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Phys. Rev. B **110**, L161103 — Published 7 October 2024

DOI: [10.1103/PhysRevB.110.L161103](https://doi.org/10.1103/PhysRevB.110.L161103)

Chern number landscape of spin-orbit coupled chiral superconductors

Matthew Bunney,^{1,2,*} Jacob Beyer,^{1,2,3,4,*} Ronny Thomale,⁴ Carsten Honerkamp,^{2,3} and Stephan Rachel¹

¹*School of Physics, University of Melbourne, Parkville, VIC 3010, Australia*

²*Institute for Theoretical Solid State Physics, RWTH Aachen University, 52062 Aachen, Germany*

³*JARA Fundamentals of Future Information Technology, 52062 Aachen, Germany*

⁴*Institute for Theoretical Physics, University of Würzburg, Am Hubland, 97074 Würzburg, Germany*

Chiral superconductors are one of the predominant quantum electronic states of matter where topology, symmetry, and Fermiology intertwine. This is pushed to a new limit by further invoking the coupling between spin and charge degrees of freedom, which fundamentally affects the principal nature of the Cooper pair wave function. We investigate the onset of superconductivity in the Rashba-Hubbard model on the triangular lattice, which is symmetry-classified by the associated irreducible representations (irrep) of the hexagonal point group. From an instability analysis by means of the truncated-unity functional renormalization group (TUFRG) we find the E_2 irrep to dominate a large fraction of phase space and to lead up to an energetically preferred gapped, chiral superconducting state. The topological phase space classification associated with the superconducting order parameter obtained from TUFRG reveals a fragmentation of the E_2 domain into different topological sectors with vastly differing Chern numbers. It hints at a potentially applicable high sensitivity and tunability of chiral superconductors with respect to topological edge modes and phase transitions.

Introduction.—Topological superconductors (TSCs) are amongst the most desired states of topological matter^{1,2}, as their ability to host Majorana zero modes is believed to be key for future implementation of topological quantum computing platforms. This immense technological potential is hindered by the lack of fundamental understanding of TSCs: (i) Theoreticians are still unable to provide recipes or guidance for crystal synthesis or to predict materials hosting TSC phases. (ii) Experimental discoveries of TSC candidate materials often come as a surprise; and often enough after some initial excitement, subsequent studies suggest that trivial superconducting states might be a more likely explanation. To remedy the former, it is essential that the many-body techniques which predict unconventional superconducting ground states are also capable of shedding light on the topological features of these many-body instabilities.

We are particularly interested in topological superconductivity in the two-dimensional triangular lattice as the simplest case with hexagonal symmetry. Here we can also draw on extensive and noteworthy literature on unconventional superconductivity in the triangular lattice Hubbard model, motivated by the doped Mott insulator κ -(BEDT-TTF)₂Cu₂(CN)₃³ as well as water-intercalated sodium cobaltates Na_xCoO₂·yH₂O^{4–7}. Another important class of materials are the $\sqrt{3} \times \sqrt{3}$ reconstructed adatom lattices on semiconductor substrates such as Sn/Si(111), Pb/Si(111) and Sn/SiC(0001). These are believed to be good realizations of one-orbital Hubbard models, and their observed ground states range from spin and charge density waves to chiral superconductivity^{8–18}. Most recently, the advent of twisted two-dimensional materials led to a renewed interest in unconventional superconductivity on the triangular lattice^{19–22}.

Inversion-symmetry breaking is ubiquitous in materials, e.g. due to crystalline absence of an inversion center, as in so-called non-centrosymmetric materials, or

heavy atom superlattices, heterostructures and surface systems. Nevertheless, the effect of inversion symmetry breaking, usually manifested as Rashba spin-orbit coupling (RSOC)^{23–25} on the correlated and superconducting phase diagrams of such materials and corresponding models remains relatively unexplored.

There have been a few studies on the paradigmatic square lattice case^{15,26,27}. As we will see later in the group-theoretical discussion, the possible mixed states in the triangular lattice case are significantly more interesting. Notable exceptions where spin-orbit effects were considered in interacting triangular lattice systems are works on $\sqrt{3} \times \sqrt{3}$ -Pb/Si(111)^{9,13,16,28}, on $\sqrt{3} \times \sqrt{3}$ -Sn/SiC(0001)⁸ as well as on twisted bilayer systems WSe₂²⁹ and PtSe₂³⁰.

In this Letter, we close this gap in the literature by presenting a paradigmatic case study for the triangular-lattice Rashba-Hubbard model (TLRHM). Despite being the most basic model, it turns out to feature a surprisingly rich phase diagram.

We employ the truncated unitary extension of the functional renormalization group (TUFRG). TUFRG allows the investigation of competing many-body instabilities in a way that is unbiased to any one particular instability, treating particle-particle and particle-hole instabilities on equal footing. (For more details, see Supplementary Material³¹, and Refs.^{32–37} contained therein.) This method is particularly suited to study the effect of RSOC on the interacting phase diagram, as the truncated unity approach retains the details of the involved lattice harmonics of the superconducting pairing. That allows us to easily extract the degree of singlet-triplet mixing of superconducting states (expected when inversion symmetry is broken²³). By feeding the TUFRG output into a Bogoliubov-de Gennes (BdG) formalism, we derive the topological properties of the unconventional superconducting phases.

