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Interplay between structure and electronic states in step arrays explored with curved surfaces

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Atomic staircases in noble metal surfaces are model one-dimensional superlattices, where free-electron-like surface states transform into superlattice bands with sizeable quantum size shifts and gaps. At critical step spacings \( d = n \times (\lambda_F / 2) \), such superlattice gaps lie at the Fermi energy, affecting the electronic energy, and hence the structural stability of the step lattice, which is held by weak elastic interactions. We use Cu, Ag, and Au curved crystals to smoothly tune the superlattice constant \( d \) in Angle Resolved Photoemission (ARPES) and Scanning Tunneling Microscopy (STM) experiments. With ARPES we accurately quantify terrace size effects and determine the superlattice potential, which increases from Ag to Cu and to Au. With STM we analyze the \( d \)-dependent terrace width distribution for Cu and Ag, and observe non-linear variations in Cu. On the grounds of simple electronic and elastic models, we conclude that terrace width distribution instabilities and electronic energy variations at \( d = n \times (\lambda_F / 2) \) have the same order of magnitude for Cu. In contrast, the weak superlattice potential in Ag, i.e., its smoother band structure modulation is not sufficient to alter the step lattice.

I. INTRODUCTION

Metallic superlattices, such as atomic step arrays, are actively investigated in the context of nanostructure growth, as suitable templates to achieve self-organized, dense nanoparticle arrays with 1-10 nm periodicities. At this length scale long range interactions, namely elastic, dipole and substrate-electron-mediated, compete to determine the equilibrium structure in templates and arrays. Understanding the hierarchy of such interactions is thus of obvious importance. To this aim, step lattices are particularly attractive since they are the simplest one-dimensional (1D) systems. The nanoscale order in a step lattice is driven by long-range, elastic repulsion between steps. Compared to atomic bonds, such elastic interactions are weak, and hence the 1D superlattice is likely to be influenced by surface electronic states. In particular in noble metals, where surface electrons scatter strongly at crystal defects, such as steps, giving rise to a significant \( 2k_F (\lambda_F / 2) \) response at the Fermi energy. The question that arises is whether surface electron scattering in step arrays is enough to trigger 1D charge-density-wave-like phenomena, i.e., structural/electronic instabilities with lattice constant (step spacing) \( d = n \times (\lambda_F / 2) \).

The existence of electronic/structural interplay associated to surface states on noble metals has been shown for adatoms and molecules adsorbed at cryogenic temperatures. In the most striking case Ce adatoms form a hexagonal lattice on Ag(111) with parameter \( d \) that matches the surface state Fermi wavelength \( \sim \lambda_F / 2 \), leading to the so-called nesting of the Fermi surface with superlattice wave vectors. However, adatom lattices are delicate systems that become unstable at room temperature, and hence have little interest for applications. In dislocation patterns and arrays of steps electronic/structural instabilities have been claimed at 300 K. In the dislocation network induced by one monolayer of Ag on Cu(111), the two-dimensional Fermi surface nesting and subsequent gapping may explain the extra compression observed in the Ag atomic lattice. In step arrays in vicinal Cu(111), the Fermi surface gap is observed with \( d = \lambda_F / 2 \), where a disorder instability is invoked, but not proved.

In order to test the existence of electronic/structural interplays, superlattices with tunable lattice constant are extremely useful. For step arrays, the tunability is readily achieved with curved surfaces. In this work we present a combined analysis of the structure and the electronic states of tunable Cu and Ag step superlattices, using curved crystals. A similar analysis was previously done for Ag, and presented elsewhere. As shown in Figs. 1 (a) and (b), our curved surfaces are defined by \( \alpha = \pm 15^\circ \) cylindrical sections around the [111] direction (\( \alpha = 0 \)). The 1D-superlattice is thus made of monatomic steps that run parallel to the [110] direction, whereas along [112] the superlattice constant \( d \) is inversely proportional to the macroscopic deviation (\( \alpha \)) from the [111] direction as:

\[
d = \frac{h}{\sin \alpha}
\]

