This is the accepted manuscript made available via CHORUS. The article has been published as:

Measurement-induced phase transitions in systems with diffusive dynamics

Hyunsoo Ha, Akshat Pandey, Sarang Gopalakrishnan, and David A. Huse

Phys. Rev. B **110**, L140301 — Published 2 October 2024

DOI: 10.1103/PhysRevB.110.L140301

Measurement-induced phase transitions in systems with diffusive dynamics

Hyunsoo Ha,[1] Akshat Pandey,[2] Sarang Gopalakrishnan,[3] and David A. Huse[1]

[1] *Department of Physics, Princeton University, Princeton, NJ 08544, USA*
[2] *Department of Physics, Stanford University, Stanford, CA 94305, USA*
[3] *Department of Electrical and Computer Engineering, Princeton University, Princeton, NJ 08544, USA*

(Dated: September 19, 2024)

The competition between scrambling and projective measurements can lead to measurement-induced entanglement phase transitions (MIPT). In this work, we show that the universality class of the MIPT can be drastically altered when the system has a diffusing conserved density. As a numerical tractable model of this, we study a 1+1d random Clifford circuit locally monitored by classically diffusing particles ("measurers"). The resulting diffusive correlations in the measurement density are a relevant perturbation to the usual space-time random MIPT critical point, producing a new universality class for this phase transition. We find "Griffiths-like" effects due to rare space-time regions where, e.g., the diffusive measurers have a low or high density, but these are considerably weaker than the Griffiths effects that occur with quenched randomness that produce rare spatial regions with infinite lifetime.

Measurement-induced phase transitions (MIPTs) are a recently discovered class of phase transitions in quantum dynamical systems subject to repeated measurements. MIPTs are transitions in the entanglement structure of typical individual *quantum trajectories*—i.e., in the state of the system conditional on a set of measurement outcomes [1–3]. They separate a "mixed" or "volume-law" phase in which the dynamics generate volume-law entanglement that is able to preserve some quantum information about the initial state, from a "pure" or "area-law" phase in which measurements rapidly extract all quantum information and prevent the buildup of entanglement [4, 5]. The MIPT in spatially local one-dimensional random quantum circuits, with local unitary gates and single-site projective measurements, has been the best-studied example [6–12]; however, MIPTs have also been explored in other geometries [13–19]. Experimental studies of MIPTs remain challenging because studying individual trajectories requires procedures (such as post-selection or classical simulations) for which the resource requirements grow exponentially with system size; nevertheless, signatures of MIPTs have been seen in recent experiments with trapped ions and superconducting qubits [20–23], and recent theoretical proposals indicate how MIPTs may be realized in larger-scale experiments [24–26].

The critical properties of MIPTs depend on the spatial dimension and range of the gates [16, 27–29]. In addition, typical trajectories are inherently random in space and time—since the measurement outcomes are random—and the critical properties are sensitive to the correlations in this randomness. The "standard" MIPT occurs in one-dimensional circuits where the unitary gates and measurement locations are chosen at random with no space-time correlations, and the measurement outcomes are set by the Born rule. A number of circuit-averaged observables at this standard MIPT seem to be governed by a nonunitary conformal field theory [9], which, in the limit of infinite on-site Hilbert space dimension, can be identified as percolation. When the measurement locations or the gate parameters have strong enough long-range correlations, the universality class changes. For example, if there is quenched spatial randomness in the measurement probabilities, the transition is instead governed by an infinite-randomness fixed point with infinitely anisotropic spacetime scaling [30, 31].

In the standard MIPT, the dynamics is not subject to any conservation laws. Even in the absence of measurements, conservation laws constrain entanglement growth in quantum circuits [32]. In this paper we explore how the presence of a diffusing conserved charge (conserved also by the measurements) affects the MIPT in 1+1 dimensions. Our analysis also extends to the $U(1)$ symmetric monitored circuits, where in addition to the MIPT (which is an *entanglement* transition), circuits with a conserved charge also exhibit a "charge-sharpening" phase transition [25, 33–35]. Note that the MIPT takes place in the charge-sharp phase, where the measurements are frequent enough to collapse and localize the charge distribution, so an individual quantum trajectory has a well-defined spacetime charge density profile that random-walks at large length scales. Generically, the critical measurement rate at the entanglement transition depends linearly on small changes in the overall charge density [36]. When this is true, as we argue below, the coupling to the diffusing charge is relevant at the critical point, and thus changes the universality class of this phase transition [37].

