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Entanglement and Absorbing-State Transitions in Interactive Quantum Dynamics

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Nascent quantum computers motivate the exploration of quantum many-body systems in nontraditional scenarios. For example, it has become natural to explore the dynamics of systems evolving under both unitary evolution and measurement. Such systems can undergo dynamical phase transitions in the entanglement properties of quantum trajectories *conditional* on the measurement outcomes. Here, we explore dynamics in which one attempts to (locally) use those measurement outcomes to steer the system toward a target state, and we study the resulting phase diagram as a function of the measurement and feedback rates. Steering succeeds when the measurement and feedback rates exceed a threshold, yielding an absorbing-state transition in the trajectory-averaged density matrix. We argue that the absorbing-state transition generally occurs at different critical parameters from the entanglement transition in individual trajectories and has distinct critical properties. The efficacy of steering depends on the nature of the target state: in particular, for local dynamics targeting long-range correlated states, steering is necessarily slow and the entanglement and steering transitions are well separated in parameter space.

Phases and criticality in quantum many-body dynamics are topics of fundamental interest. For unitary dynamics, various questions have been explored, such as the growth of entanglement [1–3], the emergence of hydrodynamics [4–6], or the effects of disorder [7–9]. Recently, motivated by advances in quantum computing hardware, “monitored” quantum systems subject to repeated local measurements have also come under intense investigation [10, 11]. These systems were shown to display the surprising phenomenon of entanglement phase transitions as a function of measurement rate [12–20]. Such phase transitions are only visible in the properties of *individual* quantum trajectories [21] corresponding to specific sequences of measurement outcomes, so they bear a prohibitive post-selection cost that is exponential in the space-time volume (with exceptions in Clifford dynamics [22, 23], spacetime-dual dynamics [24–26], or replacing measurements with swaps into an environment [27]). Monitored dynamics can be enriched by using the measurement outcomes, say, for adaptively controlling the subsequent dynamics [28–34]. Error correction is one example of such *interactive quantum dynamics* with a transition as a function of the error rate [35]. Understanding novel dynamical phenomena in interactive evolution is an active area of inquiry, especially topical in light of experimental advances in building devices capable of quantum control via measurements and feedback.

In this work, we consider interactive quantum dynamics with unitary evolution and measurements, where the measurement outcomes are used to apply local unitaries that *steer* the system towards target states [36–38]. The target states are absorbing states, so that the dynamics can evolve to the target state, but cannot leave it. We discuss steering both to trivial product states and to entangled symmetry-protected topological (SPT) states. Our dynamics only uses local feedback (in contrast to non-local classical communication), i.e., each feedback operation depends only on the measurement immediately preceding it at the same location. Therefore, the av-

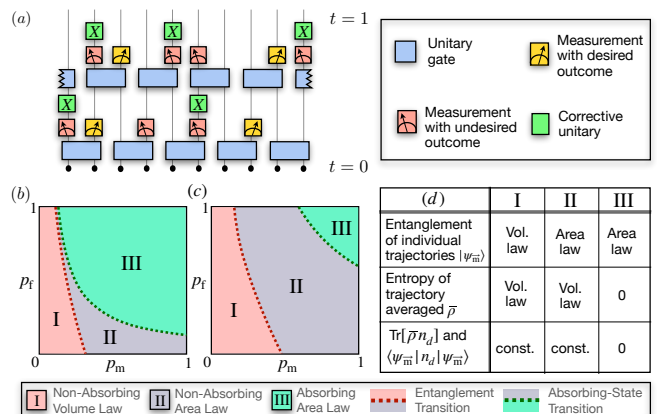


FIG. 1. Monitored quantum circuits with feedback and schematic depictions of their associated phase diagrams. (a) A quantum circuit model with $|\uparrow\uparrow\cdots\uparrow\rangle$ as its target state (see text). Sketch of the phase diagrams for interactive dynamics targeting (b) the all-up state and (c) an SPT cluster state. The two phase boundaries are close/widely separated near $p_f = 1$ in b/c respectively. The absorbing-state phase transition curves in (b,c) satisfy $p_m p_f = \text{const.}$ (d) A table of the properties of the three phases.

erage density matrix dynamics is described by a time-independent local quantum channel. Unlike the quantum channels that occur in the standard (non-adaptive) measurement-induced transition, our channel is *not* unital, so its steady state need not be the maximally mixed state. The channels we consider can in fact undergo an absorbing-state phase transition [39–46] separating an ‘absorbing phase’ in which a pure zero-entropy target state is reached in a time at most polynomial in system size, from an ‘active phase’ in which it is not.