where \( h \) is the monatomic step height. Therefore, \( d \) is accurately and smoothly varied by scanning along [112] the respective probes, i.e., the micron-size synchrotron light beam in angle resolved photoemission spectroscopy (ARPES), and the metallic tip in scanning tunneling microscopy (STM). This allows us to measure \( d \)-dependent properties directly, such as the surface band structure (Figs. 2 and 3) and the terrace confinement shift (\( \Delta E \), Fig. 4) in ARPES, as well as the terrace size distribution in STM (Figs. 5 and 6). From \( \Delta E \) we derive the superlattice potential strength (\( U_0 \) in Fig. 1 (c)), which is found to increase from Ag to Cu and Au. Such trend correlates well with the observation of very strong structural instabilities in Au (faceting), terrace size distribution variations in Cu, and apparent structural stability in Ag step lattices. Moreover, a quantitative estimation of the structural and the electronic energy variations in each case supports the existence of a delicate interplay between electronic states and lattice structure in step arrays of noble metals.
II. EXPERIMENTAL

Ag, Cu and Au single crystals (Mateck GmbH, Germany) were mechanically polished defining a $\alpha = \pm 15^\circ$ cylindrical section (11.6 mm radius) around the [111] direction ($\alpha = 0$) [Fig. 1 (a)]. Fig. 1 (b) shows a photography of the Ag crystal. As depicted in Fig. 1 (a), the two sides of the crystal correspond to A-\{100\}-oriented microfacets and B-type steps (\{111\} microfacets). All surfaces were prepared in vacuum following the standard ion sputtering plus annealing cycling used for flat crystals. The small size of the sample allows a homogeneous preparation for all miscuts, ensuring reliable $d$-dependent analysis. The low energy electron diffraction (LEED) scans display smooth and sharp $d$-dependent splitting across the sample for Cu and Ag$^{10}$. In contrast the Au crystal exhibited clear faceting in the $\sim 4^\circ$-10$^\circ$ range, as discussed elsewhere.$^9$

Locally resolved surface bands were measured with ARPES using synchrotron light from the PGM beamline of the Synchrotron Radiation Center (SRC) in Stoughton (Wisconsin). We used a hemispherical Scienta SES200 analyzer with energy and angular resolution set from 25 to 35 meV and 0.1$^\circ$, respectively, and $p$-polarized light with the polarization plane parallel to surface steps. The 100 $\mu$m diameter light spot defined an effective $\Delta \alpha \sim 0.25^\circ$, i.e., similar to the nominal polishing accuracy of crystals.$^9$ The samples were mounted with the [112] crystal direction running parallel to the analyzer entrance slit, such that the 1D step superlattice band dispersion could be directly imaged in the channelplate detector of the analyzer. For linefit analysis, channelplate images were decomposed in single energy dispersion curves (EDC) for each of the 127 channels. Peak fits were carried out using two lorentzian lines for the surface state and its umklapp band convoluted by a gaussian to account for temperature and experimental resolution. The series of fits determined peak energy, width and intensity.

The structural quality of the step superlattices as a function of $d$ is defined from the standard deviation of the lattice constant $\sigma(d)$, i.e., the terrace width distribution measured across the curved crystals with STM. Images have been systematically recorded using a variable temperature STM setup (Omicron). The description of the analytical process of the STM images, which is the same for the three crystals, was explained in detail in the previous Au work.$^9$ In summary, we perform a thorough analysis of individual frames with sizes between 400$\times$400 and 20$\times$20 nm$^2$ using the WSXM software$^{11}$. STM images are automatically processed, applying a gaussian fit to the resulting data that gives the value $\bar{d}$ in each image, as well as the standard deviation $\sigma$. The STM analysis is always limited to surface areas exhibiting homogeneous step arrays in the 1 $\mu$m scale, i.e., generally most of the surface areas with miscut angles above 2$^\circ$. For each $\sigma(d)$ point we consider an average of 5 different frames. Error bars in $d$ are estimated as $\Delta \bar{d} = \sqrt{\Delta d^2_a + \Delta d^2_b}$, where $\Delta d_a$ stands for the numerical precision determined by the fitting program, and $\Delta d_b = h(\cos \alpha/\sin^2 \alpha) \times \Delta \alpha$ accounts for the local lattice constant variability derived from the $\Delta \alpha = 0.2^\circ$ accuracy of the surface orientation. Finally, the uncertainty in $\sigma$ is defined by the fitting program. STM experiments were carried out at 300 K for Cu and Au(111), and at 150 K for Ag, the latter intended to attenuate the effect of bunching of surface steps induced by the STM tip.