Model.—In the following, we investigate the TLRHM defined as

$$H = - \sum_{\langle ij \rangle, ss'} \left[t \delta_{ss'} + \alpha (\boldsymbol{\sigma} \times \boldsymbol{\rho}_{ij})_{ss'}^z \right] c_{is}^\dagger c_{js'} + U \sum_i n_{i\uparrow} n_{i\downarrow}. \quad (1)$$

The sum $\langle ij \rangle$ runs over nearest neighbor sites, s, s' are spin indices, t is the hopping amplitude and α quantifies RSOC. The bond vector is given by $\boldsymbol{\rho}_{ij} = \mathbf{r}_i - \mathbf{r}_j$, $\boldsymbol{\sigma}$ is the vector of Pauli matrices, and $n_{is} = c_{is}^\dagger c_{is}$ is the number operator.

The band structure of (1) is readily derived from the Bloch matrix

$$h_0^{ss'}(\mathbf{k}) = \begin{pmatrix} \varepsilon_0(\mathbf{k}) & \alpha \left[\frac{\partial \varepsilon_0}{\partial k_y} + i \frac{\partial \varepsilon_0}{\partial k_x} \right] \\ \alpha \left[\frac{\partial \varepsilon_0}{\partial k_y} - i \frac{\partial \varepsilon_0}{\partial k_x} \right] & \varepsilon_0(\mathbf{k}) \end{pmatrix}_{ss'}$$

with $\varepsilon_0(\mathbf{k}) = -2t[\cos(\mathbf{a}_1 \mathbf{k}) + \cos(\mathbf{a}_2 \mathbf{k}) + \cos((\mathbf{a}_2 - \mathbf{a}_1) \mathbf{k})]$; $\mathbf{a}_1$ and $\mathbf{a}_2$ are the primitive lattice vectors.

We analyse the leading many-body instabilities of the Hubbard model through the use of TUFGRG^{32,33}. FRG interpolates between the bare Hubbard interaction and a low-energy effective two-particle interaction vertex, by means of iteratively integrating a flow equation. In the TU formalism, the interaction vertices are expressed in terms of the lattice harmonics^{34–36}, which gives the advantage of explicit formulations of leading instabilities. We can then track changes in these tight-binding parameters with changes in normal state parameters across our phase diagrams.

Superconducting instabilities on the triangular lattice can generically be written as³⁸

$$\hat{\Delta}(\mathbf{k}) = [\Psi(\mathbf{k}) \hat{1} + \mathbf{d}(\mathbf{k}) \cdot \hat{\boldsymbol{\sigma}}] i \hat{\sigma}_y, \quad (2)$$

where the Pauli matrices ($\hat{1}, \hat{\sigma}_i$) act on the spin subspace.

In the absence of RSOC, spin rotational symmetry admits spin-singlet Ψ and triplet $\mathbf{d}$ states, and all possible superconducting instabilities are then characterized according to the irreps of the point group D_{6h} .

For finite RSOC, the point group symmetry is reduced to C_{6v} due to broken inversion symmetry. The spins are also “frozen” to the lattice and must rotate with it, *i.e.*, the system is now spin-orbit coupled. Thus the superconducting states can no longer be characterized by the spatial irreps but only by *total irreps*, *i.e.*, space and spin combined. A complete discussion of group theory is presented in the Supplementary Material³¹ (and Ref.³⁹ contained therein), including derivation and the complete form of the basis functions; here we wish to emphasize that amongst the total irreps there are three different possibilities for the two-dimensional irrep E_2 , which are permitted to mix among possible tight-binding superconducting instabilities. These three states are: the standard d -wave spin-singlet $\{\Gamma_{d_{x^2-y^2}}, \Gamma_{d_{xy}}\}$; and two spin-triplet states $\{(\Gamma_f, 0, 0), (0, \Gamma_f, 0)\}$ and $\{(\Gamma_{p_x}, -\Gamma_{p_y}, 0), (\Gamma_{p_y}, \Gamma_{p_x}, 0)\}$, where the 3-vector corresponds to $\mathbf{d} = (d_x, d_y, d_z)$. We

