Quantum spinon oscillations in a finite one-dimensional transverse Ising model

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Quantum Spinon Oscillations

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The full quantum dynamics of a spinon under external magnetic fields is investigated by using the time-evolving block decimation (TEBD) method within the microcanonical picture of transport. We show that the center of the spinon oscillates back and forth in the absence of dissipation. The quantum many-body behavior can be understood in a single-particle picture of transport and Bloch oscillations, where quantum fluctuations induce finite life times. Transport, oscillations and lifetimes can be tuned to some degree separately by external fields. Other nontrivial dynamics such as resonance as well as chaos have also been discussed.

Controlling the time-evolution of quantum states and manipulating the dynamics of quantum matter have attracted considerable theoretical and experimental attention in recent years due to their relevance to fundamental physics as well as potential applications in information storage, encoding and processing. Up to now, most efforts in this field are based on solid state systems. Recently, the progress in ultracold atomic gases in optical lattices not only provides an ideal platform for simulating quantum many-body models in condensed matter physics, but also paves the way for quantum manipulation. Due to the unique properties of the ultracold atomic gases in optical lattices, such as the extremely low dissipation rates and long coherence times, we can explore the dynamics of quantum many-body systems in a clean and dissipation-less environment. This enables us to manipulate the quantum many-body state with an unprecedented degree of precision, and study new physical effects[1]. An early breakthrough in the field was the first direct observation of Bloch oscillations (BO) in a tilted optical lattice[2,3]. In solid state systems, scattering by impurities would result in strong damping of Bloch oscillations, while in an optical lattice the perfect optical crystals are free from any imperfections, and what is more, the long coherence times and high tunability of optical lattices not only enable us to observe BO directly, but also to manipulate the dynamics of the quantum particles via external fields.

Compared with the dynamics of the quantum particles, it may be more interesting to explore and manipulate the quasi-particles, which emerge as elementary excitations in quantum systems consisting of a collection of interacting particles, and may behave differently compared to the original particles. The most interesting example of this is known as fractionalization: the particles are effectively split into smaller constituent quasi-particles, which only carry a fraction of quantum numbers.

In this paper, we study the quantum dynamics of one of the most known quasi-particles: the spinon, which is an excitation separating two degenerate states of opposite magnetization in a quantum spin chain and usually carries a fractionalized quantum number (spin-$\frac{1}{2}$). Using the time-evolving block decimation (TEBD) method[4,5] within the microcanonical picture of transport[6,7], we study the time evolution from a quantum many-body initial state with a spinon, and show how to manipulate its dynamics via external fields. In particular, we find that this quantum many-body problem can be interpreted in the picture of a quantum quasiparticle at the single-particle level, which shows controllable linear transport and (Bloch) oscillation behavior depending on perpendicular external fields, where the field along the magnetization direction controls the Bloch oscillation physics and the transverse field controls the linear velocity. The broadening of the quasiparticle is also tunable by the field strength. We also find a quantum resonance behavior of the spinon under a periodically driven external field. This may pave the way to the controlled manipulation of quasiparticles, also in view of potential applications in condensed matter and ultracold atom physics.

Our Hamiltonian is a finite 1D transverse Ising model with an additional magnetic field along the z-direction:

$$H = \sum_i J\sigma_i^z \sigma_{i+1}^z + g\sigma_i^x + h\sigma_i^z \qquad (1)$$

with a ferromagnetic coupling $J = -1$, Pauli matrices $\sigma_i^{x,z}$ on sites i, and a transverse field along the x-direction that breaks magnetization conservation. Without the magnetic field $h\sigma_i^z$, the transverse Ising model can be solved exactly by a Jordan-Wigner transformation and is known as a classic paradigm of a quantum phase transition[8].