We study interactive quantum dynamics as a function of the measurement rate, p_m , and the fraction of measurements that are followed by feedback, p_f . This yields a phase diagram with two transitions: an entanglement

phase transition in individual trajectories, driven by p_m , and an absorbing-state transition in the average density matrix, driven by the total rate of feedback events $p_m p_f$ (Fig. 1). We provide numerical evidence and analytical arguments that these do not coincide in general. When we target a trivial polarized product state and use maximally efficient feedback, $p_f = 1$, the transitions occur at sufficiently close p_m that we cannot numerically distinguish the locations of the critical points. Nevertheless, we find that quantities associated with the trajectory-averaged density matrix scale with directed-percolation critical exponents that are completely different from the exponents associated with the entanglement transition in individual trajectories. This finding is natural if one posits two separate transitions, but implies a drastic violation of one-parameter scaling otherwise.

Errors above a target state with short-range correlations can be locally corrected, allowing for efficient local feedback. For target states with long-range correlations, such as a ferromagnet or a topologically ordered state, error correction requires pairing up domain walls or anyons over long distances, which requires long-range classical communication [47–49]. Absent such communication, the error correction dynamics is described by a local quantum channel, which has a light cone. Steering from a product state to a long-range correlated state therefore takes a time that diverges with system size, even at $p_m = p_f = 1$ where all trajectories are area-law entangled. To explore these consequences of locality, we study dynamics targeting a symmetry-protected topological (SPT) state using both symmetric and symmetry-breaking operations. In the former case, but not the latter, the entanglement and absorbing-state transitions are well separated, consistent with the logic above.

Polarized Absorbing State— We first consider a spin-1/2 model with the target state $|\psi_t\rangle = |\uparrow\uparrow\uparrow\cdots\uparrow\rangle$. The model comprises two-site nearest-neighbor unitary gates applied in a brickwork fashion in a 1D system of length L with periodic boundary conditions. Each gate is block-diagonal and locally leaves the $|\uparrow\uparrow\rangle$ state invariant, but acts as a Haar-random unitary $U(3)$ in the 3×3 block of states spanned by $\{|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$. The unitary dynamics leaves $|\psi_t\rangle$ invariant, but is otherwise chaotic. Each gate in the circuit is sampled independently. Unless otherwise stated, we always begin with the system in the $|\downarrow\downarrow\cdots\downarrow\rangle$ state. In between the unitary layers, we measure the Pauli Z_i operator on each site with probability p_m . If the outcome is +1, locally corresponding to the $|\uparrow\rangle_i$ state, we do nothing. If the outcome is -1, a corrective unitary X_i is applied with probability p_f to steer the system towards $|\psi_t\rangle$ [Fig. 1(a)]. As a channel, this measure-and-correct operation is equivalent to a qubit reset, which is accessible on current hardware [46]. A given quantum trajectory, $|\psi_{\vec{m}}(t)\rangle$ is labeled by the sequence of measurement outcomes, \vec{m} , encountered at each position until time t ; the label also implicitly includes the action of feedback events conditioned on the outcomes. We denote the density matrix obtained by av-

eraging over trajectories, feedback events, and choices of random circuits as $\bar{\rho}$.

We describe the phase diagram of this model in Fig. 1 using properties of trajectories and properties of $\bar{\rho}$ at times linear in system size. When the total rate of feedback operations $p_m p_f$ exceeds a critical value $p_c \approx 0.1$, $\bar{\rho}$ approaches the absorbing state (phase III). Here, every trajectory approaches the same unentangled target state. When $p_m p_f < p_c$, $\bar{\rho}$ remains mixed up to times exponential in L . However, individual trajectories exhibit two phases, even at $p_f = 0$: a volume-law phase (phase I) and an area-law phase (phase II). In phase II, each trajectory stays active and visits *different* area-law states, so $\bar{\rho}$ remains high-entropy. We will argue below that phase II intervenes between phase I and III, even when $p_f = 1$.