III. ARPES: DETERMINING THE SUPERLATTICE POTENTIAL

Figures 2 and 3 display ARPES results for the Cu and Ag curved surfaces, respectively. We show intensity plots showing the surface bands perpendicular to the step arrays in Cu and Ag [Fig. 2 (a) and 3(a)], EDC spectra at band minima for different $\alpha$ [Fig. 2 (b) and 3(c)], Fermi surfaces [Fig. 2 (c) and 3(b)] around the nesting point in Cu ($d = \lambda_F/2 \sim 17 \AA$), and also the characteristic photon-energy dependence of the band umklapps in the case of Ag [Fig. 3 (b)]. Generally measurements with two photon energies need to be combined, i.e., a low photon energy of 21 eV, which provides high intensity and optimum energy and angular resolution for individual EDC peak analysis, and a high photon energy between 33 and 46 eV, to better visualize umklapp bands. The latter are needed to define $2\pi/d$, i.e., the actual terrace size $d$ being probed in the ARPES experiment. In Fig. 2 (a) lines are Kronig-Penney fits to data (see below), whereas in Fig. 3 (a) parabolic fits mark the dispersion near the band bottom. Due to the proximity of the Fermi level, the surface state peak in Ag appears convoluted with the Fermi edge, making the band dispersion barely visible in image plots such as those of Fig. 3 (a). In Fig. 2 (c) the umklapp rings mark the intensity maxima at $E_F$, allowing to identify Fermi surface nesting. In agreement with previous measurements$^8$, the intensity drops at the crossing point of the two Fermi rings from 7$^\circ$ to 8$^\circ$ miscuts, reflecting the presence of the Fermi gap around the critical $d = \lambda_F/2 \sim 17 \AA$ value.

The EDC spectra in Figs. 2 (b) and 3 (c) correspond to the surface band bottom at different $\alpha$ angles, reflecting the characteristic upwards shift ($\Delta E$) of the surface band minimum with respect to the (111) band ($E_0$). The positive $\Delta E$ is the clearest ARPES signature of repulsive step barriers and partial confinement within $d$-wide terraces. In Ag [Fig. 3 (c)] the surface state peak is cutoff by the Fermi level at high miscut angles, and hence the peak position is only determined after line fit analysis. Since $E_0$ is simply measured at the center of each crystal, we can obtain a very consistent and accurate determination of $\Delta E$ versus $d$. In Fig. 4 we show such terrace confinement effect $\Delta E$ plotted as a function of $d$ for the three curved crystals. $\Delta E$ is a very convenient magnitude for ARPES studies of vicinal surfaces, since, unlike gaps, it is barely affected by the inherent terrace size distribution broadening$^8$. From $\Delta E$ one straightforwardly obtains the barrier strength within the simplest approach for a 1D superlattice, i.e., a periodic succession of $\delta$-like barrier potentials of strength $U_0 b$ located at step edges. The $\delta$ barrier is equivalent to the finite potential barrier of height $U_0$ and width $b$ depicted in Fig. 1 (c), with the advantage of allowing the simple 1D Kronig-Penney (KP) analysis of the superlattice$^{12}$. In the KP model, the one-dimensional band dispersion is given
by the known expression:

\[ E(k_x) = E_0 + \frac{\hbar^2}{2m^*d^2} \cos^{-1}(|T| \cos k_d - \phi)^2 \]

where \( m^* \) is the electron effective mass of the surface state and \( |T| \) and \( \phi \) respectively are the modulus and the phase of the energy-dependent, complex transmission coefficient\(^{12} \). Making \( k_d = 0 \) and inserting the analytical expressions for \( |T| \) and \( \phi \) in Eq. 2, it immediately follows the terrace size effect \( \Delta E = E(0) - E_0 \) as:

