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The thermal conductance of a two-channel-Kondo model

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A theory of thermal transport in a two-channel Kondo system, such as the one formed by a small quantum dot coupled to two leads and to a larger dot, is formulated. The interplay of the two screening constants allows an exploration of the Fermi liquid and non-Fermi liquid regimes. By using analytical, as well as numerical renormalization group methods, we study the temperature dependence of the thermal conductance and the Lorentz number. We find that in the low-temperature limit, the Lorentz number attains its universal value, irrespective of the nature of the ground state.

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I. INTRODUCTION

Quantum dots (QD) have long been recognized by experimentalists and theorists alike for their ability to embody several important theoretical paradigms. Some of the most important refer to the possibility to replicate in a tunable structure the Kondo effect. First associated with the anomalous resistivity values in metals, the Kondo effect originates in the antiferromagnetic interaction of conduction electrons with a local magnetic moment of spin 1/2. In this respect, the degenerate ground state of the electron liquid in the quantum dot functions as a magnetic impurity, while the electrons in the leads connected to the dot behave as the surrounding normal metal[1]. The multi-channel Kondo problem involves two or more electron modes in the screening of the magnetic impurity. The QD representation of this situation has been realized several years ago and is based on the different interactions established between an electron in a small dot, that functions as a magnetic impurity, with the electrons in two leads and with those located in a larger dot[2]. The relative strength of the screening realized by these two channels determines the characteristic behavior of the system.

If in the case of a single-channel Kondo (1CK) effect, at temperatures below the characteristic Kondo temperature, T_K, the system behaves like a Fermi liquid (FL) and is characterized by analytic functions of temperature, the interplay between the two screenings in the case of two channel Kondo (2CK) problem, permits the realization of a FL, when one screening dominates the other one, or of a non-Fermi liquid (NFL), when the screenings are equal. Then each reservoir is trying to screen the magnetic impurity, but in the symmetric limit $J_1 = J_2$, no one succeed. This leads to a NFL type of ground state, where, in contrast to the single-channle Kondo problem, the local impurity is only partially screened. The impact of the Kondo correlations on electronic transport has long been an area of active research. Considerably less is known about their effect on thermal transport, which only recently has been discussed in several theoretical[3–8] and experimental[9] studies that focus on the 1CK model. In this paper we extend such considerations to the study of the thermal conductivity of a double quantum dot system that can exhibit a two-channel Kondo state[10]. In this picture, the single electron in the small dot interacts with the electrons in the leads, with a screening integral J_1 and with the electrons in the bigger dot with a screening integral J_2. Besides the Kondo temperature itself, T_K, the relationship between J_1 and J_2, expressed by $K = (J_1 - J_2)/J^2$ sets another energy scale of the problem through the characteristic temperature $T^\star = K^2 T_K$, smaller than T_K. For the particular experimental setup realized in Ref. 10 the Kondo temperature ranges between 30 and 130 mK.

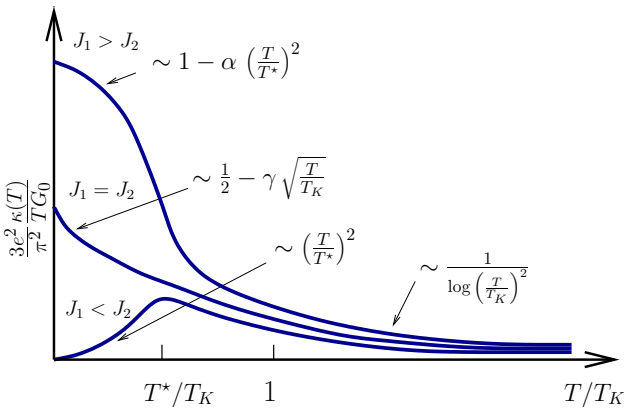

FIG. 1: (color online) The universal behavior of the thermal conductance of a 2CK model (corresponding to channel 1). Three regimes are clearly visible when $J_1 \neq J_2$. For $T \ll T^\star$ the system is in the Fermi liquid regime, followed by a crossover ($T^\star < T < T_K$) to the perturbative ($T \gg T_K$) regime. The non-Fermi fixed point is realized for $J_1 = J_2$. α and γ are two numerical constants of order 1, and G_0 is the universal conductance $G_0 = 2\,e^2/h$.

