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Persistent oscillation of Cooper pair condensate of topological defects in a nonintegrable quantum Ising chain

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We identify persistent oscillations in a nonintegrable quantum Ising chain. In the integrable chain with nearest-neighbor interactions, the nature, origin, and decay of post-transition oscillations are tied to the Kibble-Zurek mechanism. Remarkably, when coupling to the next-nearest neighbor is added, the resulting nonintegrable "zigzag" chain (still in the quantum Ising universality class) supports persistent oscillation: Topological defects (kinks) appear as a result of the quantum phase transition. However, in "zigzag" Ising chain defects can form Cooper pairs. The oscillation of the Cooper pair condensate has a frequency that depends on the binding energy gap between the paired and the unpaired defects, so it can be excited by resonant driving. While one might have expected that the integrability-breaking "zigzag" coupling causes relaxation, the oscillations we identify are persistent: Their longevity in our simulations is likely limited only by numerical accuracy. This oscillation of the Cooper pair condensate of kinks is a manifestation of quantum coherence and should be experimentally accessible.

In a recent paper [1], we have identified and characterized oscillations excited by the quench across the critical point of phase transitions in quantum Ising spin systems. These coherent many-body oscillations of the transverse magnetization are caused by quantum superpositions of broken symmetry states left behind in the wake of a nonequilibrium (nonadiabatic) quantum phase transition. Superpositions of topological defects – hence, of broken symmetry states – have been considered [2] and were recently seen in an experiment [3]. They can be understood as a flagrantly quantum consequence of the Kibble-Zurek mechanism (KZM) [4, 5] which is also responsible for the generation of topological defects in both classical [6–43] and quantum [1, 25, 44–96] phase transitions. In particular, frequencies and amplitudes of these post-critical oscillations follow the KZM scalings.

In Ref. 1, two models were studied. The simplest quantum Ising chain with only nearest-neighbor (NN) couplings is integrable and a paradigmatic example of quantum phase transitions. The Ising chain with both NN and next to NN (NNN) couplings is nonintegrable:

$$H = -\sum_{n=1}^{L} \left(g\sigma_n^x + \sigma_n^z\sigma_{n+1}^z + J_2\sigma_n^z\sigma_{n+2}^z \right). \qquad (1)$$

Here, $\sigma_n^{x,y,z}$ are the Pauli operators on site n, g is a transverse field, and J_2 is the NNN ferromagnetic coupling. It can be implemented with only NN couplings on a zigzag ladder in Fig. 1. Both models belong to the same universality class, so their behavior in the immediate vicinity of the critical point is expected to be essentially the same. Indeed, we have demonstrated [1] that both models exhibit similar oscillatory "ringdown" behavior with KZM scalings following the quench.

Dramatic differences in the behavior between the two Ising chains arise away from the transition point. In the integrable quantum Ising chain with only NN couplings, post-critical coherent oscillations gradually decay. By contrast (and to our surprise), in the nonintegrable Ising chain with both NN and NNN couplings, while the post-KZM oscillation still initially decays as in the NN chain, but before disappearing it becomes enhanced, giving rise to oscillation that does not decay on the timescales explored by our numerical simulations.