\[ \Delta E = \frac{2\hbar^2}{m^* d^2} \left[ \tan^{-1} \left( \frac{q_0}{q} \right) \right]^2 \]

where \( q_0 = (m^*/\hbar^2) \times U_0 b \) and \( q = \sqrt{(2m^*/\hbar^2)\Delta E} \). Data points in Fig. 4 correspond to \( \Delta E \) versus \( (1/d) \) measured across the three crystals. The reference energies \( E_0 \) of the (111) planes, measured at \( \alpha = 0^\circ \), are -0.056 eV for Ag(111) (100 K), -0.408 eV for Cu(111) (180 K), and -0.464 eV for Au (150 K)\(^{15} \), agreeing with the literature\(^{8,12-14} \). The lines in Fig. 4 represent fits to the data following Eq. 3, where we assume \( m^* = 0.41 m_e \) for both Cu and Ag, \( m^* = 0.27 m_e \) for Au\(^{9} \), and use \( U_0 b \) as the single fitting parameter. For the three crystals, we find an extraordinary agreement with the KP model, proving the correctness of this approach despite its simplicity.

In Fig. 4 we note that a single value \( U_0 b \) holds for each crystal at all \( d \) regimes, except in Au, where \( U_0 b \) changes at both sides of the faceting region (\( 4^\circ < \alpha < 10^\circ \)). The small 0.6 eV-\( \bar{A} \) barrier for Ag steps was also suggested in Ag/Cu(111) experiments\(^{16} \). The \( U_0 b = 1.7 \) eV-\( \bar{A} \) value found for Cu contrasts with the large 7.0 eV-\( \bar{A} \) strength claimed in Ref.\(^9 \). The systematic analysis of Fig. 4 discards such a large value. For Au and Cu, the present experiment allows us to refine the conclusions drawn from the latest data collection of flat samples\(^{13} \). \( U_0 b \) was found to fall within a constant 1-2 eV-\( \bar{A} \) range for \( d < 40 \) \( \bar{A} \), in overall agreement with Fig. 4. However, earlier data for \( d > 40 \) \( \bar{A} \) suggested a sharp barrier increase for large terraces\(^{13} \). The inset in Fig. 4 is a zoom-out of the large-\( d \) regime. The present \( \Delta E(1/d) \) data fall away from the hard wall case \( (U_0 b = \infty) \), although error bars indicate that the infinite barrier cannot be discarded for \( d > 40-50 \) \( \bar{A} \).

The accurate determination of the barrier strength for Au, Cu and Ag steps is very important in the understanding of electron scattering at surfaces, e.g., in novel topological surface states, for which noble metal surfaces are the non-topological reference\(^{17} \). STM and ARPES measurements of the step barrier in noble metals have always appeared contradictory, revealing in fact complex underlying physics\(^{4,6,8,12,13,18} \). The analysis of standing wave patterns formed around isolated steps indicate high reflectivity and partial leakage (absorption) into bulk states\(^4,6 \). By contrast, dense arrays of steps always exhibited surface band dispersion and reduced \( d \)-size effects in ARPES\(^{12,13,19} \), as also observed in Figs. 2 and 3. This is only explainable with small barriers that allow significant electron transmission.

Scattering differences between arrays with low and high density of steps were traced to the smooth transformation of surface states of large \( (111) \) terraces into bulk resonances in densely stepped surfaces\(^{12,13,18,19} \). Although fluctuations and disorder of the step lattice in Cu\(^{14} \), or surface state depopulation in Ag\(^{20} \), both occurring at \( d = \lambda_F/2 \), have been claimed to trigger the transition from surface states to surface resonances, such a transition may naturally be expected. In reciprocal space, due to the effective vanishing of the supporting band gap out of the [111] direction\(^{12} \), and in real space, due to resonance build up by surface/bulk state overlap at step edges\(^{18} \), which significantly extends inside terraces\(^{21} \). Hence, a complex bulk/surface nature characterizes electronic states in step arrays, such that the distinct sensitivity to bulk and surface may explain the differences observed between ARPES and STM. In this respect, ARPES accurately determines the whole \( k \)-dependent spectrum\(^{12,13} \), although it is affected by crystal imperfections and size distribution broadening that are avoided in STM.

