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Phonon, Electron, and Magnon Excitations in Antiferromagnetic L1₀-type MnPt

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Antiferromagnetic L1₀-type MnPt is a material with relatively simple crystal and magnetic structure, recently attracting interest due to its high Néel temperature and wide usage as a pinning layer in magnetic devices. While it is experimentally well characterized, the theoretical understanding is much less developed, in part due to the challenging accuracy requirements dictated by the small underlying energy scales that govern magnetic ordering in antiferromagnetic metals. In this work, we use density functional theory, the Korringa-Kohn-Rostoker formalism, and a Heisenberg model to establish a comprehensive theoretical description of antiferromagnetic L1₀-type MnPt, along with accuracy limits, by thoroughly comparing to available literature data. Our simulations show that the contribution of the magnetic dipole interaction to the magnetocrystalline anisotropy energy of $K_1=1.07\times 10^6$ J/m³ is comparable in magnitude to the spin-orbit contribution. Using our result for the magnetic susceptibility of 5.25×10^{-4} , a lowest magnon frequency of about 2.02 THz is predicted, confirming THz spin dynamics in this material. From our data for electron, phonon, and magnon dispersion we compute the individual contributions to the total heat capacity and show that the dominant term at or above 2 K arises from phonons. From the Landau-Lifshitz-Gilbert equation, we compute a Néel temperature of 990–1070 K. Finally, we quantify the magnitude of the magneto-optical Kerr effect generated by applying an external magnetic field. Our results provide insight into the underlying physics, which is critical for a deep understanding of fundamental limits of the time scale of spin dynamics, stability of the magnetic ordering, and the possibility of magneto-optical detection of collective spin motion.

I. INTRODUCTION

Several decades after their initial discovery,¹ antiferromagnetic materials are recently attracting great interest, owing to the successful probing and manipulation of their magnetic ordering by electrical and optical means. Electrical switching of antiferromagnetic CuMnAs was reported² in 2016 and the switching of the Néel vector was concluded from measuring the magneto-optical Voigt effect.³ Electrical read-out was demonstrated for antiferromagnetic Mn₂Au using anisotropic magnetoresistance.⁴ In addition, while ferro- or ferri-magnets are easily affected by external fields, collinear antiferromagnets are robust against such manipulation due to their vanishing net magnetization. This initially hampered applications, however, it has now become the reason for the use of antiferromagnets as excellent pinning layers: They maintain their magnetic ordering under external fields, while providing strong exchange bias on the adjacent ferromagnetic or ferrimagnetic layers.⁵

The material investigated in this work, antiferromagnetic L1₀ type MnPt, follows a similar timeline: Based on neutron powder diffraction Andersen *et al.* explained⁶ its magnetic structure as early as 1965 invoking antiferromagnetically and ferromagnetically coupled moments along the [110] and [001] directions, respectively, and a Néel vector orientation along [001]. The potential for spin-flip transitions of the magnetic alignment from [001] to [100] was recognized from two different neutron scattering experiments on powder samples.^{7,8} In addition, also single

crystal neutron scattering measurements recently confirmed this spin-flip transition between 580 K and 770 K, aligning the moments along [100].⁹ Finally, a relatively high Néel temperature of 970–975 K was measured for MnPt,^{7,10} causing its magnetic properties to be thermally stable at room temperature. Applications of antiferromagnetic MnPt include spin-valve structures with giant magnetoresistance, based on exchange bias at the interface with a ferromagnetic layer,^{11–13} and there is increased interest in this material as pinning layer in devices.¹¹ However, the fundamental exchange interactions are still under the veil, preventing detailed theoretical understanding of the Néel temperature or the wave vector dependent magnon dispersion, which also contributes to the heat capacity.

While the experimental characterization of structural and magnetic properties of MnPt is thorough, the theoretical understanding is much less developed. Metallic AFMs constitute a challenge in particular for first-principles simulations since the underlying energy scales oftentimes push the accuracy of numerical convergence to its limits. The relatively simple chemical structure and magnetic configuration make MnPt an ideal candidate to explore this issue for first-principles simulations of ground- and excited-state properties. In this work, we establish a thorough comparison between our first-principles data, other computational data from the literature, and experiments, to discuss reasons for deviations.

First, we study the atomic geometry of MnPt, its magnetic structure, and susceptibility. We then compute exchange parameters to model the magnetic structure,

confirming the early analysis by Andersen *et al.*⁶ Our simulations of the magnon gap and the Néel temperature are in good agreement with experimental data.^{7,10} For magnetocrystalline anisotropy, which helps to explain the orientation of the Néel vector in the ground state and to understand barriers against its reorientation, we compare our data with prior first-principles results and identify a so far overlooked classical contribution due to magnetic dipole interactions. Our analysis forms a basis of future studies, e.g. of the strain dependence of magnetic ordering and magnetocrystalline anisotropy, possibly helping to explain reports of non-volatile modulation of resistance using piezoelectric strain, with possible application in strain-induced switching.¹⁴

Furthermore, our simulations provide predictions that enable deeper understanding of the underlying physics of antiferromagnetic L1₀-type MnPt: This includes fundamental limits to the time scale of spin dynamics, the thermal stability of the antiferromagnetic ordering at room temperature, the relative contributions of electrons, phonons, and magnons to the heat capacity of this material, and the potential for using MnPt for magneto-optical detection of collective spin motion via MOKE measurements of precession. To this end, we derive spin dynamics and Néel temperature from the Landau-Lifshitz-Gilbert equation. We predict excited-state properties such as phonon, electron, and magnon dispersion: The electronic band structure is approximated by Kohn-Sham eigenvalues, the phonon dispersion is computed within the finite difference method, and the magnon dispersion is obtained from linear spin-wave theory. We use this data to compute the total heat capacity of the material, in good agreement with experiment,¹⁰ and also directly compare the electronic heat capacity to data from thermal relaxation experiments.¹⁰ Our results show that up to about 2 K, there are electronic contributions to the total heat capacity, but at higher temperatures most of the total heat capacity originates from phonons instead of magnons, due to the magnon gap and the low magnon density of states. This is different from materials with magnetic critical temperatures of just a few K,¹⁵ for which the magnon heat capacity can be larger than the phonon heat capacity at low temperatures. Comparing the individual contributions to the heat capacity, computed from the energy dispersion relations of phonons, electrons, and magnons, to experiment provides insight into the relative accuracy of our first-principles results. Finally, from the electronic band structure, including spin-orbit coupling, we predict optical and magneto-optical spectra of antiferromagnetic MnPt, explaining the relative importance of contributions from Mn and Pt.

After introducing the computational approaches in Sec. II, the ground state properties of antiferromagnetic L1₀-type MnPt are discussed in Sec. III, including relaxed atomic coordinates and magnetic structure, magnetocrystalline anisotropy energy, magnetic susceptibility, and exchange coupling parameters. In Sec. IV, we report first-principles results for the dispersion relations of electrons,

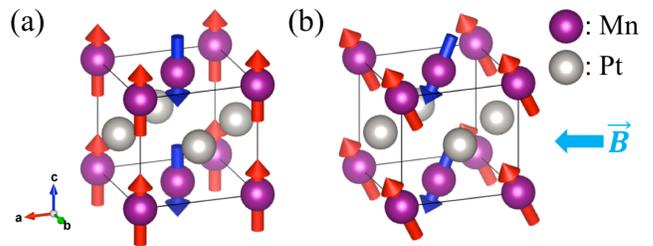


FIG. 1. (Color online.) Chemical and magnetic structure of MnPt in the (a) ground state and (b) spin-tilted state under an external magnetic field along a -axis direction. Manganese atoms are purple and platinum atoms are gray. Red and blue arrows represent antiparallel magnetic moments. The magnetic unit cell (shown above) comprises of two chemical unit cells.

phonons, and magnons, and discuss their individual contributions to the total heat capacity, which we also compare to experiment. Finally, in Sec. V we report the Néel temperature and analyze optical and magneto-optical properties in detail. We note that all units in this manuscript are in SI units unless otherwise noted explicitly.

II. COMPUTATIONAL DETAILS

First-principles simulations of MnPt are carried out within density functional theory (DFT), as implemented in the Vienna *Ab-Initio* Simulation Package (VASP).^{16–19} Exchange and correlation is described by the generalized-gradient approximation developed by Perdew, Burke, and Ernzerhof (PBE).²⁰ Kohn-Sham states are expanded into plane waves up to a kinetic-energy cutoff of 600 eV. A $15 \times 15 \times 15$ Monkhorst-Pack (MP)²¹ \mathbf{k} -point grid is used to sample the Brillouin zone for structural relaxation and optical spectrum calculations, leading to total energies that are converged to within 0.1 meV/atom. Computing the anisotropy energy requires a denser $24 \times 24 \times 24$ MP \mathbf{k} -point sampling to converge the anisotropy energy within 0.03 meV/atom. Each self-consistent calculation is performed for collinear (atomic relaxations) or non-collinear (optical properties with tilted magnetic moments) magnetic ordering first, neglecting the spin-orbit interaction. Subsequently, spin-orbit coupling is described non-selfconsistently, by using the resulting Kohn-Sham states and charge density to set up the Kohn-Sham Hamiltonian and diagonalizing it including the spin-orbit coupling term. From this, we compute ground state energies and optical properties.

We further compute phonon frequencies using the finite difference method implemented in the **phonopy** package²² for a $3 \times 3 \times 3$ supercell. For these simulations, the Brillouin zone is sampled by a $3 \times 3 \times 3$ MP \mathbf{k} -point grid, which leads to phonon frequencies converged to within less than 0.2 meV. These phonon calculations are implemented using noncollinear magnetism and include spin-orbit coupling.

We compute the exchange coefficients for antiferro-

183 magnetic L1₀-type MnPt using the spin polarized rela-232
 184 tivistic Korringa-Kohn-Rostoker (SPR-KKR) code.²³ The233
 185 electronic ground state is computed within the KKR for-234
 186 malism, based on the fully relaxed atomic structure and235
 187 using DFT-PBE²⁰ as described above. The Brillouin zone236
 188 is sampled with 1000 randomly selected \mathbf{k} points, lead-237
 189 ing to total energies converged within 0.01 meV/atom²³⁸
 190 Isotropic exchange coupling coefficients of a Heisenberg²³⁹
 191 model, 240

$$\mathcal{H}_{ex} = - \sum_{i \neq j} J_{ij} \mathbf{e}_i \mathbf{e}_j, \quad (1) \quad \begin{array}{l} 241 \\ 242 \\ 243 \\ 244 \end{array}$$

193 are then computed using Lichtenstein's approach within²⁴⁵
 194 the SPR-KKR code.²⁴ Here \mathcal{H}_{ex} is the exchange Hamilto-²⁴⁶
 195 nian and J_{ij} are exchange coupling parameters for all
 196 magnetic moments of atoms i and j and orientations \mathbf{e}_i
 197 and \mathbf{e}_j , within an interaction distance $d/a = 4.0$, where
 198 a is the lattice parameter along the a axis (see Fig. 1).

