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Detecting symmetry fractionalization in gapped quantum spin liquids by magnetic impurities

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We study the Kondo effect of spin-1/2 magnetic impurities in gapped Z_2 spin liquids on two-dimensional lattices. We find that if the impurity is placed at a high-symmetry location, a nontrivial spinon fractionalization class of the impurity site symmetry group will necessarily lead to a non-Kramers doublet in the Kondo screening regime, protected by associated crystalline symmetries. This is in sharp contrast to a featureless screening phase in the usual Kondo effect. We demonstrate this symmetry-protected topological degeneracy by an exactly solvable model and by the large-N theory. Based on this effect, we discuss how thermodynamic measurements in the limit of dilute magnetic impurities can be used to detect symmetry fractionalization in gapped Z_2 spin liquids.

I. INTRODUCTION

Quantum spin liquids (QSLs) [1–3] have attracted much interests in the past few decades due to its exotic properties transcending the Landau scheme of symmetry breaking. In particular, the presence of anyons which obey fractional statistics[4], is among the most exciting manifestations of the topological order and long-range entanglement in QSLs[5, 6], with potential applications in topological quantum computations [7]. A number of QSL candidate materials with various crystalline symmetries have been discovered experimentally [3, 8–10].

Meanwhile, there is a gap between theoretical diagnosis and experimental measurements to identify QSLs. On one hand, various theoretically computable quantities has been proposed to sharply characterize topological orders, such as the topological entanglement entropy[11, 12] and modular matrices[13, 14]. On the other hand, most existing experiments aim at ruling out long-range orders in the low temperature, deterred by the difficulty of directly probing unique features of QSLs. In particular, compared to gapless $U(1)$ spin liquids with clear signatures in inelastic neutron scattering (INS) [15] or thermal transport [3, 8], a gapped symmetric QSL is more featureless and harder to detect experimentally. While the long-range entanglement and fractional statistics, as a definitive character of topological orders, is difficult to access experimentally, the fractional symmetry quantum number[16] (formally known as symmetry fractionalization[5, 17–20]) of anyons provide extra features to characterize and identify the topological order, which is usually easier to probe experimentally. In the well known example of fractional quantum Hall effects (FQHEs), indeed the fractional charge is experimentally observed in the nineties, much earlier than the recently confirmed fractional statistics[21]. One question arises naturally: can symmetry fractionalization be experimentally detected as a direct evidence for a QSL state?

For gapped QSLs, which is the focus of this work, there are two major challenges to experimental detection of fractionalization. First of all, the fractionalized excitations such as spinons are charge neutral, therefore insensitive to charge transport probes which played a crucial role in identifying fractionalization in FQHEs[21]. Secondly, a gapped symmetric QSL usually has no features both in the bulk and on the boundary, making it very hard to access experimentally. This is unlike the $U(1)$ spin liquids, whose gapless excitations can be probed by INS in the case of emergent photons[15], or thermal transport in the case of spinon Fermi surfaces[3, 8]. Is it possible to experimentally identify a gapped QSL? Previously, INS spectroscopy has been proposed to exhibit features of fractional statistics[22] and spinon symmetry fractionalization[5, 23]. Recently two-dimensional coherent nonlinear spectroscopy has been suggested to reveal both fractionalized spinon excitations[24, 25] and their fractional statistics[26] in quantum spin liquids. In this work, we look into magnetic impurities and Kondo effects in gapped QSLs, and show that they can provide distinct thermodynamic signatures of symmetry frationalization in QSLs, in the Kondo screening regime.

