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Even-odd effect on robustness of Majorana edge states in short Kitaev chains

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We construct two-band effective models describing Majorana edge states in a short Kitaev chain. We derive an analytical formula for the effective Hamiltonian as a function of model parameters. Then, we discuss the robustness of Majorana edge states as a function of model parameters. We have found an even-odd effect on the robustness of Majorana states, which is characteristic to short Kitaev chains. It is experimentally observable as a differential conductance in quantum dot systems. We also study effects of coupling to an environment based on non-Hermitian Hamiltonians derived from the Lindblad equation. It is found that the Majorana zero-energy edge states acquire nonzero energy such as $E \propto \pm(i\Gamma)^L$ for the local dissipation, where Γ is the magnitude of the dissipation and L is the length of the chain. The even-odd effect is manifest for small L in this formula. On the other hand, the Majorana zero-energy edge states acquire nonzero energy such as $E \propto \pm i\Gamma$ for small Γ irrespective of the length L for the global dissipation.

Introduction: Majorana fermions are basic ingredient of topological quantum computation[1–6]. The Majorana edge states form a nonlocal qubit, which is robust against local perturbation due to topological protection. Thus, the qubit based on Majorana fermions will resolve the problem of decoherence in quantum computation. Majorana fermions must be materialized in topological superconductors[7–10], although its experimental realization is still controversial. A simplest model of a topological superconductor hosting Majorana fermions is the Kitaev chain[11]. Despite the simplicity of the model, it is hard to materialize it because it is hard to realize the p-wave superconducting order on the lattice. Although there are several proposals on the detection of the signature of Majorana fermions including fractional quantum Hall effects and Kitaev spin liquids, no compelling evidence of Majorana fermions has been obtained.

According to recent reports[12, 13], the two-site and three-site Kitaev chains were experimentally realized in quantum dots. They evoke studies on the minimal Kitaev chain based on double quantum dots[14–21] and a few quantum dots[22–24]. For future topological quantum computation, it is desirable to materialize Majorana states with a minimal component, where the short Kitaev chain is an ideal platform. However, the Majorana edge states in a short Kitaev chain are not topologically protected. Indeed, we need a precise tuning of the model parameters so that the Majorana edge states exist exactly at the zero energy. There are studies on the condition for the Majorana edge states to have exact zero energy[25–29]. It is intriguing to estimate the robustness of the Majorana edge states for a short Kitaev chain. It is discussed[11] that the robustness of the Majorana zero-energy states increases exponentially as a function of the length of the Kitaev chain. Eventually, if the length of the Kitaev chain is long enough, the Majorana edge states are topological protected.

The platform of the Majorana fermions such as a quantum dot system has an interaction with another system such as a substrate. In general, the coupling to bath makes the system an open quantum system. It is commonly analyzed based on the Lindblad equation[30]. The short-time dynamics is well described by a non-Hermitian Hamiltonian derived from the Lindblad equation[30]. A Kitaev chain with loss and gain has been studied in the context of non-Hermitian Hamiltonian for a sufficiently long one[31–44]. A Kitaev chain interacting with environment has also been studied for a sufficiently long one[45–47].

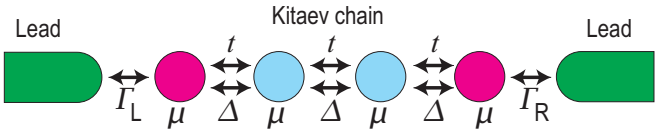


FIG. 1. Illustration of the Kitaev model with length $L = 4$ coupled with two leads, where μ is the chemical potential, $t > 0$ is the nearest-neighbor hopping strength, Δ is the p-wave pairing amplitude of the superconductor. Magenta disks indicate the Majorana edge states. $\Gamma_{\rm L}$ ($\Gamma_{\rm R}$) is a coupling between the left (right) quantum dot and the left (right) lead.

In this paper, we investigate numerically and analytically the robustness of the Majorana edge states in a short Kitaev chain. We construct an effective two-band model by using the isospectral matrix reduction method[48, 49]. First, we study the Hamiltonian in the absence of dissipation. We have found an even-odd effect on the robustness of Majorana states as a function of the length L of the Kitaev chain, which is characteristic to short chains. It is possible to observe this phenomenon by measuring the differential conductance in quantum dot systems. Next, we study the Hamiltonian in the presence of local dissipation. We show that the Majorana edge states has energy $E \propto \pm(i\Gamma)^L$, where the even-odd effect with respect to L is manifest.