199 Subsequently, we compute magnon dispersion curves
 200 within linear spin-wave theory²⁵ from the spin Hamilto-²⁴⁸
 201 nian

$$\mathcal{H} = \sum_{i,j} \mathbf{S}_i^\dagger \mathbf{J}_{ij} \mathbf{S}_j + \sum_i \mathbf{S}_i^\dagger \mathbf{A}_i \mathbf{S}_i, \quad (2) \quad \begin{array}{l} 249 \\ 250 \\ 251 \\ 252 \end{array}$$

203 that accounts for exchange and anisotropy interactions²⁵³
 204 Here \mathbf{S}_i is the 3×1 spin vector operator, \mathbf{J}_{ij} is the 3×3 ²⁵⁴
 205 exchange coupling matrix between spins at sites i and²⁵⁵
 206 j , and \mathbf{A}_i is the 3×3 anisotropy matrix. The diago-²⁵⁶
 207 nal components of \mathbf{J}_{ij} can be described by the isotropic²⁵⁷
 208 exchange coupling parameters in Eq. (1), while the off-²⁵⁸
 209 diagonal components are Dzyaloshinskii-Moriya exchange²⁵⁹
 210 parameters. Due to the inversion symmetry of antifer-²⁶⁰
 211 romagnetic L1₀-type MnPt, the Dzyaloshinskii-Moriya²⁶¹
 212 interaction²⁶ and, hence, these off-diagonal components²⁶²
 213 vanish. \mathbf{A}_i represents the anisotropy energy with two-²⁶³
 214 fold symmetry, which follows from the magnetocrystalline²⁶⁴
 215 anisotropy energy computed within DFT, including spin-²⁶⁵
 216 orbit interaction and magnetic dipole-dipole interaction²⁶⁶
 217 (see Sec. III B). For MnPt with uniaxial magnetism, all²⁶⁷
 218 components of \mathbf{A}_i vanish except for the (3,3) component²⁶⁸
 219 which is equal to $(K_1 + K_2)/n$, where K_1 and K_2 are²⁶⁹
 220 coefficients of magnetocrystalline anisotropy defined in²⁷⁰
 221 Eq. (5) and $n = 2$ is the total number of magnetic mo-²⁷¹
 222 ments in the magnetic unit cell. Subsequently, we use²⁷²
 223 the SpinW code²⁵ to compute the magnon dispersion in \mathbf{q} -
 224 space from the diagonalization of the Fourier transformed
 225 spin Hamiltonian. 273

226 Finally, we compute the Néel temperature using a
 227 Monte Carlo (MC) method to solve the stochastic Landau-²⁷⁴
 228 Lifshitz-Gilbert (LLG) equation,²⁷ 275

$$\begin{aligned} \frac{d\mathbf{m}_i}{dt} = & -\gamma_L \mathbf{m}_i \times (\mathbf{B}_i + \mathbf{B}_i^{\text{fl}}) \\ & - \gamma_L \frac{\alpha}{m_i} \mathbf{m}_i \times [\mathbf{m}_i \times (\mathbf{B}_i + \mathbf{B}_i^{\text{fl}})], \end{aligned} \quad (3) \quad \begin{array}{l} 276 \\ 277 \\ 278 \\ 279 \\ 280 \end{array}$$

230 where \mathbf{m}_i is the magnetic moment at site i consistent²⁸¹
 231 with our DFT simulations and $\gamma_L = \gamma/(1 + \alpha^2)$ is the²⁸²

renormalized gyromagnetic ratio. γ is a gyromagnetic
 ratio and we use the default value of an isotropic Gilbert
 damping constant $\alpha=0.1$ implemented in UppASD,²⁷ which
 does not affect our results, since we keep the temperature
 fixed using a heat bath and, thus, there is no damping.
 \mathbf{B}_i is the effective magnetic field as the derivative of
 the spin Hamiltonian, including exchange, anisotropy,
 and magnetic dipolar interactions, with respect to \mathbf{m}_i at
 magnetic site i . The magnetic temperature is included as
 a fluctuating magnetic field \mathbf{B}_i^{fl} based on the central limit
 theorem, using a Gaussian distribution with zero average
 and temperature dependent variance.²⁷ The magnetic
 structure at finite temperature is then calculated from Eq.
 (3) using a $15 \times 15 \times 15$ supercell and the MC approach
 implemented in the UppASD package.²⁷

III. GROUND STATE PROPERTIES

A. Atomic structure and magnetic configuration

Antiferromagnetic L1₀-type MnPt crystallizes in a
 tetragonal uniaxial structure with a chemical space group
 of $P4/mmm$ (No. 123) and magnetic space group of
 $C_{PM}'m'm$. Mn and Pt atoms occupy alternating layers
 along the c axis, which induces the tetragonal structure
 (see Fig. 1). We first compute fully relaxed lattice param-
 eters using DFT and obtain $a=3.97 \text{ \AA}$ and $c=3.71 \text{ \AA}$. These
 deviate by less than 1.5% from experimental measure-
 ments of $a=4.00 \text{ \AA}$ and $c=3.67 \text{ \AA}$ by Kren *et al.*⁷ and are
 in even better agreement with another DFT-PBE study
 by Wang *et al.*,²⁸ reporting $a=3.98 \text{ \AA}$ and $c=3.72 \text{ \AA}$. The
 collinear antiferromagnetic structure is described by a
 uniaxial magnetic unit cell with up and down magnetic
 sites along the [001] easy axis. Antiparallel magnetic
 moments are localized on Mn atoms, compensating each
 other within each layer. Our DFT results give a sub-
 lattice magnetization per Mn atom of $m=3.7 \mu_B$ along
 [001] crystalline direction. The measured value amounts
 to $m=4.3 \mu_B$ at room temperature⁷ and DFT-LDSA re-
 sults in $m=3.6 \mu_B$, reported by Umetsu *et al.*¹⁰ Our results
 and those of other experimental and theoretical work are
 compiled in Table I, from which we conclude that our
 atomic structure and magnetic configuration is in good
 agreement with literature data.

B. Magnetocrystalline anisotropy energy

The magnetocrystalline anisotropy (MCA) energy of
 an antiferromagnet originates from contributions due to
 spin-orbit interaction (SOI) and magnetic dipole-dipole
 interaction (MDD). We compute the SOI term using DFT
 total energies including spin-orbit coupling. The MDD
 contribution is a relativistic correction from transverse
 electron-electron interactions³² that is not included in the
 DFT total energy and we describe it here within classical
 electrodynamics based on the relaxed DFT ground state

TABLE I. Relaxed lattice parameters (in Å) along three crystallographic axes and magnetic moments (in μ_B) of MnPt. All theoretical results use a spin polarized description without spin-orbit coupling.

MnPt	<i>a</i>	<i>b</i>	<i>c</i>	μ_{Mn}	μ_{Pt}
This work	3.97	3.97	3.71	3.7	0.0
DFT-PBE ²⁸	3.98	3.98	3.72	3.7	–
DFT-PBE ²⁹	4.03	4.03	3.69	4.3	0.0
DFT-LDSA ³⁰	3.99	3.99	3.70	3.8	0.0
LMTO-LDSA ¹⁰	–	–	–	3.6	0.0
Exp. ⁷	4.00	4.00	3.67	4.3	–
Exp. ³¹	4.002	4.002	3.665	–	–
Exp. ⁸	–	–	–	4.0	0.4

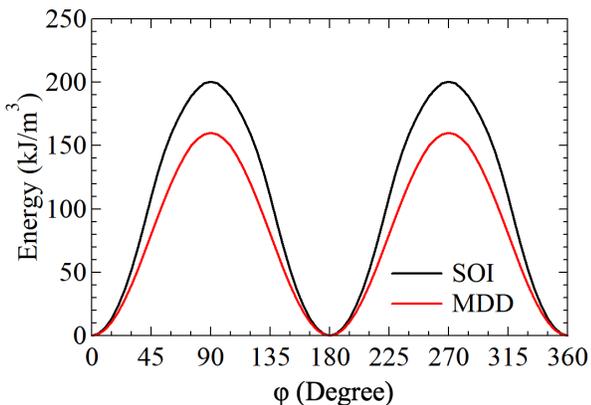


FIG. 2. (Color online.) Magnetocrystalline anisotropy of MnPt shows two-fold periodicity as a function of the tilting angle ϕ of the Néel vector with respect to the *a* axis with $\theta = 22.5^\circ$. Spin-orbit interaction (SOI, black solid line) and magnetic dipole-dipole interaction (MDD, red solid line) contributions are of comparable magnitude. V_{mag} is the volume of the magnetic unit cell.

MnPt in Fig. 2 show two-fold out-of-plane MDD and SOI contributions, confirming uniaxial magnetism. Based on the magnetocrystalline anisotropy result, the easy axis of antiferromagnetic L1₀-type MnPt is along *c*-axis, i.e. [001] direction with respect to the crystal structure, as shown in Fig. 1 (a). Using perturbation theory, the MCA energy can be expanded in terms of direction cosines,³⁴ which yields for a tetragonal crystal structure³⁵

$$\frac{E_{\text{MAE}}}{V} = K_1 \sin^2 \phi + K_2 \sin^4 \phi + K_{22} \sin^4 \phi \cos(4\theta), \quad (5)$$

where ϕ describes the angle that the Néel vector forms with the *c* axis, and θ is the angle between the *a* axis and the projection of the Néel vector to the *ab* plane. To study MCA for uniaxial magnetism, we use $\theta=22.5^\circ$ which corresponds to varying the Néel vector in the *ac* plane. We did not study MCA in the *ab* plane, because this corresponds to a hard plane. Fitting our results in Fig. 2 to Eq. (5) provides us with anisotropy coefficients that we compare to data reported in the literature in Tab. II. From this we find a significant variation of the results and note that due to their sub-meV magnitude, MCA calculations are very sensitive to details of the computational approach. In particular, the description of exchange and correlation, lattice parameters, and numerical parameters such as Brillouin zone sampling and plane-wave cutoff energy affect the results and likely explain the range of values reported in the literature. Here we converge all numerical parameters, with k-point convergence being the limiting factor, leading to a remaining error bar to be about 22%. We note that the accuracy of the magnetocrystalline anisotropy energy can further be affected by the pseudopotential and the inclusion of core electrons, which can be checked from other references in Table II. In addition, the consideration of Hubbard *U* parameters affects the electronic band structure and atomic and magnetic structure. While all resulting values are reported in Table S1³⁶ in the Supplementary Material³⁶ (see, also references^{37–39} there in), we note a particularly strong influence of the atomic coordinates on the anisotropy energy. Moreover, our results show that MDD contributions are important and only slightly depend on the atomic and magnetic structures. At last, our data illustrates that the MDD contribution to the total MCA energy of $K_1 = K_1^{\text{SOI}} + K_1^{\text{MDD}} = 1.07 \times 10^6 \text{ J/m}^3$ is as large as 68% of the SOI contribution and, hence, not negligible. In antiferromagnetic MnPt the MDD contribution is more important than in antiferromagnetic Fe₂As, where we found it to be about 50% of the SOI term.⁴⁰

atomic structure and magnetic moments. The sum of all MDD interactions in a bulk material is,³³

$$E_{\text{MDD}} = -\frac{1}{2} \frac{\mu_0}{4\pi} \sum_{i \neq j} \left(\frac{3[\mathbf{m}_i \cdot \mathbf{r}_{ij}][\mathbf{m}_j \cdot \mathbf{r}_{ij}]}{r_{ij}^5} - \frac{[\mathbf{m}_i \cdot \mathbf{m}_j]}{r_{ij}^3} \right), \quad (4)$$

where μ_0 is the vacuum permeability, \mathbf{r}_{ij} is the distance between two magnetic sites *i* and *j*, r_{ij} is the magnitude of \mathbf{r}_{ij} , and \mathbf{m}_i is the magnetic moment at site *i*. E_{MDD} decays as r_{ij}^{-3} and when numerically evaluating Eq. (4), we include interactions within a sphere with a cutoff radius of 50 Å. This converges E_{MDD} to within $10^{-9} \text{ eV/V}_{\text{mag}}$ where V_{mag} is the volume of a magnetic unit cell, allowing us to confirm that the MDD anisotropy energy in the *ab* plane is negligible.