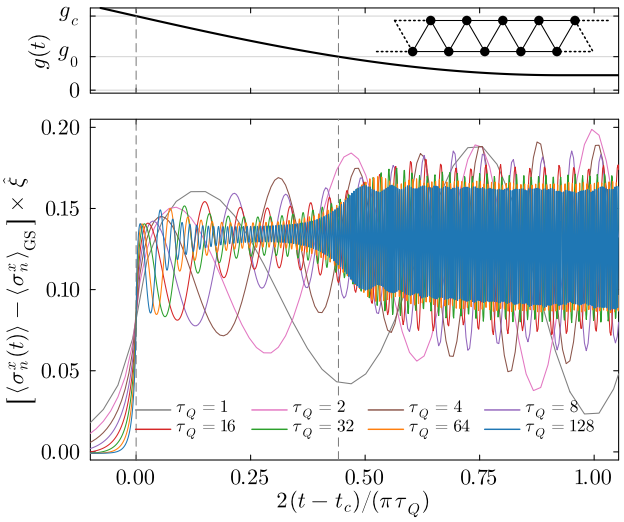


Figure 1. **Persistent coherent oscillation** in the zigzag Ising chain compared and contrasted with the decaying post-KZM oscillations after a quench into the ferromagnetic phase. Top: the KZ ramp crossing the critical point at $g_c = 2.47725$ to end in the ferromagnetic phase at non-zero $g = 0.5$. The inset shows the zigzag ladder. Bottom: Rescaled transverse magnetization $\langle\sigma_n^x\rangle - \langle\sigma_n^x\rangle_{GS}$ during the ramp as a function of time, where $\langle\sigma_n^x\rangle_{GS}$ is its value in the adiabatic ground state and $\hat{\xi} = \sqrt{\tau_Q}$ is the KZ length. Its oscillation amplitude suddenly increases in the middle of the ferromagnetic phase near $g_0 = 1.12$. See also related Fig. S5 in [97].

These reincarnated oscillations have features that are no longer subject to KZM scalings (see Fig. 1). In particular, their frequencies are no longer controlled by the size of the Ising chain gap associated with a single quasiparticle (kink).

The aim of this paper is to investigate properties and to characterize the nature of these oscillations. We trace their origin to a bosonic low energy branch of excitations (kink

"Cooper pairs") appearing for a weak transverse field. Moreover, we show how they can be efficiently excited without crossing the quantum critical point.

Integrable NN chain; $\mathbf{J_2 = 0}$. We linearly ramp the transverse field [45, 47, 53, 81, 82, 98] as

$$g(t) = g_c\,[1 - \epsilon(t)] = g_c - g_c(t - t_c)/\tau_Q, \tag{2}$$

from a strong field at $t = -\infty$, across the critical point at $g(t_c) = g_c = 1$, to $g(t_s) = 0$ when the transverse field vanishes and the ramp stops. The Jordan-Wigner transformation maps the model to a set of independent Landau-Zener systems that can be solved analytically. In particular, the final density of excited quasiparticles/kinks scales like [45, 47, 97]

$$\rho = \frac{1}{2\pi\sqrt{2\tau_Q}} \propto \hat{\xi}^{-1}. \tag{3}$$

Here, $\hat{\xi}$ is the KZ length. The KZM operates within $\pm\hat{t}$ of t_c, where $\hat{t} \propto \hat{\xi}^1$. Prior to the kink count, the final state is a quantum superposition of different numbers [53, 68] and correlated locations of kinks [79, 81, 82] that manifests in coherent oscillations of the transverse magnetization σ^x [1].

Two new results [97] make observation of coherent oscillations more feasible for the present quantum simulation platforms. The first is an analytical formula for transverse magnetization after $t_c + \hat{t}$:

$$\delta x = \langle \sigma x\rangle - \langle \sigma x\rangle_{\rm GS} \approx 2\rho + \rho 2d\frac{57\sqrt{6\pi}}{80}\cos\phi. \tag{4}$$

Here, $d = (1 + f^2)^{-3/4}$ is a dephasing factor where

$$f = \frac{3}{4\pi}\left[\gamma_E - 2\frac{t - t_c}{\tau_Q} + \ln\frac{4(t - t_c)^2}{\tau_Q}\right] \tag{5}$$

with Euler's constant γ_E, the phase of the oscillations is

$$\phi = \frac{\pi}{4} + 2\frac{(t - t_c)^2}{\tau_Q} + \frac{3}{2}\arctan f, \tag{6}$$

and $\langle\sigma^x\rangle_{\rm GS}$ is the magnetization in the adiabatic ground state. In (4), both terms are accurate to their leading order in ρ. The second result is the NN ferromagnetic correlator:

$$\begin{aligned}\delta^{zz} &= \langle\sigma^z_n\sigma^z_{n+1}\rangle - \langle\sigma^z_n\sigma^z_{n+1}\rangle_{\rm GS}\\ &\approx -2\rho - g\,\rho^2 d\frac{57\sqrt{6\pi}}{80}\cos\phi,\end{aligned} \tag{7}$$

which may be more directly accessible in some quantum simulation platforms. Together with (4), it satisfies $-\delta^{zz} - g\delta^x \approx 2(1 - g)\rho$. The excitation energy density on the left-hand side is given by the density of quasiparticles ρ times the ("dressed kink") quasiparticle gap opening with the distance from the critical point as $2(1 - g)$, as predicted by the extended quantum KZM (xQKZM) [80].