One question that arises from Fig. 4 is why \( U_0 b \) increases from Ag to Cu and to Au. The answer can shy new light on the nature of the step potential, a key issue that has been barely discussed in the past\(^{12,22} \). It is generally assumed that \( U_0 b \) is related to the local electrostatic dipole at the step edge\(^{21,22} \), although the effective barrier strength may depend on wave function properties of the surface state\(^{12,22} \). Since the latter is similar in Ag, Cu and Au one may reasonably expect \( U_0 b \) to scale with the step dipole moment. However, consistent data on dipole moments of steps for the three metals do not exist. There are only partial comparative measurements in surface/vacuum\(^{23} \) or surface/electrolyte interfaces\(^{24} \), which point to the trend found for \( U_0 b \) in Fig. 4, i.e., the step dipole in Au is found bigger than in Cu or Ag, and larger for \( A \) steps versus \( B \) steps.

Coming back to the issue of the structural/electronic interplay, electronic instabilities are expected at 1D nesting values \( d = n \times \lambda_F/2 \), for which superlattice gaps lie at \( E_F \). Gaps are clear in Au ARPES data\(^{3} \) and Cu (Fig. 2\(^{3} \)), but are hardly visible in Ag (Fig. 3), as expected from its small \( U_0 b \) barrier. In Fig. 4 we have marked the \( d = n \times \lambda_F/2 \) values determined from the 1D KP bands \( E(k_x) \) calculated using the respective \( U_0 b \) in Eq. 2. KP bands for Cu are shown superimposed to ARPES data in Fig. 2 (a), indicating overall good agreement. In Cu we obtain \( \lambda_F/2 \approx 17 \) \( \bar{A} \) and \( \lambda_F = 32 \) \( \bar{A} \), for Ag, \( \lambda_F/2 = 45 \) \( \bar{A} \), whereas in Au, \( \lambda_F/2 = 20 \) \( \bar{A} \) and \( \lambda_F = 36 \) \( \bar{A} \). Note that \( \lambda_F \) in Au lies exactly at the lower faceting onset at \( \alpha \sim 4^\circ \). Faceting is not present in Ag or Cu, which show a gradual change in lattice constant, reflected in the smooth evolution of \( \Delta E \) in Fig. 4. However, Cu in fact shows \( d \)-size distribution instabilities at \( \lambda_F/2 < d < \lambda_F \), as discussed next.

### IV. STM: ANALYSIS OF THE TERRACE WIDTH DISTRIBUTION

The curved surface approach allows a straightforward structural analysis of the step superlattice as a function of \( d \), using STM. In Au, such analysis allowed us to characterize the
faceting transition (miscut range, terrace and step structure) of the crystal, which in turn made it possible to establish a likely connection between the faceting instability and the electronic states at the $\alpha \sim 4^\circ$ onset. In Cu and Ag we have not detected faceting instabilities. Images in top panels of Fig. 5 exemplify the main STM observation, i.e., monatomic step arrays with sizeable variations in lattice quality across the curved surface, which are accounted for by means of the terrace width distribution $\sigma(d)$ analysis. At certain $d$ values (e.g. 23.5 Å) the step lattice looks sharp, whereas at others ($d=18.7$ Å) step edges appear rougher. This qualitative observation is reflected in the image analysis of the bottom panels, where we plot the statistical variation of $d$ (in entire multiples of atomic rows) across the image. The gaussian fit gives the mean terrace size $\bar{d}$, as well as the standard deviation $\sigma$ in each case.

In Figs. 6 (a) and (b) we plot the standard deviation $\sigma$ as a function of the average terrace size $\bar{d}$ in Cu and Ag, as determined from the statistical analysis of STM images extended to the whole crystals. Within the framework of the classical elastic model, the $\sigma(\bar{d})$ plot is very meaningful. In such classical theory, step interactions are assumed to be dipole-like, and hence described by an inverse square potential $U(x) = A/x^{2.25}$. $A$ refers to the step-step interaction strength, which varies as $(\sigma/d)^{-426}$. Thus, for constant $A$ there is a direct proportionality between $\sigma$ and $d$, i.e., a linear $\sigma(\bar{d})$ plot reflects an elastically stable (constant $A$) step lattice.

