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Sublattice Interference promotes Pair Density Wave order in Kagome Metals

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Motivated by the observation of a pair density wave (PDW) in the kagome metal CsV_3Sb_5 , we consider the fate of electrons near a p-type van Hove singularity (vHS) of the kagome lattice with onsite Hubbard U and the nearest neighbor repulsion V . We study the effect of such interactions on Fermi surface “patches” at the vHS. We show how a feature unique to the Kagome lattice known as sublattice interference crucially affects the form of the interactions among the patches. The renormalization group (RG) flow of such interactions results in a regime where the nearest neighbor interaction V exceed the onsite repulsion U . We identify this condition as being favorable for the formation of charge-density-wave (CDW) and PDW orders. In the weak coupling limit, we find a complex CDW order as the leading instability, which breaks time reversal symmetry. Beyond RG, we perform a Hartree-Fock study to a V -only model and find the pair-density-wave order indeed sets in at some intermediate coupling.

Introduction.— Spins and electrons on the Kagome lattice [Fig.1(a)] have long been studied due to their potential for exhibiting a panoply of exotic phases of matter. Insulating kagome compounds, for instance, are among the most prominent candidate spin liquid materials [1], and insulating phases with non-trivial topology have also been studied on the Kagome lattice [2–4]. With the discovery of a family of Kagome metals AV_3Sb_5 ($A=\text{K,Cs,Rb}$), a new wave of excitement has been elicited by the prospects for intriguing density wave and superconducting ground states in these systems [5–7].

Among the more fascinating observed phenomena in these kagome metals include new Bragg-like peaks inside the superconducting phase [8], a hallmark of an exotic superconducting order known as a pair density wave (PDW) [9–24]. PDWs are superconductors with an order parameter that varies periodically in space. Additionally, fascinating effects have been discovered in the superconducting fluctuation spectrum, including nearly condensed excited states with charge 4e, 6e superconducting fluctuations [25]. These observations call for a greater theoretical scrutiny, and invite us to make predictions for electronic phases of kagome metals.

In this Letter, motivated by these recent developments, we address the issue of whether the PDW superconducting phase can in principle arise on the Kagome lattice. Since such superconductivity requires analysis of the intermediate coupling problem, robust pairing mechanisms for PDW formation have only recently been uncovered [26]. One key requirement for PDW order is the presence of strong repulsive interactions with somewhat suppressed onsite interactions. While this is rather unusual in most solids, we show here that this requirement is met when the chemical potential crosses a so-called p-type (for “pure”) van Hove singularity (vHS) [27–30]

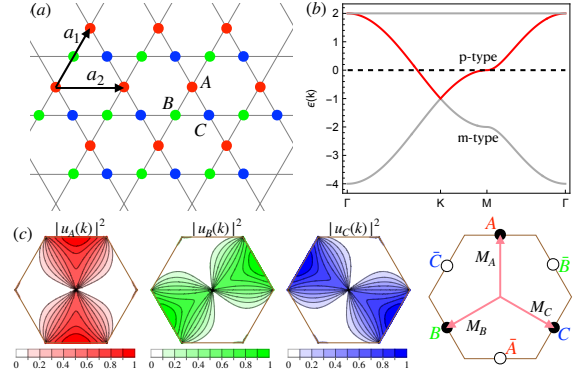


FIG. 1. Sublattice interference in the Kagome lattice. (a) Each unit cell has $\alpha = A, B, C$ different sublattice sites. (b) Band structure of the tight binding model. We focus on the middle band, highlighted with red color, which exhibits a p-type van Hove singularity. (c) For this band, the transformation matrix $u_\alpha(\mathbf{k})$, defined as $c_\alpha(\mathbf{k}) = u_\alpha(\mathbf{k})\psi_\alpha(\mathbf{k})$ where c_α and ψ_α are lattice and band fermions respectively, has a modulated distribution in the Brillouin zone.