Absorbing State Transition— The absorbing-state transition is visible in $\bar{\rho}$, and observable via conventional expectation values. In [50], we show that the diagonal elements of $\bar{\rho}$ evolve under the transfer matrix of a classical stochastic process, while the off-diagonal elements vanish after one complete time step. This mapping gives us access to much larger system sizes and allows us to convincingly demonstrate that this absorbing-state transition is in the directed percolation universality class. In the classical process, the effect of the averaged measurements is to locally send \downarrow to \uparrow with probability $p_m p_f$, while the effect of the averaged unitaries is to mix the local configurations $\uparrow\downarrow$, $\downarrow\uparrow$, and $\uparrow\uparrow$ with equal probability 1/3 [50]. Note that the averaged dynamics are now described by only a single parameter $p \equiv p_m p_f$ describing the total rate of measurements with feedback. This stochastic process has an all-up absorbing state, and undergoes an absorbing-state transition characterized by the behavior of “defects”, i.e., down-spins. The density of defects

$$n_d = \frac{1}{L} \sum_i \frac{1 - Z_i}{2}$$

serves as an order parameter that rapidly approaches zero (exponentially in time) in the absorbing phase, but reaches a long-lived nonzero value in the non-absorbing phase (for times exponential in L). In Fig. 2(a), we see that the density of defects is nonzero below $p_c \approx .1$ and vanishes above p_c . The curve $p_m p_f = p_c \approx .1$ defines the absorbing-state critical line depicted in Fig. 1(c). Note that a non-zero n_d implies an extensive entropy for $\bar{\rho}$, with the entropy density being maximal at $n_d = 0.5$ and decreasing as p_c is approached.

For a classical stochastic process with a single absorbing state, a non-disordered transfer matrix, and no additional symmetries, the critical properties are expected to be in the directed percolation (DP) universality class. We confirm this expectation in Fig. 2(b,c), using techniques discussed in Ref. [51]. We expect that n_d satisfies the following critical scaling:

$$n_d(t, L) \sim t^{-\delta} \Phi \left((p - p_c) t^{1/\nu_{||}}, \frac{t^{1/z}}{L} \right). \quad (1)$$

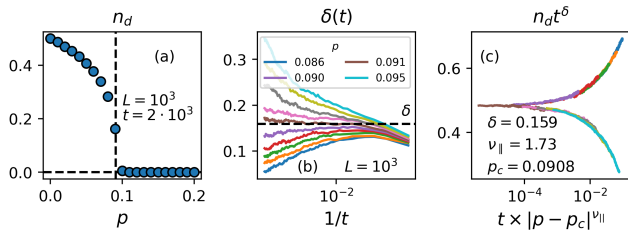


FIG. 2. Absorbing-state phase transition. (a) The sample-averaged defect density n_d as a function of $p = p_m p_f$. Between $10^3 - 2 \cdot 10^3$ samples were averaged for each data point shown. Values were computed via the mapping to a classical stochastic model and initialized with random bitstrings. (b) The running estimate of critical exponent δ as a function of inverse time $1/t$. The critical p_c is estimated to be the value of p for which the curve remains constant as $1/t \rightarrow 0$, and an estimate for δ is that constant value. (c) Scaling collapse of n_d data to the form of Eq. 1. The critical exponents are those of directed percolation.

where $\nu_{\parallel} = z\nu_{\perp}$ and z is the dynamical scaling exponent. To probe the values of δ and p_c , we define a time-dependent estimate $\delta(t)$ so that $n_d(t) \sim 1/t^{\delta(t)}$, and extract $\delta(t)$ from the quantity $\log_{10}[n_d(t)/n_d(10t)]$. We expect that at the critical point, and for $t^{1/z}/L$ sufficiently small, $\delta(t)$ is constant, while it vanishes in the non-absorbing phase and diverges in the absorbing phase. Fig. 2(b) shows a value of δ consistent with directed percolation's $\delta_{\text{DP}} = .159$, and a critical probability $p_c = .09085(5)$. We see scaling collapse of these curves in Fig. 2(c) using this p_c , δ_{DP} , and $\nu_{\text{DP}\parallel} = 1.73$. Thus, as anticipated, our model's absorbing-state transition falls under the directed percolation universality class.

Entanglement Phase Transition— We now locate and characterize the entanglement phase transition via exact simulations of quantum trajectories for $L \leq 24$. We first consider the cut $p_f = p_m$ through the phase diagram in Fig. 1(c); the feedback along this cut is weak enough that the entanglement phase transition is numerically well separated from the absorbing-state transition. We benchmark the entanglement transition against previous analyses of models without feedback. We use the tripartite quantum mutual information $I_3 = S_{Q_1} + S_{Q_2} + S_{Q_3} - S_{Q_1 \cup Q_2} - S_{Q_2 \cup Q_3} - S_{Q_3 \cup Q_1} + S_{Q_1 \cup Q_2 \cup Q_3}$, which was argued to be constant at the critical point and show a crossing [15, 52]. Here S_A is the von Neumann entanglement entropy of subsystem A , and Q_n is the n^{th} contiguous quarter of the system. We evaluate I_3 at $t = 2L$, anticipating that the entanglement transition will have dynamical exponent $z = 1$ [17, 18, 53]. The crossing of I_3 yields the critical point $p_m^c = p_f^c \cong 0.130(5)$, which is well-separated from the absorbing-state phase transition at $p_m = p_f = \sqrt{p_c} \cong \sqrt{0.09085} = 0.3014$ found above (Fig. 2). We perform a scaling collapse near the critical point, and the critical exponent $\nu \cong 1.1(2)$ obtained is consistent with previous estimates of ν for the