In Fig. 6, straight lines fit the $A$ and $B$ step data separately. In general, in both Cu and Ag, $B$ data show larger scattering than $A$ data, but the same trends. In Ag, within error bars, we observe a good linear fit, particularly in $A$ steps. In contrast, Cu data neatly deviate from the line between $\lambda_F/2$ and $\lambda_F$, defining a dip with minimum at $d=23$ Å. In fact, the step array is visually sharper in Fig. 5 (a) for $d=23.5$ Å, as compared to $d=18.7$ Å. By contrast to the Cu behavior, the data for Ag indicate, within error bars, an apparent stability, i.e., a constant $A$ value. Note that differences between Ag and Cu in $\sigma(\bar{d})$ plots correlate well with the respective superlattice potential derived from Fig. 4, i.e., weak for Ag and strong for Cu. In the following we try to discuss more quantitatively such connection.

V. DISCUSSION: ELECTRONIC AND ELASTIC ENERGY INSTABILITIES

For a quantitative evaluation of any electronic/structural interplays, one must compare the characteristic energy variations involved at critical lattice constants, i.e., electronic $\Delta E_{\text{elec}}$ versus elastic $\Delta E_{\text{elas}}$. In step arrays, these can be respectively estimated assuming the 1D KP band structure for the 1D lattice of Eq. 2, and the elastic theory.

$\Delta E_{\text{elec}}$ is related to the presence of the superlattice gaps at $E_F$ [Figs 2 (a) and (c)], which cause abrupt changes in the occupation of the surface band. To evaluate the surface state occupation, we numerically calculate the number of electronic states per surface atom $N_e$ using the following formulas for 2D systems:

$$D(\epsilon) = 2 \times \frac{1}{(2\pi)^2} \int_{E(k)=\text{const}} \frac{dS_E}{|\text{grad}_k E(k)|}$$ (4)

$$N_e = \int_{E_0}^{0} D(\epsilon) d\epsilon$$ (5)

where $\epsilon$ stands for the electron energy, $D(\epsilon)$ for the 2D density of states, and $dS_E$ is an area element of the constant energy surface perpendicular to $k$. Eqs. 4 and 5 have been numerically integrated assuming, along $k_x$, the KP band structure of Eq. 2, and along $k_y$, a free-electron-like dispersion. In Fig. 6 we plot the resulting $N_e(\bar{d})$ curve on top of the $\sigma(\bar{d})$ plots. $N_e$ increases steadily, but instabilities at successive gap crossings are clearly observed. In Cu we find $\Delta N_e \sim 0.001$ e$^-$ ripples, whereas in Ag, due to the smaller $U_0/b$ potential, $\Delta N_e$ is reduced to $\sim 0.0002$ e$^-$/atom.

The corresponding variation in electronic energy $\Delta E_{\text{elec}}$ can be estimated following the criterium applied in Ref. 8, i.e., from the difference in electronic energy between gapped KP bands and gapless free-electron bands with constant occupation $N_e$. The total electronic energy per surface atom $E_{\text{elec}}$ is derived from the known equation:

$$E_{\text{elec}} = \int_{E_0}^{0} \epsilon D(\epsilon) d\epsilon$$ (6)

The direct integration of Eq. 6 using the band structure of Eq. 2 results in the electronic energy for the KP lattice $E_{\text{elec}}^{KP}$. For the 2D free-electron gas, first we find the energy $E_0$ in Eq. 5 at which the electron density for KP bands and free electron bands are strictly the same $N_e^{KP}(E_0) = N_e^{free}(E_0)$. For such $E_0$ we integrate Eq. 6 to obtain $E_{\text{elec}}^{free}$. The electronic energy instability $\Delta E_{\text{elec}}$ is thus defined as:

$$\Delta E_{\text{elec}} = E_{\text{elec}}^{free}(E_0) - E_{\text{elec}}^{KP}(E_0).$$ (7)

Applying Eqs. 4-7 around $\lambda_F/2$ we find $\Delta E_{\text{elec}} \sim 0.3$ meV/atom in Cu in contrast to the much smaller $\Delta E_{\text{elec}} \sim 0.005$ meV/atom found in Ag.

