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Multiple conduction channels interacting with a quantum impurity – a spin in the conventional "multi-channel Kondo effect" or a topological mesoscopic device ("topological Kondo effect") – has been proposed as a platform to realize anyonic quasi-particles. However, the above implementations require either perfect channel symmetry or the use of Majorana fermions. Here we propose a Majorana-free mesoscopic setup which implements the Kondo effect of the symplectic Lie group and can harbor emergent anyons (including Majorana fermions, Fibonacci anyons, and $\mathbb{Z}_3$ parafermions) even in the absence of perfect channel symmetry. In addition to the detailed prescription of the implementation, we present the strong coupling solution by mapping the model to the multi-channel Kondo effect associated to an internal $SU(2)$ symmetry and exploit conformal field theory (CFT) to predict the non-trivial scaling of a variety of observables, including conductance, as a function of temperature. This work does not only open the door for robust Kondo-based anyon platforms, but also sheds light on the physics of strongly correlated materials with competing order parameters.

Introduction. The realization of fault tolerant quantum computation is a major goal of present day quantum research. Amongst the various hardware platforms suitable for this application, topologically ordered states with anyonic excitations are particularly appealing [1], as the robustness against noise and errors is a fundamental, intrinsic property of these quantum many-body phases. A classic platform for realizing anyons which has gained renewed interest in mesoscopic systems are frustrated and overscreened Kondo impurity models [2, 3].

The $SU(2)$ Kondo effect is a paradigmatic model of quantum many-body physics [2–6] which merges the physics of strong electronic correlations and entanglement, whilst its strong coupling physics is still amenable to non-perturbative analytical methods such as Bethe *ansatz* [7–9], CFT [10, 11], and Abelian bosonization [12]. Even though the impurity spin in the conventional Kondo effect is perfectly screened at strong coupling, the overscreened multi-channel Kondo (MCK) effect, in which $k > 2S$ electronic baths compete for screening a single spin-S, is one of the earliest examples of quantum criticality and local non-Fermi liquid (FL) behavior, and harbors a remnant zero temperature impurity entropy [13–15] $S_{\rm imp} = \ln(g_k)$ with $g_k = \sqrt{2}, (1+\sqrt{5})/2, \sqrt{3}, \ldots$ for $S = 1/2$ and $k = 2, 3, 4, \ldots$ consistent with the quantum dimensions of Ising, Fibonacci, and $\mathbb{Z}_3$ parafermionic anyons. It has thus recently been proposed to exploit these anyons for quantum information theoretical applications [16–18], but a major technical difficulty is that multichannel Kondo physics, even with $SU(N)$ and $N > 2$, is unstable with respect to unequal coupling to different electronic baths.

Stable overscreened fixed points may be achieved by using strongly interacting [19, 20] or higher spin [21] conduction electrons, or by going beyond the conventional $SU(2)$ group. A recent example of the latter is the orthogonal Kondo effect in which spin-polarized conduction electrons couple to an impurity spin transforming under the group $SO(M)$. The orthogonal Kondo effect for arbitrary M can be realized with the use of Majorana Cooper pair boxes [22–25], in which case it is called the topological Kondo effect. While fascinating, this implementation is temporarily elusive as the control over Majorana devices is still developing. Another, Majorana-free implementation for the special case $M = 5$ was recently proposed [26, 27] and it was argued that Ising anyons (Majorana) are emergent at the infrared.

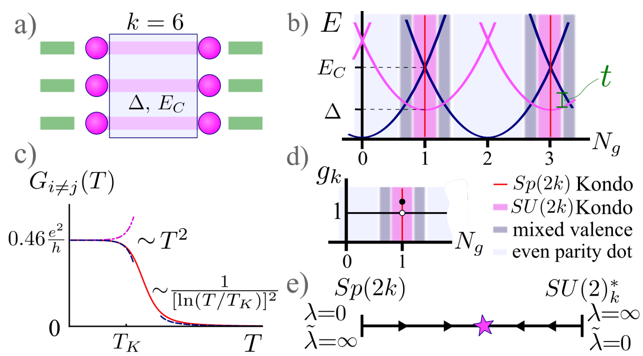

FIG. 1. a) Schematics of proposed implementation for $k = 6$ with Δ the proximity-induced gap and E_C the charging energy. Light green squares are the leads and light purple dots are the ends of 1D topological systems. The light gray square is a superconducting island. b) Energy levels as a function of gate voltage (as imposed by charge N_g; we take $\Delta = 0.4E_C$). The tunneling strength t is between the dots and the leads (nearest sites). Dark blue (light purple) curves correspond to states with even (odd) fermion parity. The background shading corresponds to different effective low-energy theories (see legend). c) Transconductance. The solid red line interpolates between the low- and high-temperature asymptotic behavior. The conductance quantization at $T = 0$ is universal, cf. Eq. (9), and equal to $(4/3)\sin^2(\pi/5) \approx 0.46e^2/h$ for $k = 3$. In the weak coupling regime, the conductance has a logarithmic temperature-dependence. d) Ground state degeneracy. e) Schematic RG flow illustrating the duality between $Sp(2k)$ Kondo effect and k-channel $SU(2)$ Kondo effect at spin $S = (k-1)/2$.

