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Probing quantum chaos through singular-value correlations in sparse non-Hermitian SYK model

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Exploring the spectral properties of non-Hermitian systems presents a substantial theoretical challenge due to the presence of a complex eigenvalue spectrum. Singular values for such systems are inherently real and non-negative, and the techniques used for Hermitian systems can be used with ease. As a prototypical example of such systems, we investigate the singular-value spectrum of a non-Hermitian extension of the sparse Sachdev-Ye-Kitaev (SYK) model, a solvable toy model of quantum chaos and quantum gravity with significant interest in digital quantum simulation. Our findings reveal a congruence between the statistics of singular values and those of the analogous Hermitian Gaussian ensembles. An increase in sparsity results in the model deviating from its chaotic behavior, a phenomenon precisely captured by both short and long-range correlations in the singular-value spectrum. These findings indicate the existence of a critical sparsity threshold, beyond which the chaotic nature of the model breaks down, suggesting a potential collapse of holographic duality in such systems.

Introduction: Spectral statistics serve as a critical tool for probing the energy levels in quantum systems, offering insights into the dynamics within quantum chaos and Random Matrix Theory (RMT) [1–8]. Short-range level statistics such as the ratio of consecutive level spacings [9] and the long-range correlations such as the Spectral Form Factor (SFF) [10–15] reveal the symmetries and randomness in the system. The latter has also been understood from the semiclassical gravity [16] and measured in many-body scenarios [17, 18]. Analogously, the upsurge in research has been directed towards unraveling the quantum chaotic properties of open quantum systems [19–21] described by non-Hermitian Hamiltonians [22–26]. This connotes a departure from conventional methods suited for Hermitian systems and necessitates new methods for spectral analysis [27–33]. One such method approaches through the Singular Value Decomposition (SVD) [34], which facilitates a straightforward generalization of the frameworks used for Hermitian systems with real and non-negative singular values. Consequently, this enables us to define the singular spacing ratios [34] and singular form factor (σ FF) [35] in a much elegant manner capturing transitions from integrability to chaos, many-body localization [35, 36], and the non-Hermitian skin effect [37–39]. Outside this domain, SVD has also found applications in generalizing entanglement entropy for non-Hermitian transition matrices in AdS/CFT correspondence [40].

In this paper, we utilize the SVD framework to study the quantum chaotic properties and their transitions in the sparse, non-Hermitian version of the Sachdev-Ye-Kitaev (SYK) model [41, 42]. Standing as a quintessential archetype for quantum chaos and possessing a dual gravitational theory at low temperatures [43], the SYK

model (see See [44, 45] for comprehensive reviews) can be adapted into sparse variants [46–50], maintaining its chaotic nature up to a critical sparsity threshold [51, 52]. This adaptation holds theoretical considerations in holography as well as the practical significance for the deployment of the SYK model on quantum processors [53, 54], where managing a sparse matrix is more viable. Our focus is particularly drawn to the influence that sparsity exerts on the average singular value spacing ratio, denoted by $\langle r_\sigma \rangle$, and the σ FF, which serves as an extension of the conventional $\langle r \rangle$ -value [9] and the SFF in Hermitian systems, respectively. Additionally, we introduce *singular complexity*, inspired by spectral complexity, a boundary dual to the volume of the Einstein-Rosen bridge in gravitational theories [55]. Our findings suggest that these metrics proficiently encapsulate the transition from chaos to integrability in the non-Hermitian SYK model, as we outline in the following sections.

Non-Hermitian sparse SYK model: The prototypical model we consider is the 4-body non-Hermitian sparse SYK model (nSYK) [41, 42], given by the following Hamiltonian

$$H_{\text{nSYK}}^{\text{sparse}} = \sum_{1 \leq a < b < c < d \leq N} x_{abcd} (J_{abcd} + iM_{abcd}) \psi_a \psi_b \psi_c \psi_d. \quad (1)$$

where J_{abcd} and M_{abcd} are the independent random couplings drawn from the Gaussian ensemble with zero mean and variance $\langle J_{abcd}^2 \rangle = \langle M_{abcd}^2 \rangle = 6/N^3$ (see [56, 57] for clean SYK model). The couplings M_{abcd} explicitly break the Hermiticity of the Hamiltonian. The variables ψ_k are the Majorana fermions obeying Clifford algebra $\{\psi_a, \psi_b\} = \delta_{ab}$. Additionally, the random variables $x_{ijkl} \in [0, 1]$ introduce an element of sparsity to