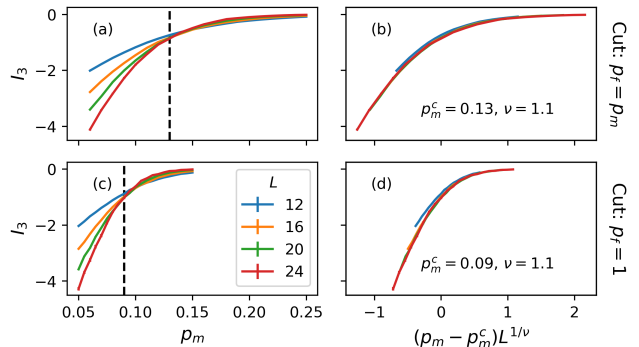


FIG. 3. Tripartite quantum mutual information. The data are averaged over $5 \cdot 10^2 - 5 \cdot 10^3$ circuit realizations and quantum trajectories in simulations of full-wavefunction evolution, and over times $t = 2L$ and $2L + \frac{1}{2}$ to avoid even-odd effects. The top row corresponds to the cut $p_f = p_m$, and the bottom to $p_f = 1$. Vertical dashed lines mark estimates of the critical points: $p_m^c \cong 0.130(5)$ (a) and $p_m^c \cong 0.090(5)$ (c). Panels (b) and (d) show the finite-size scaling collapse of the form $I_3 = f[(p_m - p_m^c)L^{1/\nu}]$, with $\nu \cong 1.1(2)$.

entanglement transition in monitored circuits with Haar-random gates [52]. With this estimate of p_m^c , we verify [50] that the dynamics at p_m^c are consistent with the exponent $z = 1$ by examining the purification of an initial mixed state [15, 32, 54]. Thus, along the line $p_f = p_m$, the entanglement and absorbing-state transitions are well separated, and the former displays the same critical properties as monitored circuits without feedback.

We now consider the case of full-strength feedback and vary p_m while $p_f = 1$ is fixed. In this case, every measure-and-feedback operation leaves the spin in the $|\uparrow\rangle$ state; hence measurements not only drive trajectories to low-entanglement states, but also specifically towards the polarized state. As a result, the entanglement and absorbing-state transitions come closer together and a similar analysis of I_3 locates the entanglement transition at $p_m^c \cong 0.090(5)$ [see Fig. 3(c)], numerically indistinguishable from the location of the absorbing-state transition found above (Fig. 2). A natural question is then whether the two phase transitions coalesce as $p_f \rightarrow 1$, or remain distinct critical phenomena. In order to see that the two transitions remain distinct, we determine their dynamical exponents z [55]. We extract z for the entanglement transition using the purification setup [50]—which examines the entropy S of a system initially in a mixture of two random orthogonal states—and for the absorbing-state transition using the density of defects n_d . In Fig. 4 we show both S and n_d data for times $t \gtrsim L^z$. The entropy continues to scale like $S \sim f(t/L)$, i.e., $z = 1.0(1)$, and n_d scales according to $\delta = 0.16(8)$ and $z = 1.6(1)$ corresponding to directed percolation. The picture that emerges is one in which the two phase transitions can become close in p_m at strong feedback ($p_f = 1$); however, they remain distinct critical phenom-

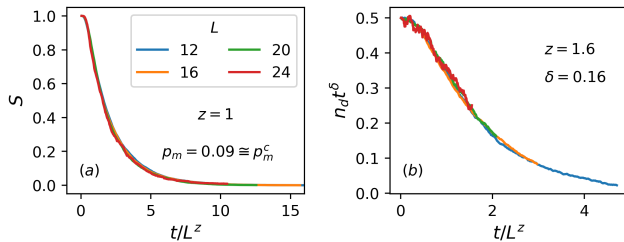


FIG. 4. Dynamical critical scaling along $p_f = 1$. The system is initialized in a mixed state with 1 bit of entropy as described in the SM. Data points are averages of $2 \cdot 10^2 - 5 \cdot 10^3$ samples. (a) The second Rényi entropy of the density matrix S vs. scaled time. Data collapse is consistent with the scaling form $S = f(t/L^z)$. (b) The scaled density of defects vs. scaled time. Data collapse is consistent with $n_d = t^{-\delta} \Phi(0, t/L^z)$ at the critical point [Eq. 1]. The z values are not consistent in (a) and (b), indicating distinct critical phenomena.

ena that happen on parametrically different time-scales as indicated by the different z values.