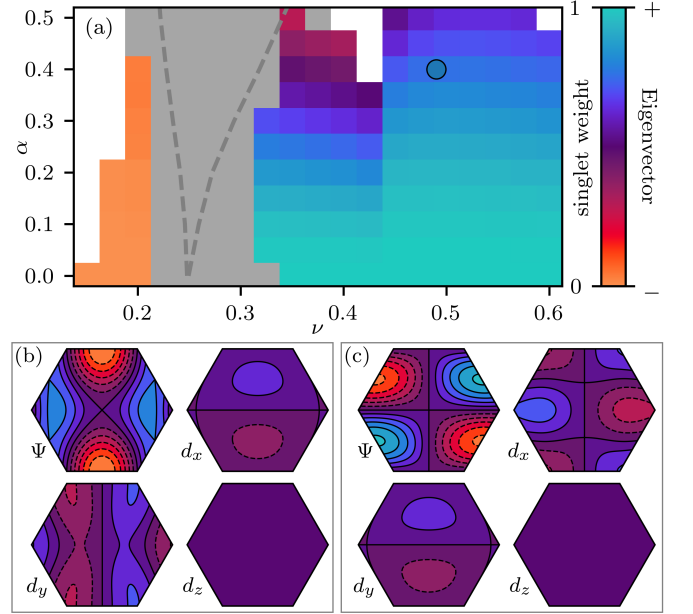


FIG. 1. (a) Phase diagram with RSOC (α) vs. filling (ν). Colored areas correspond to superconductivity, grey to magnetic order, and white to metal/Fermi liquid. Color corresponds to singlet-triplet weight of the divergent superconducting instability (see main text). The dashed line indicates van-Hove filling. (b, c) Two degenerate superconducting states corresponding to the parameters as indicated by the circle in (a) $\nu = 0.5$, $\alpha = 0.4$, realizing a superposition of the three possible E_2 superconducting states for Cooper-pairs across the nearest-neighbor bonds. The imaginary component of the two superconducting states is zero.

also point out that the superconducting instability that is parallel to the Rashba vector $\mathbf{g} = (\partial \varepsilon_0 / \partial k_y, -\partial \varepsilon_0 / \partial k_x, 0)$ is $\mathbf{d} = (\Gamma_{p_y}, -\Gamma_{p_x}, 0)$, which is a basis function of the B_2 irrep^{25,40}.

Results I: competing many-body phases.—In the following, we present our TUFGRG results for $U = 8t$ and first focus on different types of instabilities. The phase diagram Fig. 1 (a) reveals for zero RSOC ($\alpha = 0$) and filling $\nu \geq 0.34$ an extended superconducting phase with E_{2g} irrep of the symmetry group D_{6h} (spin-singlet), adjacent to a magnetically ordered phase at lower fillings. There is also a spin-triplet superconducting phase transforming under the B_{1u} irrep for fillings $\nu < 0.22$, below the van Hove singularity. The extended E_{2g} phase is not unexpected, as this is the preferred type of unconventional superconductivity on hexagonal lattices with a spherical Fermi surface.

The two-dimensional E_{2g} irrep allows for arbitrary complex superpositions of the two degenerate superconducting states that constitute the diverging instability. Free energy calculations reveal that the equal weight complex superposition “ $d+id$ ” has the lowest energy (see Supplementary Material (and Ref.²⁹) for explicit calculations), due to the largest condensation energy (as previously found in Refs. 38, 39, and 41 or often assumed

without explicit calculation). We stress that the $d + id$ SC is a topological state with chiral edge states and finite Chern number, as explicitly confirmed below. At first view, the $\alpha = 0$ results are compatible with previous work^{4,19,29}; our analysis below reveals, however, that part of it is an *extended* chiral d -wave state. Outside of the shown fillings ν , TUFGR does not diverge, signaling stability of the Fermi liquid phase.

Increasing the RSOC α does not change much on the interplay of superconducting and magnetic phases in the considered range $0 \leq \alpha < 0.5$ (see Fig. 1 a). We do find commensurate and incommensurate spin-density waves with both ferro- and antiferromagnetic ordering tendencies, but leave the detailed analysis of the magnetically ordered phase for future work.

Results II: singlet-triplet mixing.— In line with previous work in Rashba systems^{24,27,42}, we find the fraction of singlet-triplet mixing to increase with increasing RSOC, so long as the SC phase is spin singlet at $\alpha = 0$. One of the major advantages of TUFGR is to quantitatively determine this singlet-triplet mixing (see Supplementary Material³¹ for details). In particular, for $0.34 \leq \nu \leq 0.44$, finite α and thus inversion symmetry breaking, moves the chiral $d + id$ SC phase at $\alpha = 0$ to the E_2 irrep of C_{6v} , allowing an admixture of this $d + id$ spin singlet state and the two triplet E_2 states discussed above. We observe sizeable triplet contributions in the otherwise singlet-dominated phase, and for larger values of RSOC, such as $\alpha = 0.5$, singlet and triplet contributions are of similar weight. An example of this mixed singlet-triplet order parameter is shown in Fig. 1 (b,c). The superconducting order parameter was calculated for half-filling $\nu = 0.5$ and $\alpha = 0.5$, and transforms under the two-dimensional E_2 irrep. In this particular example, the d -wave singlet component Ψ is the largest, with a small mixture of the two triplet states. This mixture distinguishes all order parameters for $\alpha > 0$ from the exclusively spin-singlet chiral $d + id$. The breakdown of this state into allowed form factors is discussed further in the Supplementary Material³¹, in particular see Fig. S2.