In this paper, we propose a mesoscopic setup, see Fig. 1 realizing the symplectic Kondo effect as a platform for anyons and potentially for measurement-only topological quantum computation [28]. Following Cartan's classification of Lie groups [29, 30], we here explore the third remaining type of Lie group $Sp(2k)$, i.e. a symplectic Kondo Hamiltonian

$$H_K = \lambda \sum_{A=1}^{k(2k+1)} S^A J^A \,, \tag{1}$$

in which the symplectic impurity "spin" operators S_A transform in the fundamental $2k$-dimensional representation, and $J^A = c_0^\dagger T_A c_0$ is the symplectic spin of conduction electrons; the spinor $c_a = (c_{a,1,\uparrow}, \cdots, c_{a,k,\uparrow}, c_{a,1,\downarrow}, \cdots, c_{a,k,\downarrow})^T$ for site a has $2k$ components with $i = 1, \ldots k$ denoting the lead index and $\sigma = \uparrow, \downarrow$ the physical spin. Despite the $2k$ components of the spinor, Eq. (1) is still a one-channel Kondo model and will therefore not suffer from channel anisotropy. The $2k \times 2k$ matrices $T_A = -\sigma_y T_A^T \sigma_y$ denote $Sp(2k)$ generators in the fundamental representation [31]. We present a mesoscopic implementation of this effect for arbitrary k, the phase diagram for this nano-device, characteristic signatures in transport measurements as well as a solution of the symplectic Kondo effect in the strong coupling limit.

From the perspective of materials science, symplectic Kondo models are theoretically appealing as they allow for a proper definition of time reversal symmetry and thus for large-N descriptions of heavy fermion superconductors [32]. At the same time $SO(5) \sim Sp(4)$ theories of cuprates are popular approaches to account for competing orders [33]. From the viewpoint of quantum information theory, the symplectic Kondo effect allows for the arguably most robust way of realizing anyons in impurity models: In addition to the aforementioned stable implementation of non-trivial anyons, earlier work using CFT [26, 27, 30] demonstrates that – contrary to standard multi-channel Kondo phenomenology – the leads behave FL-like (suggesting relatively strong decoupling of anyons and conduction electrons) and that Fibonacci anyons (which are the simplest anyons allowing for universal quantum computation) can not be realized in the simplest realization of the topological Kondo effect, but are accessible in the present $Sp(6)$ setup.

Implementation of the $Sp(2k)$ Kondo model. We consider k spinful fermionic zero-energy states coupled to a floating s-wave superconductor, see Fig. 1a. These states may stem from a time-reversal symmetric higher-order topological insulator, resonant levels of quantum dots, or a set of Su-Schrieffer-Heeger chains. The low-energy Hamiltonian of our topological quantum dot is,

$$\begin{aligned} H_\text{d} =& E_C(2\hat{N}_C + \hat{n}_d - N_g)^2 \\ & - \frac{1}{2}\Delta \sum_{i=1}^{k} \sum_{\sigma\sigma'} \mathrm{e}^{-\mathrm{i}\phi} d_{i,\sigma}^\dagger (\sigma_y)_{\sigma\sigma'} d_{i,\sigma'}^\dagger + \text{H.c.}, \end{aligned} \tag{2}$$

where $\hat{n}_d = \sum_{i,\sigma} d_{i,\sigma}^\dagger d_{i\sigma}$ is the total charge in the edge states and $\hat{N}_C = -\mathrm{i}\partial_\phi$ is the number operator of the Cooper pairs of the s-wave superconductor. The Hamiltonian Eq. (2) conserves the total number of electrons $\hat{N}_\text{tot} = 2\hat{N}_C + \hat{n}_d$, controllable by the gate charge N_g. We assume that the island size exceeds the superconducting coherence length, so that crossed-Andreev reflection as well as hybridization of zero modes can be neglected, and that the proximity-induced gap Δ on the boundary states of the topological wires is less than the bulk gap, allowing us to ignore quasiparticle states of the parent superconductor in Eq. (2). We also take the gap to be smaller than the charging energy, $\Delta < E_C$, enabling a ground state with an odd number of electrons. We ignore additional mutual charging energies between the zero modes, which is a good assumption when the central superconducting island has a large normal-state conductivity [34, 35].

In the absence of Δ, each state with even N_tot is degenerate, with allowed values $n_d = 0, 2, 4, \ldots, 2k$ and all possibilities to distribute these electrons over the topological edge states. Similarly, the states with odd N_tot are also degenerate with $n_d = 1, 3, 5, \ldots, 2k-1$ allowed. The presence of Δ lifts the degeneracy as it allows to connect different states and favors a single BCS-like ground state $|\text{BCS}\rangle_d$ in the even sector (see supplement [36] for details). In the odd sector, there are $2k$ ground states given in which one of the k spin-degenerate boundary states is singly occupied, while the remaining $k-1$ are occupied by a BCS-like state, see Fig. 2, a). The ground state energy of the even sector is,

$$E_\text{even}(N_\text{tot}) = E_C(N_\text{tot} - N_g)^2 - \Delta k, \tag{3}$$

while $E_\text{odd} = E_\text{even} + \Delta$. These energies are plotted in panel b) of Fig. 1 (there, all energies E are measured with respect to $-\Delta k$).