Recently, the transverse Ising model has been realized experimentally in cobalt niobate and a E_8 symmetry has been observed in the vicinity of the quantum critical point of this model[9,10]. In the presence of a mag-
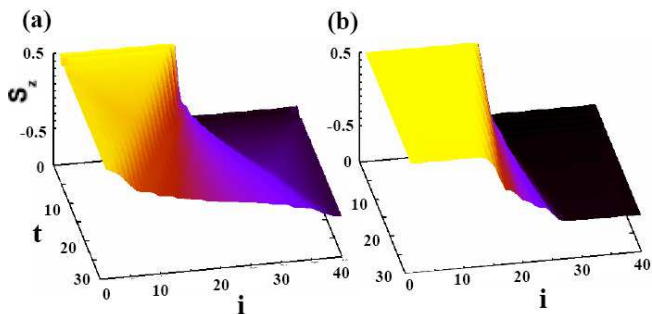
FIG. 1: The time evolution of the spinon for $h = 0$ and (a) $g = 0.1$, (b) $g = 0.3$.

netic field along the z-direction h, Eq.(1) can no longer be solved exactly by Jordan-Wigner transformation, because it would induce a nonlocal term. In this paper, h is a constant or a time-dependent field, and in both case, it will lead to nontrivial dynamics of the spinon.
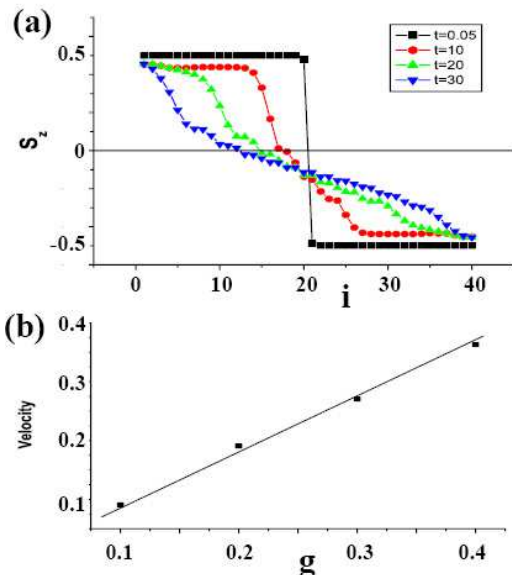

FIG. 2: (a) Magnetization S_i^z at different times (black squares: $t = 0.05$; red dots: $t = 10$; green upward triangles: $t = 20$; blue downward triangles: $t = 30$) for $h = 0$, $g = 0.3$. (b) The dependence of the spinon velocity on g.

Our many-body initial state is prepared as follows: a spinon with a finite velocity is located at the center of lattice. To realize this initial state, we can choose its wavefunction as $|\varphi\rangle_{t=0} = \frac{1}{N_0} \sum_i A_i e^{ik_0 i}|i\rangle$, where N_0 is the normalization constant and $|i\rangle$ denotes a perfect static spinon located between site i and $i + 1$: $|i\rangle = |\uparrow\rangle_1 \cdots |\uparrow\rangle_i |\downarrow\rangle_{i+1} \cdots |\downarrow\rangle_L$, where L is the length of the lattice. If we choose $A_i = 1$, the initial state is a perfect Bloch wave function with definite wave vector k_0, and the spinon is totally delocalized over the length of the lattice. To ensure that the spinon is initially located at the center of the lattice, we then choose a Gaussian
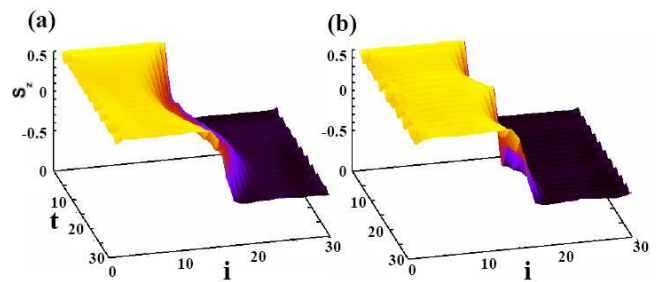

FIG. 3: The time evolution of a spinon for $g = 0.3$ and (a) $h = 0.1$, (b) $h = 0.2$.