Argument for Separate Transitions— We now argue more directly that a sliver of Phase II generically separates the two transitions, provided that the absorbing-state transition is continuous. We perform a Stinespring dilation [35] of the channel at $p_f = 1$, such that each measurement consists of swapping out one of the system spins for an ancilla spin prepared in $|\uparrow\rangle$. The steady-state entropy of $\bar{\rho}$ is the entanglement between the system and ancilla qubits; meanwhile, the entanglement entropy of trajectories is that of the system’s wavefunction once we measure all the ancillas. Consider the dynamics when $n_d \ll 1$. “Live” (spin- \downarrow) regions are dilute, and collisions between them are rare. However, processes where a qubit in the live region is swapped into the environment are common, since p_m is $O(1)$. Suppose the live region is initially entangled with the rest of the system. Before it encounters a neighbor, it undergoes many measurement/feedback processes. Therefore, the rest of the system is now entangled with the live region together with the qubits that were swapped out. Collisions are rare, so the number of swapped-out qubits exceeds the number of qubits that remain in the live region. By the decoupling principle [56], the live region is decoupled from the rest of the system: instead, the rest of the system is now entangled with the environment. Since decoupling happens between collisions, large-scale entanglement cannot build up and the system remains in an area-law phase. Crucially, the rate of collisions (set by n_d) can be made arbitrarily small relative to the rate of swaps (set by p_m), by going near the absorbing-state transition [57].

SPT Absorbing State— In principle, the approach described above can be generalized to steer a system to any pure state that is the unique state annihilated by a set of projectors. However, the efficacy of such steering depends on the nature of correlations in the target state: as noted in the introduction, local quantum chan-

nels have light cones and cannot prepare states with long-range correlations in finite time starting from a product state. Feedback operations targeting states with long-range correlations are inherently less efficient, since they generically move defects instead of eliminating them. One might expect, therefore, that the entanglement and absorbing-state transitions should be well-separated for such target states even at $p_f = 1$.

We have tested this intuition for dynamics targeting the cluster state (ground state of $H = -\sum_i Z_i X_{i+1} Z_{i+2}$), which is an SPT state; under unitary dynamics, this state can be prepared in $O(1)$ depth if the protecting $Z_2 \times Z_2$ symmetry (that is a product of X_i on even/odd sites) is broken, while it requires depth $O(L)$ in the presence of the symmetry. An appealing feature of the cluster state is that it allows steering protocols of two types: (a) those in which the feedback completely breaks the protecting symmetry, allowing individual defects to be annihilated, and (b) those in which the feedback respects a parity symmetry that constrains defects to be annihilated in pairs. The details of the interactive dynamics are described in [50]; the results can be summarized as follows. In case (a), the phase diagram and dynamics closely resemble the product-state example, while case (b) has the following salient differences. First, the entanglement and absorbing-state transitions are widely separated even for $p_f = 1$ [Fig. 1(d)]. Second, the absorbing-state transition belongs to the “parity conserving” universality class [50]. Even in the absorbing phase, the approach to steady state takes a time that scales as L^2 because of the diffusion-limited recombination of defects. This conclusion does not rely on SPT symmetry, and we expect it to generalize to other models involving pairwise annihilation of defects, including in higher dimensional systems with more exotic anyonic defects. Third, in the area-law non-absorbing regime, individual trajectories possess “spin-glass” SPT order [19, 58, 59], but the average over measurement outcomes washes out this order. Finally, with the parity symmetry, if we allow nonlocal communication so defects can be paired up more efficiently, directed-percolation universality is restored.

Discussion— In this work we explored the dynamics of single trajectories and the trajectory-averaged density matrix for a family of interactive quantum circuits with measurements and feedback. We found distinct phase transitions for single-trajectory and trajectory-averaged quantities, and argued that in general these transitions should belong to distinct universality classes and occur at distinct critical measurement rates. The feedback allowed in our setup was based on *local* information about the state; thus it was much more restrictive than the types of feedback that are allowed in general LOCC protocols or in quantum error correction. An interesting direction for future work is to explore the consequences of relaxing this locality constraint, and allowing for general forms of interactive dynamics that cannot be captured by local quantum channels (and therefore obey weaker locality constraints). It would also be interesting to under-

stand cases where the two transitions may coincide [31], and the relevance of feedback to these transitions in an RG sense.