In the $2k$-fold degenerate odd sector the quantum dot acts as an effective $Sp(2k)$ impurity. We will therefore consider N_g close to 1, where the $2k$ odd parity states with $N_\text{tot} = 1$ are lowest in energy while the lowest excited states (with $N_\text{tot} = 0, 2$) are separated by an energy gap $\Delta E_\pm = E_\text{even}(N_\text{tot} = 1 \pm 1) - E_\text{odd}(N_\text{tot} = 1)$. To derive the effective Kondo interaction, we next consider tunneling between the electrons on the dot and the first site ($a = 0$) of the lead, $H_t = -\sum_{i=1}^k \sum_{\sigma=\uparrow,\downarrow} t_i c_{0,i,\sigma}^\dagger d_{i\sigma} + \text{H.c.}$. At low temperatures and bias voltages in the weak tunneling limit, $k_B T, eV, t_i \ll \Delta E_\pm$, the dot occupation cannot change and H_t induces an effective Kondo interaction in second order perturbation theory. When we fine-tune all $t_i = t$ ($\forall i = 1, \ldots k$), we get [36],

$$H_\text{eff} = -\lambda_1 \big(d^\dagger c_0\big)\big(c_0^\dagger d\big) - \lambda_2 \big(d^\dagger \sigma_y c_0^*\big)\big(c_0^T \sigma_y d\big), \tag{4}$$

where $c_a^* = (c_a^\dagger)^T$ and Gutzwiller projection to the $2k$ $N_\text{tot} = 1$ states is understood. The coupling constants are $\lambda_1 = 2t^2/(\Delta E_-) > 0$ and $\lambda_2 = 2t^2/(\Delta E_+) > 0$. Exactly at $N_g = 1$, and after using the completeness relation of symplectic generators, $\sum_A T_A^{ij} T_A^{kl} = [\delta_{il}\delta_{jk} - (\sigma_y)_{ki}(\sigma_y)_{jl}]/2$, Eq. (4) becomes a Kondo-type interaction, Eq. (1), with (bare) coupling constant $\lambda = 2\lambda_1 = 2\lambda_2 = 4t^2/(E_C - \Delta)$. As we will see below, the anisotropy of tunneling strength t_i is irrelevant.

Weak and strong coupling. Perturbation theory in the Kondo term H_K leads to the usual logarithmic divergence

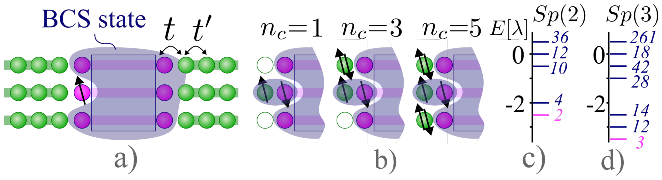

FIG. 2. a) The $2k$-degenerate ground state in the odd parity sector is given by a BCS state supplemented by one unpaired electron. b) Illustration of k charge degenerate ground states in the extreme strong coupling limit $t' = 0$ and c)-d) the corresponding energy spectrum.

at second order [37]. We therefore use the renormalization group (RG) technique to analyze Eq. (1) upon lowering the bare bandwidth/cutoff $D_0 \sim E_C - \Delta$ to a running cutoff $D = D_0 e^{-l}$ [38, 39]. We find the RG equation,

$$\frac{\mathrm{d}\lambda}{\mathrm{d}l} = (k+1)\rho_0\lambda^2, \tag{5}$$

where $\rho_0 = (\pi\hbar v_F)^{-1}$ denotes the lead density of states per spin per length and v_F is the Fermi velocity. Equation (5) implies that λ flows towards stronger coupling upon reducing the energy cutoff (set by, e.g., the temperature). We estimate the strong coupling scale to be $T_K \sim (E_C - \Delta)e^{-1/[\rho_0\lambda(D_0)2(k+1)]}$ in terms of the bare coupling.

Given that the isotropic weak-coupling fixed point ($\lambda = 0$) is unstable, with RG flow towards strong coupling, we will next investigate the stability of the strong-coupling fixed point, where the local Kondo interaction (1) is the dominant term in the Hamiltonian, and we can treat kinetic energy $t' \sim 1/\rho_0$ of the leads perturbatively. On the bare level, this corresponds to the limit $t' \ll t^2/\Delta E_\pm$ of the mesoscopic device introduced above, see Fig. 2a).

We start by finding the unperturbed ground state of Eq. (1), without kinetic terms. Similarly to Nozières' [40] description of the conventional $SU(2)$ Kondo problem, the strong coupling ground state is given by singlets formed by the impurity and the conduction electrons. We systematically derived the spectrum [36] of this problem using representation theory, and additionally explicitly constructed the singlet ground state wave functions for all k and the excited states for $k = 2, 3$, see Fig. 2 c),d). We find that the $Sp(2k)$ Kondo Hamiltonian is overscreened, with k degenerate ground states at $t' = 0$, e.g. for $k = 2$ these are the $Sp(2k)$ singlets,

$$|N=1\rangle_{\text{singlet}} = -\, i(d^\dagger\sigma_y c_0^*)\ |0\rangle_c \otimes |\text{BCS}\rangle_d\,, \tag{6a}$$

$$|N=3\rangle_{\text{singlet}} = (d^\dagger c_0)\ |4\rangle_c \otimes |\text{BCS}\rangle_d\,, \tag{6b}$$

where $|4\rangle_c$ is the state in which all electronic states on the first site of the lead are filled, while $|0\rangle_c$ is the empty state. For generic k, the degeneracy is a consequence of the symplectic symmetry associated to superconductivity[41]: Given that $|N=1\rangle_{\text{singlet}}$ is a singlet, the states $(c_0^\dagger\sigma_y c_0^*)\,|N=1\rangle_{\text{singlet}}\,,\ldots,(c_0^\dagger\sigma_y c_0^*)^{k-1}\,|N=1\rangle_{\text{singlet}}$ transform trivially under $Sp(2k)$, as well [42], see Fig. 2 b).