In an approach that combines analytical arguments with Wilson's numerical renormalization group (NRG) method, we calculate the thermal conductivity and the Lorentz number of this system, and investigate their be-

havior as a function of temperature. In this algorithm, the thermal conductance $\kappa(T)$ corresponding to channel 1 is obtained as

$$\frac{\kappa\,(T)}{T}=\frac{2}{h}\frac{\pi^2}{3}\begin{cases}\theta(K)-\mathrm{sgn}(K)\,\alpha\,(T/T^\star)^2 & \text{if } K\neq 0\\ 1/2-\gamma\sqrt{T/T_K} & \text{if } K=0\end{cases}, \tag{1}$$

where $\theta(K)$ is the Heaviside function and $\mathrm{sgn}(k)$ is the signum function; α and γ are two numerical constants[11] of order 1. A sketch of this result is presented in Fig. 1. We find that when $J_1\neq J_2$, at temperature $T<T^*$, the ground state of the system is a Fermi liquid (FL) and the thermal conductance varies as $\kappa(T)/T\propto(T/T^\star)^2$. The symmetric channel problem, when $J_1=J_2$, corresponds to a NFL behavior when the only energy scale in the problem is set by T_K. Then, $\kappa(T)/T\propto\sqrt{T/T_K}$. In the large temperature regime, when $T\gg T_K$, a $1/\log^2(T/T_K)$ behavior is obtained for $\kappa(T)/T$.

The nature of the ground state strongly affects the Lorentz number, namely, $L(T)=\kappa(T)/TG(T)$, with $G(T)$ the charge conductance. In the Fermi liquid regime, quasiparticles transport both charge and energy and the Lorentz number attains a universal value $L(T)=\pi^2k_B^2/3e^2\equiv L_0$. This statement is the essence of the Wiedemann-Franz law[12], formulated long time ago for normal metals. Deviations of $L(T)$ from this behavior are attributed to non-Fermi liquid behavior[13].

The process used in deriving Eq. (1) adopts the some of the methods that have been previously developed in investigations of the ground state and the transport properties of the 2CK model[14,15].

II. THERMAL TRANSPORT

The minimal Hamiltonian that describes the system discussed above is written as $H=H_0+H_{\rm int}$[16]. Here,

$$H_0=\sum_{r=\{L,R\},\sigma}\int d\epsilon\,\epsilon\,c^\dagger_{r,\sigma}(\epsilon)\,c_{r,\sigma}(\epsilon)\,, \tag{2}$$

represents the electrons in the non-interacting leads, where a single particle state of energy ϵ, spin σ and lead index $r=\{R,L\}$ for left, right respectively, is associated with the Fermionic operator $c^\dagger_{r,\sigma}(\epsilon)$. The interacting Hamiltonian contains two terms, $H_{\rm int}=H^{(1)}_{\rm int}+H^{(2)}_{\rm int}$, which describe the coupling of the local spin in the smaller dot with the conduction electrons in the external leads and to those in the second dot. The first term, $H^{(1)}_{\rm int}$, is

$$H^{(1)}_{\rm int}=\frac{1}{2}J_1\sum_{r,r'=L,R}\sum_{\sigma\sigma'}\eta_r\eta_{r'}\,\mathbf{S}\,\psi^\dagger_{r\sigma}\,\boldsymbol{\sigma}_{\sigma\sigma'}\,\psi_{r'\sigma'}. \tag{3}$$

Here $\boldsymbol{\sigma}$ stands for the Pauli matrices $\boldsymbol{\sigma}=\{\sigma_x,\sigma_y,\sigma_z\}$ and $\psi^\dagger_{r\sigma}$'s are the creation field operators, constructed from the creation operators of the electronic states in the leads, $\psi^\dagger_{r\sigma}=\sqrt{\varrho}\int_{-D}^{D}d\epsilon\,c^\dagger_{r,\sigma}(\epsilon)$. Here ϱ is the conduction band density of states, which is electron-hole symmetric, $\varrho(\omega)=1/2D$, $-D<\omega<D$. The coupling of the local spin with the electrons in the leads is considered to be anisotropic, and will be expressed through some dimensionless hybridization parameters, $\eta_{L/R}$, of the form, $\eta_{L/R}=v_{L/R}/(v_L^2+v_R^2)$, with $v_{L/R}$ the amplitude of the hopping between the dot and the corresponding external lead. An asymmetry parameter, ϕ, allows a more elegant description of the hybridization, $\eta_L=\cos\left(\frac{\phi}{2}\right)$ and $\eta_R=\sin\left(\frac{\phi}{2}\right)$. An even/odd basis emerges, with new annihilation operators, $\{\Psi,\tilde\Psi\}$, defined as, $\Psi=\cos\left(\frac{\phi}{2}\right)\psi_L+\sin\left(\frac{\phi}{2}\right)\psi_R$ and $\tilde\Psi=\sin\left(\frac{\phi}{2}\right)\psi_L-\cos\left(\frac{\phi}{2}\right)\psi_R$. By this unitary transformation, the local spin remains coupled only to the even channel, while the odd channel becomes decoupled, and then, can be treated as a non-interacting one.