Note Added— In the final stages of completion of this work, we became aware of related work which appeared on the arXiv and was subsequently published as [60]. Our results agree where they overlap.

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- [1] Pasquale Calabrese and John Cardy, “Evolution of entanglement entropy in one-dimensional systems,” *Journal of Statistical Mechanics: Theory and Experiment* **2005**, P04010 (2005).
- [2] Hyungwon Kim and David A. Huse, “Ballistic spreading of entanglement in a diffusive nonintegrable system,” *Phys. Rev. Lett.* **111**, 127205 (2013).
- [3] Adam Nahum, Jonathan Ruhman, Sagar Vijay, and Jeongwan Haah, “Quantum Entanglement Growth under Random Unitary Dynamics,” *Physical Review X* **7**, 031016 (2017).
- [4] Vedika Khemani, Ashvin Vishwanath, and David A. Huse, “Operator spreading and the emergence of dissipative hydrodynamics under unitary evolution with conservation laws,” *Phys. Rev. X* **8**, 031057 (2018).
- [5] Tibor Rakovszky, Frank Pollmann, and C. W. von Keyserlingk, “Diffusive hydrodynamics of out-of-time-ordered correlators with charge conservation,” *Phys. Rev. X* **8**, 031058 (2018).
- [6] Peter T. Brown, Debayan Mitra, Elmer Guardado-Sanchez, Reza Nourafkan, Alexis Reymbaut, Charles-David Hébert, Simon Bergeron, A.-M. S. Tremblay, Jure Kokalj, David A. Huse, Peter Schauf, and Waseem S. Bakr, “Bad metallic transport in a cold atom fermi-hubbard system,” *Science* **363**, 379–382 (2019).
- [7] Rahul Nandkishore and David A. Huse, “Many-body localization and thermalization in quantum statistical mechanics,” *Annual Review of Condensed Matter Physics* **6**, 15–38 (2015).
- [8] Dmitry A. Abanin, Ehud Altman, Immanuel Bloch, and Maksym Serbyn, “Colloquium: Many-body localization, thermalization, and entanglement,” *Rev. Mod. Phys.* **91**, 021001 (2019).
- [9] Fabien Alet and Nicolas Laflorencie, “Many-body localization: An introduction and selected topics,” *Comptes Rendus Physique* **19**, 498–525 (2018), quantum simulation / Simulation quantique.
- [10] Andrew C. Potter and Romain Vasseur, “Entanglement dynamics in hybrid quantum circuits,” in *Entanglement in Spin Chains: From Theory to Quantum Technology Applications*, edited by Abolfazl Bayat, Sougato Bose, and Henrik Johannesson (Springer International Publishing, Cham, 2022) pp. 211–249.
- [11] Matthew Fisher, Vedika Khemani, Adam Nahum, and Sagar Vijay, “Random quantum circuits,” *arXiv preprint arXiv:2207.14280* (2022).
- [12] Yaodong Li, Xiao Chen, and Matthew P. A. Fisher, “Quantum Zeno effect and the many-body entanglement transition,” *Physical Review B* **98**, 205136 (2018).
- [13] Brian Skinner, Jonathan Ruhman, and Adam Nahum, “Measurement-Induced Phase Transitions in the Dynamics of Entanglement,” *Physical Review X* **9**, 031009 (2019).
- [14] Soonwon Choi, Yimu Bao, Xiao-Liang Qi, and Ehud Altman, “Quantum Error Correction in Scrambling Dynamics and Measurement-Induced Phase Transition,” *Physical Review Letters* **125**, 030505 (2020).
- [15] Michael J. Gullans and David A. Huse, “Dynamical Purification Phase Transition Induced by Quantum Measurements,” *Physical Review X* **10**, 041020 (2020).
- [16] Yimu Bao, Soonwon Choi, and Ehud Altman, “Theory of the phase transition in random unitary circuits with measurements,” *Physical Review B* **101**, 104301 (2020).
- [17] Yaodong Li, Xiao Chen, and Matthew P. A. Fisher, “Measurement-driven entanglement transition in hybrid quantum circuits,” *Physical Review B* **100**, 134306 (2019).
- [18] Chao-Ming Jian, Yi-Zhuang You, Romain Vasseur, and Andreas W. W. Ludwig, “Measurement-induced criticality in random quantum circuits,” *Phys. Rev. B* **101**, 104302 (2020).
- [19] Matteo Ippoliti, Michael J. Gullans, Sarang Gopalakrishnan, David A. Huse, and Vedika Khemani, “Entanglement Phase Transitions in Measurement-Only Dynamics,” *Physical Review X* **11**, 011030 (2021).
- [20] Utkarsh Agrawal, Aidan Zabalo, Kun Chen, Justin H. Wilson, Andrew C. Potter, J.H. Pixley, Sarang Gopalakrishnan, and Romain Vasseur, “Entanglement and charge-sharpening transitions in u(1) symmetric monitored quantum circuits,” *Physical Review X* **12** (2022).
- [21] M. B. Plenio and P. L. Knight, “The quantum-jump approach to dissipative dynamics in quantum optics,” *Rev. Mod. Phys.* **70**, 101–144 (1998).
- [22] Crystal Noel, Pradeep Niroula, Daiwei Zhu, Andrew Risinger, Laird Egan, Debopriyo Biswas, Marko Cetina, Alexey V. Gorshkov, Michael J. Gullans, David A. Huse, and Christopher Monroe, “Measurement-induced quantum phases realized in a trapped-ion quantum com-