In contrast to the superconducting phase for fillings above the vH singularity, the low-filling superconducting phase remains fully spin-triplet regardless of α . However, also here we observe a mixture of SC order parameters which is not possible for $\alpha = 0$: the two triplet E_2 superconducting states mix, forming a triplet-triplet mixture, yielding the extended triplet phase. RSOC does, however, change the nature of the triplet state. At $\alpha = 0$ we find B_{1u} irrep (gapless), and for any finite α the E_2 irrep containing f -wave and p -wave pairings. Roughly speaking, the p -wave contribution, responsible for the finite gap, increases with α . At $\alpha \sim 0.4t$ this E_2 phase is dominated by p -wave pairing. We stress that there is hardly any method which can resolve spin-triplet or triplet-triplet mixing as easily as TUFGR can. To determine whether the mixed-in spin-triplet components trigger any additional topological features we need to push the analysis further.

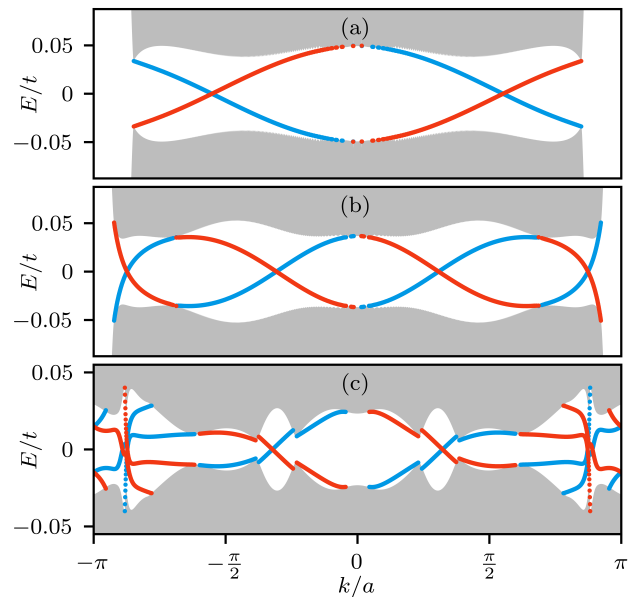


FIG. 2. Ribbon spectra from TUFGR and BdG methods. (a) $\alpha = 0$, $\nu = 0.5$ resulting in $C = 4$, as evidenced by two doubly degenerate chiral edge modes *per edge*. (b) $\alpha = 0$, $\nu = 0.4$ resulting in $C = -8$, as evidenced by four doubly degenerate chiral edge modes *per edge*. (c) $\alpha = 0.35$, $\nu = 0.4$ resulting in $C = -6$, as evidenced by six chiral edge modes *per edge*. All spectra calculated for a SC amplitude of $|\Delta| = 0.05$, on a ribbon of 900 atoms, and a k -momentum resolution of 3000 points.

Results III: BdG analysis, Chern numbers and ribbon spectra.—TUFGR provides us with details about the pairing symmetry and relative strength for shells with different distance (*e.g.* on-site pairing vs. nearest-neighbor pairing), thanks to its truncated unity extension. In the following, we directly make use of this information and feed it into a BdG Hamiltonian (see Supplementary Material³¹) where all other bandstructure parameters are kept identical to the TUFGR input. Since the superconducting states are all E_2 and doubly degenerate, we calculate the free energy to determine the stable ground state configuration^{19,30,33}. Knowledge of the BdG Hamiltonian and its eigenvectors allows us to compute Chern numbers of the corresponding gapped superconducting phases and compute ribbon (and real space) spectra to reveal topological edge states. For instance, by choosing $\alpha = 0$ and $\nu = 0.5$ we recover the earlier result⁴³ that the chiral $d + id$ superconductor (E_2 irrep) possesses a Chern number $C = 4$. The corresponding ribbon spectra is shown in Fig. 2 (a), which at $\alpha = 0$ manifests as only two distinguishable edge states due to spin degeneracy^{22,44}. When a small amount of RSOC is introduced, *i.e.*, $\alpha > 0$, the spin degeneracy is lifted and four distinguishable chiral edge states are observable.

Topological phase transitions.—By virtue of combining the TUFGR and BdG methods, we analyze the entire su-