Kitaev chain: The Kitaev p-wave superconductor model is defined on the 1D lattice as[8, 11]

$$\hat{H} = -\mu\sum_{x=1}^{L} c_x^\dagger c_x - t\sum_{x=1}^{L-1}\left(c_x^\dagger c_{x+1} + c_{x+1}^\dagger c_x\right) - \sum_{x=1}^{L-1}\left(\Delta c_x c_{x+1} + \Delta c_{x+1}^\dagger c_x^\dagger\right), \quad (1)$$

where μ is the chemical potential, $t > 0$ is the nearest-neighbor hopping strength, Δ is the p-wave pairing amplitude of the superconductor, and L is the length of the chain. Illustration of the Kitaev chain is shown in Fig.1.

This model hosts two Majorana zero-energy edge states at

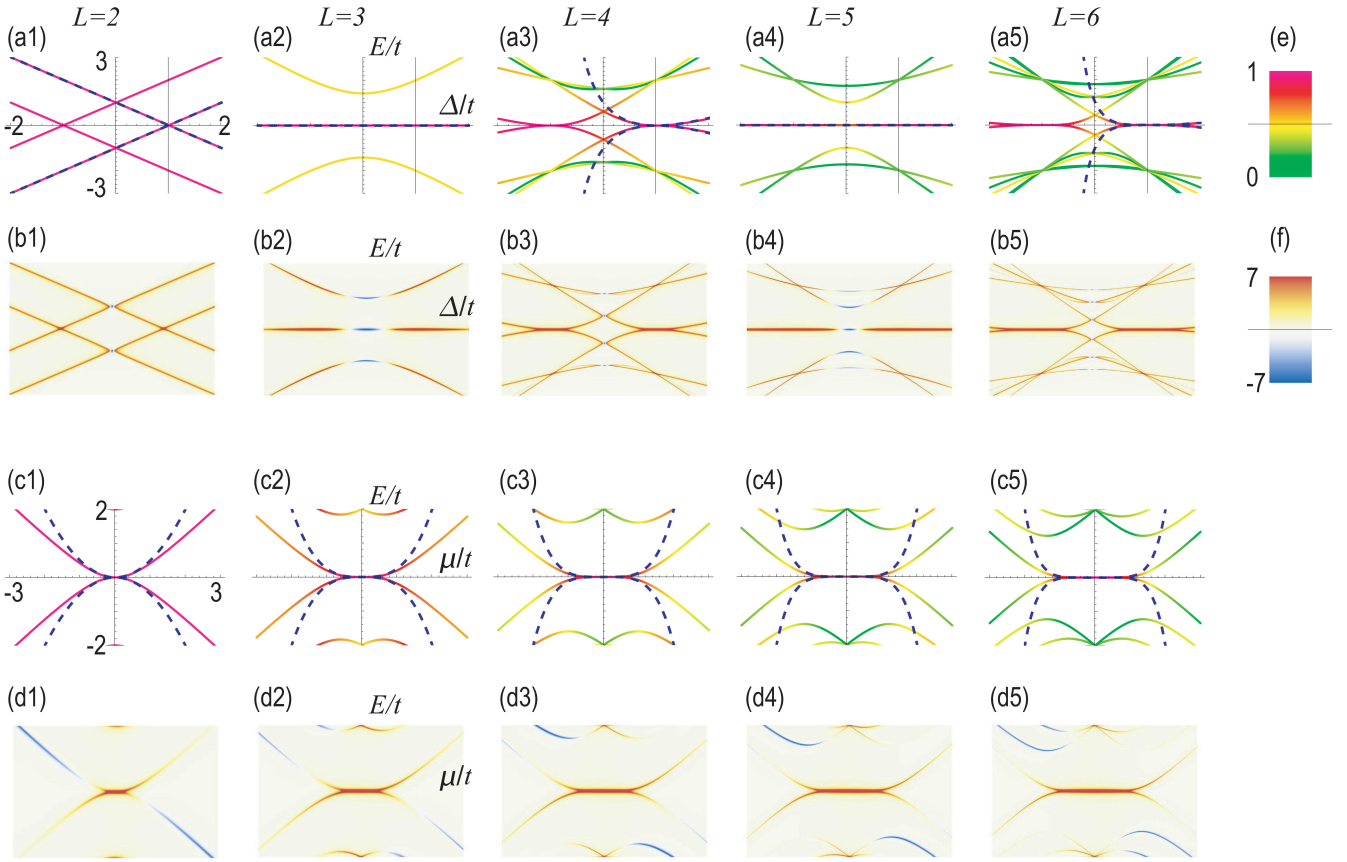


FIG. 2. (a1)∼(a5) Energy E/t as a function of Δ/t. (b1)∼(b5) Differential conductance in the Δ/t-E/t plane. We have set $\mu = 0$. (c1)∼(c5) The energy E/t as a function of μ/t. (d1)∼(d5) Differential conductance in the μ/t-E/t plane. We have set $\Delta = t$. (a1)∼(d1) $L = 2$, (a2)∼(d2) $L = 3$, (a3)∼(d3) $L = 4$, (a4)∼(d4) $L = 5$ and (a5)∼(d5) $L = 6$. Blue dotted curves are the energy derived from the effective two-band model. (e) Color palette for (a1)∼(a5) and (c1)∼(c5) indicating the amplitude at the edge sites, where red color indicates the edge states and the green color indicates the bulk states. (f) Color palette for (b1)∼(b5) and (d1)∼(d5) indicating the magnitude of the differential conductance. We have set $\Gamma_{\rm L} = \Gamma_{\rm R} = 0.1t$.

both the ends of a finite chain for $|\mu| < 2t$. The energy spectrum is exactly solvable at $|\Delta| = t$ and $\mu = 0$, where the penetration length is exactly zero. See Sec.I of Supplemental Material on this point. Hence, the Majorana edge states emerge even in the two-site system at these parameters.