For $\Delta E_{\text{elas}}$ we consider the step interaction contribution to the surface free energy $\gamma$. In a vicinal surface with monatomic steps, $\gamma$ can be expressed in terms of the miscut $\alpha$ and as a function of the temperature $T$:

$$\gamma(\alpha) = \gamma_0 + \beta \frac{\tan \alpha}{h} + \frac{B(T)}{a_0 |h^3| \tan \alpha^3}$$ (8)

where $\gamma_0$ represents the surface energy of (111) terraces, $\beta$ the step energy per unit length, $h$ the step height, $a_0$ the atomic distance parallel to the steps, and $B(T)$ the step interaction term. The latter includes both the entropic interaction $g(T)$, which is dominant at extremely low step densities, and the direct step-step interaction. For the specific case of step-step
interactions of dipole type $U(x) = \tilde{A}/x^2$, $B(T)$ can be written as:

$$B(T) = \frac{g(T)}{4} \left\{ 1 + \left[ 1 + \frac{2\pi^2 \tilde{A} a_{||}}{3g(T)} \right]^{1/2} \right\}^2$$  \hspace{1cm} (9)

For a moderate step density, which is the case of the $0^\circ$-$15^\circ$ miscut range analyzed in this work, $\tilde{A} > g(T)$, and $B(T)$ is reduced to:

$$B(T) \sim \frac{\pi^2 a_{||}}{6} \tilde{A}$$  \hspace{1cm} (10)

In systems with faceting instabilities, such as Au, two phases compete with their distinct terrace and step energies, namely the first and second terms in Eq. 8. In Cu and Ag we have a single phase, and hence lattice instabilities involve the third term of Eq. 8. Noting again that a constant interaction strength $\tilde{A}$ leads to a linear $\sigma/d$ plot, the dip observed in Fig. 6 for Cu may be assumed as due to $\Delta \tilde{A}$ changes, which in turn lead to elastic free energy changes $\Delta \gamma = \Delta E_{elas}$ as:

$$\Delta E_{elas} = \frac{\pi^2}{6h^2} \left| \tan \alpha \right|^3 \Delta \tilde{A} = \frac{\pi^2}{6d^3} \Delta \tilde{A}$$  \hspace{1cm} (11)

The strong $\sigma/d^{-4}$ dependence of $\tilde{A}$ makes it difficult to obtain reliable $\tilde{A}$ values from STM data. Among Cu(111) vicinals $\sigma/d$ is found close to 0.3, being $\tilde{A} \sim 6$ meV-Å. Assuming $\tilde{A}$ as 6 meV-Å for $d=17$ Å, the drop from $\sigma/d = 0.28$ to $\sigma/d = 0.14$ for $d=23$ Å leads to an increase of $\Delta \tilde{A}$ to 96 meV-Å. The latter value is typical for strongly interacting steps such as those in Pt(111). Thus, around $\lambda_F/2$ for Cu in Fig. 6, we estimate $\Delta \tilde{A} = 90$ meV-Å, which results (Eq. 11, $d=17-23$ Å) in $\Delta E_{elas} = 0.07-0.17$ meV/atom.

Therefore, based on the simple KP and elastic models, the same 0.1 meV order of magnitude is estimated in Cu step lattices for electronic instabilities, caused by opening the gap at $E_F$, and elastic instabilities, caused by changes in step interaction strength. In the case of Ag, note that data in Fig. 6 exhibit the same slopes as in Cu, and hence it is reasonable to expect similar elastic energy variations upon similar $\tilde{A}$ changes. Thus, the small $\Delta E_{elas} = 0.005$ meV/atom involved in the Fermi gap crossing in Ag would not be sufficient to trigger a visible $\Delta E_{elas}$ instability in Fig. 6.