Our MCA energy results for antiferromagnetic L1₀-type

C. Magnetic susceptibility

The magnetic susceptibility of a material describes how its total energy responds to a change of the magnetic structure in response to an external magnetic field. When such a field is applied to an antiferromagnetic material, magnetic moments cant towards the field direction, re-

TABLE II. Magnetocrystalline anisotropy coefficients K_1^{SOI} , K_2^{SOI} , K_1^{MDD} , and K_2^{MDD} . Other theoretical results only use the first term of Eq. (5) to calculate anisotropy, which can be compared to $K_1^{\text{SOI}} + K_2^{\text{SOI}}$ from our simulations.

$[kJ/m^3]$	K_1^{SOI}	K_2^{SOI}	K_1^{MDD}	K_2^{MDD}
This work	630	-79	438	0
DFT-LSDA ³⁰	312	-	-	-
DFT-LSDA+U ²⁹	1260	-	-	-
LMTO-LSDA-ASA ¹⁰	1400	-	-	-
GF-LMTO ⁴¹	274	-	-	-

349 ducing their antiparallel orientation that is energetically
 350 favored in the ground state. The magnetic susceptibil-
 351 ity of antiferromagnets connects tilting to a total energy
 352 change via the dependence of the exchange energy on
 353 tilting.

354 We use DFT to compute the magnetic susceptibility
 355 from the total energy change resulting from magnetic
 356 moment tilting. The total energy of the electronic system
 357 under an applied external magnetic field is⁴²

$$E_{\text{tot}} = E_0 + a\mu^2 - \mu\mathbf{B}, \quad (6)$$

359 where E_0 is the ground state total energy without mag-
 360 netic field, $a\mu^2$ describes the interaction of tilted mag-
 361 netic moments via an exchange term in a Heisenberg model,
 362 ignoring classical dipole-dipole contributions, and $-\mu\mathbf{B}$
 363 is the Zeeman energy term. \mathbf{B} is the external magnetic
 364 field vector and μ is the induced net magnetization that
 365 arises in the presence of the external field. For the small
 366 tilting studied here, the induced magnetic moments are
 367 proportional to μ . We kept all atomic positions fixed
 368 when tilting magnetic moments and found that this af-
 369 fects the resulting susceptibility by less than 0.5%. The
 370 lowest energy under an applied field minimizes Eq. (6)
 371 and corresponds to $\mathbf{B} = 2a\mu$ as discussed in Ref. 42. This
 372 yields for the magnetic susceptibility perpendicular to
 373 Néel vector

$$\chi_v = \frac{\mu_0}{2a - \mu_0}. \quad (7)$$

375 Here we compute the magnetic susceptibility for an
 376 external magnetic field along the [100] crystallographic
 377 direction as shown in Fig. 1(b). The DFT total energies
 378 for magnetic moment tilting between 0° and 10° degrees
 379 (in 1° degree increments) in Fig. 3 show a quadratic de-
 380 pendence on the resulting net magnetization. A quadratic³⁹²
 381 fit to this curve determines a in Eq. (7) and yields a unit-³⁹³
 382 less magnetic susceptibility of 5.25×10^{-4} , which is in
 383 between 4.82×10^{-4} measured at 4.2 K by Umetsu *et al.*¹⁰
 384 and 6.01×10^{-4} measured at 4.2 K by Chen *et al.*⁴³ on³⁹⁴
 385 polycrystalline samples. The magnetic susceptibility of
 386 antiferromagnets, including MnPt, is much smaller than³⁹⁵
 387 that of ferromagnets. Hence, a large external magnetic³⁹⁶
 388 field is required to induce a small amount of magnetic³⁹⁷
 389 moment tilting in antiferromagnets, illustrating the ro-³⁹⁸
 390 bustness of antiferromagnets against external fields. For³⁹⁹

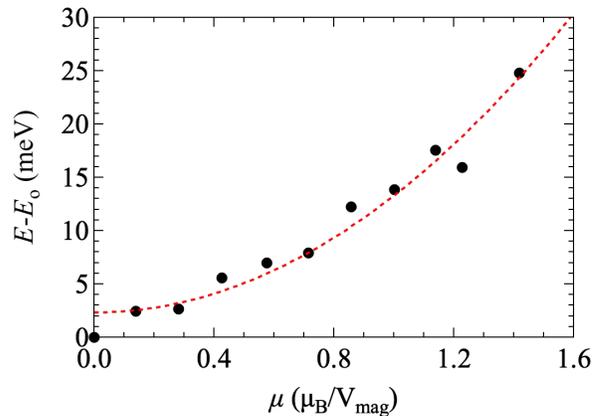


FIG. 3. (Color online.) DFT total energies for different tilting angles of magnetic moments. Each point corresponds to a tilting angle between 0° and 10° with a step size of 1°. Red dashed line shows the fit to Eq. (6) and the resulting uncertainty for the susceptibility is about 15%.

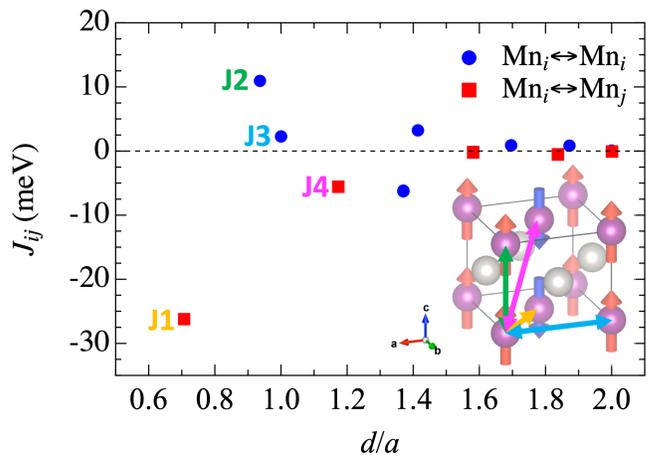


FIG. 4. (Color online.) Exchange coupling coefficients decrease with distance d (in units of the lattice parameter a). Blue circles show interactions of Mn sites with parallel moments, while red squares represent interactions of Mn sites with antiparallel moments. We include interactions up to eleventh-nearest neighbors for our magnon dispersion calculations. Colored arrows in the inset figure show the first to fourth neighboring interaction of exchange coupling.

a field oriented parallel to the Néel vector, i.e. the a -axis, the magnetic susceptibility would be zero in the limit of zero temperature.

D. Exchange coupling coefficients

Individual exchange coupling coefficients J_{ij} from a Heisenberg model can be used to explain the magnetic structure of antiferromagnetic L1₀-type MnPt in detail, in addition to the description of the collective response of the exchange coupling by the magnetic susceptibility.

400 We used the SPR-KKR code²³ to compute J_{ij} as plotted
 401 in Fig. 4. The first coefficient ($J_{ij}=-26.2$ meV) indicates
 402 antiferromagnetic coupling between any pair of nearest
 403 neighbor atoms in the ab plane (see inset of Fig. 4). The
 404 sign of the second coefficient ($J_{ij}=10.9$ meV) indicates
 405 ferromagnetic coupling between Mn atoms across the Pt
 406 layer. The third coefficient ($J_{ij}=2.3$ meV) corresponds to
 407 ferromagnetic coupling in the ab plane, while the fourth
 408 interaction ($J_{ij}=-5.5$ meV) couples two opposite mag-
 409 netic moments across the Pt layer with a $(1/2, 1/2)$ shift
 410 in the ab plane. While the Mn in-plane interaction is
 411 dominant, the Mn interlayer interaction is non-negligible.
 412 We note that the negative sign of the fifth interaction
 413 parameter represents antiferromagnetic coupling of sites
 414 with parallel magnetic moments. While this indicates
 415 magnetic frustration, the magnitude of this fifth param-
 416 eter is too small to affect the magnetic structure. Finally,
 417 we used exchange coefficients up to eleventh-neighbor
 418 atoms to compute the magnon dispersion in Sec. IV C.
 419 Using coefficients up to tenth-neighbor atoms changes the
 420 magnon dispersion by not more than 0.18 meV, which
 421 corresponds to 0.07 % of the entire magnon energy scale.

422 IV. ENERGY DISPERSION AND HEAT 423 CAPACITY

424 The energy dispersion of elementary excitations in a
 425 material allows to interpret ground state properties, such
 426 as magnetocrystalline anisotropy, and excited state prop-
 427 erties of materials, including frequency dependent optical
 428 spectra or temperature dependent heat capacity. Here
 429 we study the contributions from electrons, phonons, and
 430 magnons for antiferromagnetic MnPt and, subsequently,
 431 compute the heat capacity contributions from each ele-
 432 mentary excitation. Calculated heat capacity provides
 433 direct comparison with experiment, which is used here to
 434 validate our computational description.

435 A. Electronic structure

436 Our computed electronic band structure in Fig. 5(a) ac-
 437 counts for spin-orbit interaction and shows the metallicity
 438 of L1₀-type MnPt. While bands are crossing at the Fermi
 439 level, the density of states itself is very low and exhibits a
 440 significant dip within about 0.5 eV. This is similar to what
 441 was reported by Umetsu *et al.* from LMTO-LSDA-ASA
 442 simulations,¹⁰ and also agrees with DFT-LSDA simula-
 443 tions by Lu *et al.*³⁰ as well as DFT-PBE by Wang *et al.*²⁸
 444 and Alsaad *et al.*²⁹

445 We compute the electronic specific heat using the ther-
 446 modynamic average of the internal energy U at tempera-
 447 ture T and the Sommerfeld expansion, leading to

$$448 \quad \gamma_e = \frac{\partial U}{\partial T} = \frac{1}{3} \pi^2 k_B^2 N(E_F), \quad (8)_{450}$$