The MIPT can be studied efficiently for large systems composed of Clifford gates and Pauli measurements [3, 5, 8–10, 38]. Unfortunately, if one restricts such a random Clifford circuit to have a conserved $U(1)$ charge (e.g. $\hat{Z}_i$ summed over all qubits i) this eliminates the volume-law entangled phase and thus also the phase transition that we wish to study (see [39] for detailed discussion).

Thus we introduce a new type of model in order to efficiently numerically study this transition. In this model, a diffusing conserved *classical* charge is separate from the quantum Clifford random circuit, the circuit alone has no conservation laws, and the measurement rate at a given space-time point is set by the (classical) charge at that point. This allows us to simulate large systems and thus investigate the resulting new universality class of the MIPT. In the absence of the conserved charge, the universality class of the transition does differ in some details between Clifford and non-Clifford (e.g., Haar-random) systems, although in many respects they are qualitatively similar [8, 10]. With the conserved charge, we can again expect differences between non-Clifford and Clifford universality classes; here, we study the latter because it is much more accessible numerically by using the new type of model that we introduce. Our main results are to locate and characterize the critical point of the MIPT in this model. We also provide numerical evidence and analytical arguments to show that diffusive correlations affect the properties of the phases on either side of the MIPT.

Our model has classical particles diffusing via the symmetric simple exclusion process (SSEP) [40–42] on the same one-dimensional lattice that the qubits occupy. The particle density is equal to one particle per two qubits. These classical particles are the "measurers": at the time of measurements, each qubit that is at a site occupied by a particle is measured with probability p; see Fig. 1. This imparts diffusive correlations in space-time to the locations of the measurements. In all data presented here, the measurers have diffusivity $D = 1$, and we use periodic boundary conditions except where otherwise stated. Our systems are of length N qubits.

Generalized Harris argument— We begin by giving a generalized Harris argument [43, 44] for the relevance of diffusive correlations in the measurement rate. The standard MIPT (with uncorrelated measurements) has correlation length exponent ν and dynamical exponent $z = 1$, so for p near the critical point p_c the correlation region has space-time volume $\sim \xi^{(d+1)}(p) \sim |p - p_c|^{-(d+1)\nu}$ for $d+1$ dimensions. Space-time correlations in the measurements will be relevant if they cause the measurement rate averaged over a space-time correlation volume to vary (between different correlation volumes of the same system) by an amount δp that vanishes more slowly than $|p - p_c| \sim \xi^{-1/\nu}$ as $p \to p_c$.

If diffusive measurers are added perturbatively to the standard uncorrelated model, the number of measurers in a correlation volume does not change significantly within one correlation time; the measurer number is strongly correlated along the time direction. The variation in the number of measurers is proportional to the square root of the spatial volume, resulting in a variation of measurer density within a correlation volume proportional to $\xi^{-d/2}$. Therefore, $\delta p \sim \xi^{-d/2}(p)$, which is relevant when $\nu < 2/d$, as in the usual Harris argument [45]. In particular, for the 1+1-dimensional case that we consider here, the uncorrelated critical point has $\nu \cong 1.3 < 2/d = 2$. Therefore, diffusive correlations are indeed a relevant perturbation for the uncorrelated MIPT. Note that this relevance criterion does not change as long as the measurers move slower than ballistically (so that the number of measurers within a correlation volume does not change significantly in one correlation time in the $p \to p_c$ limit).

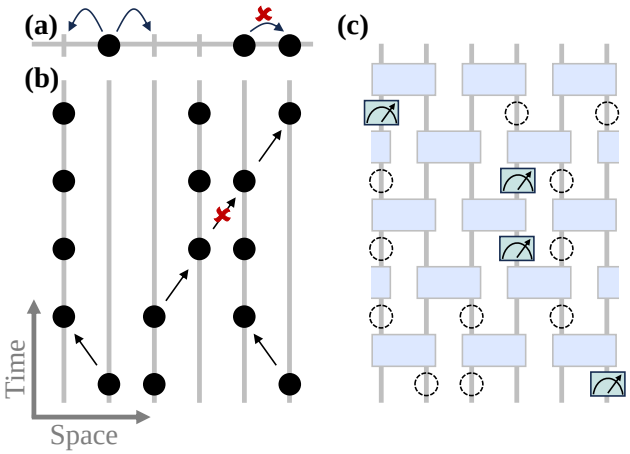


FIG. 1. Random Clifford circuit monitored by diffusing classical particles. (a) Illustration of classical SSEP rules governing measurer dynamics, where hopping to nearest-neighbor sites occurs as a continuous-time Poisson process with a specified rate and is forbidden if the adjacent site is already occupied. (b) An example of measurer trajectories. (c) A circuit with measurer locations as in (b). Two-qubit random Clifford gates are applied in a brickwork manner. At each time between layers of gates, each measurer makes a $\hat{Z}$ projective measurement of the local qubit with probability p.