The Kondo effect in QSLs has previously been studied both in theories [27–34] and in experiments [35–37], focusing on gapless QSLs. In this paper, we explore the Kondo effect in gapped Z_2 QSLs. Similar to the distinctions between Kondo effects in metals and in insulators (with a vanishing density of states), the Kondo effect in gapped Z_2 QSLs differs qualitatively from gapless QSLs. In particular, due to the energy gap for spinon excitations, there is a finite threshold of Kondo coupling strength to screen the magnetic impurity[38–43]. Most remarkably, we find that when a half-integer-spin impurity is placed at a high-symmetry location in the crystal hosting a gapped Z_2 QSL, the Kondo screening phase will feature a non-Kramers doublet localized at the impurity site, protected by fractionalized crystalline symmetries in the Z_2 QSL. This symmetry protected degeneracy lead to distinct signatures in the thermodynamics, such as specific heat, which can serve as "smoking gun" evidence for symmetry fractionalization in a gapped QSL. This phenomena is demonstrated by an exactly solvable model and large-N parton mean-field theory, which agree with each other.

II. MAIN RESULTS

We first present the major results in Fig. 1 and TABLEs I-III. Consider a gapped symmetric Z_2 spin liquid on a two-dimensional lattice, whose Hilbert space consists of a spin-

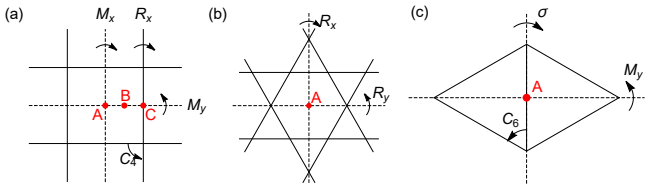


FIG. 1: High-symmetry impurity sites that can be used to detect the symmetry fractionalization of spinons in a gapped symmetric Z_2 spin liquid, on the (a) square, (b) kagome and (c) triangular lattices in two dimensions.

1/2 (or a Kramers doublet) on each lattice site. A spin-1/2 magnetic impurity located at certain high-symmetry position of the lattice can be used to diagnose the symmetry fractionalization class[17–19] of the Z_2 spin liquid phase. Specifically, when such a magnetic impurity is coupled symmetrically to the Z_2 spin liquid, in the Kondo screening regime, there may or may not be a two-fold degeneracy (a non-Kramers doublet) protected by the crystalline symmetry of the impurity site, depending on the fractionalization class of spinons in the Z_2 spin liquid.

Fig. 1 illustrates the three lattices enumerated in this work, i.e. the square, kagome and triangular lattices. In the presence of $SO(3)$ spin rotational symmetry, the classification of symmetric Z_2 spin liquids on these lattices[44, 45] are summarized in TABLE I-III. A part of the symmetry fractionalization data can be detected by presence/absence of non-Kramers doublets for Kondo-screened magnetic impurities located at different high-symmetry sites, such as a plaquette center, a nearest-neighbor link center, or on a mirror plane.

Algebraic identity	$\omega \in \mathcal{H}^2(G,\mathcal{A})$	ω^e[46]	ω^ϵ[5]	Impurity site
$(R_x)^2$	ω_{R_x,R_x}	$(-1)^{p_4}$	η_σ	-
$(M_y)^2$	ω_{M_y,M_y}	$(-1)^{p_3+p_4}$	$\eta_\sigma\eta_{xpx}$	-
$(C_4R_x)^2$	$\omega_{C_4R_x,C_4R_x}$	$(-1)^{p_4+p_7}$	$\eta_\sigma\eta_{\sigma C_4}$	-
$M_xM_yM_x^{-1}M_y^{-1}$	$\frac{\omega_{M_x,M_y}}{\omega_{M_y,M_x}}$	$(-1)^{p_1}$	η_{xy}	A
$(M_y\mathcal{T})^2$	$\omega_{M_y\mathcal{T},M_y\mathcal{T}}$	$(-1)^{p_3+p_8+1}$	$-\eta_t\eta_{xpx}$	B
$R_xM_yR_x^{-1}M_y^{-1}$	$\frac{\omega_{R_x,M_y}}{\omega_{M_y,R_x}}$	$(-1)^{p_2}$	η_{xpy}	C

TABLE I: All gapped Z_2 spin liquids of spin-1/2's on the square lattice ($G = p4m \times Z_2^{\mathcal{T}}$), characterized by 2^6 fractionalization classes[44, 45], and their realizations in the Schwinger boson[46] and Abrikosov fermion[5] representations. 3 of the 6 independent Z_2 invariants can be detected by magnetic impurities located at A, B and C sites in Fig. 1(a).

