, \tag{2}$$

where P is the Kane parameter¹⁶ and E_g and Δ are the band-gap and the spin-orbit splitting of the valence band. In GaAs $\lambda^2 \approx 5 \text{ \AA}^2$. Notice that, in writing Eq. (1) we have taken into account only the Rashba-like contribution arising from the component of the Coulomb field perpendicular to the layers. The additional term coming from the component of the Coulomb field parallel to the layers was discussed in Refs. 6, 17, and 18 and is not shown in Eq. (1).

As discussed in the introduction and in the caption of Fig. 2, a steady current driven in the active layer (2) produces, via the “spin-orbit dressed” Coulomb interaction, a quadrupolar distribution of spin in the passive layer. The spin generation rate in the passive layer can be calculated by means of the Fermi golden rule with the matrix element of interlayer electron-electron interaction given by Eq. (1)^{19,20}. To first order in λ^2 this gives

$$\begin{aligned}
\mathbf{g}_{\mathbf{k}} &= \frac{4\pi\hbar e\tau}{mk_B T} \sum_{\mathbf{k}' \mathbf{p} \mathbf{p}'} \delta_{\mathbf{k}+\mathbf{k}', \mathbf{p}+\mathbf{p}'} \delta(\mathcal{E}_{\mathbf{k}} + \mathcal{E}_{\mathbf{k}'} - \mathcal{E}_{\mathbf{p}} - \mathcal{E}_{\mathbf{p}'}) \times \\
&U_{\mathbf{k}-\mathbf{p}}[(\mathbf{p} + \mathbf{k}) \times \hat{\mathbf{z}}](\mathbf{E} \cdot (\mathbf{p} - \mathbf{k})) f_{\mathbf{k}} f_{\mathbf{k}'} (1 - f_{\mathbf{p}})(1 - f_{\mathbf{p}'}),
\end{aligned} \tag{3}$$

where $U_{\mathbf{q}} \equiv \lambda^2(q + q_s)|V_q|^2$, \mathbf{E} is the electric field acting on the carriers in the active layer, m is the effective electron mass, $k_B T$ is the temperature measured in the energy units, $\mathcal{E}_{\mathbf{k}} = \hbar^2 k^2 / (2m)$ is the electron dispersion, $f_{\mathbf{k}}$ is the Fermi-Dirac distribution function and $\hat{\mathbf{z}}$ is the unit vector normal to the layers. We have also assumed that equilibrium densities, effective masses and Fermi energies are the same in the two layers. In deriving Eq. (3) we took into account only linear-in- \mathbf{E} nonequilibrium correction to the distribution function. It is easy to show that, due to the presence of the two factors $\mathbf{E} \cdot (\mathbf{p} - \mathbf{k})$ and $(\mathbf{p} + \mathbf{k}) \times \hat{\mathbf{z}}$, the angular dependence of $\mathbf{g}_{\mathbf{k}}$ is an angular harmonic of order 2, i.e. we have $g_{x,\mathbf{k}} \propto \sin 2\varphi_{\mathbf{k}}$ and $g_{y,\mathbf{k}} \propto -\cos 2\varphi_{\mathbf{k}}$, where $\varphi_{\mathbf{k}}$ is the angle of \mathbf{k} with the x axis: this is the quadrupolar pattern of spin generation mentioned in the introduction and described in Fig. 2.

The spin dynamics in the passive layer is governed by the spin-orbit splitting of the energy spectrum. Since the first harmonic components of the spin splitting (arising from linear-in- \mathbf{k} -terms) do not result in a dc spin Hall current^{11,21,22}, we only take into account the third angular harmonics of the k^3 Dresselhaus term which is inevitable in any zincblende structure. In cubic axes with $x \parallel [100]$ and $y \parallel [010]$ these have the form

$$\Omega_{x,\mathbf{k}} = -\Omega_3 \cos 3\varphi_{\mathbf{k}}, \quad \Omega_{y,\mathbf{k}} = -\Omega_3 \sin 3\varphi_{\mathbf{k}}, \tag{4}$$

where $\Omega_3 = \gamma_c k^3 / (2\hbar)$, γ_c is the bulk Dresselhaus constant. The electron spin distribution function, $\mathbf{s}_{\mathbf{k}}$, is determined by a kinetic equation, which in the steady state takes the form¹¹

$$\mathbf{s}_{\mathbf{k}} \times \boldsymbol{\Omega}_{\mathbf{k}} + \frac{\mathbf{s}_{\mathbf{k}}}{\tau} = \mathbf{g}_{\mathbf{k}}. \tag{5}$$