The above states, Eq. (6a)–(6b), are related by particle-hole symmetry (PHS). More generally, PHS implies an inherent $SU(2)$ symmetry in Nambu space for the symplectic Kondo Hamiltonian [32] which can be made apparent by writing the $Sp(2k)$ currents as symmetric form,

$$J^A = \frac{1}{2}\begin{pmatrix} c_0^\dagger & c_0^T(i\sigma_y)\end{pmatrix}\begin{pmatrix} T_A & \mathbf{0}\\ \mathbf{0} & T_A\end{pmatrix}\begin{pmatrix} c_0\\ (-i\sigma_y)c_0^*\end{pmatrix}, \tag{7}$$

which is invariant under $SU(2)$ rotations in particle-hole space. We used here the property $T_A^T = -\sigma_y T_A \sigma_y$ of $Sp(2k)$ generators. After having established the $t' = 0$ ground states, we now incorporate the nearest-neighbor hopping $H_{\text{NN}} = -t'\sum_{i=1}^{k}\sum_{\sigma=\uparrow,\downarrow}(c^\dagger_{0,i,\sigma}c_{1,i,\sigma} + c^\dagger_{1,i,\sigma}c_{0,i,\sigma})$ as a perturbation to study the stability of the strong coupling fixed point. H_{NN} will couple the degenerate strong coupling ground states, Eqs. (6a)–(6b), in second order perturbation theory, while preserving the $SU(2)$ symmetry. Inspired by the $SU(2)$ symmetry in the particle-hole space [see Eq. (7)] and the k singlets distributing in all odd-number particle sectors [36], we thus conjecture that the strong coupling Hamiltonian takes the form of channel-isotropic k channel Kondo model,

$$H_{\text{s}} = \tilde{\lambda}\mathbf{S}\cdot\sum_{i=1}^{k}\mathbf{s}_i, \tag{8}$$

where the impurity $SU(2)$ spin-$(k-1)/2$ operator $\mathbf{S}$ acts in the $k-$dimensional subspace (spanned by Eqs. (6a)–(6b) for $k = 2$), $\mathbf{s}_i = f_i^\dagger(\boldsymbol{\sigma}/2)f_i$ and $f_i = (f_{i\uparrow}, f_{i\downarrow})^T \equiv (c^\dagger_{1,i,\uparrow}, c_{1,i,\downarrow})^T$ with $i = 1,\ldots k$ labeling the effective channel of conduction electrons. Since $S = (k-1)/2 < k/2$, the MCK Hamiltonian (8) is overscreened [43]. We have explicitly proven the conjecture for $k = 2$ ($k = 3$) by second-order perturbation theory (Schrieffer-Wolff transformation), for which virtual fluctuations into the 62 (381) excited states lead to $\tilde{\lambda} = 24t'^2/(5\lambda)$ ($\tilde{\lambda} = 128t'^2/(21\lambda)$), respectively [36]. In this context it is also worthwhile to point out a hidden (larger) $Sp(2k)$ symmetry in the $k-$channel $SU(2)$ Kondo effect [44].

Since the weak-coupling limit of the overscreened multichannel $SU(2)$ model is unstable [43, 45], the above map relating it to the strong-coupling limit of the symplectic Kondo model implies also the instability of the latter fixed point, see Fig. 1e. Together with the instability of the weak-coupling fixed point of the $Sp(2k)$ Kondo problem, see Eq. (5), these findings indicate a single stable fixed point between the two, i.e. at an intermediate coupling. Our conjecture of a single fixed point is supported by the low-temperature impurity entropy (below) which is found to have the same value, when approaching from the weak [$Sp(2k)$] and strong [k-channel $SU(2)$] coupling sides. Since $Sp(2)$ is isomorphic to $SU(2)$, our model provides an example of the level-rank duality [30] relating the weak and strong coupling theories.

Observables: thermodynamics. Above, we argued that near strong coupling, the model can be mapped to an overscreened k-channel spin-$(k-1)/2$ Kondo model, which has a stable intermediate coupling fixed point. We can use the impurity entropy [46] to characterize the effective residual ground state degeneracy g_k of this fixed point. The ground state degeneracy associated to screening a spin $(k-1)/2$ with k spin-1/2 channels is well-known, $g_k = 2\cos[\pi/(k+2)]$ [13–15, 46]. This result agrees with the impurity entropy of the $Sp(2k)$ Kondo problem, calculated using CFT [30] and Bethe Ansatz [47].

In particular, we note that the case $k = 3$ has $g_3 = (1+\sqrt{5})/2 = \varphi$, the Golden ratio, indicating an emergent Fibonacci anyon. Crucially, in our symplectic Kondo model this Fibonacci anyon occurs even in the single-channel case (in the sense that our model, Eq. (1), is of level 1) and is therefore not subject to instability due to channel anisotropy, unlike previous examples in the 3-channel Kondo [44, 48] and 2-channel topological Kondo [49] models.