$A_i = e^{-2(i-L/2)^2}$. Given the exponentially fast decay of A_i away from the center of the lattice, we can just keep $A_{L/2}$, $A_{L/2+1}$, $A_{L/2-1}$ and make all other $A_i = 0$ for constructing the initial wave function. In our calculation we furthermore choose $k_0 = \pi/2$. We also define the average position of the spinon P as the position where $S_z = 0$.
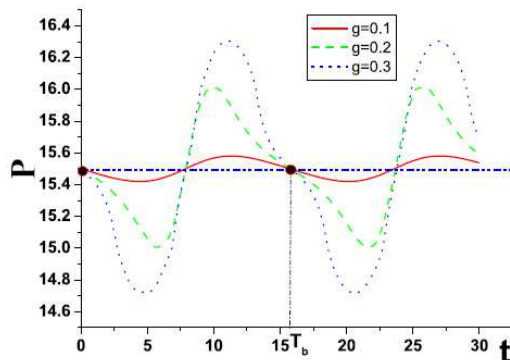

FIG. 4: The time evolution of the position of the spinon for $h = 0.2$ and different g.

As certain excitations can be modeled as quasiparticles, i.e., having similar properties to real particles, we may conjecture that this is the case here and compare the spinon to a real particle, where the field coupling to σ^x lead to the motion of the particle and where the field coupling to σ_z acts as a constant electric field along the axis in the particle picture, as the Zeeman energy induced by $h\sigma_z$ is proportional to the displacement of the spinon from its original position. To verify this picture quantitatively, we use TEBD within the microcanonical picture of transport to calculate the time evolution of the spinon from our initial state $|\varphi\rangle_{t=0}$ which is not an eigenstate of the Hamiltonian. In the course of the real time evolution we use the length of our lattice $L = 30$ and take the truncation dimension $\chi = 80$ and time step $\Delta\tau = 0.05$. The convergence is checked by taking larger χ. Times are measured in inverse hopping strengths $|J|^{-1}$ throughout the paper.

First we focus on the case $h = 0$; the real-time evolution from our initial state $|\varphi\rangle_{t=0}$ for different parameters g is shown in Fig. 1: (a) $g = 0.1$; (b) $g = 0.2$. We find
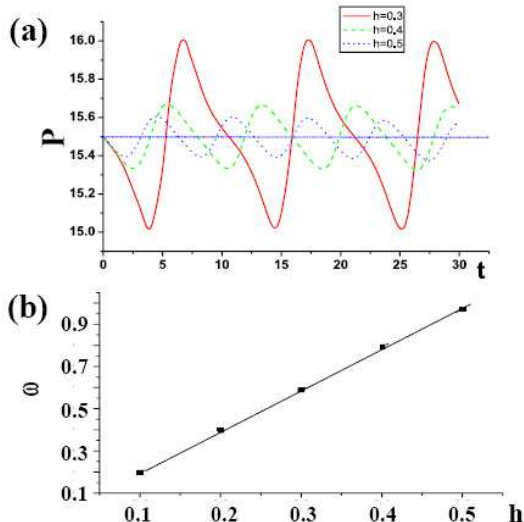
FIG. 5: (a) The time evolution of the position of the spinon for $g = 0.3$ and different h; (b) the linear relation between the frequency of BO and h.

that the spinon propagates along the $-x$-direction with a velocity proportional to g as shown in Fig. 2 (a). The linear relation between the velocity and g has been numerically verified in Fig. 2(b). The quantum fluctuations increase the width of the spinon during the time evolution, giving a lifetime to the quasiparticle. The quantum fluctuations make the quantum spinon different from its classical counterpart: the magnetic domain wall, which corresponds to a soliton solution of the nonlinear Landau-Lifshitz-Gilbert (LLG) that can preserve its shape during the propagation. Below we will see that all these results are modified when we introduce the magnetic field along the z-direction.