the Hamiltonian. They follow a Gaussian distribution with zero mean and a variance of $\langle x_{abcd}^2 \rangle = 6/(pN^3)$ [46, 51]. The parameter p governs the probability of x_{abcd} being unity, thus determining the sparsity level within the Hamiltonian. A fully *dense* model is characterized by $x_{abcd} = 1$ or $p = 1$ for all combinations of $\{a, b, c, d\}$, aligning with the original nSYK Hamiltonian [58–61]. Conversely, a completely sparse model would have $x_{abcd} = 0$ or $p = 0$ for all combinations, representing the other extreme [46]. The addition of sparsity effectively reduces the total number of independent terms in the Hamiltonian from $O(N^4)$ to $O(pN^4) \sim O(kN)$, where k is defined in (4). Yet, the probabilistic nature of x_{abcd} implies that the exact number of non-zero terms in $H_{\text{nSYK}}^{\text{sparse}}$ is not fixed but rather determined by the probability p . We follow [52] without fixing the total number of terms in the sparse Hamiltonian in each realization.

Singular value decomposition and singular value statistics: In Hermitian systems, the eigenspectrum of the Hamiltonian is a key indicator of chaotic dynamics. The eigenvalues E_n are real, allowing for the definition of consecutive level spacings $s_n = E_{n+1} - E_n$, as an ordered list. For large Hilbert spaces, the average consecutive level spacing ratio r_n is defined as $r_n = \min(s_n, s_{n+1})/\max(s_n, s_{n+1})$ with $\langle r \rangle = \text{mean}(r_n)$ being the *average r value* [9]. Its usefulness lies in simplifying the numerical analysis by bypassing the need to unfold the spectrum for level statistics. While $\langle r \rangle$ -value is well-defined for Hermitian systems, it falls short in applying for non-Hermitian systems where eigenvalues are complex, and thus an ordered sequence of level spacings is not feasible. In such scenarios, one may consider complex spacing ratios [27] and correspondingly the statistics of complex eigenvalues. However, an alternate proposal suggests focusing on singular values derived from the singular value decomposition (SVD) of the Hamiltonian [34]. SVD is expressed as $H = U\Sigma V^\dagger$, where U and V are unitary matrices, and Σ is a diagonal matrix containing the real and non-negative singular values σ_i [62]. In Hermitian matrices, singular values are simply the absolute values of the eigenvalues, whereas, in non-Hermitian matrices, their relationship is more complex [63].

The singular values of a non-Hermitian matrix H can be computed by the *Hermitization* process [64], which involves constructing a new matrix $\mathbf{H}$, twice the dimension of H . This larger matrix incorporates both H and its Hermitian conjugate $H^\dagger$ in a block structure [64]

$$\mathbf{H} = \begin{pmatrix} \mathbf{0} & H \\ H^\dagger & \mathbf{0} \end{pmatrix} \Rightarrow \mathbf{H}^2 = \begin{pmatrix} HH^\dagger & \mathbf{0} \\ \mathbf{0} & H^\dagger H \end{pmatrix}. \quad (2)$$

The eigenvalues of $\mathbf{H}$ are $\{\pm\sigma_n\}$, precisely containing the singular values of H . This is equivalent to constructing an effective Hamiltonian $H_{\text{eff}} = \sqrt{H^\dagger H}$ or $H_{\text{eff}} = \sqrt{HH^\dagger}$ so that the eigenvalues of H_{eff} represent the singular values of H [64]. Such construction of a Hermitized matrix resembles the formulation of the Wishart SYK model and

System	$N = 20$	$N = 22$	$N = 24$	$N = 26$	$N = 28$	$N = 30$
$\langle r \rangle_{\text{RMT}}$	0.6744	0.5996	0.5307	0.5996	0.6744	0.5996
$\langle r_\sigma \rangle_{\text{nSYK}}$	0.6744	0.5997	0.5307	0.5996	0.6745	0.5995

TABLE I. The Table shows the $\langle r_\sigma \rangle$ -values for the non-Hermitian *dense* SYK model for different system sizes $N = 20$ (10000), $N = 22$ (6000), $N = 24$ (2000), $N = 26$ (1000), $N = 28$ (100), and $N = 30$ (50), where the parenthesis includes the number of samples taken. Based on the $\langle r_\sigma \rangle$ values, we identify the periodicity as $N \bmod 8 = 0$ (GOE), 2, 6 (GUE), and 4 (GSE) [67]. These GOE, GUE, and GSE correspond to the eigenvalue statistics of the Hermitian matrices.