449 where k_B is the Boltzmann constant, and $N(E_F)$ is the

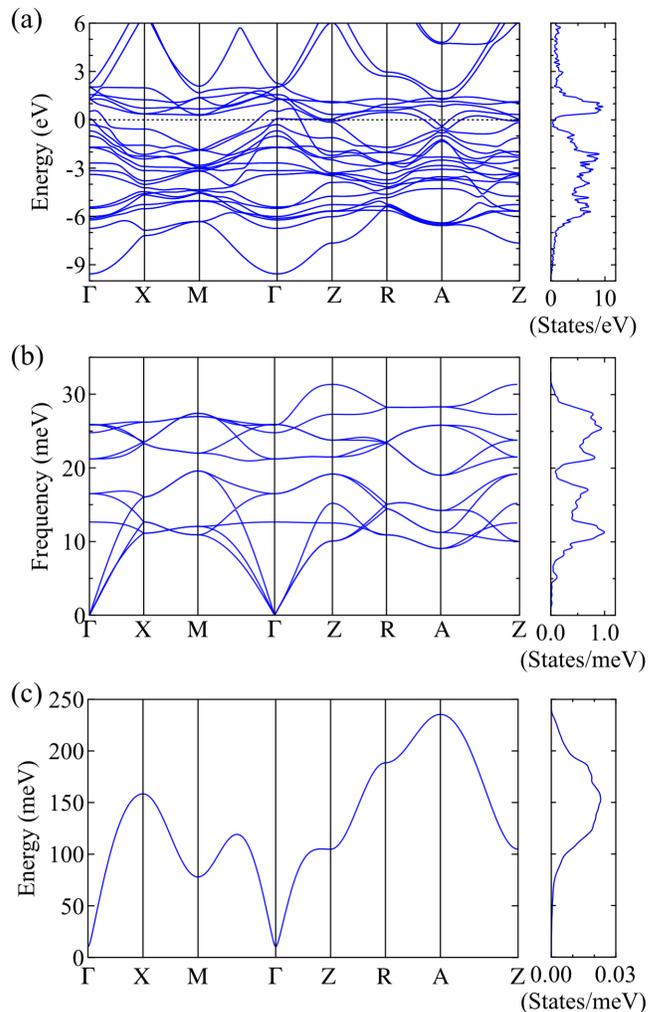


FIG. 5. (Color online.) Energy dispersion curves and densities of states (normalized per unit cell) of (a) electrons, (b) phonons, and (c) magnons. Electronic band structure (a) illustrates the metallic character with low density of states near the Fermi level at $E = 0$ eV. The energy scale of phonons is about one order of magnitude smaller than that of magnons. Magnon bands from linear spin wave theory show a magnon gap at the Γ point.

density of states at the Fermi level.¹⁰ We obtain a value of 0.32 mJ/(mol K²), which agrees very well with a measured value of 0.26 mJ/(mol K²) by Umetsu *et al.*¹⁰ and is slightly smaller than the electronic specific heat of other pure metals. Their LMTO-LSDA-ASA data¹⁰ results in 0.33 mJ/(mol K²) and the DFT-PBE result of Wang *et al.*²⁸ is somewhat lower at 0.13 mJ/(mol K²). This difference may originate from our choice of more converged Brillouin zone sampling and plane-wave cutoff that can affect the results of such a small value of the DOS near the Fermi level is computed. In addition, we note that SOC is included in our DOS simulations, while that seems not to be the case for Refs. 10 and 28.

B. Phonon dispersion

Our result for the phonon dispersion in Fig. 5(b) shows a total of 12 acoustic and optical branches, corresponding to 4 atoms per magnetic unit cell with 3 modes each, with a shallow gap in between at around 18 meV. We then use this predicted phonon dispersion to compute the phonon heat capacity from statistical mechanics with the canonical distribution and the harmonic approximation,

$$C_V^{\text{phonon}} = \sum_{\mathbf{q}\nu} k_B [\beta \hbar \omega(\mathbf{q}\nu)]^2 \frac{\exp(\beta \hbar \omega(\mathbf{q}\nu))}{[\exp(\beta \hbar \omega(\mathbf{q}\nu)) - 1]^2}, \quad (9)$$

where $\beta = 1/(k_B T)$, \mathbf{q} is a phonon wave vector, ν is an index of phonon modes, and ω is a phonon eigenvalue. We compute the phonon heat capacity using Eq. (9) and a $30 \times 30 \times 30$ \mathbf{q} -point grid. For a linear phonon dispersion near Γ , a T^3 dependence follows at low temperatures, as discussed below in Sec. IV E.

C. Magnon dispersion

Using linear spin-wave theory, the exchange coefficients from Sec. III D, and the anisotropy coefficients we discussed in Sec. III B, we compute the magnon dispersion shown in Fig. 5(c). Since antiferromagnetic MnPt has two magnetic sites, all magnon energy states are doubly degenerate. We note that the entire magnon energy range reaches up to 250 meV, which is higher than the bandwidth of phonons of about 30 meV. The magnon gap at the Γ point is 10.49 meV (=2.54 THz). Our calculated magnon dispersion (see Fig. 5) includes the anisotropy energy and, hence, it shows an energy gap at Γ . Without the anisotropy energy term, this magnon energy gap would disappear and the magnon dispersion would be linear, starting at the Γ point.

Next, we use the Kittel formula to compute the lowest magnon frequency ω_{\min} from the Landau-Lifshitz equation for $k = 0$, which provides an estimate for how fast spin dynamics occurs in MnPt. For an easy-axis antiferromagnet without external field⁴⁴ this leads to,

$$\omega_{\min} = \gamma \sqrt{2H_E H_A + H_A^2}, \quad (10)$$

where $H_E = m/\chi$ and $H_A = K/m$ are exchange field and anisotropy field, respectively, m is the magnitude of the sub-lattice magnetization, χ is the magnetic susceptibility, K is the anisotropy energy coefficient and γ is the gyromagnetic ratio ($g\mu_B/\hbar$). Since MnPt has two sites with antiparallel moments, m is identical to the magnetic moment of each of these sites, computed from ground state DFT. We use the calculated magnetic susceptibility from Sec. III C and the anisotropy energy from Sec. III B. Our result of $\omega_{\min}/2\pi = 2.02$ THz (8.97 meV) is slightly larger than the magnon gap of 7 meV measured by Hama *et al.* at 300 K for vanishing wave vector using inelastic

neutron scattering.⁹ The small difference between calculated and measured gap may be attributed to a decrease of the anisotropy energy with temperature.⁴⁵ These results also confirm the THz scale of spin dynamics for antiferromagnetic MnPt, which is faster than the GHz scale that is common for ferromagnets, such as 36 and 73 GHz in ferromagnetic Fe films under dc magnetic fields between 0 to 10 kOe⁴⁶ and 23.4 GHz for ferromagnetic garnet films doped with germanium and calcium.⁴⁷ We also note that the spin-flop transition field is closely related to the magnon energy gap, via $H_{\text{sf}} = \sqrt{2H_E H_A + H_A^2}$, resulting in $H_{\text{sf}} = 72$ T.

To compute the magnon heat capacity, we employ the same approach that we used to obtain the magnon dispersion in Fig. 5(c), to compute the magnon density of states on a $30 \times 30 \times 30$ \mathbf{q} -point grid. Since magnons are bosonic, the magnon total energy follows from

$$E_{\text{magn.}} = \sum_{\mathbf{q}\nu} \hbar \omega(\mathbf{q}\nu) \frac{1}{\exp(\beta \hbar \omega(\mathbf{q}\nu)) - 1}. \quad (11)$$

The temperature derivative of this expression leads to the magnon heat capacity

$$C_V^{\text{magn.}} = \sum_{\mathbf{q}\nu} k_B [\beta \hbar \omega(\mathbf{q}\nu)]^2 \frac{\exp(\beta \hbar \omega(\mathbf{q}\nu))}{[\exp(\beta \hbar \omega(\mathbf{q}\nu)) - 1]^2}, \quad (12)$$

which resembles the expression for the phonon heat capacity, Eq. (9). This approach is valid for the low temperature range, the so-called spin-wave region, and our result for the magnon specific heat is shown in Fig. 7. At high temperature near the critical temperature, the spin-wave description is no longer valid. Thus, we describe the critical region near the Néel temperature using a Monte Carlo approach instead, as discussed in Sec. V A.

D. Inelastic neutron scattering simulation

To facilitate comparison of our phonon and magnon dispersion data with experiment, we simulate inelastic neutron scattering (INS) intensity using the same instrument parameters as in our previous study on Fe₂As.⁴⁹ Coherent and incoherent inelastic neutron scattering is added to our phonon results using the OCLIMAX code.⁴⁸ For the magnon contribution, the dynamical spin-spin correlation function is included using the SpinW code.²⁵ Both simulated INS results are shown in Fig. 6 for the [H00] (symmetrically identical to [0K0]) and [00L] directions commonly studied in experiment. The phonon form factor is proportional to \mathbf{q}^2 , which explains the intensity increase of the phonon contribution. Conversely, the magnon contribution weakens with increasing \mathbf{q} -vector. In correspondence with Fig. 5, all phonon-related signals appear below 40 meV. The magnon signals increase as sharp linear lines beyond 40 meV and, as a result, appear only in the close vicinity of the Γ points in Fig. 6. The phonon contribution to INS along H in Fig. 6(a) shows

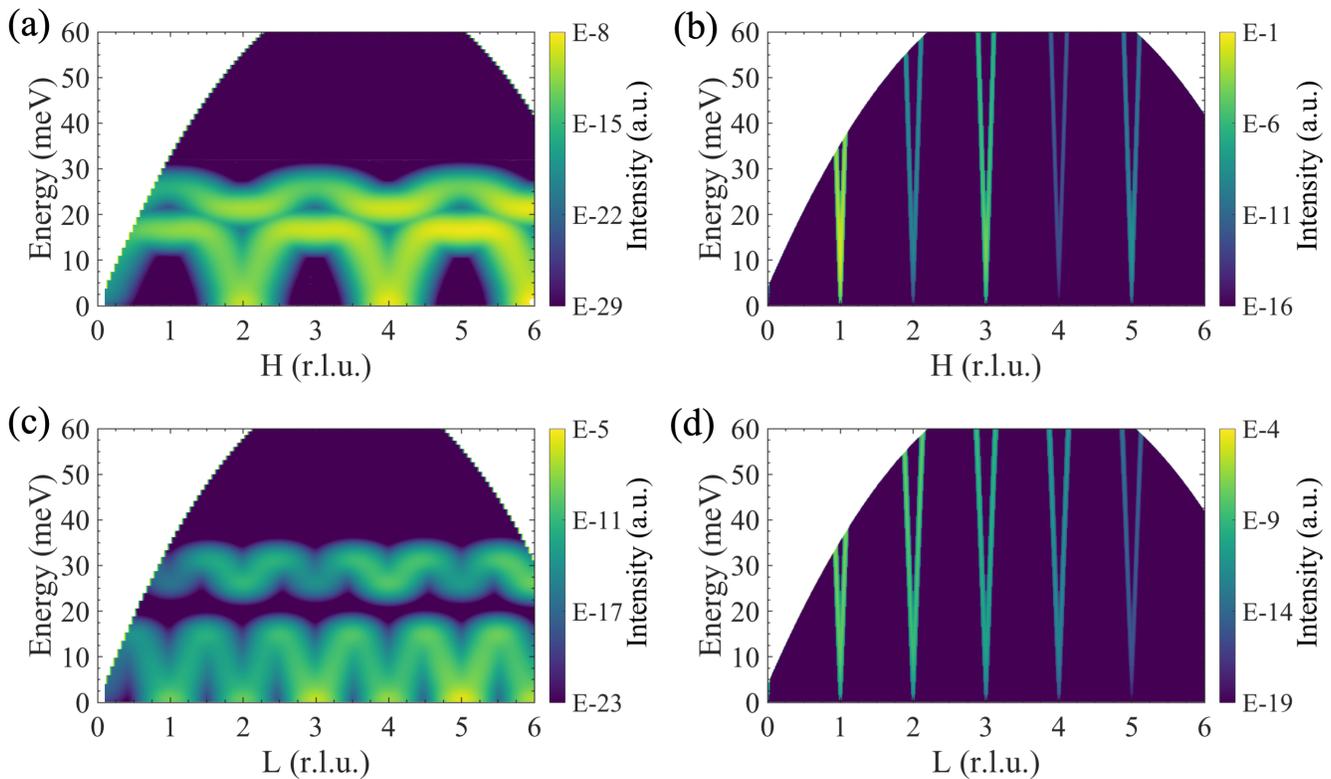


FIG. 6. (Color online.) Phonon ((a), (c)) and magnon ((b), (d)) contributions to simulated inelastic neutron scattering along $[H\ 0\ 0]$ direction ((a), (b)) and $[0\ 0\ L]$ direction ((c),(d)) in reciprocal space (in reciprocal lattice units, r.l.u.). We use a logarithmic color scale to show the upper 90% of the intensity data. Magnon curves include a Gaussian broadening of 10 meV and phonon curves are broadened using the `OCLIMAX` code⁴⁸ for a temperature of 5 K.