[see Fig.1(b)]. In an important theoretical study, one of us showed [27] that precisely at such a p-type vHS, there is the phenomena of sublattice interference (SI) - where each of the 3 distinct sublattices has non-zero support only on *one* of the 3 distinct vH points. To show this SI crucially determines the low energy effective interactions in the system, we construct a renormalization group (RG) theory based on the p-type vHS and show that the onsite repulsion runs towards weak coupling, while nearest neighbor interactions grow under the RG. This peculiarity results in a rich phase diagram including time-reversal symmetry breaking charge-density-wave (CDW) orders and various uniform superconductivity in the weak coupling limit; while in the strong coupling regime it makes

the Kagome system poised to exhibit the PDW order. While more experiments are needed to characterize precisely the normal state in this system, our analysis already establishes that the kagome motif once again provides us with an avenue towards an exotic phase of matter, in this case the PDW superconductor.

SI and the triviality of Hubbard U .— To determine the fate of effective interactions at p-type van Hove filling, we utilize parquet RG methods and restrict attention to Fermi surface “patches” in the neighborhood of the distinct van Hove points [31, 32] (similar RG analysis with two-fold of van Hove singularities is considered in Ref.[33]). Defining the fermion destruction operators in patch α as ψ_α , we encode real-space 4-fermion interactions as intra- and interpatch couplings after Fourier transformation. In a crystal with time-reversal and/or inversion symmetry, such interactions take the form $H_I = g_1 \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\alpha \psi_\beta + g_2 \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\beta \psi_\alpha + g_3 \psi_\alpha^\dagger \psi_\alpha^\dagger \psi_\beta \psi_\beta + g_4 \psi_\alpha^\dagger \psi_\alpha^\dagger \psi_\alpha \psi_\alpha$ where $\alpha \neq \beta$ and the momentum summation and a spin configuration $\sigma\sigma'\sigma'\sigma$ is assumed. Patch models have been applied to the square lattice[34], the honeycomb lattice [35, 36] and moiré systems[21, 37, 38] to capture interaction-driven electronic orders. In both cases, *all* the g_i ’s are set by the largest Hubbard onsite interaction $H_U = U \sum_i n_{i\uparrow} n_{i\downarrow}$.

However, adopting the same strategy on the Kagome lattice near a p-type vHS, one finds that the Hubbard interaction U contributes only to the g_4 . This is due to SI: Near a p-type vHS, each of the 3 sublattices (A,B,C) has non-zero support only on *one* of the 3 distinct van Hove points ($\mathbf{M}_\alpha, \alpha = A, B, C$) which is visible through the sublattice weight in Fig.1(c). The SI does not result from fine tuning, as it exists even if longer range hoppings are considered [39]. Thus, when only H_U is present for electrons near a p-type vHS, $g_1 = g_2 = g_3 = 0$, and $g_4 = U/t$. With this choice of bare couplings, g_4 weakens under RG flow, eventually reaching a trivial fixed point with $g_4^* = 0$ [39]. Thus, due to SI, the repulsive Hubbard U is irrelevant in the RG sense near the p-type vHS of the Kagome lattice [40].

Extended interactions and the 6-patch theory.— Due to the apparent irrelevance of U , it is necessary to include at least nearest neighbor interactions. The most important such term is the nearest neighbor density-density repulsion $H_V = V \sum_{\langle ij \rangle} n_i n_j$. With both H_U and H_V , the 3-patch model described above is no longer adequate since nearest neighbor interactions contain momentum dependence, which differentiates the patches at $\pm \mathbf{M}_\alpha$. Therefore, we need to consider the full 6-patch theory, for which there are 16 different symmetry allowed interactions in total, as shown in Fig.2(a). Here we adopt the convention used in Ref.[37] and define the patch fermions as $\psi_{\alpha+} = \psi(\mathbf{k} \rightarrow \mathbf{M}_\alpha)$ and $\psi_{\alpha-} = \psi(\mathbf{k} \rightarrow -\mathbf{M}_\alpha)$. The