The values are in precise agreement (within numerical accuracy) with the $\langle r \rangle_{\text{RMT}}$ -values for large N results in [9].

its supersymmetric version, comprised of non-Hermitian conserved charges or supercharges [65, 66].

Given the singular values $\{\sigma_i\}$ of the corresponding non-Hermitian Hamiltonian H , one analogously defines $\lambda_n = \sigma_{n+1} - \sigma_n$ as the *singular value spacings*. Consequently, the *singular-value-spacing ratio* $r_{\sigma,n}$ is defined analogous to the r -ratio as [34]

$$r_{\sigma,n} = \frac{\min(\lambda_n, \lambda_{n+1})}{\max(\lambda_n, \lambda_{n+1})}, \quad \langle r_\sigma \rangle = \text{mean}(r_{\sigma,n}). \quad (3)$$

Referred to as the $\langle r_\sigma \rangle$ -value, they adhere to statistical distributions akin to those observed for non-Hermitian Hamiltonians, similar to the $\langle r \rangle$ -value distributions for Hermitian matrices. This framework has been instrumental in addressing the 38-fold symmetry classification in non-Hermitian systems [34].

Table I showcases the $\langle r_\sigma \rangle$ -value for the *dense* non-Hermitian SYK model for varying system sizes N , representing the total number of fermions. The data is juxtaposed with corresponding eigenvalue statistics of Hermitian Gaussian ensembles, revealing a striking correlation emphasizing the utility of the $\langle r_\sigma \rangle$ -value in non-Hermitian systems. Similar to its Hermitian counterpart, we observe an $N \bmod 8$ periodicity using singular value statistics: $N \bmod 8 = 0$ corresponds to GOE, $N \bmod 8 = 2$ and 6 corresponds to GUE, and $N \bmod 8 = 4$ corresponds to GSE for the eigenvalue statistics of the Hermitian matrices. Figure 1 correspondingly shows the consecutive *singular level-spacing distribution* $p_\sigma(\lambda)$ for $N = 24$ (GOE), $N = 26$ (GUE) and $N = 28$ (GSE) respectively, which are *approximated* by the analogous Hermitian Wigner-Dyson distribution [1]. Note that we are comparing the singular value statistics of non-Hermitian matrices to the eigenvalue statistics of Hermitian matrices. The singular value correlations in Hermitian matrices are generally weaker than the eigenvalue correlations and exhibit different spacing ratios [34]. However, the nSYK falls into the different non-Hermitian symmetry classes, which can also be used to show the validity of the singular value statistics [59].

SVD in sparse non-Hermitian SYK model: We now turn our attention to the sparse variant of the model,

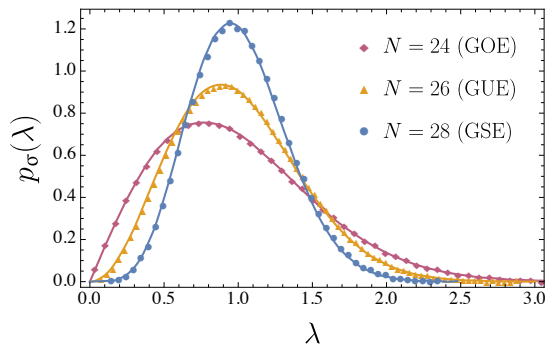


FIG. 1. The consecutive singular level-spacing distribution in the non-Hermitian dense SYK model for $N = 24$ (2000) (GOE), 26 (1000) (GUE) and 28 (100) (GSE) with the corresponding realizations shown in the brackets. The solid lines represent distribution for GOE, GUE, and GSE respectively taking matrices of size $10^4 \times 10^4$ with averaging over 1000 realizations.