In a gapped system with a vanishing density of states, a finite Kondo coupling strength is required to enter the Kondo screening regime[38–43], where the system typically reaches a featureless paramagnetic ground state. The proposed non-Kramers doublet protected by crystalline symmetries at the impurity site of the Z_2 spin liquid is therefore a striking and unusual phenomenon. Below we describe the physical picture behind this observation.

Algebraic identity	$\omega \in \mathcal{H}^2(G,\mathcal{A})$	ω^e[46]	ω^ϵ[5]	Impurity site
$(R_x)^2$	ω_{R_x,R_x}	$(-1)^{p_2+p_3}$	η_σ	-
$(R_y)^2$	ω_{R_y,R_y}	$(-1)^{p_2}$	$\eta_\sigma\eta_{\sigma C_6}$	-
$R_xR_yR_x^{-1}R_y^{-1}$	$\frac{\omega_{R_x,R_y}}{\omega_{R_y,R_x}}$	$(-1)^{p_1}$	η_{12}	A

TABLE II: All gapped Z_2 spin liquids of spin-1/2's on the kagome lattice ($G = p6mm \times Z_2^{\mathcal{T}}$), characterized by 2^3 fractionalization classes[44, 45], and their realizations in the Schwinger boson[47] and Abrikosov fermion[48] representations. 1 of the 3 independent Z_2 invariants can be detected by magnetic impurities located at A site in Fig. 1(b).

Algebraic identity	$\omega \in \mathcal{H}^2(G,\mathcal{A})$	ω^e[46]	ω^ϵ[5]	Impurity site
σ^2	$\omega_{\sigma,\sigma}$	$(-1)^{p_2}$	η_σ	-
$(M_y)^2$	ω_{M_y,M_y}	$(-1)^{p_2+p_3}$	$\eta_{\sigma C_6}$	-
$\sigma M_y\sigma^{-1}M_y^{-1}$	$\frac{\omega_{\sigma,M_y}}{\omega_{M_y,\sigma}}$	$(-1)^{p_1}$	η_{12}	A

TABLE III: All gapped Z_2 spin liquids of spin-1/2's on the triangular lattice ($G = p6mm \times Z_2^{\mathcal{T}}$), characterized by 2^3 fractionalization classes[44, 45], and their realizations in the Schwinger boson[47] and Abrikosov fermion[49] representations. 1 of the 3 independent Z_2 invariants can be detected by magnetic impurities located at A site in Fig. 1(c).

A gapped Z_2 spin liquid hosts three types of anyons (or superselection sectors): bosonic spinon e, vison m, and their bound state $\epsilon = e \times m$ known as a fermionic spinon[50]. In a symmetric Z_2 spin liquid on a lattice with an odd number of spin-1/2's in each unit cell, spinons e and ϵ must carry spin-1/2 each, while vison m is spinless[51]. As a result, when a spin-1/2 magnetic impurity is coupled to such a Z_2 spin liquid, to reach a spin-singlet ground state, it can only be screened by a spinon e or ϵ. The same conclusion holds if we replace spin-1/2 by a Kramers doublet with $\mathcal{T}^2 = -1$ in the argument.

In the presence of crystalline and time reversal symmetries, different gapped Z_2 spin liquids are distinguished by their symmetry fractionalization classes, classified by 2nd group cohomology $\mathcal{H}^2(G,\mathcal{A})$, where the symmetry group is $G = SG \times Z_2^{\mathcal{T}}$ (SG being the space group), and $\mathcal{A} = Z_2 \times Z_2$ is the fusion group of Abelian anyons in the Z_2 spin liquid[17]. Thanks to the $SO(3)$ spin rotational symmetry, the vison fractionalization class is uniquely fixed on the three lattices[52, 53], leading to the classification shown in Table I-III[44, 45], characterized by projective representations of G carried by spinons. Consequently, the singlet bound state formed by the impurity spin and the screening cloud of spinon can carry a projective representation of the impurity site symmetry group. This leads to the non-Kramers degeneracy at the impurity site, protected by both crystalline and time reversal symmetries.