560 periodicity with every two reciprocal lattice periods, while
 561 the signal along L presents the same periodicity as shown
 562 in Fig. 5. For the magnon signal in Fig. 6(b) we find
 563 alternating intensities for even and odd reciprocal lattice
 564 periods, since the magnetic unit cell comprises of two
 565 chemical unit cells in the ab plane (see Fig. 1).

566 Our analysis shows that phonon and magnon contribu-
 567 tions can be clearly distinguished in INS experimental
 568 data. In experiment, the magnon gap energy at $\mathbf{q} = 0$
 569 is determined by finding the energy where the INS in-
 570 tensity is at a maximum for (100) along the H direction
 571 in reciprocal space. The calculated INS data shown in
 572 Fig. 6 demonstrates that there is no phonon contribution
 573 at this point. Therefore, the signal clearly originates from
 574 magnons, as reported in the INS study of Hema *et al.*,⁹
 575 confirming that the corresponding gap energy is a magnon
 576 gap.

577 E. Total heat capacity

578 In Fig. 7 we show the total heat capacity and partition
 579 it into electron, phonon, and magnon contributions. This
 580 illustrates that at temperatures below 2 K, the electronic
 581 contribution is dominant and at higher temperatures the
 582 phonon contribution takes over with a T^3 dependence,

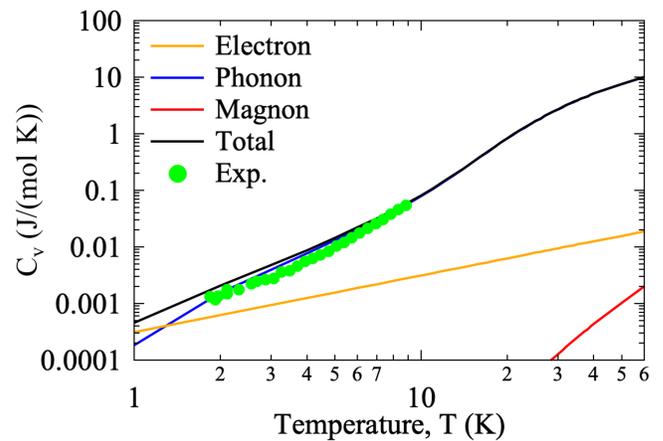


FIG. 7. (Color online.) Temperature dependence of electron, phonon, and magnon contributions to the total heat capacity. Our first-principles results for electron, phonon, and magnon contributions to the total specific heat agree well with experimental data from Ref. 10 at low temperatures.

which is consistent with a linear phonon dispersion near Γ .^{33,50} The prefactor of the T^3 term due to phonons is 0.090 mJ/(mol K⁴). The onset of the magnon contribution appears at non-zero temperature due to the Γ point gap

of the magnon dispersion that results from the nonzero anisotropy energy. The low magnon contribution to the heat capacity at low temperatures further results from the low magnon density of states in the energy range below 50 meV. The total number of phonon modes is twelve per four-atom magnetic unit cell, while that of magnons is two per four-atom magnetic unit cell with two magnetic moments. Finally, the electronic specific heat contributes linearly with T , which determines the total heat capacity near 0 K as shown in Fig. 7. We note that our computed total heat capacity in Fig. 7 agrees well with measured results and show that the overall temperature dependence is thus dominated by the *phonon* contribution in the low temperature range. The lower magnon density of states leads to a lower magnon heat capacity, compared to phonons, see Figs. 5(b) and (c).

V. NÉEL TEMPERATURE AND MAGNETO-OPTICAL PROPERTIES

Excited state properties are popularly used as materials selection criteria, to identify materials well-suited for specific applications, and provide insight into the physics of the AFM since they derive from the electronic band structure including spin-orbit coupling. In particular, first-principles studies can predict the Néel temperature, which determines thermal stability of the AFM configuration, and magneto-optical effects, that play a role for magnetic characterization.

A. Néel temperature

We compute the Néel temperature from thermodynamic observables using a Monte-Carlo solution of the stochastic LLG equation, Eq. (3), parametrized by our calculated exchange interactions. We use this approach since, near the critical temperature, the linear-spin wave approach discussed in Sec. IV C is not applicable. In the MC approach, the average sub-lattice magnetization is typically studied as a function of temperature, and should be zero at the critical temperature. However, due to finite size effects in our simulations, this transition cannot be easily detected and is not very sharp in Fig. 8.

Instead, the Binder cumulant, the isothermal susceptibility, and the specific heat are thermodynamic observables that provide a clearer picture.²⁷ The fourth-order Binder cumulant U_4 was specifically developed to correct the finite size problem for second-order phase transitions,⁵¹

$$U_4 = 1 - \frac{\langle m^4 \rangle}{3 \langle m^2 \rangle^2}, \quad (13)$$

where m is the magnitude of the sub-lattice magnetization. In this work, m is identical to the magnetic moment at each magnetic site because MnPt has two sites with

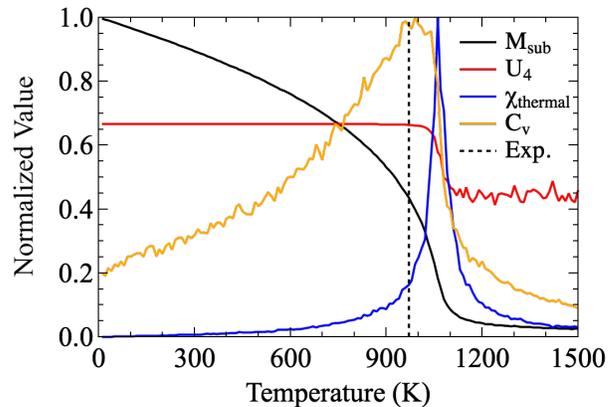


FIG. 8. (Color online.) Temperature dependence of sub-lattice magnetization (M_{sub}), isothermal susceptibility (χ_{thermal}), and heat capacity (C_V). These are normalized using the respective maximum values in this temperature range, i.e., the ground-state sub-lattice magnetization for M_{sub} , and the peak values at the critical temperature for χ_{thermal} and C_V . The fourth order Binder cumulant U_4 is computed from Eq. (13) and shown as red solid line.

antiparallel moments. The value of the cumulant changes at the Néel temperature from $U_4 \approx 0.444$ for $T > T_N$ to $U_4 \approx 0.667$ for $T < T_N$. From this, we compute a transition temperature of around 1070 K. The isothermal susceptibility of a sub-lattice susceptibility

$$\chi_{\text{thermal}} = \frac{\langle m^2 \rangle - \langle m \rangle^2}{k_B T} \quad (14)$$

is another thermodynamic observable which describes the response of the magnetization to temperature, where m is the sub-lattice magnetization, k_B is the Boltzmann constant, and T is temperature. It peaks at around 1060 K, which is close to the value from the Binder cumulant. Lastly, the heat capacity

$$C_V = \frac{\langle E^2 \rangle - \langle E \rangle^2}{k_B T^2} \quad (15)$$

can be computed from the variance of the MC total energy E and leads to a peak of the specific heat around 990 K. These results agree well with measured Néel temperatures of 975 K⁷ and 970 K.¹⁰

In addition, the Néel temperature can be computed without the MC approach, via integration of the adiabatic magnon dispersion in Fig. 5(c). Two methods are commonly used in the literature,⁵² one is based on the mean-field approximation (MFA)

$$k_B T_N^{\text{MFA}} = \frac{m}{3} \left[\frac{1}{N} \sum_{\mathbf{q}=0}^{\text{BZ}} \omega(\mathbf{q}) \right], \quad (16)$$

and another on the random phase approximation (RPA)

$$k_B T_N^{\text{RPA}} = \frac{m}{3} \left[N \sum_{\mathbf{q}=0}^{\text{BZ}} \frac{1}{\omega(\mathbf{q})} \right]^{-1}. \quad (17)$$

661 In both expressions m stands for the sub-lattice magneti-
 662 zation and N is the total number of magnon energies sam-
 663 pled by the \mathbf{q} point grid. Here we use a $30 \times 30 \times 30$ \mathbf{q} -point
 664 grid to evaluate these expressions and obtain $T_N^{\text{MFA}}=1250$
 665 K and $T_N^{\text{RPA}}=1190$ K. Both values are slightly larger than
 666 measured values of 975 K⁷ and 970 K,¹⁰ or another DFT
 667 result, using exchange coefficients, of 989 K.²⁹ A similar
 668 overestimation of the experimental result on the order
 669 of 25 % by this approach is also reported e.g. for ferro-
 670 magnetic BCC Fe⁵³ and antiferromagnetic NiO.⁵² The
 671 MC approach shows better agreement with experiment
 672 because the methods based on magnon dispersion assume
 673 the spin-wave regime, which is only appropriate at low
 674 temperatures relative to the Néel temperature.

675 B. Optical response

676 In this work we use the Kohn-Sham electronic structure,
 677 including spin-orbit coupling effect, to compute optical
 678 spectra of MnPt. First, we compute the imaginary part
 679 of the interband contribution to the complex, frequency-
 680 dependent dielectric tensor¹⁸ in CGS units using

$$681 \quad \varepsilon_{\alpha\beta}^{(2)} = \frac{4\pi^2 e^2}{\Omega} \lim_{q \rightarrow 0} \frac{1}{q^2} \sum_{c,v,\mathbf{k}} 2w_{\mathbf{k}} \delta(\epsilon_{c\mathbf{k}} - \epsilon_{v\mathbf{k}} - \omega) \times \quad (18)$$

$$\times \langle u_{c\mathbf{k}+e_{\alpha}q} | u_{v\mathbf{k}} \rangle \langle u_{v\mathbf{k}} | u_{c\mathbf{k}+e_{\beta}q} \rangle,$$

682 where α and β are Cartesian indices, Ω is the unit cell
 683 volume, $w_{\mathbf{k}}$ is the symmetry weight of each \mathbf{k} -point, c
 684 and v index conduction and valence bands, $\epsilon_{c\mathbf{k}}$ and $\epsilon_{v\mathbf{k}}$
 685 are Kohn-Sham eigenvalues, and $u_{c\mathbf{k}}$ and $u_{v\mathbf{k}}$ are the cell
 686 periodic part of the Kohn-Sham orbitals. The real part,
 687 $\varepsilon_{\alpha\beta}^{(1)}$, follows from the imaginary part, $\varepsilon_{\alpha\beta}^{(2)}$, via Kramers-
 688 Kronig transformation.