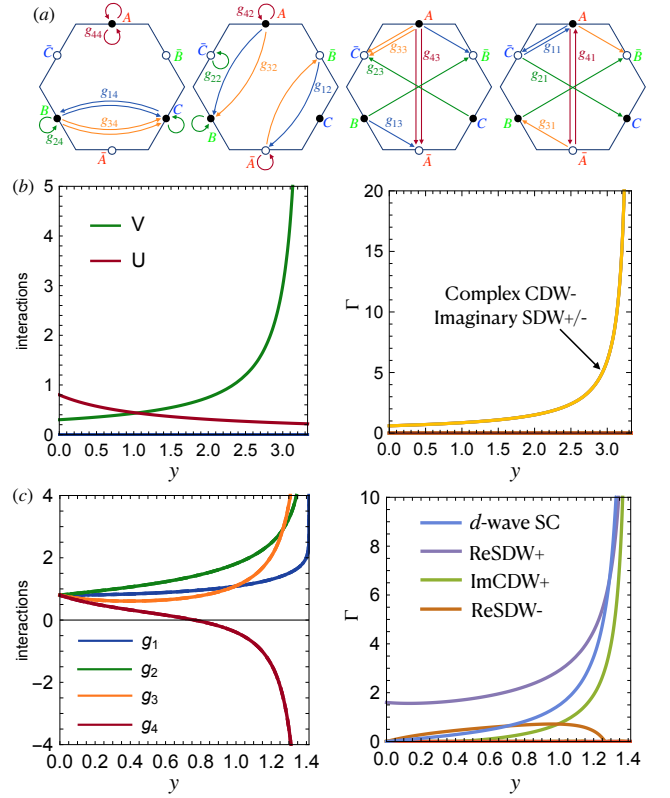


FIG. 2. (a) All symmetry allowed interactions in the six patch model.(b) RG flow for the interactions and the order parameter vertex Γ with the SI effect. We set $U(0) = 0.8t$ and $V(0) = 0.3t$ and starting from some intermediate energy scale we have $V \gg U$. The resulting weak coupling instabilities are degenerate complex CDW₋ and imaginary SDW_{+/-}. (c) The same RG flow but without SI effect. In this case all the $g_{ij,0}$ are set by the largest Hubbard U , and there does not exist a constant map U, V and $g_{ij}(y)$. The leading weak coupling instability is the d -wave SC[35, 36].

interactions in this six patch model can be written as

$$H_I = \sum_{i,j=1}^4 \sum_{\{\alpha_j, \tau_j\}} g_{ij} \psi_{\alpha_1 \tau_1}^\dagger \psi_{\alpha_2 \tau_2}^\dagger \psi_{\alpha_3 \tau_3} \psi_{\alpha_4 \tau_4}, \quad (1)$$

with g_{ij} being the dimensionless interaction strengths. Momentum conservation constrains the indices as follows: the patch indices satisfy $\alpha_1 = \alpha_3 \neq \alpha_2 = \alpha_4$ for $i = 1$, $\alpha_1 = \alpha_4 \neq \alpha_2 = \alpha_3$ for $i = 2$, $\alpha_1 = \alpha_2 \neq \alpha_3 = \alpha_4$ for $i = 3$, and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$ for $i = 4$. The ‘valley’ indices $\tau_i = \pm$, labeling whether the patch is at $\mathbf{M}_\alpha$ or $-\mathbf{M}_\alpha$, obey the same rule associated with j . That is, $\tau_1 = \tau_3 \neq \tau_2 = \tau_4$ for $j = 1$ et cetera.

Once again, SI crucially influences the initial conditions of the RG flows. A straightforward calculation [39] shows that U contributes only to g_{4j} - a direct generalization of the 3-patch theory - while V only contributes

to g_{2j} :

$$g_{4j,0} = \frac{U}{t}; g_{22,0} = g_{24,0} = -g_{21,0} = -g_{23,0} = \frac{2V}{t}. \quad (2)$$

The subscript 0 above denotes bare interactions before running RG. From microscopics, it is natural to expect that U (and therefore $g_{4j,0}$) should be the largest.

CDW at weak coupling.— We first investigate the weak coupling limit. In this case the fate of the fermions can be described by the one loop RG equations of g_{ij} , for which we keep the most divergent “log squared” terms in perturbation theory, i.e. the particle-hole bubble $\Pi_{ph}^{(0)}$ at momenta $\mathbf{M}_\alpha$ and the particle-particle bubble $\Pi_{pp}^{(0)}$ at zero momentum, and set the running parameter as the latter: $y = \Pi_{pp}^{(0)}(0)$ [37, 39]. With this convention, the RG equations for the interactions take the form $dg_{ij}/dy = g_{mn}R_{mn,kl}g_{kl}$. It’s easy to see that close to the critical value y_c all g_{ij} behaves in a similar manner, namely $g_{ij} = G_{ij}/(y_c - y)$. If some G_{ij} is nonzero, the corresponding g_{ij} thus diverges at y_c . A typical order parameter flows as $d\Delta_i/dy = d\Gamma_i\Delta_i$ under renormalization, where $d = 1$ for superconducting orders and $d = d_1 \leq \frac{1}{2}$ for density wave orders, and Γ_i is certain linear combination of g_{ij} [39]. From this we can integrate to obtain the behavior of the corresponding susceptibility χ_i which scales as $\chi_i \sim (y_c - y)^{\gamma_i}$. The leading instabilities will be those with the most negative γ_i , and accordingly the most divergent Γ_i . We mainly consider the following weak coupling instabilities,