Next, we apply a constant magnetic field h along the z-direction. By interpreting the dynamics of the spinon in terms of a real particle in a constant external field suggests the observation of Bloch oscillations. First we fix $g = 0.3$ and investigate the dependence of the spinon dynamics on h. The time evolution of the spinon under different magnetic fields h is shown in Fig. 3(a) $h = 0.1$; (b) $h = 0.2$. For Bloch oscillations (BO) the frequency of the BO is only dependent on the strength of the external constant field, which can also be verified via our numerical result, as shown in Fig. 4, where we fix $h = 0.2$. We can find that for different g, the period of the BO is the same ($T_b = 15.5$), while the amplitude of BO A is determined by g. If we fix $g = 0.3$, the time evolution of the spinon for different h is shown in Fig. 5(a). Following the interpretation in terms of a Bloch oscillation, we expect that the oscillation frequency to be proportional to the strength of the 'external field' h, which can be verified numerically (Fig. 5(b)).

Next, we study the time evolution of width of the spinon. The width of the spinon Δ can be defined similarly to the definition of the width of a wavepacket. We
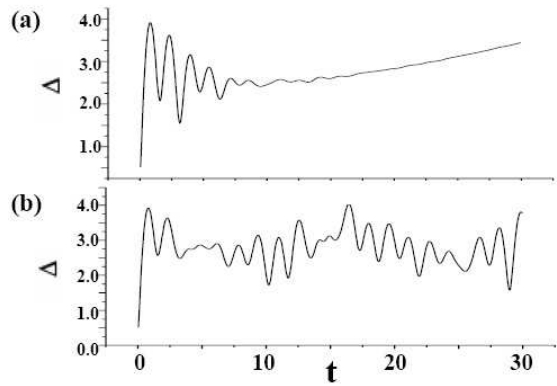

FIG. 6: The time evolution of the width Δ of the spinon for $g = 0.1$ and (a) $h = 0.0$; (b) $h = 0.2$.

introduce $n_i = 0.5 - |S_i^z|$ and $\Delta = \sqrt{\langle n \rangle^2 - \langle n^2 \rangle}$, , where $\langle O \rangle = \frac{\sum_i i O_i}{\sum_i O_i}$. The result is shown in Fig. 6. Without the magnetic field along the z-direction, $h = 0$, we observe (Fig. 6 (a)) that the width oscillates strongly at first whereas after some relaxation time, it grows linearly in time, which means the spinon would become wider and wider in the process of propagation of the spinon. The situation is different for $h \neq 0$, as shown in Fig.6 (b), where after some short-time behavior the width of the spinon oscillates around an average value instead of diverging, which corresponds to a solitonic breather mode: not only is the position of the spinon confined to oscillate around the center of the lattice by the magnetic field, but its width oscillates as well around an average value.
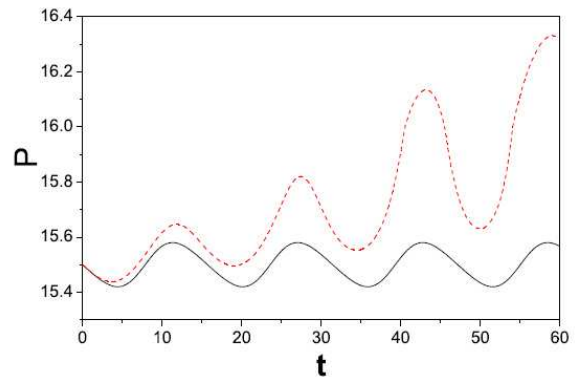

FIG. 7: The time evolution of the position of the spinon for $g = 0.1$, $h = 0.2$ (solid line) and $h = 0.2 + 0.05 \cos(\omega_0 t)$.