We show the energy spectrum of Eq.(1) as a function of Δ/t for $L = 2, 3, 4, 5, 6$ in Fig.2(a1)∼(a5) and that as a function of μ/t in Fig.2(b1)∼(b5). Especially, the energy has a linear dependence $E = |\Delta - t|$ for the two-site Kitaev chain with $\mu = 0$ as shown in Fig.2(a1). On the other hand, the energy has a parabolic dependence $E \propto \mu^2/t$ for the two-site Kitaev chain with $\Delta = t$ as shown in Fig.2(b1).

The real part of the energy is exactly zero for the model with odd L as shown in Fig.2(a2) and (a4). We will analytically verify these results by deriving an effective two-band model in the following.

The energy of free Majorana fermions should be exactly zero in topological superconductors. However, it acquires a nonzero real energy for a finite length chain as in Fig.2 in general, because there is an overlap between the left and right edge states. There is an exceptional case of $|\Delta| = t$ and $\mu = 0$, where the penetration length of the Majorana edge state is exactly zero irrespective of the value L, as we review in Sec.I of Supplemental Material. Since the energy is exactly zero, the Majorana states are robust in this exceptional case. We note that the robustness is estimated by the amount how their energy is different from zero.

Effective two-band model: In order to reveal the behavior of the energy of Majorana states analytically near the exactly solvable point with the parameters $|\Delta| = t$ and $\mu = 0$, we construct an effective two-band model based on the isospectral matrix reduction method[48, 49]. As we give a detailed derivation in Sec.II of Supplemental Material, we obtain

$$H_{\rm eff} = \frac{(-1)^{L+1}}{(\Delta + t)^{L-1}} \sum_{m=0}^{\lfloor L/2 \rfloor} \begin{pmatrix} L-m \\ m \end{pmatrix} \mu^{L-2m} \left(\Delta^2 - t^2\right)^m \sigma_y \tag{2}$$

for $L \geq 2$, where $\begin{pmatrix} L-m \\ m \end{pmatrix}$ is a binomial coefficient.