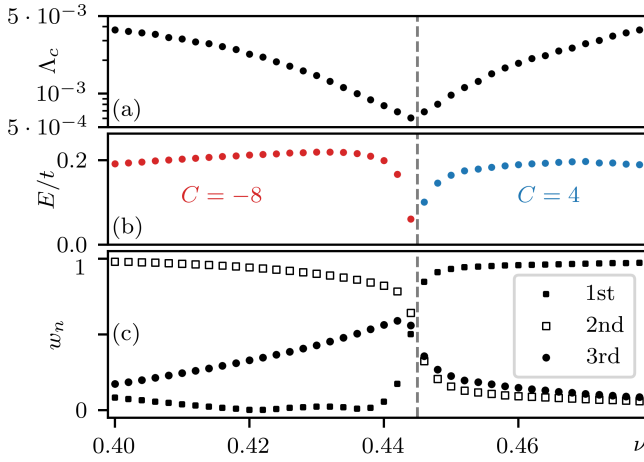


FIG. 3. Topological transition in filling at $\alpha = 0$. (a) Critical TUFGR scale Λ_c as a function of filling ν . (b) BdG energy E as a function of filling ν , Chern numbers color-coded relative to Fig. 4. (c) Relative weights in the TUFGR superconducting instability for the n -th nearest neighbor pairing lengths as a function of filling. In all plots, the topological transition at filling $\nu = 0.445$ is shown by the dashed grey line.

perconducting phase space. First, we focus on constant RSOC α and vary the filling ν . At $\nu_c = 0.445$ (irrespective of α), we observe a dip in critical scale Λ_c (see Fig. 3(a)). We note that this feature should also be observable within other FRG approaches. We observe this non-analytical behavior also in other quantities such as the gap energy of the BdG Hamiltonian (Fig. 3(b)), as well as a shift in the spectral weight of the Cooper pairing from second- and third-nearest neighbor bonding to primarily nearest-neighbor. We visualize this by calculating the spectral weight on each interaction shell w_n in Fig. 3(c) (for details see Supplementary Material³¹). We find that for fillings larger than ν_c the BdG Hamiltonian possesses a Chern number $C = 4$, however, for fillings smaller than ν_c we obtain $C = -8$; see the corresponding ribbon spectra in Fig. 2 (a, b), revealing a topological phase transition.

We can understand this topological phase transition by focusing on the purely singlet chiral $d + id$ -wave superconducting phases at $\alpha = 0$. In this limit, the gap in the BdG spectrum closes when the Fermi surface intersects vortices in the superconducting pairing $\Gamma_{d_{x^2-y^2}} + i\Gamma_{d_{xy}}$. The location and number of these vortices depends on the precise mix of the three interaction shell pairings. With a longer pairing, higher harmonics are introduced into the superconducting pairing function, which means more vortices are introduced for longer range pairing. These vortices can then be shifted in reciprocal space by changing the admixture of the superconducting pairings, with the total number changing when vortices recombine and are moved in and out the Brillouin zone. That is, the number of vortices can change and thus the topological invariant can change too. Bottomline: even in the ab-

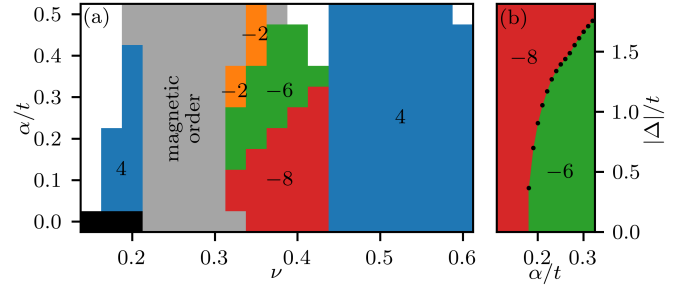


FIG. 4. (a) Topological phase diagram: superconducting phases from Fig. 1a with their Chern numbers. White regions correspond to the Fermi liquid, grey is magnetic ordering, and black states are gapless. (b) Varying Chern number with increasing RSOC α and superconducting order parameter amplitude $|\Delta|$, for fixed filling $\nu = 0.4$.

sence of RSOC, topological phase transitions are to be expected, as demonstrated above, although we find E_2 irreps on both sides of the phase transition. We note that a similar phase transition was observed at $\alpha = 0$ in extended Hubbard and tJ models^{6,22,45}. Our work establishes the plain-vanilla onsite Hubbard model as the minimal model to find such a phase transition, as well as its stability towards RSOC.

We also encounter topological phase transitions when varying α . For instance, for fixed $\nu = 0.4$, we previously found $C = -8$ at $\alpha = 0$ (see a corresponding ribbon spectra in Fig. 2 (b)) and for larger $\alpha = 0.35$ we find $C = -6$, as corroborated by Fig. 2 (c). The entire analysis of Chern number and edge states from ribbon spectra culminates in the topological phase diagram Fig. 4 (a).

Clear critical signatures as those shown in Fig. 3(a) are absent, however, the BdG energy gap clearly reveals the topological phase transition when changing α . This difference can be traced to the nature of the BdG gap closing. Unlike the purely singlet case, the characteristic polynomial of the BdG matrix has some constant terms, which mean that the BdG energies can have zeros and the BdG gap can close where the superconducting amplitude $|\Delta|$ is not zero; Fig. 4 (b) plots the relationship between α and $|\Delta|$. If we assume that $|\Delta|/t < 0.1$ (which is reasonable as we follow the educated guess $|\Delta| \propto \Lambda_c$) then this fixes $\alpha_c = 0.17$ to two significant figures regardless how small $|\Delta|$ will be. This also justifies omitting an additional self-consistent treatment of the magnitude of $|\Delta|$. We then compute topological invariants and derive ribbon spectra resulting in Fig. 4 (a).