The coupling with the larger dot,

$$H^{(2)}_{\rm int}=\frac{1}{2}J_2\sum_{\sigma\sigma'}\mathbf{S}\,\psi^\dagger_{2\sigma}\boldsymbol{\sigma}_{\sigma\sigma'}\psi_{2\sigma'} \tag{4}$$

is considered to be isotropic, with a dimensionless amplitude J_2. The field operators, $\psi^\dagger_{2\sigma}$, describe the electron-hole excitations in the larger dot.

Our theory permits the calculation of the thermal conductance of a 2CK system by starting from the heat operator, $Q^{(Q)}$,

$$Q^{(Q)}=\sum_\sigma\int d\epsilon\,\epsilon\left(c^\dagger_{L,\sigma}(\epsilon)c_{L,\sigma}(\epsilon)-(c^\dagger_{R,\sigma}(\epsilon)(c_{R,\sigma}(\epsilon)\right), \tag{5}$$

which describes the heat transfer under a temperature gradient between the leads. We implicitly assume that the leads are in equilibrium, $\mu_L=\mu_R=0$. The heat current is defined through the usual expression $I^{(Q)}=i\left[H^{(1)}_{\rm int},Q^{(Q)}\right]$. We introduce the so-called composite fermion operators,[17,18] $\mathcal{F}_\sigma=\sum^\dagger_{\sigma'}\Psi^\dagger_{\sigma'}\boldsymbol{\sigma}_{\sigma'\sigma}\mathbf{S}$, and $\tilde h^\dagger_\sigma=2\,\varrho^{3/2}\sqrt{3}\int_{-D}^{D}d\epsilon\,\epsilon\,\tilde c^\dagger(\epsilon)$, to rewrite the heat current as

$$I^{(Q)}=\frac{1}{2\sqrt{3}}J_1\,\sin(\phi)\sum_\sigma\left(\mathrm{i}\,\mathcal{F}^\dagger_\sigma\,\tilde h_\sigma+\mathrm{h.c.}\right). \tag{6}$$

The prefactor in the definition of $\tilde h_\sigma$ was fixed, such that it satisfies the canonical anticommutation relations. Moreover, $\tilde h_\sigma$ describes electrons in the odd-channel. Since this channel is decoupled it may be treated as a non-interacting Fermi system.

When the system approaches the equilibrium, the heat current through the dot can be calculated with the Kubo formalism. For that purpose, we consider a temperature gradient being applied between the external leads. At temperature T, the Hamiltonian acquires an additional term, $H_T=Q^{(Q)}\nabla T/T$, with T the temperature itself, and ∇T the temperature gradient between the external

leads. Then the average heat current, in its most general form, can be expressed as

$$\left\langle I^{(Q)}(t)\right\rangle = \int_{-\infty}^{t} T\kappa(T,t-t')\frac{\nabla T}{T}(t')\,dt', \qquad (7)$$

with $\kappa(T,t)$ denoting the thermal conductance. While this formalism is general enough to allow the calculation of the ac-thermal conductance, $\kappa(T,\omega)$, here, we will focus on the dc-limit only. Algebraic manipulation of Eq.(7) permits expressing the thermal conductance in terms of heat current operators,

$$T\kappa(T) = -\mathrm{i}\int_0^\infty dt\left\langle\left[I^{(Q)}(0), I^{(Q)}(t)\right]\right\rangle. \qquad (8)$$

Finally a simple, analytical form can be derived,

$$T\kappa(T) = \frac{2}{h}\frac{\sin^2(\phi)}{4\sqrt{3}}\sum_\sigma\int d\omega\,\omega^2\Im m\left\{\mathcal{T}(\omega,T)\right\}\frac{\partial f(\omega,T)}{\partial\omega} \qquad (9)$$

In Eq. (9) we can identify the rescaled Green's function of the composite operator as the T-matrix, $\mathcal{T}(\omega) = J_1^2\mathcal{G}^R_{\mathcal{F}_\sigma}(\omega)$.