If $\mu = 0$ in (2), we have $H_{\rm eff} = 0$ for odd L, which well describes the energy near the zero energy as shown in Fig.2(a2)

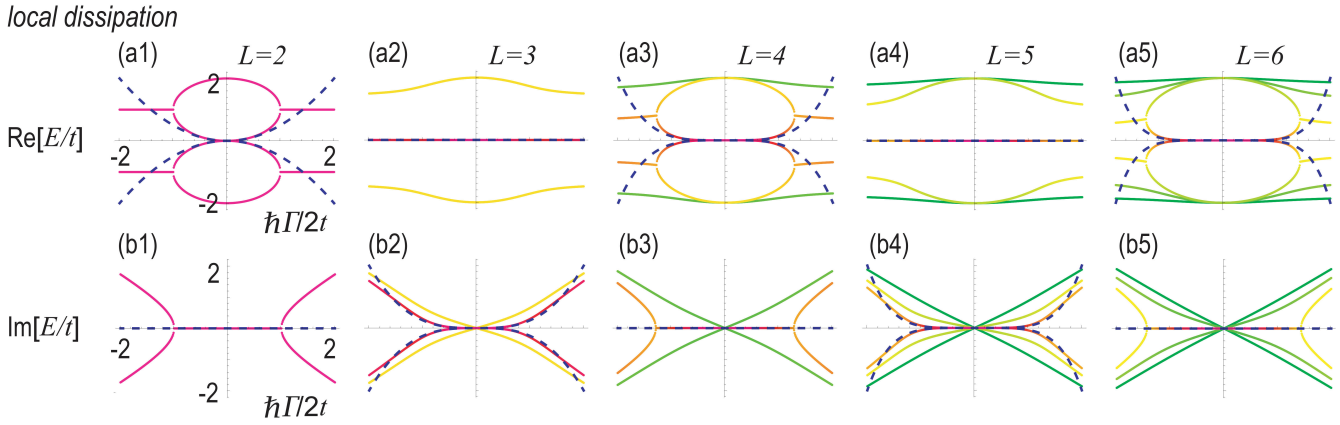


FIG. 3. (a1)∼(a4) Real part and (b1)∼(b4) imaginary part of the energy as a function of the local dissipation $\hbar\Gamma/2t$. (a1) and (b1) $L = 2$, (a2) and (b2) $L = 3$, (a3) and (b3) $L = 4$, (a4) and (b4) $L = 5$, (a5) and (b5) $L = 6$. We have set $\Delta = t$ and $\mu = 0$. Blue dotted curves are the energy derived from the effective two-band model. See the color palette in Fig.2.

and (a4). On the other hand, we have

$$H_{\text{eff}} = -\frac{(\Delta - t)^{L/2}}{(\Delta + t)^{L/2-1}}\sigma_y \tag{3}$$

for even L, which well describes the energy near the zero energy as shown in Fig.2(a1), (a3) and (a5). The energy spectrum in the vicinity of the zero energy is significantly different between even and odd length of the chain. This is the even-odd effect, which is negligible for longer chain. According to Eq.(3) the energy is almost zero for small $|\Delta - t|$ and large L, implying that the Majorana edge states are robust.

If $\Delta = t$ in (2), we have

$$H_{\text{eff}} = -\frac{\mu^L}{(2t)^{L-1}}\sigma_y. \tag{4}$$

It well fits the energy near the zero energy as shown in Fig.2(b1)∼(b5). It is almost zero for small μ and large L, implying that the Majorana edge states are robust.

Differential conductance: Differential conductance is an experimentally measurable quantity in quantum dot systems[12, 13]. It is calculated based on the S matrix theory in the wide-band limit[12, 50]. The S matrix is calculated as

$$\begin{aligned} S(E) &= \begin{pmatrix} S_{\text{ee}}(E) & S_{\text{eh}}(E) \\ S_{\text{he}}(E) & S_{\text{hh}}(E) \end{pmatrix} \\ &= 1 - iW^\dagger \left(E - H + \frac{i}{2}WW^\dagger\right)^{-1} W, \end{aligned} \tag{5}$$

where

$$W \equiv \text{diag.}\left\{\sqrt{\Gamma_{\text{L}}}, 0, \cdots, 0, \sqrt{\Gamma_{\text{R}}}, -\sqrt{\Gamma_{\text{L}}}, 0, \cdots, 0, -\sqrt{\Gamma_{\text{R}}}\right\} \tag{6}$$

is the tunnel matrix, with Γ_α being the tunnel coupling strength between the dot α and the lead α (α =L,R). See Fig.1. The zero-temperature differential conductance is obtained as[12, 50]

$$G_{\alpha\beta}(E) = \frac{dI_\alpha}{dV_\beta} = \frac{e^2}{h}\left(\delta_{\alpha\beta} - \left|S_{\text{ee}}^{\alpha\beta}(E)\right|^2 + \left|S_{\text{he}}^{\alpha\beta}(E)\right|^2\right). \tag{7}$$