Critical point: numerical results— The generalized Harris argument establishes that diffusive correlations drive the system away from the spacetime-uncorrelated critical point. To study the new critical point that *does* occur, we turn to numerical simulations of these Clifford circuits. We begin by estimating the critical measurement rate p_c based on the peak of the antipodal mutual information (AMI) defined as $S_A + S_B - S_{AB}$, where A and B are subsystems of length $N/8$ qubits located at opposite ends of the system [inset of Fig. 2(a)]. Here, $S_{A(B)}$ is the bipartite entanglement entropy of subsystem $A(B)$. The AMI vanishes as $N \to \infty$ in both the volume-law and area-law phases and is expected to exhibit a peak at p_c under fairly general conditions [3]. As shown in Fig. 2(a), the peak in the AMI for accessible sizes suggests $p_c \cong 0.33$. Interpreted conservatively, our data on the AMI constrains p_c to lie in the interval $(0.31, 0.35)$: for p within this interval, the AMI grows with L for all accessible sizes, so any value in this range could be the critical point. The estimated p_c for this model is approximately twice

that in the standard random-Clifford MIPT. This factor of roughly two is plausible because the number of measurers is $N_p = N/2$, so the average measurement rate over all spacetime is $p/2$. In [39], we attempt to extract the correlation-length exponent ν at this critical point by collapsing the AMI as well as a different observable, the tripartite mutual information. This gives us a rough estimate of $\nu = 1.6(5)$. The relatively poor quality of the collapse indicates that despite the large system sizes we are considering, finite-size effects are still severe.

To explore dynamical scaling at the critical point, we turn to the purification dynamics of an initially maximally mixed state for measurement rates p around this critical window. We specifically investigate the purification time τ_P when the maximally mixed state purifies to a pure state. At the critical point, we expect that the timescale for purifying a system of size N scales as $\tau_P(N) \sim N^z$, where z is the dynamical critical exponent. For the standard MIPT, $z = 1$, and for the case of quenched randomness, $z = \infty$ [30]. In the diffusive case, our numerical results [Fig. 2(b)] clearly indicate that $z \geq 2$. Assuming that $N = 256$ is larger than any relevant microscopic scale, the slope $d(\log \tau_P)/d(\log N)$ at $p = 0.35$ lower bounds the value of z. This lower bound is close to the diffusive value $z = 2$, which, as we will discuss, is the largest value that is theoretically well-motivated. If we instead focus on the apparent p_c—given by the peak in the AMI—we find a slope that is clearly drifting to larger values with increasing N. This would imply $z > 3.5$ and would be most consistent with a formally infinite z, with power-law dynamical scaling replaced by some form of activated scaling, as at the infinite-randomness critical point. Indeed, as shown in the inset to Fig. 2(b), an activated scaling ansatz with scaling $\log \tau_P \sim N^\psi$ with $\psi \cong 0.44$ is consistent with the purification data.

Thus, our numerical results are consistent with at least two possibilities for the critical point. One possible scenario, suggested by a straightforward analysis of the numerics, gives $p_c \approx 0.33$ and $z > 3.5$. Although consistent with the numerics, this scenario seems implausible, by the following logic: Imagine running a renormalization-group scheme that gives an exponent $z > 2$. After many steps of this scheme, the system will have $O(1)$ remaining degrees of freedom per $\ell \times \ell^z$ patch for some large ℓ. The remaining disorder at this scale is spatially uncorrelated, so if the renormalized description of the circuit resembles the original microscopic description, the flow beyond this scale will be governed by the standard MIPT and z will decrease. We emphasize that this argument makes some nontrivial assumptions about the nature of the flow, and does not rule out the $z > 2$ scenario. A simpler possibility, however, is that the AMI is showing strong finite-size effects and the true critical point lies near the upper end of the allowed range $(0.31, 0.35)$ and has $z = 2$, so the apparent values of $z > 2$ are actually in the volume-law