which is governed by the sparsity parameter p . Figure 2 illustrates the dependence of p on $\langle r_\sigma \rangle$ -value for different system sizes. Each system size, representing a distinct ensemble, is depicted in a unique color. The dotted lines mark the expected values for Poisson statistics and the Hermitian ensemble, as specified in Table I. It is observed that the Hamiltonian (1) retains its chaotic nature up to a critical level of sparsity denoted by p_{crit} . This chaotic behavior is also corroborated by the $\langle r_\sigma \rangle$ -ratio presented in Table I. Consequently, we define a transition point p_{crit} beyond which the Hamiltonian (1) ceases to exhibit chaotic dynamics. Such transition point is obtained by considering approximately 99% of the corresponding $\langle r_\sigma \rangle$ value at $p = 1$ [52]. Notably, as the system size increases, this transition point migrates towards lower sparsity levels. When the sparsity is sufficiently low, the $\langle r_\sigma \rangle$ -value aligns with the Poisson limit, signifying the Hamiltonian's transition to complete integrability. Note that when a term of the Hamiltonian very weakly breaks a degeneracy, the resulting spectrum has r_σ values close to zero, which can cause the $\langle r_\sigma \rangle$ value to fall below the Poisson limit.

In conjunction with the above, the relationship between the critical sparsity p_{crit} and the system size N is also noteworthy. The critical value diminishes as the system size increases. This reduction follows a power-law trend, best described by fitting the data to the curve [52]

$$p_{\text{crit}} = kN \binom{N}{4}^{-1} \approx \frac{24k}{N^3}, \quad (4)$$

in the large N limit. where the decrease is proportional to $1/N^3$ and the constant k is approximately 1.68, as determined by the fitting process. This scaling behavior is consistent with that observed for the sparse Hermitian SYK model, as reported in [51, 52].

Spectral and Singular form factor: Contrary to the

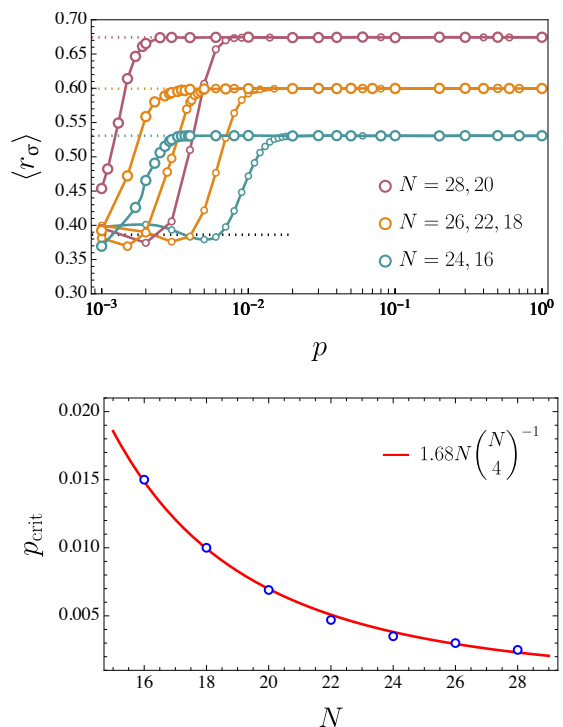


FIG. 2. (Top) Variation of $\langle r_\sigma \rangle$ -value with various level of sparsity p . The size of the circle varies with N i.e., the largest N corresponds to the leftmost plot for each color. At low sparsity, the $\langle r_\sigma \rangle$ matches with the corresponding random matrix ensemble until a threshold p_{crit} is reached, after which $\langle r_\sigma \rangle$ drops to the Poissonian value, indicating a hint of integrability. The estimated errors for the values obtained are smaller than the markers in the plot. (Bottom) The finite-size dependence of the threshold or the critical value p_{crit} . It decreases as the system size increases. The decrease scales as $1/N^3$ with large system size, as given in (4).