We show the differential conductance G_{LL} as a function of Δ/t and E/t in Fig.2(b1)∼(b5) and that as a function of μ/t and E/t in Fig.2(d1)∼(d5). They well agree with the energy spectra shown in Fig.2(a1)∼(a5) and Fig.2(c1)∼(c5), respectively. Hence, the energy spectrum is an experimentally observable in quantum dots.

Open quantum system: Effects of the coupling between the system and an environment are described by the Lindblad equation[30] for the density matrix ρ as

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[\hat{H}, \rho\right] + \left(\sum_\alpha L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\left\{L_\alpha^\dagger L_\alpha, \rho\right\}\right), \tag{8}$$

where $\hat{H}$ is given by Eq.(1) and L is the Lindblad operator describing the dissipation.

This equation is rewritten in the form of

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left(\hat{H}_{\text{total}}\rho - \rho\hat{H}_{\text{total}}^\dagger\right) + \sum_\alpha L_\alpha \rho L_\alpha^\dagger, \tag{9}$$

where $\hat{H}_{\text{total}}$ is a non-Hermitian effective Hamiltonian defined by $\hat{H}_{\text{total}} \equiv \hat{H} + \hat{H}_{\text{dissipation}}$ with the dissipation Hamiltonian

$$\hat{H}_{\text{dissipation}} \equiv -\frac{i\hbar}{2}\sum_\alpha L_\alpha^\dagger L_\alpha. \tag{10}$$

It describes a short-time dynamics[30]. We show a derivation of Eqs.(9) and (10) in Sec.III of Supplemental Material.

Local dissipation: We study the local dissipation[42, 51], where the Lindblad operators are given by

$$L_x^- = \sqrt{\Gamma_-}c_x, \quad L_x^+ = \sqrt{\Gamma_+}c_x^\dagger, \tag{11}$$

where $\Gamma_\pm$ represent the dissipation. They describes the effect that the particle is coming in and out of a single site. There is a relation[51],

$$\frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+} = \tanh\frac{\mu\beta}{2}, \tag{12}$$

where $\beta \equiv 1/k_{\rm B}T$ is the inverse temperature. The corresponding dissipation Hamiltonian reads

$$\hat{H}_{\rm dissipation} = -\frac{i\hbar}{2}\sum_{x=1}^{L}\left[\Gamma c_x^\dagger c_x + \Gamma_+\right], \tag{13}$$

where $\Gamma \equiv \Gamma_- - \Gamma_+$. We give a derivation of Eq.(13) in Sec.III of Supplemental Material.

By introducing a complex chemical potential

$$\tilde{\mu} = \mu + \frac{i\hbar}{2}\Gamma, \tag{14}$$

the effect of the local dissipation is fully taken into account. We show the energy spectrum as a function of Γ in Fig.3 by diagonalizing the Hamiltonian (1) together with (13), where we have omitted the constant term $-i\hbar L\Gamma_+/2$ in (13). The real part of the energy for odd length is exactly zero as shown in Fig.3(a2) and (a4). On the other hand, the imaginary part of the energy for even length is exactly zero if the dissipation is smaller than a certain critical value $|\Gamma| < |\Gamma_{\rm critical}|$.

The flat region of the zero-energy Majorana edge states expands for longer chains. These properties are explained based on the effective model with local dissipation. By inserting Eq.(14) into (2), we obtain

$$H_{\rm eff} = -\frac{(i\hbar\Gamma/2)^L}{(2t)^{L-1}}\sigma_y \tag{15}$$

for $L \geq 2$, $\Delta = t$ and $\mu = 0$. This formula well explains the fact that the real [imaginary] part of the energy is zero for even [odd] L as depicted by blue dotted curves in Fig.3(a2) and (a4), [(b1) and (b3)]. The even-odd effect is manifest also in the presence of the local dissipation. The Majorana edge states becomes robust for a long chain provided $|\hbar\Gamma/2t| < 1$.