$$\begin{aligned} \Delta_{CDW\pm} &= \langle \psi_{\alpha+}^\dagger \psi_{\beta+} \pm \psi_{\alpha-}^\dagger \psi_{\beta-} \rangle, \quad \Delta_{SC} = \langle f_\alpha \psi_{\alpha+} \psi_{\alpha-} \rangle, \\ \Delta_{SDW\pm} &= \langle \psi_{\alpha+}^\dagger \sigma \psi_{\beta+} \pm \psi_{\alpha-}^\dagger \sigma \psi_{\beta-} \rangle. \end{aligned} \quad (3)$$

Note that the form factor $f_\alpha = \pm 1$ in Δ_{SC} determines the pairing symmetry. For the density wave orders, their real and imaginary parts flow differently, so depending on whether their imaginary parts are zero or not, the system could either preserve or break the time reversal symmetry.

In Fig.2(b) we show the RG results in the presence of SI, i.e. the initial values of g_{ij} are set by Eq.(2). Interestingly, we find that this map between lattice interactions and g_{ij} persists for all $y < y_c$, and it is this constant map that enables us to extract the RG flows for U and V . We see the U decays as before, while V increases. The leading instability in this case is the degenerate complex Δ_{CDW-} and imaginary $\Delta_{SDW\pm}$. The presence of imaginary parts for these density wave orders indicate the time reversal symmetry is broken[41–47], which is due to the SI effect. For comparison, we also show in Fig.2(c) the RG analysis for the model but with no SI effect. In this case, all g_{ij} are set by the largest interaction U , and the resulting weak coupling instability is the d -wave uniform superconductivity, consistent with Ref.[35, 36].

PDW at intermediate coupling.—Having established the

way different interactions get renormalized in the presence of the SI effect, we now discuss how the PDW order can emerge at some intermediate energy scale. Based on the observation discussed above [see Fig.2(b) for example], we consider an effective model where only a large V is kept. Model similar this has also been studied recently in Ref.[48]. We consider a sufficiently large V , and perform a Hartree-Fock mean-field study[49–53] for the corresponding orders[54]. Without loss of generality, we discuss the AB bond only, as the other bonds follow via C_3 rotation. On the AB bond, the relevant interaction is $2V \cos(\mathbf{q} \cdot \boldsymbol{\alpha}) \psi_{A\sigma}^\dagger(\mathbf{k}) \psi_{B\sigma'}^\dagger(\mathbf{k}') \psi_{B\sigma'}(\mathbf{k}' + \mathbf{q}) \psi_{A\sigma}(\mathbf{k} - \mathbf{q})$ where $\boldsymbol{\alpha} = \mathbf{a}_1/2$ and $\mathbf{a}_1$ is the vector connecting two adjacent A and B sites [see Fig.1 (a)]. Whether this is repulsive or attractive depends on the momentum transfer $\mathbf{q}$. In the Cooper channel, we can write it as $2V \cos[(\mathbf{k} + \mathbf{k}' - \mathbf{q}) \cdot \boldsymbol{\alpha}] \psi_{A\sigma}^\dagger(\mathbf{k}) \psi_{B\sigma'}^\dagger(-\mathbf{k} + \mathbf{q}) \psi_{B\sigma'}(\mathbf{k}') \psi_{A\sigma}(-\mathbf{k}' + \mathbf{q})$, which is attractive when $\cos[(\mathbf{k} + \mathbf{k}' - \mathbf{q}) \cdot \boldsymbol{\alpha}] < 0$. In the patch model this condition is met when $\mathbf{k}$ is around $\mathbf{M}_A$, $-\mathbf{k} + \mathbf{q}$ is around $-\mathbf{M}_B$, and $\mathbf{k}'$ is around $-\mathbf{M}_A$. The interaction then becomes $-2V \psi_A^\dagger \psi_B^\dagger \psi_B \psi_A$. When V is large, we can use