singular value statistics, the SFF probes the long-range eigenvalue correlations. It is defined as the Fourier transform of two-point eigenvalue correlation pairs [12–14]. For systems exhibiting quantum chaos, the SFF displays a characteristic dip-ramp-plateau pattern. The initial dip, evident in the early-time regime, is a non-universal trait influenced by the detailed nature of the energy spectrum [10]. This dip extends until a timescale known as the *Thouless time* (t_{Th}) [68, 69] or *ramp time* [15] beyond which the energy difference surpasses the inverse of t_{Th} , referred to as the *Thouless energy* [70]. Subsequently, a ramp emerges at t_{Th} , indicative of eigenvalue repulsion, a phenomenon that mirrors the SFF behavior predicted by RMT [14, 15] and underscores the chaotic dynamics of the Hamiltonian. Moreover, this ramp hints at the presence of a holographic dual to the corresponding theory. Beyond the *Heisenberg time* [71, 72], which scales inversely to the mean level spacing, the SFF reaches a plateau that depends on the system size. In contrast, integrable systems typically bypass the ramp stage and

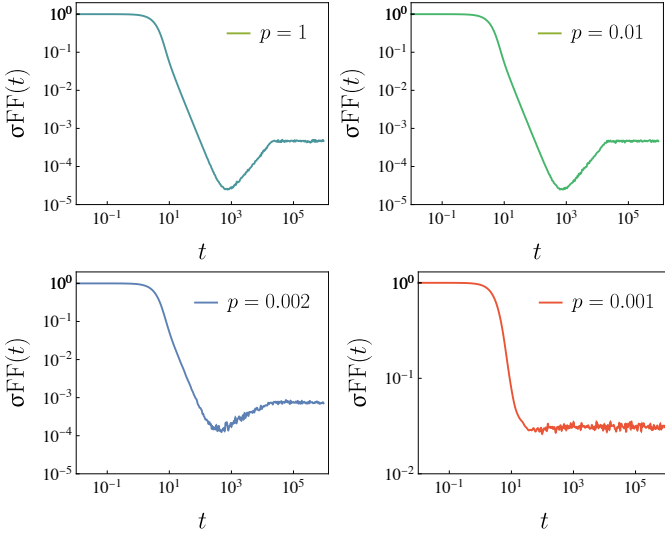


FIG. 3. Behavior of $\sigma\text{FF}(t)$ for various sparsity parameter p . The linear ramp ceases to exist as the sparsity increases. The system parameters are $N = 26$ (GUE) with 1000 Hamiltonian realizations. Here we choose the Gaussian filter with $\alpha = 3.27$, which lies within the prescribed range in [52].

saturate directly from the dip to the plateau regime. SFF in sparse SYK models has been thoroughly investigated in recent literature [47, 48, 51, 52]. Several other generalizations for SFF in non-Hermitian and open systems have also been proposed [31–33].

Analogous to the SFF, the singular form factor (σFF) is defined as [35]

$$\sigma\text{FF}(t) = \frac{1}{L^2} \langle |Z_\sigma(it)|^2 \rangle = \frac{1}{L^2} \left\langle \left| \sum_n e^{-i\sigma_n t} \right|^2 \right\rangle, \quad (5)$$

where $Z_\sigma(it) = \sum_n e^{-i\sigma_n t}$ is the infinite-temperature partition function of singular values, which can be further expressed as the overlap between the time-evolved right or left singular vectors governed by the effective Hamiltonian $\sqrt{H^\dagger H}$ or $\sqrt{H H^\dagger}$ respectively [35]. Here the angle brackets indicate an ensemble average, which is particularly relevant in the context of a disordered system since σFF is not self-averaging [72].

Figure 3 shows the distinct dip-ramp-plateau pattern exhibited by σFF for the Hamiltonian (1), across varying degrees of sparsity. In this work, we consider a system size of $N = 26$ and analyze the outcomes from 1000 independent realizations of the Hamiltonian to ensure robust statistical analysis. To reduce the oscillations in the dip region, we use the filter function $Y_\sigma(\alpha, t)$ [68] such that

$$\sigma\text{FF}(t) = \left\langle \frac{|Y_\sigma(\alpha, t)|^2}{|Y_\sigma(\alpha, 0)|^2} \right\rangle, \quad (6)$$

Here the Gaussian filter function for the singular values is defined as $|Y_\sigma(\alpha, t)|^2 = |\sum_n e^{-\alpha \sigma_n^2} e^{-i\sigma_n t}|^2$. Similar

to the eigenspectrum [68, 69], it ensures that the dominant contribution is received from the bulk of the singular value spectrum. Here α is a N -dependent parameter that controls the Gaussianity of the spectrum. This filtering is performed for the usual spectrum, without unfolding it. For the latter case, an equivalent of *connected unfolded* SFF [69, 73] can be defined. We employ the filter function on the singular values by choosing $\alpha = 3.27$, which lies inside the range given in [52] for $N = 26$. Filter functions for SFF have also recently been shown to be associated with quantum channels [74].