Outlook.—There are many examples of studying unconventional superconductivity as the groundstate of correlated electron systems in the literature. In particular, for hexagonal lattices with a circular Fermi surface, the E_2 irrep is the most likely candidate. While it is well-known that such a state usually corresponds to the chiral $d + id$ superconductor, here we show that the phenomenology is much richer, in particular, in the presence of spin-orbit coupling. We find Chern num-

bers $C = 4, -2, -6, -8$ within an otherwise *homogeneous* E_2 irrep phase. Only the employed TUFGR formalism allows to reveal these topological features and the associated topological phase transition within that phase. In the light of ongoing experimental activities with the prospect of unconventional superconductivity in triangular lattice materials such as B-doped Sn/Si(111) and related compounds, we hope that our work will spark further activity for these systems since different topological states and possibly even topological phase transitions might be experimentally observable.

ACKNOWLEDGMENTS

The authors thank L. Classen, M. Dürrnagel, A. Fischer, L. Klebl, J. B. Profe, A. Sanders, A. Schnyder, T. Schwemmer and C. Timm for helpful discussions. This work was funded by the Australian Research Council

(ARC) through Grant No. DP200101118, by the German Research Foundation (DFG) through RTG 1995 and the Priority Program SPP 2244 “2DMP” and by the Würzburg–Dresden Cluster of Excellence on Complexity and Topology in Quantum Matter – ct.qmat (EXC 2147, project id 390858490). This research was supported in part by the National Science Foundation under Grant No. NSF PHY-1748958. The authors gratefully acknowledge the scientific support and HPC resources provided by the Erlangen National High Performance Computing Center (NHR@FAU) of the Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU) under the NHR project “k102de-FRG”. NHR funding is provided by federal and Bavarian state authorities. NHR@FAU hardware is partially funded by the DFG – 440719683. The authors further acknowledge the resources from the National Computational Infrastructure (NCI Australia), an NCRIS enabled capability supported by the Australian Government.

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- * These authors contributed equally.
- ¹ C. Kallin and J. Berlinsky, “Chiral superconductors,” *Rep. Prog. Phys.* **79**, 054502 (2016).
 - ² M. Sato and Y. Ando, “Topological superconductors: a review,” *Rep. Prog. Phys.* **80**, 076501 (2017).
 - ³ M. Vojta and E. Dagotto, “Indications of unconventional superconductivity in doped and undoped triangular antiferromagnets,” *Phys. Rev. B* **59**, R713–R716 (1999).
 - ⁴ C. Honerkamp, “Instabilities of interacting electrons on the triangular lattice,” *Phys. Rev. B* **68**, 104510 (2003).
 - ⁵ H. Ikeda, Y. Nisikawa, and K. Yamada, “Possibility of f -wave spin-triplet superconductivity in the CoO superconductor: A case study on a 2D triangular lattice in the repulsive Hubbard model,” *J. Phys. Soc. Jpn.* **73**, 17–20 (2004).