$$\psi_A^\dagger \psi_B^\dagger \psi_B \psi_A \approx \langle \psi_A^\dagger \psi_B^\dagger \rangle \psi_B \psi_A + \psi_A^\dagger \psi_B^\dagger \langle \psi_B \psi_A \rangle \quad (4)$$

for mean field analysis, and the gap function, defined as $\Delta_{\mathbf{Q}} \sim \langle \psi_B \psi_A \rangle$, is apparently a PDW order with momentum $\mathbf{M}_C$. We note that, due to the presence of SI, the uniform superconductivity is not a competing order in the large V model, since such order couples to, e.g. $\psi_A^\dagger \psi_A$, which can not be obtained by decomposing Eq.(4).

Similarly, the onsite CDW does not arise since this order parameter couples to, e.g. $\psi_A^\dagger \psi_A$, and effectively becomes the chemical potential. However, the bond charge density wave (bond CDW) order can indeed arise when V becomes strong [55], and compete with PDW. This occurs also when $\cos(\mathbf{q} \cdot \boldsymbol{\alpha}) < 0$ and the density interaction becomes $-2V \psi_A^\dagger \psi_B^\dagger \psi_B \psi_A$ with $V > 0$. We can, however, consider

$$-\psi_A^\dagger \psi_B^\dagger \psi_B \psi_A \approx \langle \psi_A^\dagger \psi_B \rangle \psi_B^\dagger \psi_A + \psi_A^\dagger \psi_B \langle \psi_B^\dagger \psi_A \rangle \quad (5)$$

to absorb the minus sign and manifestly show it contains strong repulsion in the particle-hole channel. Moreover, the spin indices on the left hand side of Eq. 5 can be included explicitly, namely $-\psi_{A\alpha}^\dagger \psi_{B\beta}^\dagger \psi_{B\gamma} \psi_{A\delta} \delta_{\alpha\delta} \delta_{\beta\gamma}$, and using the $SU(2)$ identity $\delta_{\alpha\delta} \delta_{\beta\gamma} = (\sigma_{\alpha\gamma} \delta_{\beta\delta} + \sigma_{\alpha\gamma} \sigma_{\beta\delta})/2$, it is easy to see that the order parameter $\Delta_{\mathbf{Q}} = \langle \psi_A^\dagger \psi_B \rangle$ represents a bond CDW/SDW order which are directly related to those density waves that arise in the weak coupling limit. In fact, the degeneracy is an artifact of the oversimplification of the patch model. Considering that the SDW order has to break a global $SU(2)$ symmetry, and upon taking the whole Fermi surface into account, we assume for the time being that the CDW order is

more likely to occur than SDW. In the following we thus constrain ourselves to CDW order competing with PDW at large V .

To inspect the competition between PDW and CDW at large $V > 0$, we can calculate the susceptibilities for both of the two orders in random-phase-approximation (RPA). The result is $\chi_{PDW} = \Pi_{pp}^0(\mathbf{Q})/[1 - 2V\Pi_{pp}^0(\mathbf{Q})]$ and $\chi_{CDW} = \Pi_{ph}^0/[1 - 2V\Pi_{ph}^0(\mathbf{Q})]$. Here the bare susceptibilities $\Pi_{pp}^0 > 0$ and $\Pi_{ph} > 0$ are obtained using free fermion propagators. The ordering condition is given when the denominators vanish. The problem is now reduced to comparing the strength of the bare $\Pi_{pp}^{(0)}$ and $\Pi_{ph}^{(0)}$. Within the patch model, the particle-particle and particle-hole bubbles can be obtained in a straightforward way, and the results are[56]

$$\begin{aligned}\Pi_{pp}^{(0)}(\mathbf{Q}) &= \frac{1}{4\sqrt{3}\pi^2 t} \ln \frac{\Lambda}{\max\{T, |\mu|\}} \\ \Pi_{ph}^{(0)}(\mathbf{Q}) &= \frac{1}{8\sqrt{3}\pi^2 t} \ln \frac{\Lambda}{\max\{T, |\mu|\}} \ln \frac{\Lambda}{\max\{T, \mu, |t'|\}},\end{aligned}\quad (6)$$