689 Since antiferromagnetic MnPt is metallic (see
 690 Sec. IV A), intraband contributions to the dielectric ten-
 691 sor need to be included, in addition to the interband
 692 contributions in Eq. (18). We use the Drude equation,

$$693 \quad \varepsilon(\omega) = -\frac{\omega_p^2}{\omega^2 + i\omega\Gamma_D}, \quad (19)$$

694 where ω_p is the plasma frequency and Γ_D is the line width
 695 originating from the finite electron lifetime. We compute
 696 the plasma frequency from our Kohn-Sham electronic
 697 structure,⁵⁴ using

$$698 \quad \omega_{p,\alpha\beta}^2 = \frac{4\pi e^2}{\Omega \hbar^2} \sum_{n,\mathbf{k}} 2g_{\mathbf{k}} \frac{\partial f(\epsilon_{n\mathbf{k}})}{\partial \epsilon} \left(\mathbf{e}_{\alpha} \frac{\partial \epsilon_{n\mathbf{k}}}{\partial \mathbf{k}} \right) \left(\mathbf{e}_{\beta} \frac{\partial \epsilon_{n\mathbf{k}}}{\partial \mathbf{k}} \right), \quad (20)$$

699 where $g_{\mathbf{k}}$ is the \mathbf{k} -point weighting factor, $f(\epsilon_{n\mathbf{k}})$ is an⁷⁰⁹
 700 occupation number at energy state $\epsilon_{n\mathbf{k}}$, n is a band index,⁷¹⁰
 701 and \mathbf{e}_{α} is a unit vector along α -direction. Various scat-
 702 tering mechanisms affect the electron lifetime, including⁷¹²
 703 electron-electron and electron-phonon scattering, and this⁷¹³
 704 value is challenging to compute from first principles.^{55,56}⁷¹⁴
 705 Instead, here we use the electric resistivity of 21 $\mu\Omega\text{-cm}$ ⁷¹⁵

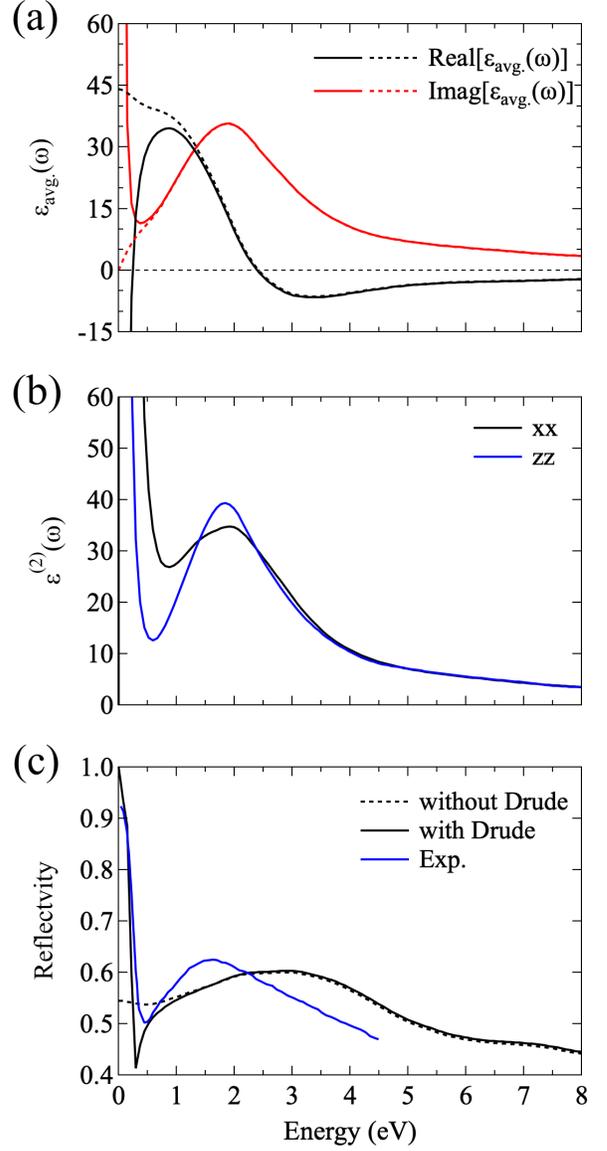


FIG. 9. (Color online.) (a) Real (black) and imaginary (red) part of the complex dielectric tensor, averaged over the Cartesian components. (b) Imaginary part of the two diagonal elements of the complex dielectric tensor. (c) Reflectivity of antiferromagnetic MnPt, experimental results are from Kubota *et al.*⁵⁷ Solid and dashed lines show our simulation results with and without intraband Drude contribution, respectively.

at 300 K measured by Umetsu *et al.*¹⁰ and our value for the averaged plasma frequency of $\omega_p=5.29$ eV to estimate the electron scattering time quasi-classically as $\Gamma_D=1/(\epsilon_0\omega_p^2\rho)=7.44$ fs, which corresponds to a lifetime broadening of 0.56 eV.

Comparing our calculated frequency dependent dielectric functions with and without Drude contribution in Fig. 9(a) illustrates that the intraband contribution predominantly affects the low energy range below 1 eV. In particular, the interplay of intra- and interband contri-

716 bution leads to a valley of $\varepsilon_{\alpha\beta}^{(2)}$ for a photon energy of
 717 about 0.5 eV. Anisotropic dielectric functions along three
 718 crystallographic axes directions are shown in Fig. 9 (b).
 719 Due to the tetragonal crystal structure of this material,
 720 the xx and yy components of the dielectric tensor show
 721 identical spectra, and the zz component differs.

722 We also compute the reflectivity⁵⁸ from

$$723 \quad R = \left| \frac{\tilde{n} - 1}{\tilde{n} + 1} \right|^2 = \left| \frac{\sqrt{\tilde{\varepsilon}} - 1}{\sqrt{\tilde{\varepsilon}} + 1} \right|^2, \quad (21)$$

724 where \tilde{n} is the complex refractive index which is the square
 725 root of the averaged diagonal components of complex relative
 726 dielectric constant, $\tilde{n}^2 = \tilde{\varepsilon}$. The resulting reflectivity
 727 spectra are plotted in Fig. 9(c) and compared to exper-
 728 imental data from Kubota *et al.*⁵⁷ We find that the overall
 729 spectrum agrees well between experiment and simulation,
 730 but the position of the low-energy reflectivity minimum
 731 differs between experiment (0.45 eV) and simulation (0.29
 732 eV). Also the position of a broad higher energy reflectivity
 733 peak disagrees between 1.63 eV (experiment) and 2.95
 734 eV (simulation). In addition, the comparison of the re-
 735 flectivity with and without Drude contribution confirms
 736 that the high reflectivity at low photon energies originates
 737 from intraband transitions.

738 C. Linear Magneto-Optical Kerr Effect

739 While most collinear antiferromagnets do not show linear
 740 magneto-optical effects,⁵⁹ it is possible to generate
 741 such signals using spin precession⁶⁰ or external stimulation
 742 e.g. via an electric field.⁶¹ Applying an external magnetic
 743 field also can break the $C_{2v}m'm'$ magnetic space group
 744 symmetry of antiferromagnetic MnPt, leading to non-zero
 745 linear magneto-optical Kerr effect. Here, we introduce
 746 such a field perpendicular to Néel vector (a -axis direc-
 747 tion) by tilting the magnetic moments between 0° and
 748 3° in steps of 1° , inducing a small net magnetization,
 749 see Fig. 1(b). We then follow Ref. 62 and compute the
 750 frequency-dependent polar magneto-optical Kerr effect
 751 (PMOKE) using

$$752 \quad \Psi_K(\omega) = \theta_K(\omega) + i\gamma_K(\omega) = \frac{-\epsilon_{xy}}{(\epsilon_{xx} - 1)\sqrt{\epsilon_{xx}}}. \quad (22)$$

753 All calculations include the Drude contribution, assuming⁷⁶⁶
 754 the constant electron lifetime discussed in Sec. V B.⁷⁶⁷

755 Figure 10(a) and (b) show the resulting PMOKE rota-⁷⁶⁸
 756 tion and ellipticity spectra. The field strength is computed⁷⁶⁹
 757 from the tilting angle using the magnetic susceptibility⁷⁷⁰
 758 discussed in Sec. III C. Due to the small magnetic suscepti-⁷⁷¹
 759 bility compared to ferromagnetic materials, tilting angles⁷⁷²
 760 of 1° correspond to an external magnetic field of 52 T for⁷⁷³
 761 antiferromagnetic MnPt. From Fig. 10 we find maximum⁷⁷⁴
 762 Kerr rotation and ellipticity in the visible spectral range⁷⁷⁵
 763 near 1.40 eV and 1.91 eV, respectively.⁷⁷⁶

764 Our results also show that the interband PMOKE sig-⁷⁷⁷
 765 nal, at energies larger than about 1 eV, is approximately⁷⁷⁸

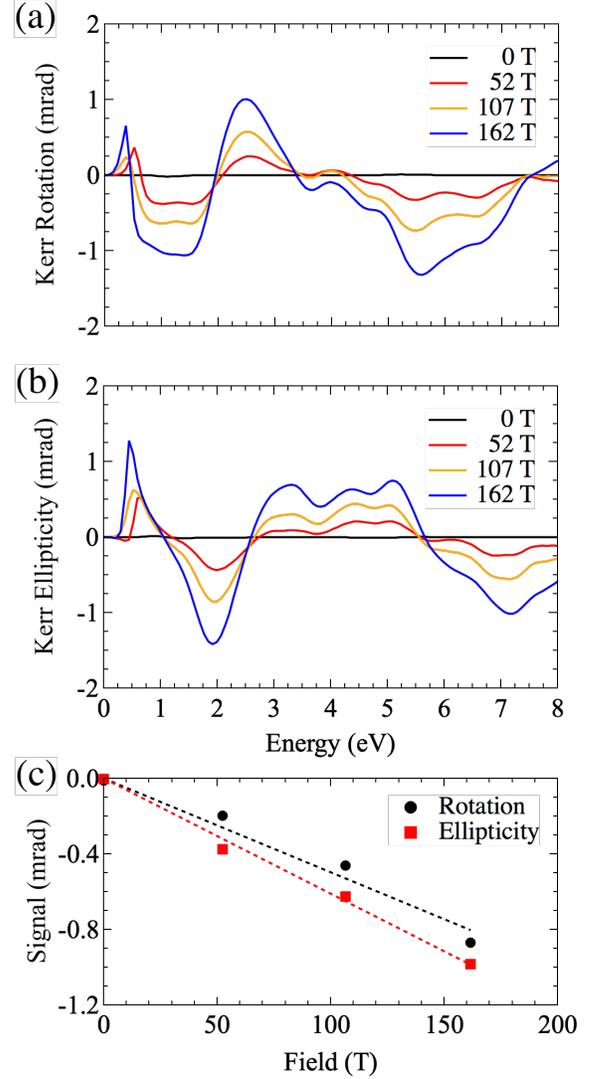


FIG. 10. (Color online.) Optical polar magneto-optical Kerr (a) rotation and (b) ellipticity spectra for different external magnetic fields. Maxima of Kerr rotation and ellipticity occur at 1.40 eV and 1.91 eV, respectively. In (c) the linear dependence of the Kerr signals on the magnetic field is shown for a wave length of 785 nm (= 1.58 eV).

proportional to the external magnetic field. This can be understood from a Taylor expansion of the dielectric function with respect to net magnetization.⁶³ For small tilting angles and small net magnetization the proportionality of linear MOKE with B is valid, i.e. $\Delta\varepsilon \propto \boldsymbol{\mu} \propto \mathbf{B}$. The approximately linear dependence of the Kerr signals on the magnetic fields studies in this work is explicitly shown for a laser wave length of 785 nm in Fig. 10(c). From interpolating the linear fit to this data to a magnetic field of 1 T, we find a Kerr rotation and ellipticity of $-6.1 \mu\text{rad}$ and $-5.0 \mu\text{rad}$, respectively.