There are two types of impurity sites of particular interests to this work: (i) the impurity site lies at the intersection of two mirror planes, such as site A and C in Fig.1; (ii) the impurity site lies on a mirror plane, such as site B in Fig. 1. In case (i), since the two mirror symmetries M_x and M_y commute in the Hilbert space of the impurity spin, if they

anticommute on the spinon screening the impurity spin (i.e. $\omega_{M_x,M_y}/\omega_{M_y,M_x} = -1$), the bound state of impurity spin-1/2 and spinon will carry a projective representation of the site symmetry group $G_s = Z_2^{M_x} \times Z_2^{M_y}$, leading to a 2-fold degeneracy protected by the mirror symmetries. On the other hand, if the two mirror actions commute on the spinon (i.e. $\omega_{M_x,M_y}/\omega_{M_y,M_x} = +1$), the bound state will carry a linear representation of the site symmetry and hence no degeneracy in the screening regime. In case (ii), the impurity spin-1/2 on a mirror (M_x) plane carries a projective representation $(M_x\mathcal{T})^2 = -1$. If the spinon screening the impurity carry a linear representation with $\omega_{M_x\mathcal{T},M_x\mathcal{T}} = +1$, their bound state forms a projective representation of the site symmetry group $G_s = Z_2^{M_x} \times Z_2^{\mathcal{T}}$, leading to a 2-fold non-Kramers degeneracy protected by both mirror M_x and time reversal symmetries. In contrary, if the spinon carries a projective representation with $\omega_{M_x\mathcal{T},M_x\mathcal{T}} = -1$, the bound state would instead form a linear representation $(M_x\mathcal{T})^2 = +1$ with no degeneracy in the screened phase. This shows how the response of a Z_2 spin liquid to impurity spin-1/2's can diagnose a part of the spinon fractionalization data in the spin liquid phase, as summarized in Table I-III. While this manuscript focuses on Kondo impurities in Z_2 spin liquids, the theory framework can be applied to the impurity problem in a general symmetry enriched topological order[17–19], which we discussed in section A.3 of the supplemental materials[54] (see also the reference [55] therein).

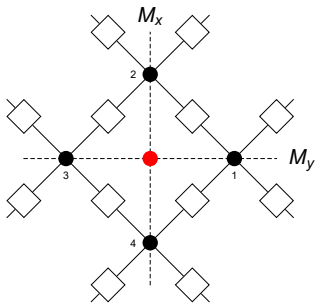


FIG. 2: An exactly solvable model for a spin-1/2 impurity in a Z_2 spin liquid with $SU(2)$ symmetry. The squares on link centers are qubits in the toric code[50]. Each black dot on a star/vertex represents a Hilbert space of spin $0 \oplus 1/2$. The red dot denotes the spin-1/2 impurity with a site symmetry $G_s = Z_2^{M_x} \times Z_2^{M_y}$[54].

III. METHODS

We use two methods to demonstrate the symmetry-protected non-Kramers degeneracy induced by a spin-1/2 impurity in Z_2 spin liquids, in the Kondo screening regime. We focus on case (i) of impurity site symmetry $G_s = Z_2^{M_x} \times Z_2^{M_y}$, where the impurity spin-1/2 lies at the intersection of two mirror planes $M_{x,y}$. First we construct an exactly solvable model by modifying the toric code, to show the exact degeneracy protected by two mirror symmetries. Next we use the large-N approach to solve the Kondo problem in a symmetric Z_2 spin liquid, and to compute the temperature dependence of thermodynamic quantities.