- puter,” *Nature Physics* **18**, 760–764 (2022).
- [23] Yaodong Li, Yijian Zou, Paolo Glorioso, Ehud Altman, and Matthew Fisher, “Cross entropy benchmark for measurement-induced phase transitions,” *arXiv preprint arXiv:2209.00609* (2022).
- [24] Matteo Ippoliti and Vedika Khemani, “Postselection-free entanglement dynamics via spacetime duality,” *Phys. Rev. Lett.* **126**, 060501 (2021).
- [25] Matteo Ippoliti, Tibor Rakovszky, and Vedika Khemani, “Fractal, Logarithmic, and Volume-Law Entangled Non-thermal Steady States via Spacetime Duality,” *Physical Review X* **12**, 011045 (2022).
- [26] Tsung-Cheng Lu and Tarun Grover, “Spacetime duality between localization transitions and measurement-induced transitions,” *PRX Quantum* **2**, 040319 (2021).
- [27] Zack Weinstein, Shane P Kelly, Jamir Marino, and Ehud Altman, “Scrambling transition in a radiative random unitary circuit,” *arXiv preprint arXiv:2210.14242* (2022).
- [28] Piotr Sierant, Giuliano Chiriacò, Federica M. Surace, Shraddha Sharma, Xhek Turkeshi, Marcello Dalmonte, Rosario Fazio, and Guido Pagano, “Dissipative Floquet Dynamics: from Steady State to Measurement Induced Criticality in Trapped-ion Chains,” *Quantum* **6**, 638 (2022).
- [29] Shuohang Wu and Zi Cai, “Feedback-induced interactive dynamics: unitary but dissipative evolution,” *arXiv preprint arXiv:2211.09291* (2022).
- [30] SJ Garratt, Z Weinstein, and E Altman, “Measurements conspire nonlocally to restructure critical quantum states,” *arXiv preprint arXiv:2207.09476* (2022).
- [31] Michael Buchhold, Thomas Mueller, and Sebastian Diehl, “Revealing measurement-induced phase transitions by pre-selection,” *arXiv preprint arXiv:2208.10506* (2022).
- [32] Thomas Iadecola, Sriram Ganeshan, JH Pixley, and Justin H Wilson, “Dynamical entanglement transition in the probabilistic control of chaos,” *arXiv preprint arXiv:2207.12415* (2022).
- [33] Max McGinley, Sthitadhi Roy, and S. A. Parameswaran, “Absolutely stable spatiotemporal order in noisy quantum systems,” *Phys. Rev. Lett.* **129**, 090404 (2022).
- [34] Aaron J. Friedman, Oliver Hart, and Rahul Nandkishore, “Measurement-induced phases of matter require adaptive dynamics,” *arXiv preprint arXiv:2210.07256* (2022).
- [35] Michael A. Nielsen and Isaac L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition* (Cambridge University Press, 2010).
- [36] Sthitadhi Roy, J. T. Chalker, I. V. Gornyi, and Yuval Gefen, “Measurement-induced steering of quantum systems,” *Phys. Rev. Res.* **2**, 033347 (2020).
- [37] B. Kraus, H. P. Büchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, “Preparation of entangled states by quantum markov processes,” *Phys. Rev. A* **78**, 042307 (2008).
- [38] Leo Zhou, Soonwon Choi, and Mikhail D. Lukin, “Symmetry-protected dissipative preparation of matrix product states,” *Phys. Rev. A* **104**, 032418 (2021).
- [39] Haye Hinrichsen, “Non-equilibrium critical phenomena and phase transitions into absorbing states,” *Advances in Physics* **49**, 815–958 (2000).
- [40] M. Henkel, H. Hinrichsen, and S. Lübeck, *Non-Equilibrium Phase Transitions: Volume 1: Absorbing Phase Transitions*, Theoretical and Mathematical Physics (Springer Netherlands, 2008).
- [41] Matteo Marcuzzi, Michael Buchhold, Sebastian Diehl, and Igor Lesanovsky, “Absorbing state phase transition with competing quantum and classical fluctuations,” *Phys. Rev. Lett.* **116**, 245701 (2016).
- [42] Michael Buchhold and Sebastian Diehl, “Background field functional renormalization group for absorbing state phase transitions,” *Phys. Rev. E* **94**, 012138 (2016).