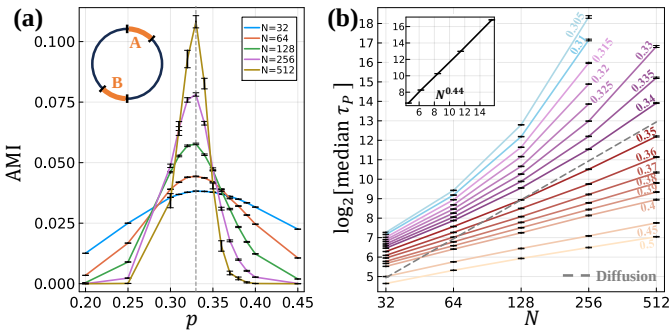


FIG. 2. Entanglement and purification dynamics at and near the critical point. (a) Antipodal mutual information in the long-time steady state distribution of pure states shows that the critical point p_c is located near $p = 0.33$. (b) Purification time τ_P. The median over samples is presented as a representative of a typical sample. The gray dotted line is the relaxation time scale for the diffusive measurers with $z = 2$. The inset shows the apparent activated dynamical scaling at $p = 0.33$, with the horizontal axis re-scaled to $N^{0.44}$.

phase. We present some tentative numerical support for this possibility in [39].

Rare region effects— We now turn to the phases on either side of this new MIPT. As in the infinite-randomness case, we expect that some properties of these phases are dominated by rare spacetime regions that are locally "in the other phase". To influence dynamics on a timescale t, such a rare configuration must persist for time t. In the standard MIPT, the measurement probabilities are drawn independently at each time, so this probability decays exponentially in time. However, in the diffusive case, a rare region of size $\ell \gtrsim \sqrt{Dt}$ typically persists for time t. The probability "cost" of such a region is stretched-exponential in t, so rare-region effects can dominate late-time dynamics of certain quantities whose typical behavior is exponential decay. The quantities that show these effects are different in the two phases:

In the volume-law phase, we find that regions with anomalously high density of measurers behave as bottlenecks that suppress *entanglement growth* due to the resulting high density of measurements. We prepare a random pure product initial state, evolve with open boundary conditions until time t, bipartition the system with a single cut at position x near the midpoint ($x = N/2$) and obtain the entanglement entropy that we label $S_{N/2}(t)$. We define the total probability $P^{<}_{S_{N/2}}(t)$ at time t of having $S_{N/2}(t) < \bar{S}_{N/2}(t)/2$: the probability of the entanglement entropy for a single instance being suppressed by at least a factor of two from its mean over many instances at time t.

In the usual MIPT, the most probable way to produce such a large suppression is due to a space-time path of time duration $\sim t$ that has substantially suppressed en-

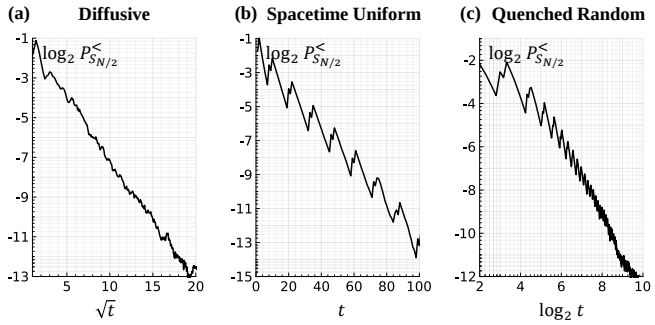


FIG. 3. Rare region effects in the volume-law phase in models with (a) diffusing measurers, (b) space-time uncorrelated measurements, and (c) immobile measurers having quenched randomness. Note, the horizontal axis is different in each case. The presented measurement rates are $p \cong p_c/2$, thus well into the volume-law phase. The system size is $N = 1024$, and cuts are sampled within a distance of 64 sites of the center of the chain.

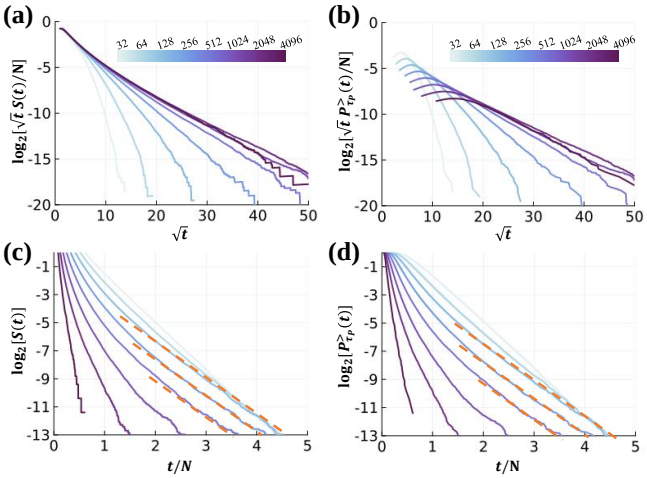


FIG. 4. Rare region effects in the area-law phase. (a,b) Numerical demonstrations of Eqs. (1) at intermediate times, where the longest-living rare region determines the decay of the total entropy and the tail of the purification time distribution. (c,d) At later times $S(t)$ and $P^>_{\tau_P}(t)$ decay exponentially with t/N. Color scale indicates N.