Eq. (9) allows us to estimate the thermal conductivity of the 2CK model. We define the Kondo temperature T_K through the equation, $\Im m\mathcal{T}(\omega = T_K) = \frac{1}{2}\Im m\mathcal{T}(\omega = 0)$, at the symmetric point, $J_1 = J_2$. In this situation, the screening of the local spin in the smaller dot is equally performed by both channels, their competition leading to an NFL quantum critical state. Away from the symmetric point, $J_1 \neq J_2$, the screening is realized by the stronger coupling channel.

In the following, we will always focus on the temperature dependence of the thermal conductivity for channel 1. In the FL regime, for $T \ll T^\star$, the T-matrix is analytical[16],

$$\Im m\mathcal{T}(\omega,T) \simeq \frac{1}{\pi\varrho}\left(\theta(K) - \mathrm{sgn}(K)\frac{3\omega^2+\pi^2T^2}{T^{\star 2}}\right). \qquad (10)$$

At the symmetrical point, and finite temperatures, the non-Fermi liquid character is manifested by a $\sqrt{T}$ contribution,

$$\Im m\mathcal{T}(\omega,T) \simeq \frac{1}{2\pi\varrho}\left(1 - a\sqrt{\omega/T_K} - b\sqrt{T/T_K} + \ldots\right). \qquad (11)$$

Here a and b are two universal constants. In this case the T-matrix is purely imaginary and is reduced by half from the unitary value in the Fermi liquid regime. Simple integrations leads to the qualitative behavior presented in Fig. 1.

Although this is a perturbative calculation, in the FL regime, the ratio of the thermal conductance and the electrical conductance recovers the Lorentz number exactly in the $T \to 0$ limit. It is interesting to point out that this also happens for $J_1 = J_2$, when again, for $\lim_{T\to 0}$, $L(T) = L_0$. This result is somewhat contradictory to the general conclusion that attributes deviations of the Lorentz number from its constant value L_0 to non-Fermi liquid behavior.

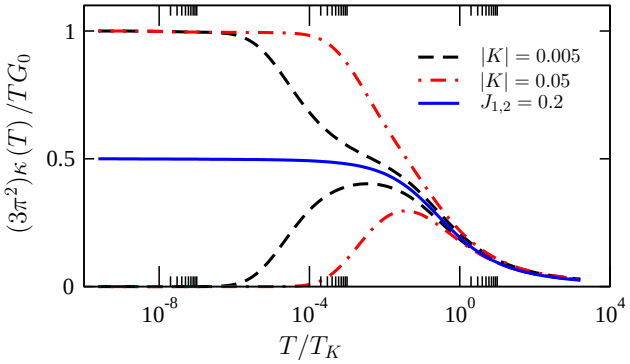

FIG. 2: (color online) Thermal conductance as function of T/T_K on a logarithmic scale. The NFL fixed point result, $J_1 = J_2$, corresponds to the solid line, while dashed and dash-dotted lines correspond to $J_1 \neq J_2$. K is the anisotropy factor $(J_1 - J_2)/J^2$. In both cases the average coupling is $J = (J_1 + J_2)/2 = 0.2$.

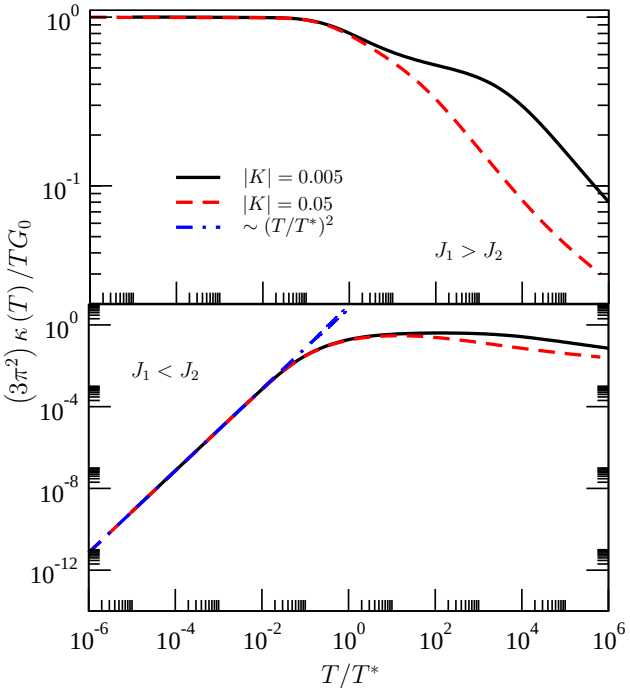

FIG. 3: (color online) The thermal conductance in the Fermi liquid regime for $K > 0$ (upper panel) and for $K < 0$ (lower panel). $\kappa(T)/T$ given by Eq. (1) is plotted as a function of T/T^* on a scaled temperature axis.