A comment is in order. The Majorana edge state acquires nonzero pure imaginary energy for odd L in Eq.(15). It means that the life time of the Majorana state becomes finite. Even in the presence of the non-Hermitian term, particle-hole symmetry is preserved and Majorana states are intact[39].

We can also study the adjacent and the global dissipation, whose results are shown in Sec. IV and V of Supplemental Material, respectively. In particular, the Majorana zero-energy edge states acquire nonzero energy such as $E \propto \pm i\Gamma$ for small Γ irrespective of the length L for the global dissipation.

Discussion: We have proposed and analytically studied two-band effective models describing the Majorana edge states in a chain with length L, motivated by recent experiments on quantum dots. We have predicted the even-odd effect on the robustness of the Majorana states in short Kitaev chain. Especially, we have found that the Majorana states in the Kitaev chain with $L = 3$ is more robust than that with $L = 4$ as a function of Δ, which is an unexpected result. It will be observed in the system of quantum dot systems by measuring the differential conductance.

The status of experiments on quantum dots is still in the beginning stage. The four- and five-site Kitaev models will be realized based on four- and five-quantum dots soon. Our results provide a criteria how much it is necessary to tune the parameters and dissipations to increase the robustness of Majorana states in a short Kitaev chain with various L. In addition, we can check whether the Kitaev model is actually realized in experiments by observing the behavior of the energy by way of intentionally tuning away from the exactly solvable parameters $\Delta = t$ and $\mu = 0$. They will be useful for future experimental works of the short Kitaev chain based on quantum dots.

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[1] S. B. Bravyi and A. Yu. Kitaev, Fermionic Quantum Computation, Annals of Physics 298, 210 (2002).

[2] D. A. Ivanov, Non-Abelian statistics of half-quantum vortices in p-wave superconductors, Phys. Rev. Lett. 86, 268 (2001).

[3] A. Kitaev, Fault-tolerant quantum computation by anyons, Ann. Phys. 303, 2 (2003).

[4] S. Das Sarma, M. Freedman, and C. Nayak, Topologically protected qubits from a possible non-Abelian fractional quantum Hall state, Phys. Rev. Lett. 94, 166802 (2005).

[5] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Non-Abelian anyons and topological quantum computation, Rev. Mod. Phys. **80**, 1083 (2008).

[6] Motohiko Ezawa, Systematic construction of topological-nontopological hybrid universal quantum gates based on many-body Majorana fermion interactions arXiv:2304.06260

[7] X.-L. Qi, S.-C. Zhang, Topological insulators and superconductors, Rev. Mod. Phys. **83**, 1057 (2011).

[8] J. Alicea, New directions in the pursuit of Majorana fermions in solid state systems, Rep. Prog. Phys. 75, 076501 (2012).

[9] M. Sato and Y. Ando, Topological superconductors: a review, Rep. Prog. Phys. **80**, 076501 (2017).

[10] J. Alicea, Y. Oreg, G. Refael, F. von Oppen and M.P.A. Fisher, Non-Abelian statistics and topological quantum information processing in 1D wire networks, Nat. Phys. **7**, 412 (2011).

[11] A. Yu Kitaev, Unpaired Majorana fermions in quantum wiresm Phys.-Usp. 44 131 (2001).

[12] Tom Dvir, Guanzhong Wang, Nick van Loo, Chun-Xiao Liu, Grzegorz P. Mazur, Alberto Bordin, Sebastiaan L. D. ten Haaf, Ji-Yin Wang, David van Driel, Francesco Zatelli, Xiang Li, Filip K. Malinowski, Sasa Gazibegovic, Ghada Badawy, Erik P. A. M. Bakkers, Michael Wimmer and Leo P. Kouwenhoven, Realization of a minimal Kitaev chain in coupled quantum dots, Nature 614, 445 (2023).