First we present an exactly solvable model illustrated in Fig. 2. The bulk Z_2 spin liquid is described by:

$$\begin{aligned}\hat{H}_{bulk} = &-\sum_s A_s - \sum_p B_p \\ &- \sum_s \Delta[\frac{(A_s+1)}{2} P_s(S=0) + \frac{(1-A_s)}{2} P_s(S=1/2)]\end{aligned} \tag{1}$$

where A_s and B_p are the star and plaquette operators in Kitaev's toric code[50]. In addition to one qubits on each link, there is a 3-dimensional Hilbert space of spin $0 \oplus \frac{1}{2}$ on each site/vertex (see Fig. 2). In the limit $\Delta \gg 1$, each e and ϵ particle will carry spin-1/2 of the site/vertex Hilbert space, while m particles are spinless. The fractionalization class associated with the impurity site symmetry group $G_s = Z_2^{M_x} \times Z_2^{M_y}$ is given by[17]

$$\frac{\omega^e_{M_x,M_y}}{\omega^e_{M_y,M_x}} = \frac{\omega^m_{M_x,M_y}}{\omega^m_{M_y,M_x}} = +1, \quad \frac{\omega^\epsilon_{M_x,M_y}}{\omega^\epsilon_{M_y,M_x}} = -1. \tag{2}$$

Next we introduce a spin-1/2 impurity located at the center of plaquette (1234) in Fig. 2, which is coupled to the bulk spin liquid as follows:

$$H_{imp} = J \sum_{i=1}^{4} \vec{S}_i \cdot \vec{S}_{imp} + E_c(A_1+A_2+A_3+A_4-3)^2 + \Delta_\epsilon B_p[1234] \tag{3}$$

In addition to the usual Kondo coupling J, we also introduce (i) a Coulomb repulsion E_c for spinons, which makes sure the impurity is screened by one spinon, and (ii) an energy Δ_ϵ coupled to the plaquette opeartor on plaquette [1234], to control which type of spinons (e vs. ϵ) will screen the impurity spin. Assuming $E_c \gg 1, J$, the Kondo screening regime happens when $J > 4/3$, leading to 4 degenerate states in the low energy manifold where a single spinon is located at one neighboring site (out of $1, 2, 3, 4$). When $\Delta_\epsilon < 1$, the bosonic spinons cost less energy and will screen the impurity, and the 4-fold degenerate can be completely lifted with a unique paramagnetic ground state without breaking any symmetry[54]. This is consistent with the trivial fractionalization class of e particles in (2). When $\Delta_\epsilon > 1$, however, the fermionic spinons cost lower energy and are responsible for the Kondo screening. As detailed in supplemental materials[54], the 4-dimensional low energy space can be split into two doublets, each of which form an irreducible projective representation of the impurity site symmetry group $G_s = Z_2^{M_x} \times Z_2^{M_y}$. As a result, a 2-fold degeneracy protected by two mirror symmetries $M_{x,y}$ will emerge in the Kondo screening regime, as indicated by the nontrivial fractionalization class (2) of ϵ particles. Therefore we have demonstrated the correspondence between nontrivial fractionalization class of spinons screening the impurity, and protected 2-fold degeneracy in the Kondo screening regime.

Next we use a large-N mean field theory to solve the Kondo problem in symmetric Z_2 spin liquids. Both the bulk and impurity spins are represented by fermonic partons with $Sp(2N)$