tanglement growth along the entire path. Therefore one expects $-\log P^<_{S_{N/2}} \sim t$. In our diffusive circuit, the tail is due to the rare regions of spatial width $O(\sqrt{Dt})$ such that they can survive until time t. Therefore, we expect $-\log P^<_{S_{N/2}} \sim \sqrt{Dt}$. With time-independent quenched randomness, the rare region is a bottleneck of the entanglement growth with a rate exponentially decaying with its length. In this case the most probable way to suppress the entanglement is due to a nearby bottleneck that is of spatial width $O(\log t)$, leading to the Griffiths power law: $-\log P^<_{S_{N/2}} \sim \log t$ [30]. We numerically verified the results are consistent with expectations for all three models (Fig. 3). This probability also oscillates in time as it decays because entanglement in Clifford circuits is integer-valued [39].

In the area-law phase, rare regions with low measurement density slow down *purification* from the maximally mixed initial state. We focus on two quantities: the mean total entropy $S(t)$ (entropy of the mixed state averaged over samples) and the tail distribution of the purification time τ_P, denoted as $P^>_{\tau_P}(t) \equiv P(\tau_P \geq t)$. These two quantities behave similarly at very late times when $S(t) < 1$, as both diagnose the last bit of entropy to be purified.

In the intermediate time regime of $1 \ll Dt < O(N^2)$, the entropy remains in rare regions with a lower density of measurers that were already present at time zero. For these to remain until time t they must be of spatial length $\ell \gtrsim \sqrt{Dt}$. The probability of having a rare region decays exponentially in its length ℓ, and once the probability is rare, there are approximately N/ℓ independent locations where such rare regions may occur. Therefore, we expect the following intermediate-time scaling:

$$\log\left(\frac{S(t)}{N/\sqrt{Dt}}\right) \sim -\sqrt{Dt}, \qquad \log\left(\frac{P^>_{\tau_P}(t)}{N/\sqrt{Dt}}\right) \sim -\sqrt{Dt}\ , \tag{1}$$

which are clearly observed in Fig. 4(a,b). The typical purification time diverges with system size as $\sim (\log N)^2/D$: The largest rare region length is typically $O(\log N)$, and the typical purification time is determined by the diffusive lifetime ($z = 2$) of this rare region.

Beyond the diffusive time scale $t \gg O(N^2/D)$, almost all the rare samples in the area-law phase that have not fully purified have entropy one bit. This logical operator has survived in spite of the strong measurements due to a sequence of rare spacetime regions. The most probable way for a logical to survive for such long times is for there to be a rare region of size of order N: the density of measurers is low over some fraction of order one half of the full system. Such a pattern occurs with a probability that is exponentially small in N. This rare pattern would normally disappear after a time $\sim N^2/D$, so for it to persist even longer to time $t \gg N^2/D$ means it did not decay away of order Dt/N^2 times. Thus the probability of the system not purifying to these late times is

$$\log P^>_{\tau_P}(t) \cong \log S(t) \sim (-N) \times \frac{Dt}{N^2} = -\frac{Dt}{N}. \tag{2}$$

This is consistent with Fig. 4(c,d), where for large t/N both $S(t)$ and $P^>_{\tau_P}(t)$ decay exponentially with t/N (orange dotted lines are parallel).

Conclusion— We have shown that diffusive correlations due to a conservation law change the universality

class of the MIPT, as well as the long-time asymptotics of the phases on either side of it—specifically, entanglement dynamics in the volume-law phase and purification dynamics in the area-law phase. We have provided analytic arguments to show that the universality class of the transition must change, and to estimate the "Griffiths-like" effects from diffusive correlations in the phases. To isolate the effect of diffusive correlations while still reaching large system sizes, we numerically explored a model in which the diffusive correlations are put in by hand. However, our analytic arguments do not rely on the specifics of this model, and we expect that our main conclusions are general, although critical exponents might differ between Clifford and generic circuits.