To determine the origin of features in the Kerr rotation spectrum, we decompose it according to contributions

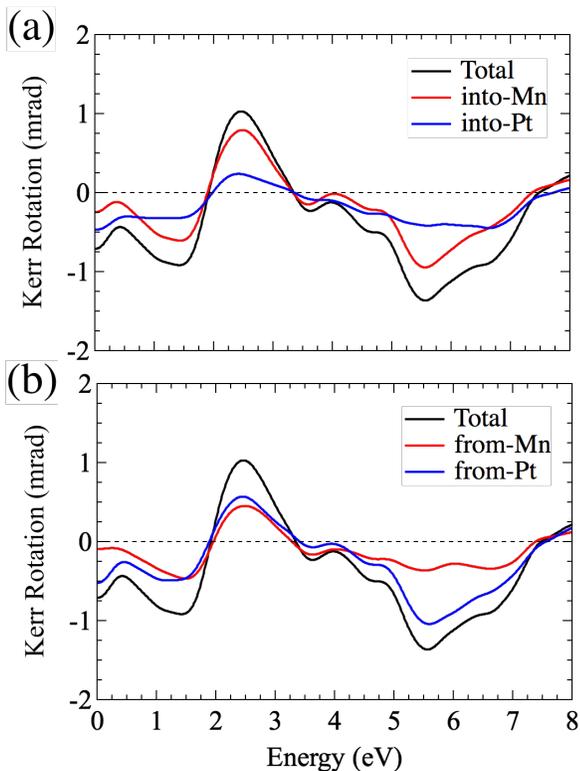


FIG. 11. (Color online.) Projected element orbital decomposition of PMOKE Kerr rotation at 3° tilting angle contributed by (a) transitions from all valence states to conduction states of specific atom element and (b) transitions from valence states of specific atom element to all conduction states. Spectrum in this figure does not include the intraband transition contribution. Black solid line shows total Kerr rotation spectrum from interband transitions.

779 from valence and conduction electrons of Mn and Pt states⁸³⁴
 780 using the scheme described in Refs. 42 and 64. The data⁸³⁵
 781 in Fig. 11(a) illustrates that peaks at 1.42 eV, 2.46 eV, and⁸³⁶
 782 5.57 eV feature large contributions due to transitions into⁸³⁷
 783 empty Mn states. Transition originating in Mn valence⁸³⁸
 784 states contribute about the same to the PMOKE spectrum⁸³⁹
 785 across the entire spectral range as those from Pt valence⁸⁴⁰
 786 states, except for the peak at 5.57 eV, that is dominated by⁸⁴¹
 787 Pt valence states. Furthermore, our orbital decomposition⁸⁴²
 788 concludes that transitions among d orbitals are the main⁸⁴³
 789 source of the PMOKE spectrum, which is consistent with⁸⁴⁴
 790 the majority of states near the Fermi level exhibiting⁸⁴⁵
 791 d orbital characters (see Fig. S5 in the Supplementary⁸⁴⁶
 792 Material,³⁶ see, also references^{37–39} there in).⁸⁴⁷

793 VI. CONCLUSIONS

794 As the interest in antiferromagnetic spintronics in⁸⁵²
 795 creases, fundamental properties of antiferromagnetic met⁸⁵³
 796 als, and their accurate prediction from first principles⁸⁵⁴
 797 become increasingly important. Here we report a compre⁸⁵⁵

798 hensive first-principles computational study of antiferro-
 799 magnetic $L1_0$ type MnPt. For the lattice geometry, and
 800 electronic and magnetic structure we find very good agree-
 801 ment with earlier experimental and theoretical results.
 802 Similarly, our prediction of the magnetic susceptibility
 803 agrees well with experimental data. We then compute the
 804 previously unknown exchange coupling coefficients and
 805 discuss how these explain the ground-state magnetic struc-
 806 ture. Using these coefficients, we predict the magnon dis-
 807 persion of MnPt, including the lowest magnon frequency
 808 of 8.97 meV, which is critical for a deep understanding
 809 of fundamental limits of the time scale of spin dynam-
 810 ics. The corresponding gap at the Γ -point of the magnon
 811 dispersion agrees well with the lowest magnon frequency
 812 computed using spin-wave theory and we also find very
 813 good agreement with an experimentally reported value.

814 Having established the accuracy of our first-principles
 815 description, we proceed to compute electron, phonon, and
 816 magnon dispersion data that we use to derive the indi-
 817 vidual contributions to the heat capacity of MnPt. We
 818 unambiguously show that the temperature dependence of
 819 the heat capacity is dominated by phonon contributions
 820 at low temperatures, and the magnon contribution re-
 821 mains small, owing to the sizable magnon gap and the low
 822 magnon density of states. Using our data, we individually
 823 predict phonon and magnon contributions to inelastic
 824 neutron scattering, which will facilitate identification of
 825 each contribution in experiment. Phonon inelastic neu-
 826 tron scattering shows a periodicity over two reciprocal
 827 lattice units along the H direction, while the magnon
 828 signal presents alternating intensities with a periodicity
 829 of one reciprocal lattice unit. This is because two mag-
 830 netic Mn atoms are placed along $[100]$ when viewed along
 831 $[010]$. The broader energy range and characteristic lin-
 832 ear magnon dispersion curves that originate from every
 833 reciprocal lattice unit allow distinguishing phonons and
 magnons experimentally.

In order to explore the stability of the magnetic order-
 ing and the possibility of reorienting the Néel vector,
 we compute the magnetocrystalline anisotropy energy,
 and find confirmation of the uniaxial antiferromagnetic
 structure of the material. We explicitly include a classical
 contribution to this energy that accounts for magnetic
 dipole interactions and previously was ignored for anti-
 ferromagnets. Our simulations provide clear evidence for
 the importance of this contribution to the K_1 anisotropy
 coefficient, as it amounts to about 2/3 of the commonly
 studied term due to the spin-orbit interaction. In addition,
 we employed the Monte Carlo method with our calculated
 exchange and anisotropy coefficients to compute three
 thermodynamic observables from atomistic spin dynam-
 ics, from which we estimate the Néel temperature to be
 990–1070 K, which is within 100 K from experimental val-
 ues. The high Néel temperature around 1000 K indicates
 the thermal stability of the magnetic structure, possibly
 enabling magnetic devices at room temperature.

Finally, we compute the optical and magneto-optical
 properties of MnPt via the dielectric function and the

reflectivity spectrum including intra- and interband contributions, to provide insight into the underlying physics and the possibility of magneto-optical detection of collective spin motion. From this, we predict the generation of polar magneto-optical effects of antiferromagnetic MnPt when applying an external magnetic field. Our simulations show a polar MOKE signal on the order of μrad for an external field of 1 T. We find this to be in the linear response regime and our data can provide guidance for maximizing the polar MOKE signal in experiments with a few hundreds nrad of resolution through linear interpolation.

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- 1 M. L. Néel, *Ann. Phys.* **12**, 137 (1948), URL <https://doi.org/10.1051/anphys/194812030137>.
 - 2 P. Wadley, B. Howells, J. Železný, C. Andrews, V. Hills, R. P. Champion, V. Novák, K. Olejník, F. Maccherozzi, S. S. Dhesi, et al., *Science* **351**, 587 (2016), ISSN 0036-8075, URL <https://science.sciencemag.org/content/351/6273/587.full.pdf>, URL <https://science.sciencemag.org/content/351/6273/587>.
 - 3 V. Saidl, P. Němec, P. Wadley, V. Hills, R. P. Champion, V. Novák, K. W. Edmonds, F. Maccherozzi, S. S. Dhesi, B. L. Gallagher, et al., *Nature Photonics* **11**, 91 (2017), URL <https://doi.org/10.1038/nphoton.2016.255>.
 - 4 M. Meinert, D. Graulich, and T. Matalla-Wagner, *Phys. Rev. Applied* **9**, 064040 (2018), URL <https://link.aps.org/doi/10.1103/PhysRevApplied.9.064040>.
 - 5 J. Noguchi and I. K. Schuller, *Journal of Magnetism and Magnetic Materials* **192**, 203 (1999), ISSN 0304-8853, URL <http://www.sciencedirect.com/science/article/pii/S0304885398002662>.
 - 6 A. F. Andresen, A. Kjekshus, R. MÅžllerud, and W. B. Pearson, *The Philosophical Magazine: A Journal of Theoretical Experimental and Applied Physics* **11**, 1245 (1965), URL <https://doi.org/10.1080/14786436508224933>, URL <https://doi.org/10.1080/14786436508224933>.
 - 7 E. Krén, G. Kádár, L. Pál, J. Sólyom, P. Szabó, and T. Tarnóczy, *Phys. Rev.* **171**, 574 (1968), URL <https://link.aps.org/doi/10.1103/PhysRev.171.574>.
 - 8 C. S. Severin, C. W. Chen, and C. Stassis, *Journal of Applied Physics* **50**, 4259 (1979), URL <https://doi.org/10.1063/1.326458>, URL <https://doi.org/10.1063/1.326458>.
 - 9 H. Hama, R. Motomura, T. Shinozaki, and Y. Tsunoda, *Journal of Physics: Condensed Matter* **19**, 176228 (2007), URL <https://doi.org/10.1088/0953-8984/19/2F17/2F176228>.
 - 10 R. Y. Umetsu, K. Fukamichi, and A. Sakuma, *MATERIALS TRANSACTIONS* **47**, 2 (2006).
 - 11 G. W. Anderson, Y. Huai, and M. Pakala, *Journal of Applied Physics* **87**, 5726 (2000), URL <https://doi.org/10.1063/1.372502>, URL <https://doi.org/10.1063/1.372502>.
 - 12 M. F. Toney, M. G. Samant, T. Lin, and D. Mauri, *Applied Physics Letters* **81**, 4565 (2002), URL <https://doi.org/10.1063/1.1528279>, URL <https://doi.org/10.1063/1.1528279>.
 - 13 M. Rickart, P. P. Freitas, I. G. Trindade, N. P. Barradas, E. Alves, M. Salgueiro, N. Muga, J. Ventura, J. B. Sousa, G. Proudfoot, et al., *Journal of Applied Physics* **95**, 6317 (2004), URL <https://doi.org/10.1063/1.1713022>, URL <https://doi.org/10.1063/1.1713022>.
 - 14 H. Yan, Z. Feng, S. Shang, X. Wang, Z. Hu, J. Wang, Z. Zhu, H. Wang, Z. Chen, H. Hua, et al., *Nature Nanotechnology* **14**, 131 (2019).
 - 15 P. Khuntia and A. V. Mahajan, **22**, 296002 (2010), URL <https://doi.org/10.1088/0953-8984/22/29/296002>.
 - 16 G. Kresse and J. Furthmüller, *Phys. Rev. B* **54**, 11169 (1996).
 - 17 G. Kresse and D. Joubert, *Phys. Rev. B* **59**, 1758 (1999).
 - 18 M. Gajdoš, K. Hummer, G. Kresse, J. Furthmüller, and F. Bechstedt, *Phys. Rev. B* **73**, 045112 (2006).
 - 19 S. Steiner, S. Khmelevskiy, M. Marsmann, and G. Kresse, *Phys. Rev. B* **93**, 224425 (2016).
 - 20 J. P. Perdew, K. Burke, and M. Ernzerhof, *Phys. Rev. Lett.* **77**, 3865 (1996).
 - 21 H. J. Monkhorst and J. D. Pack, *Phys. Rev. B* **13**, 5188 (1976).
 - 22 A. Togo and I. Tanaka, *Scripta Materialia* **108**, 1 (2015), ISSN 1359-6462, URL <http://www.sciencedirect.com/science/article/pii/S1359646215003127>.
 - 23 H. Ebert, D. Ködderitzsch, and J. Minár, *Reports on Progress in Physics* **74**, 096501 (2011), URL <https://doi.org/10.1088/2F0034-4885/74/2F96501>.
 - 24 A. I. Liechtenstein, M. I. Katsnelson, and V. A. Gubanov, *Journal of Physics F: Metal Physics* **14**, L125 (1984), URL <https://doi.org/10.1088/2F0305-4608/14/2F7/2F007>.
 - 25 S. Toth and B. Lake, *Journal of Physics: Condensed Matter* **27**, 166002 (2015), URL <https://doi.org/10.1088/0953-8984/27/2F16/2F166002>.