The dynamics of the spinon is even more interesting when the system is driven by a time-dependent magnetic field $h(t)$. In our case, we choose a periodic driving magnetic field: $h(t) = h_0 + \delta \cos(\omega t)$. Classically, it is known that if the frequency of the driving force can match the intrinsic frequency of the system, the system can accumulate vibrational energy and even a small periodic driving force can produce large amplitude oscillations, in another words, a resonance occurs. This phenomenon can also be

observed in our quantum system, where the intrinsic frequency corresponds to the frequency of Bloch oscillations of the spinon. As shown in Fig.7, the amplitude of the oscillation of spinon is drastically enhanced when the frequency of the driving potential is comparable to ω_0, the frequency of BO corresponding to $h = h_0$. Highly nontrivial dynamics may emerge when the frequencies of the external driving force and the Bloch oscillation become incommensurate, which could induce chaotic dynamics of the spinon.

Now we discuss several possible trapped ultracold atoms systems that could realize our Hamiltonian (1) experimentally. Likely candidates are Rydberg atoms[11], which have been already used in quantum simulations and manipulation and have attracted lot of attention in recent years[12,13]. Albeit highly excited, the lifetime of Rydberg atoms can reach even hundreds of microseconds, which is much longer than the typical lifetime of the first excited state of alkali metal atoms. Two possible states of a Rydberg atom (excited state and ground state) on each site reduce the problem to that of a pseudo-spin system, and the strongly dipole-dipole interactions contribute to the interaction between the pseudo-spins ($J\sigma_i^z\sigma_j^z$), and the external magnetic fields σ_i^x and σ_i^z can be realized by a laser with single atom Rabi frequency Ω_0 and detuning Δ respectively. Notice that the dipole-dipole interaction drops off as r^{-3}, meaning that the next-nearest neighbor interaction will still be appreciable, however it is much weaker than the NN neighbor interactions. Therefore, we do not expect that this changes the physics qualitatively for a ferromagnetic interaction. An alternative way would be provided by ultracold dipolar atoms or molecules with two internal states in optical lattices in the hardcore limit, with an externally driven intra-species transition (g-field) and a Zeeman term[14]. The recent experimental progress in observing ultracold atoms at the single atom level (or even single spin level)[15–17] would be helpful to detect the dynamics of the single quasiparticles (spinons). The classic counterpart, a magnetic domain wall, has been realized experimentally as a super cold atom thermometer[18]. More recently, both our Hamiltonian and the magnetic domain wall have been experimentally realized in optical lattice[19]. Considering

the rapid development of the field, we expect the nontrivial spinon dynamics proposed in this paper should be accessible experimentally in various setups soon.

Our results could also be illuminating for condensed matter physics, especially for the domain wall motion in magnetic nanowires. Classically, the magnetization dynamics of magnetic systems is governed by LLG equation[20], where the spin is considered as a classic vector and its dynamics is reduced to a classic nonlinear equation. However, with advances in the miniaturization of magnetic structures one has to anticipate that the classical and phenomenological LLG equation will eventually be inadequate, necessitating a full microscopic quantum description of the magnetization dynamics. Our result actually paves the way to study the motion of the domain wall in the quantum situation. While dissipation would be very weak in a quantum optical implementation of our model, it would be more relevant in a solid. For the classic domain wall, the dissipation is known to play a key role in determining its velocity[21]. Under the assumption of a memory-free bath, dissipation could be modeled in the framework of a Lindblad quantum master equation or a stochastic Schr̈odinger equation[22]; the numerical method used here can be extended to simulate this model, both in an matrix product operator (MPO)[23,24] or quantum jump approach[25]. Depending on the detailed nature of the physical realization (and hence the bath), much richer physics could emerge due to the interplay between interactions and dissipation making it an interesting topic for future studies.

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