VI. SUMMARY

In summary, using curved crystals we carried out a consistent surface state and structure analysis of Ag, Cu, and Au step lattices with smoothly varying $d$, using ARPES and STM. With ARPES we measured the superlattice band structure and the terrace confinement effect as a function of $d$. From the latter we accurately obtained the superlattice potential, which remains constant upon $d$ variation, but increases from Ag to Cu and to Au. With STM we measured the $d$-size distribution in Cu and Ag. We detect step lattice instabilities in the case of Cu within the $\lambda_F/2 < d < \lambda_F$ range. The quantitative estimation of both lattice (terrace width) and electronic (Fermi energy gap) instabilities based on classical elastic theory and Kronig-Penney band structure give the same (0.1 mev/atom) order of magnitude in Cu, suggesting the presence of electronic/structural interplays. By contrast, the weak electron potential in Ag appears insufficient to promote instabilities in the step lattice, which in fact appears structurally stable in STM.

VII. ACKNOWLEDGMENTS

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In Au, $E_0$ corresponds to the reconstructed (111) plane, and hence it may not be the appropriate reference at high miscuts. This was taken into account in a fit to the whole surface band\textsuperscript{15}, but the $U_0b$ values obtained were similar to those in Fig. 2.


FIG. 1: (a) Schematic description of the Au, Cu, and Ag curved crystal samples and (b) photography of the $6\times6$ mm$^2$ Ag crystal used in this work. $\alpha$ stands for the miscut deviation with respect to the (111) surface at the center of the crystal. (c) Side-view sketch of the 1D periodic potential in a step superlattice with lattice constant $d$ and $U_0 b$ barriers at step edges.

FIG. 2: (a) Photoemission intensity plots showing the surface band dispersion measured around $d = \lambda_F/2$ at different miscut angles in curved Cu. The solid lines are Kronig-Penney bands that indicate the presence of a Fermi gap. (b) EDC spectra for band minima at different angles, from which the $d$-size effect ($\Delta E$) is straightforwardly determined. (c) Photoemission intensity at the Fermi energy for different miscuts in Cu. The rings mark the intensity maxima and hence the Fermi surface. For 7.2° and 8.1° miscuts the intensity weakens at the ring crossing region, indicating the presence of a gap.
FIG. 3: (a) Photoemission intensity plots showing the surface state band in Ag at different miscut angles. (b) Photon-energy-dependence of the surface state band in Ag at $\alpha = 3.6^\circ$. Umklapps are better observed near the $\Gamma$ point of the bulk band structure, probed with 33-36 eV photon energy. (c) Surface state peak fits carried out at surface band minima in curved Ag at $\alpha = 0^\circ$ ([111] surface) and $\alpha = 8.5^\circ$. In the latter, the peak is cutoff by the Fermi edge, which needs to be deconvoluted by line fitting.

FIG. 4: Terrace-size confinement effect for Ag, Cu, and Au step lattices. Solid lines are fits to the respective sets of data using the Kronig-Penney model to extract the step barrier strength $U_0 b$. Subsequently, one obtains the $d = n \times \lambda_F / 2$ values indicated. The black line is calculated for $U_0 b = \infty$. The gap of data beyond $\lambda_F / 2$ in Au corresponds to its faceting range. The inset is a blow out of the large-$d$ region, where only Cu data are shown. The accuracy at this range does not allow to discard the hard wall case found in STM.
FIG. 5: Characteristic STM images (top) from Cu curved surfaces showing (a) relatively broad (18.7 Å) and (b) sharp (23.5 Å) lattices. The respective statistical image analysis (bottom) renders the average terrace size \( d \) and the standard deviation \( \sigma \).

FIG. 6: Terrace width variation \( \sigma \) as a function of the average lattice constant \( d \) for (a) Cu and (b) Ag, as determined with STM. A linear function is expected in lattices with inverse square \( A/x^2 \) step interactions, but constant \( \tilde{A} \) strength. Straight lines fit separately \( A \) and \( B \) step data in each crystal. For Cu, a dip (dotted line) is observed between \( \lambda_F/2 \) and \( \lambda_F \), suggesting a structural instability triggered by surface states. The surface electron density curves shown on top are calculated with the Kronig Penney model and the \( U_0b \) values of Fig. 4. They exhibit ripples at critical \( n \times d = \lambda_F/2 \) points.