Here we have assumed that all spin-independent scattering processes can be characterized by a single relaxation time τ . The solution of Eq. (5) is

$$s_{z,\mathbf{k}} = \frac{(\boldsymbol{\Omega}_{\mathbf{k}} \times \mathbf{g}_{\mathbf{k}})_z}{\Omega_{\mathbf{k}}^2 + 1/\tau^2}. \tag{6}$$

We note that such a simple form of the solution results from keeping only the third angular harmonics in $\Omega_{\mathbf{k}}$. The solution in the general case can be constructed following Refs. 11 and 23: in such a case the overall spin will be smaller due to faster spin relaxation caused by the linear-in- \mathbf{k} Dresselhaus term, and the spin distribution may have more complicated form due to the anisotropy of the spin splitting.

By definition, the current density of spin z component is

$$\mathbf{J}^z = \sum_{\mathbf{k}} s_{z,\mathbf{k}} \mathbf{v}_{\mathbf{k}}, \quad (7)$$

where $\mathbf{v}_{\mathbf{k}} = \hbar \mathbf{k} / m$ is the electron velocity. Substituting the expression for $\mathbf{g}_{\mathbf{k}}$, obtained from Eq. (3), into Eq. (6) for $s_{z,\mathbf{k}}$, and then $s_{z,\mathbf{k}}$ in Eq. (7), we arrive at our main result

$$\begin{aligned} \mathbf{J}^z = & -\frac{2\pi\hbar e\tau^3}{mk_B T} \sum_{\mathbf{k}\mathbf{k}'\mathbf{p}\mathbf{p}'} \mathbf{v}_{\mathbf{k}} \delta_{\mathbf{k}+\mathbf{k}', \mathbf{p}+\mathbf{p}'} \delta(\mathcal{E}_{\mathbf{k}} + \mathcal{E}_{\mathbf{k}'} - \mathcal{E}_{\mathbf{p}} - \mathcal{E}_{\mathbf{p}'}) \\ & 2U_{\mathbf{k}-\mathbf{p}} [\Omega_{\mathbf{k}} \cdot (\mathbf{p} + \mathbf{k})] [\mathbf{E} \cdot (\mathbf{p} - \mathbf{k})] \\ & f_{\mathbf{k}} f_{\mathbf{k}'} (1 - f_{\mathbf{p}}) (1 - f_{\mathbf{p}'}). \end{aligned} \quad (8)$$

Equation (8) can be recast in the standard form:

$$J_i^z = G_{ij} E_j, \quad (9)$$

where G_{ij} is the spin drag conductivity. Our system is characterized by the C_{2v} point symmetry group because we take into account (i) the Dresselhaus field, and (ii) the Rashba-like interaction associated with the perpendicular-to-plane component of the Coulomb field. In the basis of the principal axes $x_1 \parallel [1\bar{1}0]$ and $y_1 \parallel [110]$ Eq. (9) can be written in terms of two independent constants G and G_1 as

$$J_{x_1}^z = (G + G_1) E_{y_1}, \quad J_{y_1}^z = (G - G_1) E_{x_1}. \quad (10)$$

Equation (10) gives the full phenomenological picture of the spin drag in the presence of the spin-orbit interaction. The drag mechanism described here produces $G \neq 0$ and $G_1 = 0$. This rather peculiar situation implies that the spin current flows parallel to the electric field if the latter is applied along the $[100]$ or $[010]$ directions, but, in general, it has both a parallel and a perpendicular component. It is only when the electric field is along one of the principal axes that we get a pure transverse current, but even in this case it is in a sharp contrast with the spin Hall drag, namely an electric field along x_1 produces a spin current along y_1 , but an electric field along y_1 produces a spin current along x_1 , rather than $-x_1$. These unique symmetry-related features will help distinguishing the predicted new effect.