Despite this appearance of the same anyon-like ground-state degeneracies and an unstable strong coupling fixed point which is equivalent to the k-channel spin-$(k-1)/2$ $SU(2)$ Kondo model, we emphasize that in our model due to PHS, not all operators of the $SU(2)$ Kondo model are effective. For example, the symplectic susceptibility involves the excitation of states outside the low-energy manifold Eqs. (6a),(6b), leading to less singular behavior than for the $SU(2)_k$ susceptibility [36]. More generally, we expect that the irrelevant operator of scaling dimension $1+2/(2+k)$ is forbidden for the dual Kondo problem, Eq. (8). This implies Fermi-liquid like temperature and field dependence of thermodynamic quantities, consistent with results [26, 27, 30, 47] based on the weak coupling Hamiltonian, Eq. (1).

We note that CFTs in which certain operators are symmetry disallowed are well known in the theory of (e.g. confinement-deconfinement) phase transitions in gauge theories and usually denoted by an asterisk [50–52]. In view of the relationship between deconfining gauge theories and overscreened Kondo impurities [53], we borrow the notation employed for the latter phenomena and denote the boundary CFT describing the dual Kondo problem, Eq. (8), as $SU(2)^*_k$.

Observables: transport. We propose to test the non-trivial nature of the symplectic Kondo effect in a charge transport experiment across the mesoscopic island. As we explicitly demonstrate [36] using the CFT method [11, 46, 54–60], the fixed point off-diagonal conductance, Eq. (9), of $Sp(2k)$ Kondo model and the spin-1/2, k-channel $SU(2)$ charge Kondo model [61, 62] are identical up to normalization [63]. Nevertheless, we emphasize that our result is valid far from the charge degeneracy points, in the regime of elastic cotunneling akin to spin Kondo effect [64]. At low temperatures, $T \ll T_K$, near the intermediate coupling fixed point, the off-diagonal $Sp(2k)$ charge conductance is

$$G_{i\neq j}(T) = \frac{4e^2}{hk}\sin^2\left(\frac{\pi}{k+2}\right)\left[1+c_{ij}\left(\frac{T}{T_K}\right)^2\right], \quad (9)$$

where the $T = 0$ value is obtained in Ref. [36]. For $k = 2$ we have exactly half of the maximum conductance, analogous to halving of the conductance in the spin 2-channel Kondo effect [65] and also similar to the conductance in the quarter-filling $SU(4)$ Kondo model [66]. The finite-temperature correction with its dimensionless coefficient c_{ij} and the Kondo temperature T_K are determined from the microscopic physics, see below Eq. (5) for the latter. The temperature dependent transconductance (including larger temperature regimes) is plotted in Fig. 1 c). The exponent in the finite-temperature correction to $G_{ij}(T = 0)$ is determined by the scaling dimension Δ_{LIO} of the leading irrelevant operator. Importantly, in the 1-channel $Sp(2k)$ model the leading irrelevant operator [26, 27, 30] is local density-density interaction with $\Delta_{\text{LIO}} = 2$, giving a FL-like temperature dependence while it is non-FL like for $SU(2)_k$. As explained above, also for $SU(2)^*_k$ the operator responsible for non-FL power-laws is absent and we expect the exponent in Eq. (9) to be the same regardless of whether we approach the stable intermediate $(T = 0)$ fixed point from weak or strong coupling. The exotic zero-temperature conductance value $G_{i\neq j}(0)$ reminiscent of the multi-channel charge Kondo effect [3, 4, 61, 62, 67] together with FL corrections to it are unique signatures of the $Sp(2k)$ intermediate fixed point.

Effect of anisotropy and PHS breaking. When deriving the $Sp(2k)$ Kondo interaction, we required fine tuning of the tunneling strengths $t_i = t$ $(\forall\, i = 1,\cdots,k)$ and particle-hole symmetry $N_g = 1$. Although these parameters can be controlled in experiments, we will discuss next what happens when we deviate from the requirements. We show that the former requirement can be relaxed but the deviation from PHS will drive the system towards an $SU(2k)$ Kondo fixed point. Let us first discuss the anisotropy of the tunneling amplitudes, while keeping the system PHS [36]. When we consider the anisotropic version of the effective Hamiltonian Eq. (4), the anisotropic tunneling strength $t_i > 0$ can be absorbed into the operators $\tilde{d} = \eta d$ and $\tilde{c}_0 = \eta c_0$, where $\eta = \mathbb{I}\otimes \text{diag}(\sqrt{t_1},\cdots,\sqrt{t_k})/\sqrt{t}$, where t now denotes the geometric mean of t_i. The anisotropic Hamiltonian then takes the same form as Eq. (4), with the replacement $d, c_0 \to \tilde{d}, \tilde{c}_0$. Upon using the completeness relation and restoring the physical operators d, c_0, we obtain transformed generators $\eta T_A \eta$ in the operators S^A and J^A. The transformed generators are still $Sp(2k)$ generators because the matrix η commutes with $(\sigma_y\otimes\mathbb{I})$, thus $(\sigma_y\otimes\mathbb{I})(\eta T_A\eta)^T(\sigma_y\otimes\mathbb{I}) = -\eta T_A \eta$ according to the properties of $Sp(2k)$ generators [31]. Then, we can expand the transformed generators by the original generators: $\eta T_A \eta = \sum_B \kappa_{AB} T^B$. From this we see that the anisotropy of tunneling amplitudes is equivalent to the "exchange" anisotropy of the $Sp(2k)$ Kondo model, $H_K = \lambda\sum_{A,B}\kappa_{AB}S^A J^B$. Using the generalized version [36], weak anisotropies $|\kappa_{AB} - \delta_{AB}| \ll 1$, can be shown to be irrelevant on general grounds. The same situation occurs with $SO(M)$, in the topological Kondo model [22, 23, 68–70], where the isotropic direction dominates the RG flow. We note however that in the effective strong coupling multichannel $SU(2)$ model, Eq. (8), time-reversal symmetric tunneling anisotropy (unequal t'_i) corresponds to channel anisotropy which is a relevant perturbation. Thus, the strong coupling multichannel Kondo physics requires fine-tuning of the $Sp(2k)$ symmetry.