 - ⁶ S. Zhou and Z. Wang, “Nodal $d + id$ Pairing and Topological Phases on the Triangular Lattice of $\text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O}$: Evidence for an Unconventional Superconducting State,” *Phys. Rev. Lett.* **100**, 217002 (2008).
 - ⁷ K. S. Chen, Z. Y. Meng, U. Yu, S. Yang, M. Jarrell, and J. Moreno, “Unconventional superconductivity on the triangular lattice Hubbard model,” *Phys. Rev. B* **88**, 041103 (2013).
 - ⁸ S. Glass, G. Li, F. Adler, J. Aulbach, A. Fleszar, R. Thomale, W. Hanke, R. Claessen, and J. Schäfer, “Triangular Spin-Orbit-Coupled Lattice with Strong Coulomb Correlations: Sn Atoms on a SiC(0001) Substrate,” *Phys. Rev. Lett.* **114**, 247602 (2015).
 - ⁹ D. I. Badrtdinov, S. A. Nikolaev, M. I. Katsnelson, and V. V. Mazurenko, “Spin-orbit coupling and magnetic interactions in Si(111): C, Si, Sn, Pb,” *Phys. Rev. B* **94**, 224418 (2016).
 - ¹⁰ F. Ming, S. Johnston, D. Mulugeta, T. S. Smith, P. Vilmercati, G. Lee, T. A. Maier, P. C. Snijders, and H. H. Weitering, “Realization of a Hole-Doped Mott Insulator on a Triangular Silicon Lattice,” *Phys. Rev. Lett.* **119**, 266802 (2017).
 - ¹¹ X. Cao, T. Ayral, Z. Zhong, O. Parcollet, D. Manske, and P. Hansmann, “Chiral d -wave superconductivity in a triangular surface lattice mediated by long-range interaction,” *Phys. Rev. B* **97**, 155145 (2018).
 - ¹² T. Nakamura, H. Kim, S. Ichinokura, A. Takayama, A. V. Zotov, A. A. Saranin, Y. Hasegawa, and S. Hasegawa, “Unconventional superconductivity in the single-atom-layer alloy Si(111)- $\sqrt{3} \times \sqrt{3}$ -(Tl, Pb),” *Phys. Rev. B* **98**, 134505 (2018).
 - ¹³ C. Tresca, C. Brun, T. Bilgeri, G. Menard, V. Cherkez, R. Federicci, D. Longo, F. Debontridder, M. D’angelo, D. Roditchev, G. Profeta, M. Calandra, and T. Cren, “Chiral Spin Texture in the Charge-Density-Wave Phase of the Correlated Metallic Pb/Si(111) Monolayer,” *Phys. Rev. Lett.* **120**, 196402 (2018).
 - ¹⁴ F. Adler, S. Rachel, M. Laubach, J. Maklar, A. Fleszar, J. Schäfer, and R. Claessen, “Correlation-Driven Charge Order in a Frustrated Two-Dimensional Atom Lattice,” *Phys. Rev. Lett.* **123**, 086401 (2019).
 - ¹⁵ S. Wolf, D. Di Sante, T. Schwemmer, R. Thomale, and S. Rachel, “Triplet Superconductivity from Nonlocal Coulomb Repulsion in an Atomic Sn Layer Deposited onto a Si(111) Substrate,” *Phys. Rev. Lett.* **128**, 167002 (2022).
 - ¹⁶ T. Machida, Y. Yoshimura, T. Nakamura, Y. Kohsaka, T. Hanaguri, C.-R. Hsing, C.-M. Wei, Y. Hasegawa, S. Hasegawa, and A. Takayama, “Superconductivity near the saddle point in the two-dimensional Rashba system Si(111)- 3×3 -(Tl, Pb),” *Phys. Rev. B* **105**, 064507 (2022).
 - ¹⁷ M. Biderang, M.-H. Zare, and J. Sirker, “Topological superconductivity in Sn/Si(111) driven by nonlocal Coulomb interactions,” *Phys. Rev. B* **106**, 054514 (2022).
 - ¹⁸ F. Ming, X. Wu, C. Chen, K. D. Wang, P. Mai, T. A. Maier, J. Strokoz, J. W. F. Venderbos, C. Gonzalez, J. Ortega, S. Johnston, and H. H. Weitering, “Evidence for chiral superconductivity on a silicon surface,” *Nat. Phys.* **19**, 500 (2023).
 - ¹⁹ L. Classen, C. Honerkamp, and M. M. Scherer, “Competing phases of interacting electrons on triangular lattices in moiré heterostructures,” *Phys. Rev. B* **99**, 195120 (2019).