The oscillations (4) are damped by dephasing of non-interacting quasiparticles. Breaking the integrability with $J_2 = 1$, one might expect them to become even more scrambled and attenuated, but, surprisingly, they are reincarnated, as a persistent oscillation with a larger amplitude (see Fig. 1).

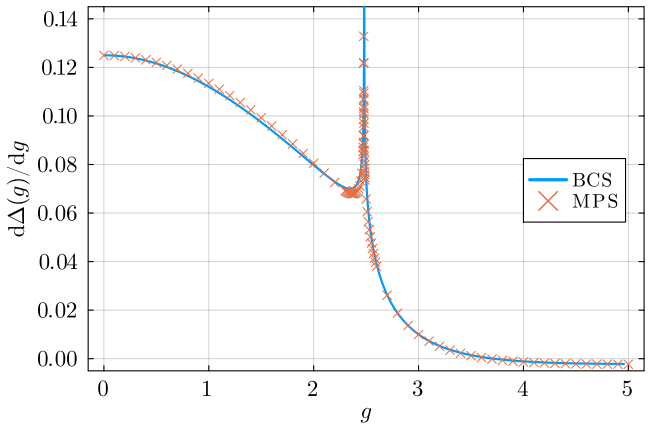


Figure 2. **BCS versus MPS.** The first derivative of the gap function Δ with respect to the field g in the ground state from the BCS theory and matrix product states (MPS). Estimates of g_c from BCS and MPS are $g_c = 2.48135$ and $g_c = 2.47725$, respectively.

Kink Cooper pair condensate in a "zigzag" Ising chain: $\mathbf{J_2 = 1}$. With a *fermionic* operator that annihilates a kink on a bond between sites n and $n + 1$ [82],

$$f_n = \left(\prod_{l\le n}\sigma^x_l\right)\frac{\sigma^z_n - \sigma^z_{n+1}}{2i}, \tag{8}$$

one can rewrite the Hamiltonian (1) with $J_2 = 1$ as

$$\begin{aligned}H = -2L + 6\sum_n f^\dagger_n f_n - 4\sum_n f^\dagger_n f_n f^\dagger_{n+1} f_{n+1}\\ -g\sum_n\left(f^\dagger_n f_{n+1} + f_{n+1}f_n + \text{h.c.}\right).\end{aligned} \tag{9}$$

The first term is the energy of the ferromagnetic state, and the second is that of individual kinks in this state. The transverse field in the second line can create/annihilate a pair of kinks on NN bonds by flipping the spin between them. A spin flip can also make a kink hop between NN bonds. The integrability is broken by the quartic attraction between NN kinks.

In the BCS theory, the ground state is assumed to be a fermionic Gaussian state $|0\rangle$. After a Fourier transform

$$f_n = e^{i\pi/4}\sum_k \frac{e^{ik(n+1/2)}}{\sqrt{L}} f_k, \tag{10}$$

and a Bogoliubov transformation,

$$f_k = u_k\gamma_k + v^*_{-k}\gamma^\dagger_{-k}, \tag{11}$$

that introduces fermionic quasiparticles annihilated(created) by $\gamma^{(\dagger)}$, the energy is minimized by a vacuum, $\gamma_k|0\rangle = 0$, where (u_k, v_k) solve the Bogoliubov-de Gennes equations:

$$\omega_k\begin{pmatrix}u_k\\ v_k\end{pmatrix} = \begin{pmatrix}H_k & D_k\\ D^*_k & -H_k\end{pmatrix}\begin{pmatrix}u_k\\ v_k\end{pmatrix}. \tag{12}$$

Here, $H_k = 2(3 - 4\rho) - 2(g - 4t_f)\cos k$, $D_k = -2(g + 4\Delta)\sin k$, and the quasiparticle dispersion $\omega_k =$