A notable observation in Fig. 3 is a parallelism between the behavior of σFF in non-Hermitian systems and the SFF in Hermitian systems. This similarity underscores the efficacy of σFF as a robust indicator for quantum chaos in non-Hermitian frameworks. In regimes of lower sparsity, the presence of the ramp is evident, signifying the retention of chaotic dynamics within this parameter space. It remains pronounced until a critical sparsity threshold is reached. Beyond this juncture, the linear ramp—a signature trait of non-integrable systems—ceases to manifest, indicating a shift towards integrability and a departure from chaotic behavior.

Figure 4 shows the variation of Thouless time (t_{Th}), for $N = 26$, with sparseness parameter p . The t_{Th} is calculated by considering the time at which the σFF of the system considered first intersects the SFF of the GUE random matrices [15]. To determine the intersection point precisely, we consider the fractional error function $\epsilon(t) := \left| \frac{\sigma\text{FF}(t) - \sigma_{\text{ramp}}(t)}{\sigma_{\text{ramp}}(t)} \right|$ [68] indicating the deviation of the computed σFF to the linear-fitted function for the ramp $\sigma_{\text{ramp}}(t)$ for $p = 1$. We consider the value t as the Thouless time or ramp time t_{Th} , where the fractional error $\epsilon(t)$ reaches 20%. The red line shows the fitting $t_{\text{Th}} \approx a/p^b + c$ with $a = 1.923$, $b = 0.8767$ and $c = 0.007$. Curiously, this fitting follows closely to the $1/p$ scaling in contrast to the $1/p^2$ scaling in Hermitian systems [52]. Such distinction is possible since the singular values and the eigenvalues are inherently different (see the Supplemental Material [75] S1). Yet, we do not rule out the possibility of the scaling of the exponent b with N .

Spectral complexity for singular values - Singular complexity: Within the framework of the AdS/CFT correspondence [76, 77], a particular spectral measure has been proposed to capture the essence of long-range spectral correlations. This measure, known as *spectral complexity*, is proposed to be a dual quantity of the Einstein-Rosen bridge [55] in the gravitational theory, serving as an analog for the computational and circuit complexity [78]. In a similar spirit, we define *singular complexity* at infinite temperature as

$$C_\sigma(t) = \frac{1}{L^2} \sum_{\sigma_i \neq \sigma_j} \left[\frac{\sin(t(\sigma_i - \sigma_j)/2)}{(\sigma_i - \sigma_j)/2} \right]^2, \quad (7)$$

designed to be suitable for non-Hermitian systems. At

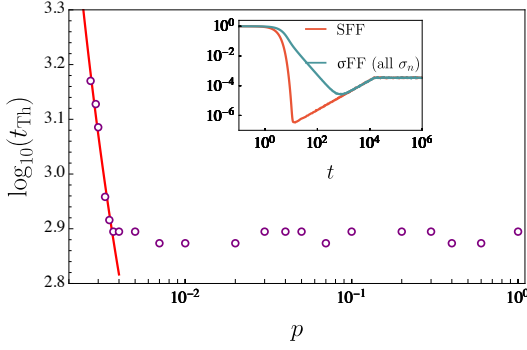


FIG. 4. (Main plot) Thouless time (ramp time) with respect to sparsity parameter. The red line is the fit $t_{\text{Th}} \approx a/p^b + c$ with $a = 1.923$, $b = 0.8767$ and $c = 0.007$. (Inset) The behavior of SFF and the σFF for the Hermitian dense SYK₄ model. The system parameters are $N = 26$ with 1000 random Hamiltonian realizations taken. The σFF (green line) is computed considering all the positive singular values. It reduces to the SFF after taking the appropriate rescaling of the singular vectors and singular values, which otherwise equals the eigenvalues (with appropriate signs).