III. NUMERICAL RESULTS

The analytical results obtained in Eq. (1) above were subjected to a numerical test produced with the numerical renormalization group (NRG) method. Within the NRG framework, the spectral function of the T-matrix is given as a weighted sum of δ-functions of the form $\Im m\mathcal{T}(\omega,T) \sim \sum_i w_i\delta(\omega-\omega_i)$. This expression replaces the integral over frequency in Eq. (9). The numerical

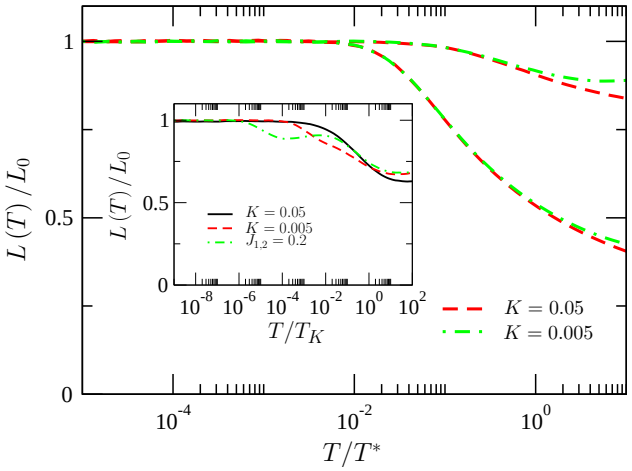

FIG. 4: (color online) The universal behavior of the Lorentz number as function of $T/T^\star$ in the Fermi liquid regime. In the low temperature limit $T \ll T^\star$, $L(T)$ converges to L_0. The inset presents $L(T)$ as function of T/T_K. At the NFL fixed point, for $T \ll T_K$ the same universality is recovered.

results were obtained with the Flexible-DMNRG code[19], which allows the explicit use of the symmetries of the Hamiltonian. In our particular case we built in the $SU_{c1}(2) \otimes SU_{c2}(2) \otimes SU_s(2)$ with $SU_{ca}(2)$ the charge $SU(2)$ symmetry in channel a and $SU_s(2)$ the global spin symmetry. For every run we have kept 1000 multiplets and we fixed $\Lambda = 2$. In Fig. 2 we represent the results for the thermal conductance $\kappa(T)$ as function of T/T_K. At the symmetric point, $J_1 = J_2$, the curves for different J's are scaling one on top of each other and we have found that the thermal conductance is a universal function of T/T_K, with the behavior predicted by Eq. (1). In Fig. 3 we have represented $\kappa(T)/T$ as function of $T/T^\star$. For temperatures $T \ll T^\star$, the universal behavior $\kappa(T)/T \propto (T/T^\star)^2$, obtained analytically in Eq. (1) was also recovered. Thus the numerical results confirm the universal behavior found analytically.

Finally, the universality of the Lorentz number was checked numerically. In Fig. 4, main panel, we present the results for the Lorentz number as function of temperature in the FL regime. The conductance $G(T)$ employed in the calculation was previously shown[14] to have a universal behavior for $T \ll T^\star$, when $G(T) \propto G(T/T^\star)$. Since both the electric and thermal conductance have a similar behavior, $L(T) \propto L(T/T^\star)$ is found to be a universal function. At the NFL fixed point, when $G(T) \simeq G_0 \left(1 - \sqrt{\pi\, T/T_K}\right)/2$, the Lorentz number becomes a universal function of T/T_K. As can be seen in the inset of Fig. 4, this universal behavior is valid well below T_K, while the scaling in the FL regime goes up to temperatures as large as $T^\star$. In the limit $T \to 0$, $L(T)/L_0 \to 1$ in all cases. Similarly, a large violation of the WF law is observed at large temperatures, $T \gg T_K$, mostly on account of the suppression of the thermal transport.

In conclusion we have constructed a general framework for the calculation of the thermal conductance. The method can be easily extended to the computation of the thermopower, as well as the ac-thermal properties of any quantum dot system. The NRG method was used to numerical evaluate the conductance, thermal conductance and the Lorenz number in the 2CK model as function of temperature. Our findings point towards a universal behavior of all these quantities, similar with the electrical conductance, studied previously[14]. In the limit of zero temperature we have found that $L(T \to 0) = L_0$ both in the FL and NFL ground states.

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