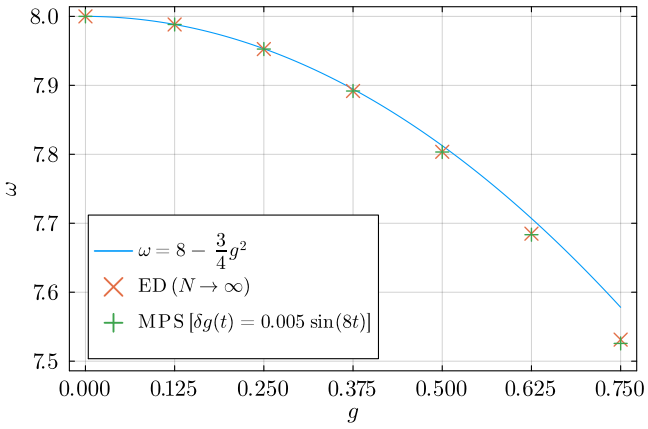


Figure 3. **Oscillation frequency.** The plot compares (16) with the gap between the ground and first excited state obtained with exact diagonalization (ED) extrapolated to infinite periodic chain size ($L \to \infty$), and transverse oscillation frequency of the Cooper pair condensate of topological defects after the periodic drive (18) with small amplitude obtained by a fit to MPS simulations. The ED and MPS error bars are on the order of the marker stroke width.

$\sqrt{H_k^2 + D_k^2}$. Here, $\rho = \langle 0| f_n^\dagger f_n |0\rangle$ is density of kinks, $\Delta = \langle 0| f_{n+1} f_n |0\rangle$ is their anomalous density, and $t_f = \langle 0| f_{n+1}^\dagger f_n |0\rangle$ is a correction to the kink hopping rate. The BCS solution compares well with the matrix product states (MPS) [99] (see Figs. S1 and S2 in [97]). The critical $g_c = 2.48135$ by BCS lies within 0.2% of $g_c = 2.47725$ obtained by MPS algorithm. The quasiparticle (dressed kink) dispersion is

$$\omega_k = \omega_\gamma - 2t_\gamma \cos k - 2t'_\gamma \cos 2k + \mathcal{O}(g^3), \qquad (13)$$

where $\omega_\gamma = 6 + g^2/8$, $t_\gamma = g$ and $t'_\gamma = 3g^2/16$, see [97].

The quartic attraction in (9) motivates the introduction of NN pair annihilation operators

$$b_n = \gamma_{n+1}\gamma_n. \qquad (14)$$

They are approximately bosonic at low pair density where an effective Hamiltonian can be obtained as [97]

$$H_b = \omega_b \sum_n b_n^\dagger b_n - t_b \sum_n \left(b_n^\dagger b_{n+1} + \text{h.c.}\right) - t'_b \sum_n \left(b_n^\dagger b_{n+2} + \text{h.c.}\right) + \mathcal{O}\left(g^3\right). \qquad (15)$$

Here, $\omega_b = 8 - g^2/4$, $t_b = g^2/8$, and $t'_b = g^2/8$. A pair has minimal energy for zero pair quasimomentum:

$$\omega = \omega_b - 2t_b - 2t'_b = 8 - \frac{3}{4}g^2, \qquad (16)$$

which agrees with the gap from exact diagonalization in Fig. 3. The Hamiltonian (15) becomes accurate when the pairs are the lowest branch of excitations, i.e. the pair gap in (16) is less than twice the minimal quasiparticle "dressed kink" gap in (13), $\omega < 2\omega_0$, below $g_0 \approx 1.12$. The terms neglected in (15), that are breaking kink pairs or changing the number of fermions, are suppressed by the pair gap and the gap between the pair branch and the fermionic quasiparticles – dressed kinks.

Within a dynamic BCS theory, the KZ ramp results in a fermionic Gaussian state where pairs of γ-quasiparticles are not bound but free to move along the chain [82]. Bound pairs would not affect the long-range order, while the actual state has finite ferromagnetic domains of typical size $\hat{\xi}$. The free quasiparticles occupy quasimomenta up to $\propto \hat{\xi}^{-1}$. Their nontrivial dispersion results in different quasimomenta, contributing a continuum of different frequencies to the oscillations. Inevitably, this forces them to go out of phase (dephase), thus suppressing their amplitude.

However, as g is slowly ramped further down below g_0, the individual quasiparticles become unstable towards the lower energy branch of pair excitations, see Fig. 4. The pair dispersion is weaker than the dispersion of quasiparticles ($\propto g^2$ in (15) versus $\propto g$ in (13), respectively) but not weak enough to explain the absence of dephasing that we observe for times much longer than $\propto g^{-2}$. The pair dispersion is made irrelevant by a Bose-enhanced transfer of quasiparticles into pairs with zero quasimomentum. Unlike fermions, the bosonic kink pairs can occupy the same quasimomentum state where they all have the same oscillation frequency of the Cooper pair condensate of topological defects. The appearance of pairs manifests in Fig. 1 by the amplification of oscillations near g_0, as can be explained with a simple example: for $|\psi\rangle = \alpha |0\rangle + \beta b_{n-1}^\dagger |0\rangle e^{-i\omega t}$ we obtain