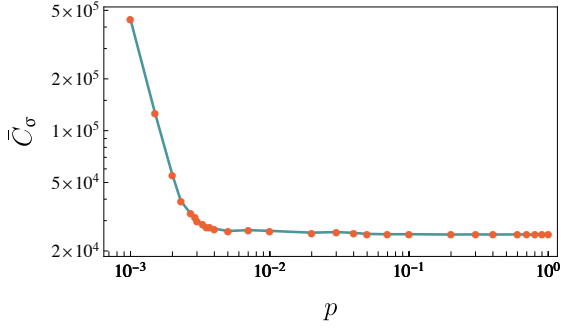


FIG. 5. Behaviour of the saturation of the singular complexity with different sparseness for $N = 26$ with 1000 random Hamiltonian realizations.

early times, it grows quadratically, followed by a linear growth and plateau regime (see Supplemental Material [75] S2 for more details). The early time behavior mirrors the behavior of spread complexity [79, 80] based on the Krylov space approach [81]. The plateau value can be obtained analytically by taking the long-time average

$$\bar{C}_\sigma = \lim_{t_f \rightarrow \infty} \frac{1}{t_f} \int_0^{t_f} C_\sigma(t) dt = \frac{2}{L^2} \sum_{\sigma_i \neq \sigma_j} \frac{1}{(\sigma_i - \sigma_j)^2}, \quad (8)$$

which only depends on the difference between the singular values. In the Hermitian case, such saturation values have been studied for quantum billiards [82] and mixed-field Ising model [83], capturing the *spectral rigidity*. Integrable systems saturate at higher values compared to chaotic ones due to the presence of level repulsion, as evident from the equivalent eigenspectrum of (8).

Figure 5 depicts the saturation of singular complexity at varying levels of sparsity, which maintains a magnitude

order of $O(10^4)$ until it encounters a critical threshold. Beyond this threshold, there is a marked escalation at lower sparsity levels signaling a level-repulsion between the singular values. This critical value is dependent on the system size N , and for $N = 26$, it occurs within the sparsity regime $0.003 \lesssim p_{\text{crit}} \lesssim 0.004$. This value resembles the expected $\langle r_\sigma \rangle$ -value shown in Fig. 2.

Exhibiting chaotic dynamics is considered to be a crucial constraint that possesses a holographic dual geometry. While this discussion does not explore the specifics of such geometry, some proposals exist in the literature [58, 84–86]. The breakdown of chaotic nature suggests that such dual geometry corresponding to a non-Hermitian system may be compromised at low sparsity.

Conclusion and Outlook: Drawing on the foundational work [34] on the applicability of SVD in non-Hermitian systems, our study addresses the quantum chaotic attributes of the sparse SYK model across varying levels of sparsity. The complex eigenvalues inherent to non-Hermitian Hamiltonians pose challenges to the generalization of traditional eigenvalue statistics. This complexity persists despite strides made with complex spacing ratios [27] and the DSFF [28, 29]. Nonetheless, recent advancements propose a novel perspective by focusing on singular values as opposed to eigenvalues within the non-Hermitian framework. Unlike their complex counterparts, singular values are inherently real and non-negative, aligning with the real eigenvalues in Hermitian systems upon appropriate alignment with singular vectors, as detailed in the Supplemental Material [75] S1.

Our investigation underscores that the statistical analysis of singular values, which elucidates short-range correlations, coupled with the σFF and singular complexity, illuminating long-range correlations among singular values, serve as potent indicators of the chaos within non-Hermitian systems. As such, exploring the holographic bulk dual of singular complexity may offer insights into a potential holographic counterpart for non-Hermitian systems, especially the effective range of k in (4) in relation to $2d$ de Sitter (dS₂) gravity [84], which we leave for future work. Moreover, these tools adeptly reflect the shift towards integrability as sparsity intensifies. Such transition differs from other chaotic-integrable transitions by two-point interactions observed in the SYK model [87]. We envision that these features will be exhibited in other correlation measures such as higher spacing ratios [88–90] and the number variance [31, 68, 91, 92].

Parallel research avenues have explored the efficacy of sparse Hamiltonians [46, 47] within quantum processors [53, 54], maintaining chaotic dynamics to a substantial degree of sparsity. Notably, the SYK model retains its holographic duality even at minimal sparsity levels [51, 52]. Our study contributes to the burgeoning field of non-Hermitian systems in the realm of open-system dynamics, proposing a framework for the extrapolation of quantum chaotic properties within these systems. This might serve

as the benchmark results for the simulation of sparse non-Hermitian systems on quantum processors.

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