- ²⁰ J. Venderley and E.-A. Kim, “Density matrix renormalization group study of superconductivity in the triangular lattice Hubbard model,” *Phys. Rev. B* **100**, 060506 (2019).
- ²¹ N. Gneist, L. Classen, and M. M. Scherer, “Competing instabilities of the extended Hubbard model on the triangular lattice: Truncated-unity functional renormalization group and application to moiré materials,” *Phys. Rev. B* **106**, 125141 (2022).
- ²² M. M. Scherer, D. M. Kennes, and L. Classen, “Chiral superconductivity with enhanced quantized Hall responses in moiré transition metal dichalcogenides,” *npj Quantum Mater.* **7**, 1–9 (2022).
- ²³ L. P. Gor’kov and E. I. Rashba, “Superconducting 2D System with Lifted Spin Degeneracy: Mixed Singlet-Triplet State,” *Phys. Rev. Lett.* **87**, 037004 (2001).
- ²⁴ O. Vafek and L. Wang, “Spin-orbit coupling induced enhancement of superconductivity in a two-dimensional repulsive gas of fermions,” *Phys. Rev. B* **84**, 172501 (2011).
- ²⁵ M. H. Fischer, M. Sigrist, D. F. Agterberg, and Y. Yanase, “Superconductivity and Local Inversion-Symmetry Breaking,” *Ann. Rev. Con. Mat. Phys.* **14**, 153–172 (2023).
- ²⁶ A. Greco and A. P. Schnyder, “Mechanism for Unconventional Superconductivity in the Hole-Doped Rashba-Hubbard Model,” *Phys. Rev. Lett.* **120**, 177002 (2018).
- ²⁷ J. Beyer, J. B. Hauck, L. Klebl, T. Schwemmer, D. M. Kennes, R. Thomale, C. Honerkamp, and S. Rachel, “Rashba spin-orbit coupling in the square-lattice Hubbard model: A truncated-unity functional renormalization group study,” *Phys. Rev. B* **107**, 125115 (2023).
- ²⁸ M. Biderang, M.-H. Zare, and J. Sirker, “Proximity-driven ferromagnetism and superconductivity in the triangular Rashba-Hubbard model,” *Phys. Rev. B* **105**, 064504 (2022).
- ²⁹ L. Klebl, A. Fischer, L. Classen, M. M. Scherer, and D. M. Kennes, “Competition of density waves and superconductivity in twisted tungsten diselenide,” *Phys. Rev. Res.* **5**, L012034 (2023).
- ³⁰ L. Klebl, Q. Xu, A. Fischer, L. Xian, M. Claassen, A. Rubio, and D. M. Kennes, “Moiré engineering of spin-orbit coupling in twisted platinum diselenide,” *Electron. Struct.* **4**, 014004 (2022).
- ³¹ See Supplementary Material [url], which includes Refs 46–49, for further details about the TUFGR implementation, group theoretical analysis and breakdown of the SC order parameter, the BdG analysis, which includes the topological analysis via Chern number calculations and calculation of the ribbon spectra.
- ³² W. Metzner, M. Salmhofer, C. Honerkamp, V. Meden, and K. Schoenhammer, “Functional renormalization group approach to correlated fermion systems,” *Rev. Mod. Phys.* **84**, 299–352 (2012).
- ³³ C. Platt, W. Hanke, and R. Thomale, “Functional renormalization group for multi-orbital Fermi surface instabilities,” *Adv. Phys.* **62**, 453–562 (2013).
- ³⁴ J. Lichtenstein, D. Sánchez de la Peña, D. Rohe, E. Di Napoli, C. Honerkamp, and S. Maier, “High-performance functional Renormalization Group calculations for interacting fermions,” *Comput. Phys. Commun.* **213**, 100–110 (2017).
- ³⁵ C. Husemann and M. Salmhofer, “Efficient parametrization of the vertex function, Ω scheme, and the (t, t') Hubbard model at van Hove filling,” *Phys. Rev. B* **79**, 195125 (2009).
- ³⁶ W.-S. Wang, Y.-Y. Xiang, Q.-H. Wang, F. Wang, F. Yang, and D.-H. Lee, “Functional renormalization group and variational Monte Carlo studies of the electronic instabilities in graphene near 1/4 doping,” *Phys. Rev. B* **85**, 035414 (2012).
- ³⁷ J. Beyer, J. B. Profe, and L. Klebl, “Reference results for the momentum space functional renormalization group,” *Eur. Phys. J. B* **95**, 65 (2022).
- ³⁸ M. Sigrist and T. M. Rice, “Symmetry classification of states in high temperature superconductors,” *Z. Phys. B Con. Mat.* **68**, 9–14 (1987).
- ³⁹ M. Sigrist and K. Ueda, “Phenomenological theory of unconventional superconductivity,” *Rev. Mod. Phys.* **63**, 239–311 (1991).
- ⁴⁰ P. A. Frigeri, D. F. Agterberg, A. Koga, and M. Sigrist, “Superconductivity without inversion symmetry: MnSi versus CePt₃Si,” *Phys. Rev. Lett.* **92**, 097001 (2004).
- ⁴¹ V. Kuznetsova and V. Barzykin, “Pairing state in multicomponent superconductors,” *Europhys. Lett.* **72**, 437 (2005).
- ⁴² S. Wolf and S. Rachel, “Spin-orbit coupled superconductivity: Rashba-Hubbard model on the square lattice,” *Phys. Rev. B* **102** (2020).
- ⁴³ A. Crépieux, E. Pangburn, L. Haurie, O. A. Awoga, A. M. Black-Schaffer, N. Sedlmayr, C. Pépin, and C. Bena, “Superconductivity in monolayer and few-layer graphene. ii. topological edge states and chern numbers,” *Phys. Rev. B* **108**, 134515 (2023).
- ⁴⁴ A. M. Black-Schaffer, “Edge Properties and Majorana Fermions in the Proposed Chiral d -Wave Superconducting State of Doped Graphene,” *Phys. Rev. Lett.* **109**, 197001 (2012).
- ⁴⁵ V. V. Val’kov, T. A. Val’kova, and V. A. Mitskan, “Superconducting phase with the $(d + id)$ order parameter in an ensemble of Hubbard fermions on the triangular lattice,” *JETP Lett.* **102**, 361–366 (2015).
- ⁴⁶ A. Szasz, J. Motruk, M. P. Zaletel, and J. E. Moore, “Chiral spin liquid phase of the triangular lattice Hubbard model: A density matrix renormalization group study,” *Phys. Rev. X* **10**, 021042 (2020).
- ⁴⁷ M. F. Holst, M. Sigrist, and M. H. Fischer, “Role of topology and symmetry for the edge currents of a two-dimensional superconductor,” *Phys. Rev. Res.* **4**, 013244 (2022).
- ⁴⁸ T. Fukui, Y. Hatsugai, and H. Suzuki, “Chern numbers in discretized Brillouin zone: efficient method of computing (spin) Hall conductances,” *J. Phys. Soc. Jpn.* **74**, 1674–1677 (2005).
- ⁴⁹ R. B. Laughlin, “Quantized Hall conductivity in two dimensions,” *Phys. Rev. B* **23**, 5632 (1981).