We now come to the quantitative evaluation of G . The expression (8) for the spin current density is similar to the expression encountered in the calculation of the ordinary drag current¹³. The main difference is that the integrand depends not only on the momentum transfer $\mathbf{q} = \mathbf{p} - \mathbf{k}$, but also on the sum of the initial and final momenta, $\mathbf{p} + \mathbf{k}$. For simplicity we focus here on the case of well separated layers, $k_F L \gg 1$, where the momentum transfer is typically small ($q \sim L^{-1} \ll k_F$), and therefore one can replace $\mathbf{k} + \mathbf{p} \approx 2\mathbf{k}_F$, making an error of order q/k_F . With this approximation, the standard method of evaluation can be applied, and, after converting the wave vector sums into integrals, and for low enough temperatures $k_B T \ll E_F$, we obtain

$$G = -\frac{\hbar^3 e \Omega_3 k_F \tau^3}{2\pi m^2 k_B T} \int \frac{d\mathbf{q}}{(2\pi)^2} \int_{-\infty}^{+\infty} d\omega \frac{\text{Im}[\chi_0(q, \omega)]^2}{\sinh^2\left(\frac{\hbar\omega}{2k_B T}\right)} U_q q^2, \quad (11)$$

where $\chi_0(q, \omega)$ is the non-interacting density-density response function of the two-dimensional electron gas²⁴. Integrating over frequency we get

$$G = -\frac{\lambda^2 \Omega_3 e^5 k_F^4 \tau^3}{4\pi^2 \kappa^2 \hbar^2} \left(\frac{k_B T}{E_F}\right)^2 I_2(q_s L) \quad (12)$$

with

$$I_2(x) = \frac{2\pi^2}{3k_F L} \int_0^\infty \frac{y^3(y+x)e^{-2y}}{[(x+y)^2 - x^2 e^{-2y}]^2} dy, \quad (13)$$

for $k_F L \gg 1$. For $q_s L \gg 1$ the integral Eq. (13) is evaluated to be $I_2(q_s L) \approx 2.96/(k_F q_s^3 L^4)$, as a result $G \propto 1/L^4$ as in the ordinary drag effect^{12,13}.

Results and discussion – It is instructive to estimate the drag resistivity which controls the observable spin accumulation. This is determined as follows. First, we express the external field E via the current density generated by this field in the active layer, $j = \sigma E$, where $\sigma = ne^2\tau/m$ is the Drude conductivity of the electrons and n is their density. Then, we observe that the spin current is associated with some effective electric “spin” field E_s

$$\frac{2e}{\hbar} J^z = \sigma E_s,$$

As a result we obtain

$$E_s = \frac{2eJ^z}{\hbar\sigma} = \frac{2eG}{\hbar\sigma^2} j = \rho_s j.$$

Hence, the drag resistivity is given by

$$\rho_s = -\frac{2\hbar}{e^2} (\lambda k_F)^2 \Omega_3 \tau \left(\frac{k_B T}{E_F} \right)^2 \left(\frac{e^2}{\kappa \hbar v_F} \right)^2 I_2(q_s L), \quad (14)$$

where v_F is the Fermi velocity. Notice that, at variance with the (side-jump) spin Hall drag resistivity, the present result is proportional to the momentum relaxation time τ and therefore it is parametrically dominant in the “clean” limit, $\tau \rightarrow \infty$.

For the following values of the parameters: electron density $n = 2 \times 10^{11} \text{ cm}^{-2}$, $\tau = 40 \text{ ps}$ (which corresponds to a mobility $\mu = 10^6 \text{ cm}^2/\text{Vs}$ in a GaAs quantum well with $m = 0.067m_0$), bulk Dresselhaus splitting constant $\gamma_c = 20 \text{ eV } \text{\AA}^3$, $\lambda^2 = 5 \text{ \AA}^2$, and $\kappa = 13$, we obtain $(\lambda k_F)^2 \approx 6 \times 10^{-4}$, $\Omega_3 \tau \approx 0.9$, and $e^2/\kappa \hbar v_F \approx 1$. Thus, taking $I_2 \sim 1$, we have $\rho_s \sim 4 \left(\frac{k_B T}{E_F} \right)^2 \text{ Ohm}$, which for $(k_B T/E_F) = 0.1$ is 0.04 Ohm , i.e., about two times larger than the conservative estimate for the spin Hall drag resistivity from side jump given in Ref. 6. In InAs based structures the spin-orbit coupling parameter λ^2 is about an order of magnitude higher than in GaAs and Ω_3 is also larger. This makes narrow-band semiconductor bi-layers particularly suitable for the observation of the spin current injection.

In conclusion, we have described a new coupling mechanism, partly Coulomb and partly spin-orbit, through which a spin current can be injected, or a spin accumulation induced, in an electron layer by a regular electric current flowing in an adjacent layer. The new coupling can play a role in the design of circuits in which an electric current must be converted into a spin current and viceversa.

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