$$\langle\psi| \sigma_n^x |\psi\rangle \approx \langle\psi| \gamma_{n-1}^\dagger\gamma_n + \gamma_n^\dagger\gamma_{n-1} - \gamma_{n-1}\gamma_n - \gamma_n^\dagger\gamma_{n-1}^\dagger |\psi\rangle = \alpha^*\beta e^{-i\omega t} + \alpha\beta^* e^{+i\omega t}. \qquad (17)$$

As the pair is a reversed n-th spin (dressed in quantum fluctuations), it manifests in oscillations of the spin-reversing σ_n^x.

Periodic drive. The KZ ramp is not the most efficient way to excite coherent oscillations of the Cooper pair condensate of topological defects. Starting from a ground state at g, the pair condensate can also be readily excited by a time-dependent perturbation to the Hamiltonian

$$\delta H = \delta g(t) \sum_n \sigma_n^x. \qquad (18)$$

Assuming a small density of pairs, where their hard-core nature can be ignored, and there is a gap between paired and unpaired quasiparticles ($g \ll g_0$), we can approximate the transverse magnetization as

$$\sum_n \left(\sigma_n^x - \langle\sigma_n^x\rangle_{GS}\right) \approx \sum_n \left(b_n + b_n^\dagger\right) = \sqrt{L}\left(\tilde{b}_0 + \tilde{b}_0^\dagger\right), \qquad (19)$$

where $\tilde{b}_k = e^{i\pi/4} b_k$, and the pair Hamiltonian (15) becomes

$$H'_b = \sum_k \omega_{b,k} \tilde{b}_k^\dagger \tilde{b}_k + \delta H. \qquad (20)$$

Here, $\omega_{b,k} = \omega_b - 2t_b \cos k - 2t'_b \cos 2k$. Since only $k = 0$ is driven by δH in (19), it reduces to a driven oscillator:

$$H'_{b,0} = \omega \tilde{b}_0^\dagger \tilde{b}_0 + \delta g(t)\sqrt{L}\left(\tilde{b}_0 + \tilde{b}_0^\dagger\right). \qquad (21)$$

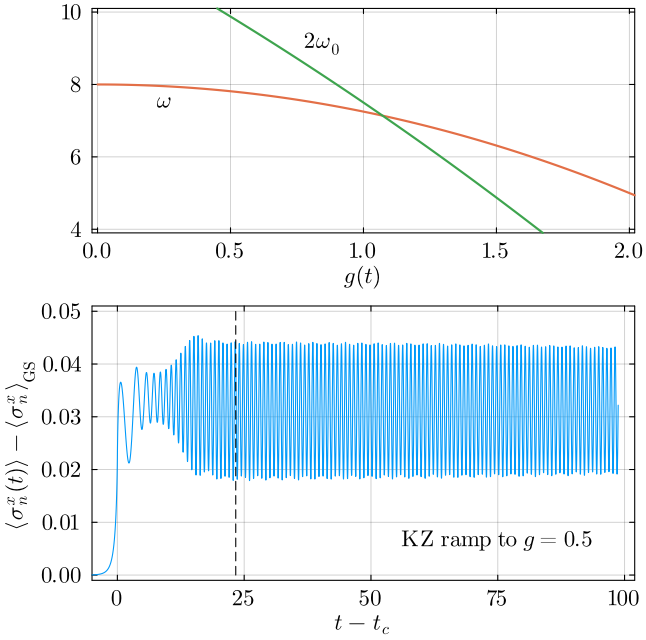


Figure 4. **Oscillations of the Cooper pair condensate of topological defects after KZ ramp.** Top: the energy gap for creating a pair of fermionic quasiparticles, $2\omega_0$, versus that for their bound pair, ω. They crossover near $g_0 = 1.12$. Bottom: oscillations after a smooth KZ ramp to $g = 0.5$ with $\tau_Q = 16$, using a smooth ramp $g(t) = g_c[2-(1-0.5/(2g_c))(1+\sin(t/\tau_Q))], t \in [-\tau_Q \times \pi/2, \tau_Q \times \pi/2]$. The dashed line indicates the end of the ramp, followed by a free evolution at $g = 0.5$. A quality factor Q (a decay time divided by a period of oscillation) during the free evolution increases from $Q{\approx}4012$ for $\tau_Q = 16$ to $Q \approx 8000$ for $\tau_Q = 32$. The estimates of Q here and below may well be significantly affected by numerical imperfections.