- [43] Ricardo Gutiérrez, Cristiano Simonelli, Matteo Archimi, Francesco Castellucci, Ennio Arimondo, Donatella Ciampini, Matteo Marcuzzi, Igor Lesanovsky, and Oliver Morsch, “Experimental signatures of an absorbing-state phase transition in an open driven many-body quantum system,” *Phys. Rev. A* **96**, 041602 (2017).
- [44] Igor Lesanovsky, Katarzyna Macieszczak, and Juan P Garrahan, “Non-equilibrium absorbing state phase transitions in discrete-time quantum cellular automaton dynamics on spin lattices,” *Quantum Science and Technology* **4**, 02LT02 (2019).
- [45] Federico Carollo, Edward Gillman, Hendrik Weimer, and Igor Lesanovsky, “Critical behavior of the quantum contact process in one dimension,” *Phys. Rev. Lett.* **123**, 100604 (2019).
- [46] E Chertkov, Z Cheng, AC Potter, S Gopalakrishnan, TM Gatterman, JA Gerber, K Gilmore, D Gresh, A Hall, A Hankin, *et al.*, “Characterizing a non-equilibrium phase transition on a quantum computer,” *arXiv preprint arXiv:2209.12889* (2022).
- [47] Lorenzo Piroli, Georgios Styliaris, and J. Ignacio Cirac, “Quantum circuits assisted by local operations and classical communication: Transformations and phases of matter,” *Physical Review Letters* **127** (2021).
- [48] Tsung-Cheng Lu, Leonardo A. Lessa, Isaac H. Kim, and Timothy H. Hsieh, “Measurement as a shortcut to long-range entangled quantum matter,” *arXiv preprint arXiv:2206.13527* (2022).
- [49] Nathanan Tantivasadakarn, Ashvin Vishwanath, and Ruben Verresen, “A hierarchy of topological order from finite-depth unitaries, measurement and feedforward,” *arXiv preprint arXiv:2209.06202* (2022).
- [50] See the Supplemental Material.
- [51] J. Ricardo G. Mendonça, “Monte carlo investigation of the critical behavior of stavskaya’s probabilistic cellular automaton,” *Phys. Rev. E* **83**, 012102 (2011).
- [52] Aidan Zabalo, Michael J. Gullans, Justin H. Wilson, Sarang Gopalakrishnan, David A. Huse, and J. H. Pixley, “Critical properties of the measurement-induced transition in random quantum circuits,” *Physical Review B* **101**, 060301 (2020).
- [53] Yaodong Li, Xiao Chen, Andreas W. W. Ludwig, and Matthew P. A. Fisher, “Conformal invariance and quantum nonlocality in critical hybrid circuits,” *Physical Review B* **104**, 104305 (2021).
- [54] Michael J. Gullans and David A. Huse, “Scalable Probes of Measurement-Induced Criticality,” *Physical Review Letters* **125**, 070606 (2020).
- [55] While the I_3 data along $p_f = 1$ still collapses well with $\nu \cong 1.1(2)$ [Fig. 3(d)], this value of ν is also consistent with the value $\nu_{\perp} = 1.1$ for directed percolation and thus the absorbing-state transition.
- [56] Benjamin Schumacher and Michael D. Westmoreland, “Approximate quantum error correction,” *Quantum Information Processing* **1**, 5–12 (2002).
- [57] In our numerics, n_d at the critical point is still relatively

high at the accessible system sizes, so this asymptotic separation might not be visible.

- [58] Ali Lavasani, Yahya Alavirad, and Maissam Barkeshli, “Measurement-induced topological entanglement transitions in symmetric random quantum circuits,” *Nature Physics* **17**, 342–347 (2021).
- [59] Shengqi Sang and Timothy H. Hsieh, “Measurement-protected quantum phases,” *Physical Review Research* **3** (2021).
- [60] Vikram Ravindranath, Yiqiu Han, Zhi-Cheng Yang, and Xiao Chen, “Entanglement steering in adaptive circuits with feedback,” *Phys. Rev. B* **108**, L041103 (2023).