If $z > 2$, the correlation length exponent has a weaker lower bound, as the single correlation volume lasts for ξ^{z-2} correlation times of the measurers on scale ξ. Hence, the rms variation in the measurement rate between such space-time correlation volumes is $\delta p \sim \xi^{(2-d-z)/2}$, matching to the $\sim \xi^{d/2}$ for $z \leq 2$. This gives a generalized Chayes inequality $\nu \geq 2/(z+d-2) = 2/(z-1)$ for $1+1$ dimension and $z \geq 2$. Therefore, if the system is behaving on some scale with apparent $z > 2$, the apparent ν we extract may only obey this generalized inequality and "explain" our apparent $\nu < 2$ (although it has a large uncertainty).

The critical point we have explored is one of a large family of new critical points that arise naturally in the dynamics of quantum and/or classical information. The sharpening transition is another example in this vein; in one dimension disorder is irrelevant at this transition, but in higher dimensions (or for symmetry groups other than $U(1)$) it might be a relevant perturbation. The dynamics of out-of-time-order correlators in classical diffusive spin chains is another closely related example [44]. It was also recently shown that the coupling between quantum and classical circuits leads to nontrivial entanglement growth [46]. These critical points involve an information-theoretic quantity, such as entanglement, charge fluctuations, or an out-of-time-order correlator, coupled nonreciprocally to one or more hydrodynamic modes. Developing a framework for describing these new phase transitions is an important topic for future work.

H.H. thanks Grace Sommers and Su-un Lee for advice on numerics, Zack Weinstein, Ahana Chakraborty, Vir Bulchandani, and Rhine Samajdar for fruitful discussions. A.P. is grateful to Aditya Cowsik and Nicholas O'Dea for discussions. S.G. and D.A.H. thank Jed Pixley, Drew Potter, and Romain Vasseur for helpful discussions and collaborations on related topics. D.A.H. was supported in part by NSF QLCI grant OMA-2120757. This material is based upon work supported by the U.S. Department of Energy, Office of Science, National Quantum Information Science Research Centers, Co-design Center for Quantum Advantage (C2QA) under contract number DE-SC0012704 (theoretical analysis performed by S.G.). Simulations presented in this work were performed on computational resources managed and supported by Princeton Research Computing. The open-source `QuantumClifford.jl` package was used for Clifford simulations [47].

[1] Y. Li, X. Chen, and M. P. A. Fisher, Quantum zeno effect and the many-body entanglement transition, Phys. Rev. B **98**, 205136 (2018).
[2] B. Skinner, J. Ruhman, and A. Nahum, Measurement-induced phase transitions in the dynamics of entanglement, Phys. Rev. X **9**, 031009 (2019).
[3] Y. Li, X. Chen, and M. P. A. Fisher, Measurement-driven entanglement transition in hybrid quantum circuits, Phys. Rev. B **100**, 134306 (2019).
[4] S. Choi, Y. Bao, X.-L. Qi, and E. Altman, Quantum error correction in scrambling dynamics and measurement-induced phase transition, Phys. Rev. Lett. **125**, 030505 (2020).
[5] M. J. Gullans and D. A. Huse, Dynamical purification phase transition induced by quantum measurements, Phys. Rev. X **10**, 041020 (2020).
[6] R. Vasseur, A. C. Potter, Y.-Z. You, and A. W. W. Ludwig, Entanglement transitions from holographic random tensor networks, Phys. Rev. B **100**, 134203 (2019).
[7] Y. Bao, S. Choi, and E. Altman, Theory of the phase transition in random unitary circuits with measurements, Phys. Rev. B **101**, 104301 (2020).
[8] A. Zabalo, M. J. Gullans, J. H. Wilson, S. Gopalakrishnan, D. A. Huse, and J. H. Pixley, Critical properties of the measurement-induced transition in random quantum circuits, Phys. Rev. B **101**, 060301 (2020).
[9] Y. Li, X. Chen, A. W. W. Ludwig, and M. P. A. Fisher, Conformal invariance and quantum nonlocality in critical hybrid circuits, Phys. Rev. B **104**, 104305 (2021).
[10] A. Zabalo, M. J. Gullans, J. H. Wilson, R. Vasseur, A. W. W. Ludwig, S. Gopalakrishnan, D. A. Huse, and J. H. Pixley, Operator scaling dimensions and multifractality at measurement-induced transitions, Phys. Rev. Lett. **128**, 050602 (2022).
[11] A. C. Potter and R. Vasseur, Entanglement dynamics in hybrid quantum circuits, in *Entanglement in Spin Chains: From Theory to Quantum Technology Applications*, edited by A. Bayat, S. Bose, and H. Johannesson (Springer International Publishing, Cham, 2022) pp. 211–249.
[12] M. P. Fisher, V. Khemani, A. Nahum, and S. Vijay, Random quantum circuits, Annual Review of Condensed Matter Physics **14**, 335 (2023).
[13] X. Turkeshi, R. Fazio, and M. Dalmonte, Measurement-induced criticality in (2+1)-dimensional hybrid quantum circuits, Phys. Rev. B **102**, 014315 (2020).
[14] J. Lopez-Piqueres, B. Ware, and R. Vasseur, Mean-field entanglement transitions in random tree tensor networks, Phys. Rev. B **102**, 064202 (2020).
[15] S. Vijay, Measurement-driven phase transition within a volume-law entangled phase, arXiv preprint arXiv:2005.03052 (2020).
[16] O. Lunt, M. Szyniszewski, and A. Pal, Measurement-induced criticality and entanglement clusters: A study