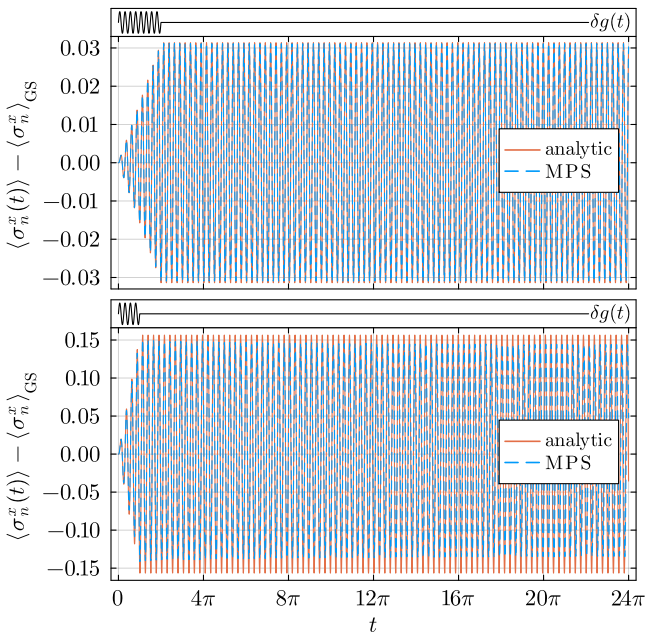


Figure 5. **Oscillations of the Cooper pair condensate of topological defects after periodic driving.** Top: coherent transverse field oscillations at $g = 0.25$ after driving $\delta g(t) = 0.005\sin(8t)$ close to resonance for time 2π. The plot compares (22) with MPS simulations. The maximum percentage difference between (22) and MPS simulations is less than 0.8%. Bottom: the same at $g = 0.5$ after driving $\delta g(t) = 0.05\sin(8t)$ for time π. The Q factors of the top and bottom plots are $Q \approx 461504$ and $Q \approx 8307$, respectively.

The driving generates a coherent state of $\tilde{b}_0$ bosons. Starting from the ground state, the transverse magnetization evolves as

$$\left(\frac{d^2}{dt^2} + \omega^2\right)(\langle\sigma_n^x\rangle - \langle\sigma_n^x\rangle_{GS}) = -2\omega\,\delta g(t). \qquad (22)$$

The pairs manifest themselves also in the correlators:

$$-\sum_n \langle\sigma_n^z\sigma_{n+1}^z\rangle \approx -\sum_n \langle\sigma_n^z\sigma_{n+2}^z\rangle \approx$$
$$-L + 4\sum_n \langle b_n^\dagger b_n\rangle + \frac{g}{2}\sqrt{L}\left\langle \tilde{b}_0 + \tilde{b}_0^\dagger\right\rangle. \qquad (23)$$

A weak driving for a finite time results in persistent oscillations with a well-defined frequency. Fig. 5 shows their examples for $g = 0.25, 0.5$. Fig. 3 demonstrates the frequency to be consistent with the gap estimate from exact diagonalization and the perturbative formula (16).

Conclusion. In the "zigzag" quantum Ising chain, individual topological defects (dressed kinks – fermionic Bogoliubov quasiparticles that appear below the critical transverse magnetic field) give rise to tightly bound Cooper pairs as the lowest branch of excitations.

The oscillation of the condensate of Cooper pairs of kinks can be excited either by the KZ ramp into the ferromagnetic broken symmetry state or by periodic driving. Despite the non-integrability of the "zigzag" Ising chain, the oscillation of the Cooper pair condensate of topological defects is persistent – once excited, either by the quench or by resonant driving, it does not appear to decay.

There are other phenomena (like quantum many-body scars [100–103] or quantum time crystals [104, 105]) where oscillation is seen or expected. What is unique in the "zigzag" Ising chain is that the oscillation is both a manifestation of Cooper pairing of topological defects and of quantum coherence of the resulting BCS-like condensate.

The data used for the figures in this article are openly available from the RODBUK repository at Ref. 106.

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