[13] Alberto Bordin, Xiang Li, David van Driel, Jan Cornelis Wolff,

Qingzhen Wang, Sebastiaan L.?D. ten Haaf, Guanzhong Wang, Nick van Loo, Leo P. Kouwenhoven, and Tom Dvir, Crossed Andreev Reflection and Elastic Cotunneling in Three Quantum Dots Coupled by Superconductors, Phys. Rev. Lett. 132, 056602 (2024)

[14] Athanasios Tsintzis, Ruben Seoane Souto and Martin Leijnse, Creating and detecting poor man's Majorana bound states in interacting quantum dots Phys. Rev. B 106, L201404 (2022).

[15] Chun-Xiao Liu , Guanzhong Wang , Tom Dvir , and Michael Wimmer, Tunable Superconducting Coupling of Quantum Dots via Andreev Bound States in Semiconductor-Superconductor Nanowires, Phys. Rev. Lett. 129, 267701 (2022).

[16] Chun-Xiao Liu, Haining Pan, F. Setiawan, Michael Wimmer and Jay D. Sau, Fusion protocol for Majorana modes in coupled quantum dotsm Phys. Rev. B 108, 085437 (2023).

[17] Rouven Koch, David van Driel, Alberto Bordin, Jose L. Lado and Eliska Greplova, Adversarial Hamiltonian learning of quantum dots in a minimal Kitaev chain, Phys. Rev. Applied 20, 044081 (2023)

[18] Athanasios Tsintzis, Ruben Seoane Souto, Karsten Flensberg, Jeroen Danon and Martin Leijnse, Majorana Qubits and Non-Abelian Physics in Quantum Dot?Based Minimal Kitaev Chains, PRX Quantum 5, 010323 (2024)

[19] D. Michel Pino, Ruben Seoane Souto and Ramon Aguado, Minimal Kitaev–transmon qubit based on double quantum dots, Phys. Rev. B 109, 075101 (2024)

[20] William Samuelson, Viktor Svensson and Martin Leijnse, A minimal quantum dot-based Kitaev chain with only local superconducting proximity effect, Phys. Rev. B 109, 035415 (2024)

[21] Chun-Xiao Liu, A. Mert Bozkurt, Francesco Zatelli, Sebastiaan L.D. ten Haaf, Tom Dvir, Michael Wimmer, Enhancing the excitation gap of a quantum-dot-based Kitaev chain, arXiv:2310.09106

[22] Mahan Mohseni, Hassan Allami, Daniel Miravet, David J. Gayowsky, Marek Korkusinski, Pawel Hawrylak, Majorana excitons in a Kitaev chain of semiconductor quantum dots in a nanowire, arXiv:2307.00100

[23] Ruben Seoane Souto, Athanasios Tsintzis, Martin Leijnse and Jeroen Danon , Probing Majorana localization in minimal Kitaev chains through a quantum dot, Phys. Rev. Research 5, 043182 (2023)

[24] Sebastian Miles, David van Driel, Michael Wimmer and Chun-Xiao Liu, Kitaev chain in an alternating quantum dot-Andreev bound state array, arXiv:2309.15777

[25] Hsien-chung Kao, Phys. Rev. B 90, 245435, Chiral zero modes in superconducting nanowires with Dresselhaus spin-orbit coupling, Phys. Rev. B 90, 245435 (2014).

[26] Suraj Hegde, Vasudha Shivamoggi, Smitha Vishveshwara and Diptiman Sen, Quench dynamics and parity blocking in Majorana wires New J. Phys. 17 053036 (2015).

[27] A. A. Zvyagin, Majorana bound states in the finite-length chain Low Temp. Phys. 41 625 (2015).

[28] Chuanchang Zeng, Christopher Moore, Apparao M. Rao, Tudor D. Stanescu, and Sumanta Tewari, Analytical solution of the finite-length Kitaev chain coupled to a quantum dot Phys. Rev. B 99, 094523 (2019).

[29] Nico Leumer, Magdalena Marganska, Bhaskaran Muralidharan and Milena Grifoni, Exact eigenvectors and eigenvalues of the finite Kitaev chain and its topological properties J. Phys.: Condens. Matter 32 445502 (2020).

[30] G. Lindblad, On the generators of quantum dynamical semigroups, Commun. Math. Phys. 48, 119 (1976).