While at weak coupling anisotropy in the tunnel-couplings is harmless, the $Sp(2k)$ is more sensitive to breaking of PHS. We first consider $N_g \neq 1$ ($\lambda_1 \neq \lambda_2$) in Eq. (4), whilst still requiring $t \ll \Delta E_\pm$ (regime of pink shading of Fig. 1 b)). Then, we can rewrite Eq. (4) as a potential scattering term for conduction electrons and an anisotropic $SU(2k)$ Kondo interaction. This $SU(2k)$ Kondo model is exactly screened and has a FL fixed point, and thus the non-FL fixed point of the $Sp(2k)$ Kondo model will be unstable. An example with $k = 2$ has been discussed in Ref. [27]. Also, the term arising from $\lambda_1 \neq \lambda_2$ maps to an effective magnetic field in the $SU(2)$ Kondo model in the strong coupling regime, similar to the case in charge Kondo [67]. Near the intermediate coupling fixed point such a perturbation is relevant, with a scaling dimension $\Delta_H = 2/(2 + k)$, and drives the system to a FL fixed point [44]. Hence, we conclude that the PHS breaking anisotropy ($N_g \neq 1$) is relevant. As N_g is further detuned from unity to a regime $t \sim \Delta E_\pm$ and further, we first enter an $SU(2k)$ mixed valence regime (dark gray in Fig. 1 b), in which odd and even parity states are of comparable energy, and ultimately reach the regime in which the impurity ground state is non-degenerate. In the infrared, FL behavior persists, see Fig. 1 d).

Summary and Conclusions. In summary, we proposed a mesoscopic implementation of the symplectic Kondo effect, in which the group $Sp(2k)$ naturally describes spin-1/2 fermions in k orbitals in a Coulomb blockaded island hosting k spinful topological zero-energy Andreev states. We couple each zero-energy state to a spinful fermion lead and found the symplectic Kondo Hamiltonian Eq. (1) for an odd-parity charge state of the Coulomb blockaded island.

Interesting open questions about the symplectic Cooper pair box setup include the Coulomb blockaded transport beyond $N_g = 1$ and complementary analytical, numerical and experimental studies which should help shed light on the anyonic signatures and their quantum-information theoretic potential.

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[1] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das-Sarma, Non-abelian anyons and topological quantum computation, Reviews of Modern Physics **80**, 1083 (2008).
[2] R. M. Potok, I. G. Rau, H. Shtrikman, Y. Oreg, and D. Goldhaber-Gordon, Observation of the two-channel Kondo effect, Nature (London) **446**, 167 (2007).
[3] Z. Iftikhar, A. Anthore, A. K. Mitchell, F. D. Parmentier, U. Gennser, A. Ouerghi, A. Cavanna, C. Mora, P. Simon, and F. Pierre, Tunable quantum criticality and super-ballistic transport in a "charge" Kondo circuit, Science **360**, 1315 (2018).
[4] Z. Iftikhar, S. Jezouin, A. Anthore, U. Gennser, F. D. Parmentier, A. Cavanna, and F. Pierre, Two-channel Kondo effect and renormalization flow with macroscopic quantum charge states, Nature (London) **526**, 233 (2015).
[5] A. J. Keller, L. Peeters, C. P. Moca, I. Weymann, D. Mahalu, V. Umansky, G. Zaránd, and D. Goldhaber-Gordon, Universal Fermi liquid crossover and quantum criticality in a mesoscopic system, Nature (London) **526**, 237 (2015).
[6] W. Pouse, L. Peeters, C. L. Hsueh, U. Gennser, A. Cavanna, M. A. Kastner, A. K. Mitchell, and D. Goldhaber-Gordon, Quantum simulation of an exotic quantum critical point in a two-site charge Kondo circuit, Nature Physics , 1 (2023).
[7] N. Andrei, Diagonalization of the kondo hamiltonian, Physical Review Letters **45**, 379 (1980).
[8] P. Vigman, Exact solution of sd exchange model at t= 0, Soviet Journal of Experimental and Theoretical Physics Letters **31**, 364 (1980).
[9] A. Tsvelick and P. Wiegmann, Exact results in the theory of magnetic alloys, Advances in Physics **32**, 453 (1983).
[10] A. Tsvelick, The transport properties of magnetic alloys with multi-channel kondo impurities, Journal of Physics: Condensed Matter **2**, 2833 (1990).
[11] I. Affleck and A. W. Ludwig, Critical theory of overscreened kondo fixed points, Nuclear Physics B **360**, 641 (1991).
[12] V. J. Emery and S. Kivelson, Mapping of the two-channel kondo problem to a resonant-level model, Phys. Rev. B **46**, 10812 (1992).
[13] N. Andrei and C. Destri, Solution of the multichannel kondo problem, Phys. Rev. Lett. **52**, 364 (1984).
[14] A. Tsvelick and P. Wiegmann, Exact solution of the multichannel kondo problem, scaling, and integrability, Journal of Statistical Physics **38**, 125 (1985).
[15] A. Tsvelick, The thermodynamics of multichannel kondo problem, Journal of Physics C: Solid State Physics **18**, 159 (1985).
[16] P. L. S. Lopes, I. Affleck, and E. Sela, Anyons in multichannel kondo systems, Phys. Rev. B **101**, 085141 (2020).
[17] Y. Komijani, Isolating kondo anyons for topological quantum computation, Phys. Rev. B **101**, 235131 (2020).
[18] D. Gabay, C. Han, P. L. S. Lopes, I. Affleck, and E. Sela, Multi-impurity chiral kondo model: Correlation functions and anyon fusion rules, Phys. Rev. B **105**, 035151 (2022).
[19] G. A. Fiete, W. Bishara, and C. Nayak, Multichannel kondo models in non-abelian quantum hall droplets, Phys. Rev. Lett. **101**, 176801 (2008).
[20] E. J. König, P. Coleman, and A. M. Tsvelik, Spin magnetometry as a probe of stripe superconductivity in twisted bilayer graphene, Phys. Rev. B **102**, 104514 (2020).
[21] A. M. Sengupta and Y. B. Kim, Overscreened single-channel kondo problem, Phys. Rev. B **54**, 14918 (1996).
[22] B. Béri and N. R. Cooper, Topological kondo effect with majorana fermions, Phys. Rev. Lett. **109**, 156803 (2012).
[23] B. Béri, Majorana-klein hybridization in topological superconductor junctions, Phys. Rev. Lett. **110**, 216803 (2013).
[24] A. Altland, B. Béri, R. Egger, and A. M. Tsvelik, Multichannel kondo impurity dynamics in a majorana device, Phys. Rev. Lett. **113**, 076401 (2014).
[25] E. Eriksson, C. Mora, A. Zazunov, and R. Egger, Non-fermi-liquid manifold in a majorana device, Phys. Rev. Lett. **113**,