of one-dimensional and two-dimensional clifford circuits, Phys. Rev. B **104**, 155111 (2021).
[17] A. Nahum, S. Roy, B. Skinner, and J. Ruhman, Measurement and entanglement phase transitions in all-to-all quantum circuits, on quantum trees, and in landau-ginsburg theory, PRX Quantum **2**, 010352 (2021).
[18] P. Sierant, M. Schirò, M. Lewenstein, and X. Turkeshi, Measurement-induced phase transitions in $(d+1)$-dimensional stabilizer circuits, Phys. Rev. B **106**, 214316 (2022).
[19] X. Feng, B. Skinner, and A. Nahum, Measurement-induced phase transitions on dynamical quantum trees, PRX Quantum **4**, 030333 (2023).
[20] C. Noel, P. Niroula, D. Zhu, A. Risinger, L. Egan, D. Biswas, M. Cetina, A. V. Gorshkov, M. J. Gullans, D. A. Huse, *et al.*, Measurement-induced quantum phases realized in a trapped-ion quantum computer, Nature Physics **18**, 760 (2022).
[21] J. M. Koh, S.-N. Sun, M. Motta, and A. J. Minnich, Measurement-induced entanglement phase transition on a superconducting quantum processor with mid-circuit readout, Nature Physics **19**, 1314 (2023).
[22] G. Q. AI and Collaborators, Measurement-induced entanglement and teleportation on a noisy quantum processor, Nature 10.1038/s41586-023-06505-7 (2023).
[23] H. Kamakari, J. Sun, Y. Li, J. J. Thio, T. P. Gujarati, M. P. A. Fisher, M. Motta, and A. J. Minnich, Experimental demonstration of scalable cross-entropy benchmarking to detect measurement-induced phase transitions on a superconducting quantum processor (2024), arXiv:2403.00938 [quant-ph].
[24] Y. Li, Y. Zou, P. Glorioso, E. Altman, and M. P. A. Fisher, Cross entropy benchmark for measurement-induced phase transitions, Phys. Rev. Lett. **130**, 220404 (2023).
[25] F. Barratt, U. Agrawal, A. C. Potter, S. Gopalakrishnan, and R. Vasseur, Transitions in the learnability of global charges from local measurements, Phys. Rev. Lett. **129**, 200602 (2022).
[26] S. J. Garratt and E. Altman, Probing postmeasurement entanglement without postselection, PRX Quantum **5**, 030311 (2024).
[27] T. Minato, K. Sugimoto, T. Kuwahara, and K. Saito, Fate of measurement-induced phase transition in long-range interactions, Phys. Rev. Lett. **128**, 010603 (2022).
[28] M. Block, Y. Bao, S. Choi, E. Altman, and N. Y. Yao, Measurement-induced transition in long-range interacting quantum circuits, Phys. Rev. Lett. **128**, 010604 (2022).
[29] T. Müller, S. Diehl, and M. Buchhold, Measurement-induced dark state phase transitions in long-ranged fermion systems, Phys. Rev. Lett. **128**, 010605 (2022).
[30] A. Zabalo, J. H. Wilson, M. J. Gullans, R. Vasseur, S. Gopalakrishnan, D. A. Huse, and J. H. Pixley, Infinite-randomness criticality in monitored quantum dynamics with static disorder, Phys. Rev. B **107**, L220204 (2023).
[31] G. Shkolnik, A. Zabalo, R. Vasseur, D. A. Huse, J. H. Pixley, and S. Gazit, Measurement induced criticality in quasiperiodic modulated random hybrid circuits, Phys. Rev. B **108**, 184204 (2023).
[32] T. Rakovszky, F. Pollmann, and C. W. von Keyserlingk, Sub-ballistic growth of rényi entropies due to diffusion, Phys. Rev. Lett. **122**, 250602 (2019).
[33] U. Agrawal, A. Zabalo, K. Chen, J. H. Wilson, A. C. Potter, J. H. Pixley, S. Gopalakrishnan, and R. Vasseur, Entanglement and charge-sharpening transitions in u(1) symmetric monitored quantum circuits, Phys. Rev. X **12**, 041002 (2022).