where $\mu = 0$ at the van Hove filling, and t' is the next nearest neighbor hopping which can spoil the Fermi surface nesting in the particle-hole channel. In Fig. 3(a) we compare both $\Pi_{pp}^{(0)}$ and $\Pi_{ph}^{(0)}$ as a function of temperature T at $\mu = 0$. We see that the $\ln^2$ divergence of $\Pi_{ph}^{(0)}$ becomes dominant at small T for the case of perfect nesting ($t' = 0$), while at larger T , $\Pi_{pp}^{(0)}$ is larger even with perfect nesting. The consequence is that, in the weak coupling limit, one has to go to small T to see the instability, where CDW wins over PDW. For large V , however, a relatively smaller $\Pi_{pp}^{(0)}$ or $\Pi_{ph}^{(0)}$ is enough to induce the instability, which could be in the regime where $\Pi_{pp}^{(0)} > \Pi_{ph}^{(0)}$ and PDW is the leading instability. Note that Eq.(6) is only a crude estimation about the relative strength between $\Pi_{pp}^{(0)}(\mathbf{Q})$ and $\Pi_{ph}^{(0)}(\mathbf{Q})$. Away from the van Hove filling ($\mu \neq 0$), we expect that both bubbles are reduced, and therefore one needs to reach smaller T in order to reach the instability. This results in the schematic picture we show as the inset of Fig.2(a).

Other interactions. We note that other than U and V , further lattice interaction might be worth analyzing. For instance, we can consider the bond singlet pair hopping term, for which, again taking AB bond as an example, the on-bond contribution is $J_0 \sum_i P_{AB}^\dagger(\mathbf{r}_i) P_{AB}(\mathbf{r}_i)$, where the pair operator at position $\mathbf{r}_i$ is given by $P_{AB}^\dagger(\mathbf{r}_i) = c_{A\uparrow}^\dagger(\mathbf{r}_i + \boldsymbol{\alpha}) c_{B\downarrow}^\dagger(\mathbf{r}_i) - c_{A\downarrow}^\dagger(\mathbf{r}_i + \boldsymbol{\alpha}) c_{B\uparrow}^\dagger(\mathbf{r}_i)$ and $\boldsymbol{\alpha} = \mathbf{a}_1/2$ is the vector connecting two adjacent A and B sites. In fact, the J_0 term just reads as the exchange interaction. The site pair hopping term such as $K_0 \sum_{\langle ij \rangle} P_i^\dagger P_j$ with $P_i^\dagger = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger$ is also possible. With the SI effect, it is easy to realize that J_0 contributes to both g_{1j} and g_{2j} . In particular, $g_{11}(0) = g_{14}(0) = -g_{12}(0) = -g_{13}(0) = 2J_0$, and $-g_{21}(0) = -g_{23}(0) =$