076404 (2014).
[26] A. K. Mitchell, A. Liberman, E. Sela, and I. Affleck, SO(5) Non-Fermi Liquid in a Coulomb Box Device, Phys. Rev. Lett. **126**, 147702 (2021).
[27] A. Liberman, A. K. Mitchell, I. Affleck, and E. Sela, SO(5) critical point in a spin-flavor Kondo device: Bosonization and refermionization solution, Phys. Rev. B **103**, 195131 (2021).
[28] P. Bonderson, M. Freedman, and C. Nayak, Measurement-only topological quantum computation, Phys. Rev. Lett. **101**, 010501 (2008).
[29] H. Georgi, *Lie algebras in particle physics: from isospin to unified theories* (Taylor & Francis, 2000).
[30] T. Kimura, Abcd of kondo effect, Journal of the Physical Society of Japan **90**, 024708 (2021).
[31] The generators are $\mathbb{I}\otimes \mathrm{i}\vec{A}$, $\sigma_x\otimes\vec{S}$, $\sigma_y\otimes\vec{S}$ and $\sigma_z\otimes\vec{S}$ where $\vec{A}$ are $k(k-1)/2$ real $k\times k$ antisymmetric matrices, $\vec{S}$ are $k(k+1)/2$ real $k\times k$ symmetric matrices and $\sigma_{x,y,z}$ are Pauli matrices. They satisfy $(\sigma_y\otimes\mathbb{I})(T_A)^T(\sigma_y\otimes\mathbb{I}) = -T_A$. We normalize the generators as $\mathrm{Tr}(T_AT_B)=\delta_{AB}$.
[32] R. Flint, M. Dzero, and P. Coleman, Heavy electrons and the symplectic symmetry of spin, Nature Physics **4**, 643 (2008).
[33] E. Demler, W. Hanke, and S.-C. Zhang, $SO(5)$ theory of antiferromagnetism and superconductivity, Rev. Mod. Phys. **76**, 909 (2004).
[34] C. Knapp, J. I. Väyrynen, and R. M. Lutchyn, Number-conserving analysis of measurement-based braiding with majorana zero modes, Phys. Rev. B **101**, 125108 (2020).
[35] Z. Shi, P. W. Brouwer, K. Flensberg, L. I. Glazman, and F. von Oppen, Long-distance coherence of majorana wires, Phys. Rev. B **101**, 241414(R) (2020).
[36] See supplementary materials for details.
[37] A. C. Hewson, *The Kondo problem to heavy fermions*, 2 (Cambridge university press, 1997).
[38] P. W. Anderson, A poor man's derivation of scaling laws for the kondo problem, Journal of Physics C: Solid State Physics **3**, 2436 (1970).
[39] E. Kogan, Poor man's scaling and lie algebras, Journal of Physics Communications **3**, 125001 (2019).
[40] P. Nozieres, A "fermi-liquid" description of the kondo problem at low temperatures, Journal of low température physics **17**, 31 (1974).
[41] A. Altland and M. R. Zirnbauer, Nonstandard symmetry classes in mesoscopic normal-superconducting hybrid structures, Phys. Rev. B **55**, 1142 (1997).
[42] Under $Sp(2k)$: $c_a\to Uc_a$ with $U^T\sigma_yU=\sigma_y$.
[43] Nozières, Ph. and Blandin, A., Kondo effect in real metals, J. Phys. France **41**, 193 (1980).
[44] I. Affleck, A. W. W. Ludwig, H.-B. Pang, and D. L. Cox, Relevance of anisotropy in the multichannel kondo effect: Comparison of conformal field theory and numerical renormalization-group results, Phys. Rev. B **45**, 7918 (1992).
[45] C. Kolf and J. Kroha, Strong versus weak coupling duality and coupling dependence of the kondo temperature in the two-channel kondo model, Phys. Rev. B **75**, 045129 (2007).
[46] I. Affleck, Conformal field theory approach to the kondo effect, Acta Phys. Polon. **B26**, 1869 (1995).
[47] E. König and A. Tsvelik, Exact solution of the symplectic topological kondo problem, arXiv:2211.00034.
[48] C. Han, Z. Iftikhar, Y. Kleeorin, A. Anthore, F. Pierre, Y. Meir, A. K. Mitchell, and E. Sela, Fractional entropy of multichannel kondo systems from conductance-charge relations, Phys. Rev. Lett. **128**, 146803 (2022).
[49] G. Li, Y. Oreg, and J. I. Väyrynen, Multichannel Topological Kondo Effect, Phys. Rev. Lett. **130**, 066302 (2023).