[34] F. Barratt, U. Agrawal, S. Gopalakrishnan, D. A. Huse, R. Vasseur, and A. C. Potter, Field theory of charge sharpening in symmetric monitored quantum circuits, Phys. Rev. Lett. **129**, 120604 (2022).
[35] S. Majidy, U. Agrawal, S. Gopalakrishnan, A. C. Potter, R. Vasseur, and N. Y. Halpern, Critical phase and spin sharpening in su(2)-symmetric monitored quantum circuits, Phys. Rev. B **108**, 054307 (2023).
[36] A. Chakraborty, K. Chen, A. Zabalo, J. H. Wilson, and J. H. Pixley, Charge and entanglement criticality in a u(1)-symmetric hybrid circuit of qubits, Phys. Rev. B **110**, 045135 (2024).
[37] At special points where the critical measurement rate does not depend linearly on the density, the coupling to diffusion may be irrelevant. We will not consider this case in the present work.
[38] M. J. Gullans and D. A. Huse, Scalable probes of measurement-induced criticality, Phys. Rev. Lett. **125**, 070606 (2020).
[39] See the Supplemental Material for additional details (see also references [48–52] therein).
[40] T. M. Liggett, *Stochastic Interacting Systems: Contact, Voter and Exclusion Processes*, 1st ed. (Springer Berlin, Heidelberg, 1999).
[41] C. Kipnis, S. Olla, and S. S. Varadhan, Hydrodynamics and large deviation for simple exclusion processes, Communications on Pure and Applied Mathematics **42**, 115 (1989).
[42] G. Eyink, J. L. Lebowitz, and H. Spohn, Hydrodynamics of stationary non-equilibrium states for some stochastic lattice gas models, Communications in mathematical physics **132**, 253 (1990).
[43] T. Vojta and R. Dickman, Spatiotemporal generalization of the harris criterion and its application to diffusive disorder, Phys. Rev. E **93**, 032143 (2016).
[44] H. Ando and D. A. Huse, Conserved quantities in models of classical chaos, arXiv:2307.11733 (2023).
[45] A. B. Harris, Effect of random defects on the critical behaviour of ising models, Journal of Physics C: Solid State Physics **7**, 1671 (1974).
[46] K. Klocke, J. E. Moore, and M. Buchhold, Power-law entanglement and hilbert space fragmentation in nonreciprocal quantum circuits, Phys. Rev. Lett. **133**, 070401 (2024).
[47] S. Krastanov, P. Viswanathan, S. Pardis, A. Micciche, J. Lapeyre, F. Ahmad, Rabqubit, gsommers, T. Hofmann, A. Bhatt, A. Meligrana, Benzillaist, C. Zhao, IsaacP1234, ShuGe-MIT, T. Holy, T. Dang, adrianariton, and ismoldayev, Quantumsavory/quantumclifford.jl: v0.9.9 (2024).
[48] N. Kawashima and N. Ito, Critical behavior of the three-dimensional ±j model in a magnetic field, Journal of the Physical Society of Japan **62**, 435 (1993), https://doi.org/10.1143/JPSJ.62.435.
[49] S. Bravyi and B. Terhal, A no-go theorem for a two-dimensional self-correcting quantum memory based on stabilizer codes, New Journal of Physics **11**, 043029 (2009).
[50] M. Ippoliti, M. J. Gullans, S. Gopalakrishnan, D. A. Huse, and V. Khemani, Entanglement phase transitions

in measurement-only dynamics, Phys. Rev. X **11**, 011030 (2021).
[51] A. Nahum, J. Ruhman, S. Vijay, and J. Haah, Quantum entanglement growth under random unitary dynamics, Phys. Rev. X **7**, 031016 (2017).
[52] S. Torquato, Hyperuniform states of matter, Physics Reports **745**, 1 (2018).