- 26 T. Moriya, Phys. Rev. **120**, 91 (1960), URL <https://link.aps.org/doi/10.1103/PhysRev.120.91>.
- 27 O. Eriksson, A. Bergman, L. Bergqvist, and J. Hellsvik, *Atomistic Spin Dynamics: Foundations and Applications* (Oxford University Press, 2017), ISBN 9780198788669, URL <https://books.google.com/books?id=d0EWDgAAQBAJ>.
- 28 J. Wang, A. Gao, W. Chen, X. Zhang, B. Zhou, and Z. Jiang, Journal of Magnetism and Magnetic Materials **333**, 93 (2013), ISSN 0304-8853, URL <https://www.sciencedirect.com/science/article/pii/S0304885312010360>.
- 29 A. Alsaad, A. Ahmad, and T. S. Obeidat, Helveticum **6**, e03545 (2020), ISSN 2405-8440, URL <https://www.sciencedirect.com/science/article/pii/S240584402030390X>.
- 30 Z. Lu, R. V. Chepulskii, and W. H. Butler, Phys. Rev. B **81**, 094437 (2010), URL <https://link.aps.org/doi/10.1103/PhysRevB.81.094437>.
- 31 W. B. Pearson, *Pearson's handbook of crystallographic data for intermetallic phases* (American Society for Metals, Metals Park, Oh, 1985), ISBN 0871702177.
- 32 A. Schrön, C. Rödl, and F. Bechstedt, Phys. Rev. B **86**, 115134 (2012), URL <https://link.aps.org/doi/10.1103/PhysRevB.86.115134>.
- 33 A. W. N. Ashcroft, N. Mermin, N. Mermin, and B. P. Compary, *Solid State Physics*, HRW international editions (Holt Rinehart and Winston, 1976), ISBN 9780030839931, URL <https://books.google.com/books?id=1C9HAQAIAAJ>.
- 34 L. Landau and E. Lifshitz, in *Electrodynamics of Continuous Media (Second Edition)* (Pergamon, Amsterdam, 1984), vol. 8 of *Course of Theoretical Physics*, pp. 130–179, second edition ed., ISBN 9780080-08-030275-1, URL <https://www.sciencedirect.com/science/article/pii/B9780080302751500114>.
- 35 M. Herak, M. Miljak, G. Dhalenne, and A. Revcolevschi, Journal of Physics: Condensed Matter **22**, 026006 (2009), URL <https://doi.org/10.1088/0953-8984/22/2/026006>.
- 36 See Supplemental Material at [URL will be inserted by publisher] for the investigation on the impact of different effective U parameters in DFT+U method to lattice parameters, phonon dispersion, and exchanging coefficients, the broadening effect of the density of states at the Fermi level, the decomposition of magneto-optical Kerr effect spectra based on *d*-orbital contributions, and the magnetocrystalline anisotropy with different effective U parameters in DFT+U method.
- 37 S. L. Dudarev, G. A. Botton, S. Y. Savrasov, C. Humphreys, and A. P. Sutton, Phys. Rev. B **57**, 1505 (1998), URL <https://link.aps.org/doi/10.1103/PhysRevB.57.1505>.
- 38 H. Yang, J.-y. Yang, C. N. Savory, J. M. Skelton, B. Morgan, D. O. Scanlon, and A. Walsh, The Journal of Physical Chemistry Letters **10**, 5552 (2019), pMID: 31475830, URL <https://doi.org/10.1021/acs.jpclett.9b02073>, URL <https://doi.org/10.1021/acs.jpclett.9b02073>.
- 39 S. C. Pandey, X. Xu, I. Williamson, E. B. Nelson, and L. Li, Chemical Physics Letters **669**, 1 (2017), ISSN 0009-2614, URL <https://www.sciencedirect.com/science/article/pii/S000926141630954X>.
- 40 K. Yang, K. Kang, Z. Diao, M. H. Karigerasi, D. Shoemaker, A. Schleife, and D. G. Cahill, Phys. Rev. B **102**, 064415 (2020), URL <https://link.aps.org/doi/10.1103/PhysRevB.102.064415>.
- 41 P.-H. Chang, I. A. Zhuravlev, and K. D. Belashchenko, Phys. Rev. Materials **2**, 044407 (2018), URL <https://link.aps.org/doi/10.1103/PhysRevMaterials.2.044407>.
- 42 K. Kang, K. Yang, K. Puthalath, D. G. Cahill, and A. Schleife, Phys. Rev. B **105**, 184404 (2022), URL <https://link.aps.org/doi/10.1103/PhysRevB.105.184404>.
- 43 C. W. Chen and R. W. Buttry, AIP Conference Proceedings **24**, 437 (1975), URL <https://aip.scitation.org/doi/pdf/10.1063/1.29941>, URL <https://aip.scitation.org/doi/abs/10.1063/1.29941>.
- 44 S. M. Rezende, A. Azevedo, and R. L. Rodríguez-Suárez, Journal of Applied Physics **126**, 151101 (2019), URL <https://doi.org/10.1063/1.5109132>, URL <https://doi.org/10.1063/1.5109132>.
- 45 P. Pincus, Phys. Rev. **113**, 769 (1959), URL <https://link.aps.org/doi/10.1103/PhysRev.113.769>.
- 46 B. Heinrich, S. T. Purcell, J. R. Dutcher, K. B. Urquhart, J. F. Cochran, and A. S. Arrott, Phys. Rev. B **38**, 12879 (1988), URL <https://link.aps.org/doi/10.1103/PhysRevB.38.12879>.
- 47 Y. He and P. Wigen, Journal of Magnetism and Magnetic Materials **53**, 115 (1985), ISSN 0304-8853, URL <https://www.sciencedirect.com/science/article/pii/0304885385901398>.
- 48 Y. Q. Cheng, L. L. Daemen, A. I. Kolesnikov, and A. J. Ramirez-Cuesta, Journal of Chemical Theory and Computation **15**, 1974 (2019), pMID: 30735379, URL <https://doi.org/10.1021/acs.jctc.8b01250>, URL <https://doi.org/10.1021/acs.jctc.8b01250>.
- 49 M. H. Karigerasi, K. Kang, G. E. Granroth, A. Banerjee, A. Schleife, and D. P. Shoemaker, Phys. Rev. Materials **4**, 114416 (2020), URL <https://link.aps.org/doi/10.1103/PhysRevMaterials.4.114416>.
- 50 C. Kittel, *Introduction to Solid State Physics* (Wiley, 2004), ISBN 9780471415268, URL <https://books.google.com/books?id=kym4QgAACAAJ>.
- 51 K. Binder, Zeitschrift für Physik B Condensed Matter **43**, 119 (1981), URL <https://doi.org/10.1007/BF01293604>.
- 52 F. Essenberg, S. Sharma, J. K. Dewhurst, C. Bersier, F. Cricchio, L. Nordström, and E. K. U. Gross, Phys. Rev. B **84**, 174425 (2011), URL <https://link.aps.org/doi/10.1103/PhysRevB.84.174425>.
- 53 M. Pajda, J. Kudrnovský, I. Turek, V. Drchal, and P. Bruno, Phys. Rev. B **64**, 174402 (2001), URL <https://link.aps.org/doi/10.1103/PhysRevB.64.174402>.
- 54 J. Harl, G. Kresse, L. D. Sun, M. Hohage, and P. Zeppenfeld, Phys. Rev. B **76**, 035436 (2007), URL <https://link.aps.org/doi/10.1103/PhysRevB.76.035436>.
- 55 M. Bernardi, D. Vigil-Fowler, J. Lischner, J. B. Neaton, and S. G. Louie, Phys. Rev. Lett. **112**, 257402 (2014), URL <https://link.aps.org/doi/10.1103/PhysRevLett.112.257402>.
- 56 M. Bernardi, J. Mustafa, J. B. Neaton, and S. G. Louie, Nature Communications **6**, 7044 (2015), ISSN 2041-1723, URL <https://doi.org/10.1038/ncomms8044>.
- 57 M. Kubota, K. Ono, R. Y. Umetsu, H. Akinaga, A. Sakuma, and K. Fukamichi, Applied Physics Letters **90**, 091911 (2007), URL <https://doi.org/10.1063/1.2561008>, URL <https://doi.org/10.1063/1.2561008>.
- 58 M. Fox, *Optical Properties of Solids*, Oxford Master Series in Physics (OUP Oxford, 2010), ISBN 9780199573363, URL <https://books.google.com/books?id=5WkVDAQAQBAJ>.
- 59 L. Šmejkal, R. González-Hernández, T. Jungwirth, and J. Sinova, Science Advances **6** (2020), URL <https://>

- 1093 advances.sciencemag.org/content/6/23/eaaz8809. 1103
1094 ⁶⁰ A. V. Kimel, B. A. Ivanov, R. V. Pisarev, P. A. Usachev, A. Kirilyuk, and T. Rasing, Nature Physics **5**, 727 (2009), ISSN 1745-2481, URL <https://doi.org/10.1038/nphys1369>. 1104
1095
1096
1097
1098 ⁶¹ N. Sivadas, S. Okamoto, and D. Xiao, Phys. Rev. Lett. **117**, 267203 (2016), URL <https://link.aps.org/doi/10.1103/PhysRevLett.117.267203>. 1109
1099
1100
1101 ⁶² D. Sangalli, A. Marini, and A. Debernardi, Phys. Rev. B **86**, 125139 (2012), URL <https://link.aps.org/doi/10.1103/PhysRevB.86.125139>. 1110
1102
1103
1104
1105
1106
1107
1108
1109
1110
1111
1112 ⁶³ S. A. Siddiqui, J. Sklenar, K. Kang, M. J. Gilbert, A. Schleife, N. Mason, and A. Hoffmann, Journal of Applied Physics **128**, 040904 (2020), <https://doi.org/10.1063/5.0009445>, URL <https://doi.org/10.1063/5.0009445>.
⁶⁴ J. Leveillee, C. Katan, L. Zhou, A. D. Mohite, J. Even, S. Tretiak, A. Schleife, and A. J. Neukirch, Phys. Rev. Materials **2**, 105406 (2018), URL <https://link.aps.org/doi/10.1103/PhysRevMaterials.2.105406>.