[31] Sebastian Diehl, Enrique Rico, Mikhail A. Baranov and Peter Zoller, Topology by dissipation in atomic quantum wires, Nature Physics 7, 971 (2011).

[32] Xiaohui Wang, Tingting Liu, and Ye Xiong, Spontaneous PT -symmetry breaking in non-Hermitian Kitaev and extended Kitaev models, Phys. Rev. A 92, 012116 (2015).

[33] C. Yuce, Majorana edge modes with gain and loss, Phys. Rev. A 93, 062130 (2016).

[34] Qi-Bo Zeng, Baogang Zhu, Shu Chen, L. You, and Rong Lu, Non-Hermitian Kitaev chain with complex on-site potentials, Phys. Rev. A 94, 022119 (2016).

[35] Kohei Kawabata, Yuto Ashida, Hosho Katsura, Masahito Ueda, Parity-time-symmetric topological superconductor, Phys. Rev. B 98, 085116 (2018).

[36] Kohei Kawabata, Sho Higashikawa, Zongping Gong, Yuto Ashida and Masahito Ueda, Topological unification of time-reversal and particle-hole symmetries in non-Hermitian physics Nat. Com. 10, 297 (2019).

[37] Kohei Kawabata, Ken Shiozaki, Masahito Ueda and Masatoshi Sato, Symmetry and Topology in Non-Hermitian Physics, Phys. Rev. X 9, 041015 (2019).

[38] Naoyuki Shibata and Hosho Katsura, Dissipative spin chain as a non-Hermitian Kitaev ladder, Phys. Rev. B B 99, 174303 (2019).

[39] Motohiko Ezawa, Braiding of Majorana-like corner states in electric circuits and its non-Hermitian generalization, Phys. Rev. B 100, 045407 (2019).

[40] Chunhui Zhang, Li Sheng and Dingyu Xing, Non-Hermitian disorder-driven topological transition in a dimerized Kitaev superconductor chain, Phys. Rev. B 103, 224207 (2021).

[41] Xiao-Ming Zhao, Cui-Xian Guo, Su-Peng Kou, Lin Zhuang, and Wu-Ming Liu Phys. Rev. B 104, 205131, Defective Majorana zero modes in a non-Hermitian Kitaev chain (2021).

[42] Simon Lieu, Non-Hermitian Majorana modes protect degenerate steady states, Phys. Rev. B 100, 085110 (2019).

[43] A Maiellaro and R Citro, Non-Hermitian topological phases in an extended Kitaev model, J. Phys.: Conf. Ser. 1548 012026 (2020).

[44] Sharareh Sayyad and Jose L. Lado, Topological phase diagrams of exactly solvable non-Hermitian interacting Kitaev chains, Phys. Rev. Res. 5, L022046 (2023).

[45] Fabio L. Pedrocchi and David P. DiVincenzo, Majorana Braiding with Thermal Noise, Phys. Rev. Lett. 115, 120402 (2015).

[46] Fabio L. Pedrocchi, N. E. Bonesteel, and David P. DiVincenzo, Monte Carlo studies of the self-correcting properties of the Majorana quantum error correction code under braiding, Phys. Rev. B 92, 115441 (2015).

[47] Ying Hu, Zi Cai, Mikhail A. Baranov, and Peter Zoller, Majorana fermions in noisy Kitaev wires, Phys. Rev. B 92, 165118 (2015).

[48] Lumen Eek, Anouar Moustaj, Malte Rontgen, Vincent Pagneux, Vassos Achilleos, and Cristiane Morais Smith, Emergent non-Hermitian models, Phys. Rev. B 109, 045122 (2024)

[49] Malte Rontgen, Xuelong Chen, Wenlong Gao, Maxim Pyzh, Peter Schmelcher, Vincent Pagneux, Vassos Achilleos and Antonin Coutant, Latent Su–Schrieffer–Heeger models, arXiv:2310.07619

[50] S. Datta, Electronic Transport in Mesoscopic Systems (Cambridge University Press, Cambridge, England, 1995): Quantum Transport: Atom to Transistor (Cambridge University Press, England, 2005).

[51] Pavel Kos and Tomaž Prosen, Time-dependent correlation functions in open quadratic fermionic systems J. Stat. Mech. 123103 (2017).