[50] M. Schuler, S. Whitsitt, L.-P. Henry, S. Sachdev, and A. M. Läuchli, Universal signatures of quantum critical points from finite-size torus spectra: A window into the operator content of higher-dimensional conformal field theories, Phys. Rev. Lett. **117**, 210401 (2016).
[51] S. Sachdev, Topological order, emergent gauge fields, and fermi surface reconstruction, Reports on Progress in Physics **82**, 014001 (2018).
[52] M. Vojta, Frustration and quantum criticality, Reports on Progress in Physics **81**, 064501 (2018).
[53] E. J. König, P. Coleman, and Y. Komijani, Frustrated kondo impurity triangle: A simple model of deconfinement, Phys. Rev. B **104**, 115103 (2021).
[54] I. Affleck and A. W. W. Ludwig, Universal noninteger "ground-state degeneracy" in critical quantum systems, Phys. Rev. Lett. **67**, 161 (1991).
[55] I. Affleck, A current algebra approach to the kondo effect, Nuclear Physics B **336**, 517 (1990).
[56] I. Affleck and A. W. Ludwig, The kondo effect, conformal field theory and fusion rules, Nuclear Physics B **352**, 849 (1991).
[57] I. Affleck and A. W. W. Ludwig, Exact conformal-field-theory results on the multichannel kondo effect: Single-fermion green's function, self-energy, and resistivity, Phys. Rev. B **48**, 7297 (1993).
[58] A. W. Ludwig and I. Affleck, Exact conformal-field-theory results on the multi-channel kondo effect: Asymptotic three-dimensional space-and time-dependent multi-point and many-particle green's functions, Nuclear Physics B **428**, 545 (1994).
[59] A. W. W. Ludwig and I. Affleck, Exact, asymptotic, three-dimensional, space- and time-dependent, green's functions in the multichannel kondo effect, Phys. Rev. Lett. **67**, 3160 (1991).
[60] O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta, Overscreened multichannel $\mathrm{SU}(N)$ Kondo model: Large-N solution and conformal field theory, Phys. Rev. B **58**, 3794 (1998).
[61] H. Yi and C. L. Kane, Quantum brownian motion in a periodic potential and the multichannel kondo problem, Phys. Rev. B **57**, R5579 (1998).
[62] H. Yi, Resonant tunneling and the multichannel kondo problem: Quantum brownian motion description, Phys. Rev. B **65**, 195101 (2002).
[63] The natural normalization is to use the maximal conductance of a multiterminal junction, which in the $Sp(2k)$ case is $G^{Sp(2k)}_{\max}=4e^2/(hk)$. Due to spin degeneracy, $G^{Sp(2k)}_{\max}$ is twice as large as the maximum conductance in the $SO(k)$ topological Kondo effect and four times as large as in the k-channel $SU(2)$ charge Kondo effect (the additional factor 2 is due to the inelastic cotunneling nature of the latter [62, 67]).
[64] M. Pustilnik and L. Glazman, Kondo effect in quantum dots, Journal of Physics: Condensed Matter **16**, R513 (2004).
[65] Y. Oreg and D. Goldhaber-Gordon, Non-fermi-liquid in a modified single electron transistor (2002).
[66] M.-S. Choi, R. López, and R. Aguado, Su(4) kondo effect in carbon nanotubes, Phys. Rev. Lett. **95**, 067204 (2005).
[67] A. Furusaki and K. A. Matveev, Theory of strong inelastic cotunneling, Phys. Rev. B **52**, 16676 (1995).
[68] L. Herviou, K. Le Hur, and C. Mora, Many-terminal majorana island: From topological to multichannel kondo model, Phys. Rev. B **94**, 235102 (2016).
[69] J. I. Väyrynen, A. E. Feiguin, and R. M. Lutchyn, Signatures of topological ground state degeneracy in majorana islands, Phys. Rev. Research **2**, 043228 (2020).
[70] K. Snizhko, F. Buccheri, R. Egger, and Y. Gefen, Parafermionic

generalization of the topological kondo effect, Phys. Rev. B **97**, 235139 (2018).