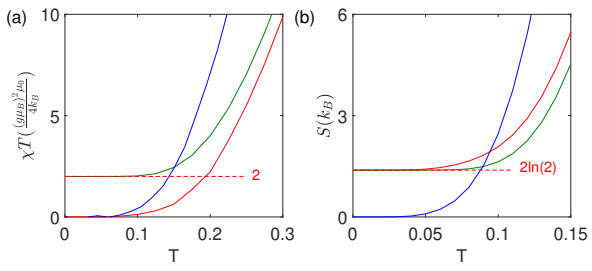


FIG. 3: The temperature dependence of (a) uniform magnetic susceptibility $\chi(T)$ and (b) entropy $S(T)$ contributed by Kondo impurities in different regimes: the unscreened regime of free moments at the impurity sites (green), the Kondo screening regime in Z_2 spin liquids with a trivial (blue) vs. nontrivial (red) spinon fractionalization class. The calculations are performed for two distant impurities with site symmetry $G_s = Z_2^{M_x} \times Z_2^{M_y}$ on a 20×20 lattice[54].

symmetry[56]:

$$S^{ab+} = \frac{1}{2}(c_\uparrow^{a\dagger} c_\downarrow^b + c_\uparrow^{b\dagger} c_\downarrow^a), \quad S^{ab,z} = \frac{1}{2}(c_\uparrow^{a\dagger} c_\uparrow^b - c_\downarrow^{b\dagger} c_\downarrow^a) \quad (4)$$

with $1 \le a, b \le N$. They reduce to the familiar $SU(2)$ spin-1/2 case when $N = 1$. The model consists of a Z_2 spin liquid in the bulk described by parton mean-field ansatz of $Sp(2N)$ partons

$$H_{bulk} = \sum_{a=1}^{N} \sum_{i,j} J_{i,j} \psi_i^{a,\dagger} u_{i,j} \psi_j^a + h.c. \quad (5)$$

where we denote $\psi_i^a = (c_{i,\uparrow}^a, c_{i,\downarrow}^{a\dagger})^T$, and the Kondo coupling between a $Sp(2N)$ impurity spin and its neighboring spins:

$$H_{imp} = \sum_{\langle j,imp \rangle} \frac{J}{N} \mathbf{S}_j^{ab} \cdot \mathbf{S}_{imp}^{ba} + \frac{J'}{N^3} (\mathbf{S}_j^{ab} \cdot \mathbf{S}_{imp}^{ba})^2 \quad (6)$$

As detailed in supplemental materials[54] (see also the reference [57] therein), the bulk parton ansatz can be exactly realized in solvable models in analogy to Kitaev's honeycomb model[58], and choosing different link parameters $\{u_{ij}\}$ can lead to either trivial or nontrivial fractionalization classes for fermionic spinons $\{\psi_i^a\}$, with $M_x M_y M_x^{-1} M_y^{-1} = \pm 1$. A self-consistent mean-field calculation, which becomes exact in the large N limit, reveals a Kondo screening phase separated from the unscreened phase by a Kondo temperature $T_K(J)$, for Kondo couplings beyond a finite threshold $J > J_c$[54]. In the Kondo screening regime with $T < T_K(J)$, while the trivial fractionalization class ($M_x M_y M_x^{-1} M_y^{-1} = +1$) shows a unique paramagnetic ground state, the nontrivial fractionalization class ($M_x M_y M_x^{-1} M_y^{-1} = -1$) exhibits two degenerate ground states which cannot be mixed by any local perturbations preserving mirror symmetries $M_{x,y}$[54]. This again demonstrated our conclusion, that a nontrivial spinon fractionalization class will lead to symmetry protected zero modes localized at high-symmetry impurity sites.