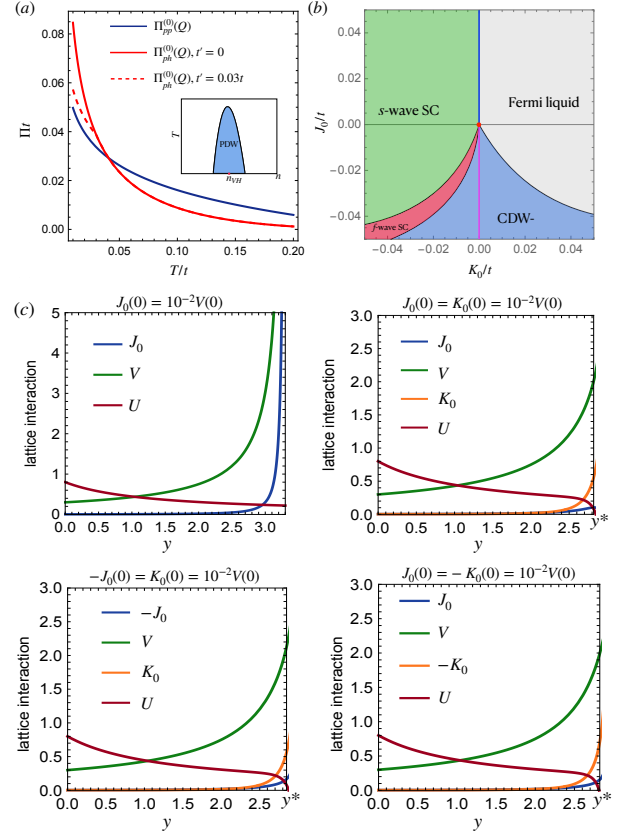


FIG. 3. (a) Comparison between $\Pi_{pp}^{(0)}(\mathbf{Q})$ and $\Pi_{ph}^{(0)}(\mathbf{Q})$ as a function of temperature T . The fact that $\Pi_{pp}^{(0)}(\mathbf{Q}) > \Pi_{ph}^{(0)}(\mathbf{Q})$ at larger T indicates that PDW order may win over CDW order when interaction is sufficiently strong. (b) The weak coupling instabilities (at perfect nesting) when other interactions such as J_0 and K_0 are taken into account. (c) RG flows for the lattice interactions for $U(0) = 0.8t$ and $V(0) = 0.3t$ and for various J_0 and K_0 . In all cases there is a wide range in energy scale where V is the largest.

$g_{22}(0) = g_{24}(0) = 2V + 2J_0$. The K_0 term only contributes to g_{3j} : $g_{31}(0) = g_{32}(0) = -g_{33}(0) = -g_{34}(0) = 2K_0$. We expect that J_0 and K_0 are of the same order, and both are orders of magnitudes smaller than U and V [57–60]. In the presence of J_0 and K_0 , both of them increase under RG, leading to different orders in the weak coupling limit which we present in Fig.3(b). These include s - and f -wave uniform SC, the real CDW order with odd parity (CDW₋). Interestingly, if we keep $K_0 = 0$ there is still a constant map between U, V, J_0 and g_{ij} for all $y < y_c$, and the weak coupling instabilities are degenerate imaginary $\Delta_{SDW\pm}$ for $J_0 > 0$, and a complex Δ_{CDW-} for $J_0 < 0$ [see the blue and purple line in Fig.3(b)]. A finite K_0 spoils the constant map between lattice interactions and g_{ij} at y near y_c , but at some intermediate $y^* < y_c$ we can still approximately determine the flow of U, V, J_0 and K_0 . Fig.3(c) (see also in [39]) shows that V can still be the largest at intermediate energy scale for various cases, which justifies the appli-

cability of our effective V model discussed above. We close by noting the possibility that when a sizable K_0 is present initially, K_0 could become comparable to V even at $y < y^*$. Since K_0 is related to the formation of uniform SC, and the uniform pairing susceptibility $\Pi_{pp}^{(0)}(0)$ scales as $\ln^2(\Lambda/T)$, one could expect that uniform SC may become leading order. However, in a system with small $K_0(0)$ and thus can be accurately described by the U - V model, our results are applicable.

Discussion.- We have focused our analysis entirely on the case of electrons subject to instantaneous repulsive interactions on the Kagome lattice near p-type van Hove filling. At the current level of understanding, it is not yet settled to which extent the kagome metals AV_3Sb_5 ($A=K,Cs,Rb$) are faithfully represented by this simplified model. Previous attempts to improve on the microscopic rigor by taking into account the multi-orbital nature at the Fermi level [28, 61], combined with the relevance of phononic contributions [62], suggest the need for further analysis to quantitatively approach the experimental setup in AV_3Sb_5 , and further experiments involving Kagome metals are necessary to validate the nature of the CDW observed at scales as high as $T \sim 90K$. In particular, such order would of course affect the reconstructed bands out of which the superconductivity ultimately develops. A coherent theory of such orders, including the effects of phonons, will only be feasible once the precise nature of the CDW order in this system is well-characterized.

Our analysis does, however, make a rare *microscopically founded* statement about the realization of PDW in a physically sensible model, specifically in a kagome metal nearby a p-type vHS with local and nearest neighbor Coulomb repulsion. This is because SI crucially affects the renormalization of repulsive interactions in Kagome metals, and thus the extended repulsive forces are enhanced relative to the onsite Hubbard repulsion. As a result, there is an increased tendency towards PDW and bond CDW order, which we identified without any need for a potentially biased mean field analysis. Further input from experiment is needed at this stage to bridge the ideas described in this paper with the thus far concluded phase diagram of Kagome metals.

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