IV. EXPERIMENTAL IMPLICATIONS

The large-N mean field theory also allows us to predict distinct experimental signatures of the anomalous Kondo screening phase described above. The temperature dependence of uniform magnetic susceptibility $\chi(T)$ and the entropy $S(T) = \int_0^T \frac{C_v(t)}{t} dt$ are shown in Fig. 3. The shown impurity contribution shown in the figure susceptibility $\chi(T)$ and specific heat $C_v(T)$, by subtracting the bulk contribution of Hamiltonian H_{bulk} from the total amount of $H_{bulk} + H_{imp}$. Three different regimes can be differentiated from each other by inspecting the susceptibility and entropy (by integrating the specific heat) at low temperatures: (i) In the unscreened regime, the magnetic impurity behaves as a free moment, leading to $\chi(T) \sim 1/T$ and a finite entropy of $k_B \ln 2$ per impurity, colored green in Fig. 3. This scenario also applies to the situations where the magnetic impurity is under-screened or over-screened, leaving a residual local moment near the impurity. (ii) In the Kondo screening regime, for Z_2 spin liquid with trivial fractionalization class, Kondo screening leaves a unique paramagnetic ground state below the Kondo temperature, and therefore exponentially decaying thermodynamic responses $\chi(T), C_v(T) \sim e^{-\Delta/k_B T}$, as colored blue in Fig. 3. (iii) A Z_2 spin liquid with a nontrivial fractionalization class, on the other hand, features a symmetry protected non-Kramers doublet (2-fold degeneracy) localized at each impurity site in the Kondo screening regime. As a result, while the susceptiblity vanishes exponentially at low temperatures $\chi(T) \sim e^{-\Delta/k_B T}$, there is a low energy entropy of $k_B \ln 2$ per impurity below the Kondo temperature, as colored red in Fig. 3. The sharp differences between the three scenarios provide clear experimental features to detect a gapped Z_2 spin liquid with a nontrivial symmetry fractionalization class for spinons.

Theoretically we only discussed the case of isolated impurities in the large-N self-consistent mean-field theory described earlier. In real materials, we expect our predictions in Fig. 3 to hold in the case of *dilute magnetic impurities*, where the average distance r between neighboring impurities is much larger than the bulk correlation length ξ of the gapped spin liquid. In this case, the splitting of the symmetry protected zero modes localized at impurity sites will be small $\sim Je^{-C_0 r/\xi}$, where C_0 is a constant of order one. This will lead to a peak in specific heat at low temperature $T \sim Je^{-C_0 r/\xi}$. The splitting of protected degeneracy can also be caused by crystalline symmetry breaking due to e.g. Jahn-Teller effect, which can smear out the plateau of residue entropy in Fig. 3(b). Detailed calculations regarding broken crystalline symmetry are shown in section C.5 in supplemental materials[54].

Among the candidate materials of quantum spin liquids, Herbertsmithite [59] is most relevant to our proposal. Nuclear magnetic resonance measurements[60] pointed to a spin liquid ground state with a finite gap of around 10 K, a small fraction of the intralayer Cu-Cu exchange interaction $J \simeq 200$ K. Previous resonant X-ray scattering results revealed 15% intersite impurities[61] between the kagome layers of Cu^{2+} ions, where excessive spin-1/2 Cu^{2+} ions occupy the interlayer A sites in Fig. 1(b) ("intersites") of Zn^{2+} ions. Neutron scattering results[62] suggested a coupling strength of about 10 K

between the intersite Cu impurities, comparable to the bulk gap in kagome layer. Therefore, Herbertsmithite is a potential material candidate our theory proposal can apply to.

V. CONCLUDING REMARKS

In this work, we show that when magnetic impurities with half integer spins are coupled to symmetric Z_2 spin liquids in an isotropic magnet with $SU(2)$ symmetry, a spinon will form a singlet bound state with the impurity in the Kondo screening regime. This bound state will feature a symmetry protected 2-fold degeneracy, if the spinon fractionalization class of the impurity site symmetry is nontrivial, therefore leading to a non-Kramers doublet localized at the impurity site. We further show that this local degeneracy in the Kondo screening regime can be distinguished from other scenarios by the low temperature behaviors of magnetic susceptibiltiy and specific heat, hence unveiling a new way to detect symmetry fractionalization in QSLs.

In the future, it will be insightful to apply the new angle proposed in this work to examine the candidate materials of QSLs, where magnetic impurities are known to exist at high symmetry sites e.g. in Herbertsmithite[60]. Theoretically, this work also provides a new idea to probe fractionalization in models of strongly correlated electrons, by studying the impurity problem e.g. using numerical methods.

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