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Multi-dome superconductivity in charge density wave kagome metals

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Motivated by the recent experiments on the kagome metals AV_3Sb_5 with $A = K, Rb$, and Cs , which see the charge density wave (CDW) at ~ 100 K and the superconductivity at ~ 1 K, we explore the onset of the superconductivity, taking the perspective that it descends from a parent CDW. We argue that viewing the superconductivity as a weak-coupling instability of a reconstructed (by the CDW) Fermi surface naturally explains the experimentally observed ‘multi-dome’ non-monotonic dependence on pressure, with the ‘peaks’ in the superconducting critical temperature being associated to the Van Hove singularities of the reconstructed Fermi surface. This ‘parent-child relationship’ also naturally explains the large separation of energy scales between the superconductivity and the CDW. We discuss different possible pairing mechanisms, and speculate that the CDW/reconstructed Pomeranchuk fluctuations may mediate the pairing interaction.

Introduction.—The correlated phenomena in the recently uncovered kagome metals AV_3Sb_5 with $A = K, Rb$, and Cs have drawn enormous attention. The charge density waves (CDW) were observed at high critical temperature $T_{CDW} \sim 100$ K [1–29]. With the simultaneous ordering at three commensurate momenta, these $3Q$ orders are believed to be driven by the Fermi surface nesting, further enhanced by the Van Hove singularity (VHS) [30]. The bond density modulations form the (inverse) star-of-David patterns [31–39] with possible higher-order topology [40]. Meanwhile, the giant anomalous Hall effects indicate the band topology with time-reversal symmetry breaking [2, 7], which may be attributed to the loop currents [5, 17, 20, 26, 32–35, 37, 41–46]. On the other hand, the C_3 symmetry breaking is generally observed and is possibly induced by the three-dimensional orders [35, 38, 39].

More exotic observations appeared in the superconductivity (SC) [3, 6, 8, 10, 12, 22–24, 27, 29, 47–55]. The superconductivity develops at much lower critical temperature $T_{SC} \sim 1$ K with the C_3 symmetry breaking persisting [52, 53]. Remarkably, the superconductivity exhibits a double-dome structure under pressure, which extends above the critical pressure of the CDW [6, 8, 24, 27]. New superconducting domes can even appear at much higher pressure [8, 51, 55]. Despite the rich experimental observations, a thorough theoretical understanding remains elusive. Given the proximity to the VHS, the superconductivity may be treated as a competing instability with the CDW [56]. This scenario may explain the opposite trends of T_{CDW} and T_{SC} in certain experimental regimes, such as under pressure [6, 8, 24, 27, 51, 55] and uniaxial strain [29]. However, the scenario does not fit the large separation of energy scales between the superconductivity and the CDW, neither does it explain the double-dome superconductivity under the monotonically suppressed CDW. An alternative scenario is thus suggested for the origin of the superconductivity.

In this Letter, we propose an alternative scenario that appears consistent with the salient experimental facts. We adopt the perspective that the CDW is a ‘parent’

phase, and that the ‘child’ superconductivity emerges from the reconstructed Fermi surface thereof. This ‘parent-child relationship’ not only naturally explains the large separation of energy scales between the superconductivity and the CDW, but it also predicts a multi-dome structure for the superconductivity (Fig. 1) [6, 24, 27], with the critical temperature enhanced near the (reconstructed) VHSs. We further discuss possible pairing mechanisms, and speculate that the CDW/reconstructed Pomeranchuk (RPOM) fluctuations may mediate the pairing interaction.

CDW kagome metal.—Our analysis focuses on the VH CDW on the middle band of the kagome lattice. At the VHS, the Fermi surface is a hexagon in the hexagonal Brillouin zone (BZ). The corner saddle points sit at the zone edge centers $\mathbf{M}_{\alpha=1,2,3}$ [Figs. 2(a) and 2(b)], where the particle-type dispersion energy reads $\varepsilon_{\alpha\mathbf{k}} =$

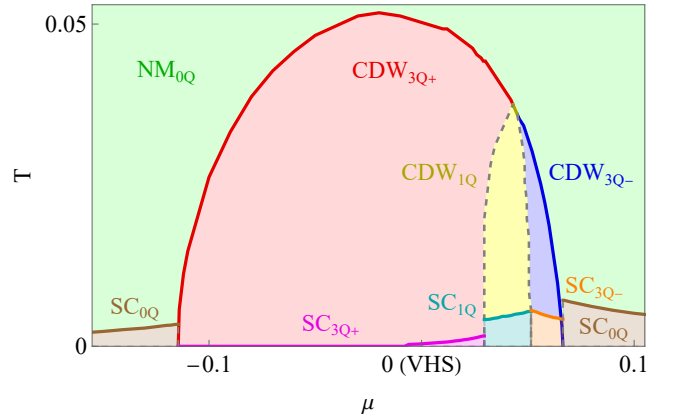


FIG. 1. Phase diagram of the parent CDW and the multi-dome child superconductivity. The CDW is computed from the free energy (1) with $\Lambda_k = 1$. The dispersion coefficients $A_+ = 3.15$ and $A_- = 1.45$ are set according to the tight-binding model for Figs. 2 and 3. The interaction $V_{CDW} = 25$ is chosen to approach the large gap in the experiment [19]. The critical temperature of the superconductivity T_{SC} [Eqs. (8) and (15)] is estimated with $\Lambda_{SC} = 0.01$ and $C_{SC} = 2$.

$A_+k_{\alpha+}^2 - A_-k_{\alpha-}^2$ with $A_{\pm} > 0$ for small relative momenta ($k_{\alpha+}, k_{\alpha-}$) from $\mathbf{M}_{\alpha}$. The density of states experiences the logarithmic VHS at these points [30], which enhances the Fermi liquid instabilities in various channels [35, 46, 57–64]. In particular, the Fermi surface nesting can enhance the CDW at three commensurate nesting momenta $\mathbf{Q}_{\alpha} \equiv \mathbf{M}_{\alpha}$. Due to the doubled periodicity $2\mathbf{Q}_{\alpha} \equiv 0$, the bands are folded quadruply onto the $1/2 \times 1/2$ reduced BZ. The CDW then opens the gaps and shrinks the reconstructed Fermi surface.

We choose the real CDW $\vec{\Delta} = (\Delta_1, \Delta_2, \Delta_3)$ specifically to exemplify our analysis. The energetically favored ground states can be determined by minimizing the mean-field free energy [65]

$$f[\vec{\Delta}] = \frac{2}{V_{\text{CDW}}} - \sum_n \int_{\mathbf{k}} \ln[1 + e^{-(E_{n\mathbf{k}}[\vec{\Delta}] - \mu)/T}]. \quad (1)$$

Each energy $E_{n\mathbf{k}}[\vec{\Delta}]$ is an eigenvalue of the effective Hamiltonian

$$H_{\text{CDW},\mathbf{k}}[\vec{\Delta}] = \begin{pmatrix} \varepsilon_{1\mathbf{k}} & -\Delta_3 & -\Delta_2 \\ -\Delta_3 & \varepsilon_{2\mathbf{k}} & -\Delta_1 \\ -\Delta_2 & -\Delta_1 & \varepsilon_{3\mathbf{k}} \end{pmatrix}, \quad (2)$$

which characterizes a low-energy patch model with the radial cutoff Λ_k around each saddle point [Fig. 2(c)]. V_{CDW} defines the projected interaction in the CDW channel. The phase diagram (Fig. 1) [35] is consistent with the intuition from the Ginzburg-Landau theory [33–35].

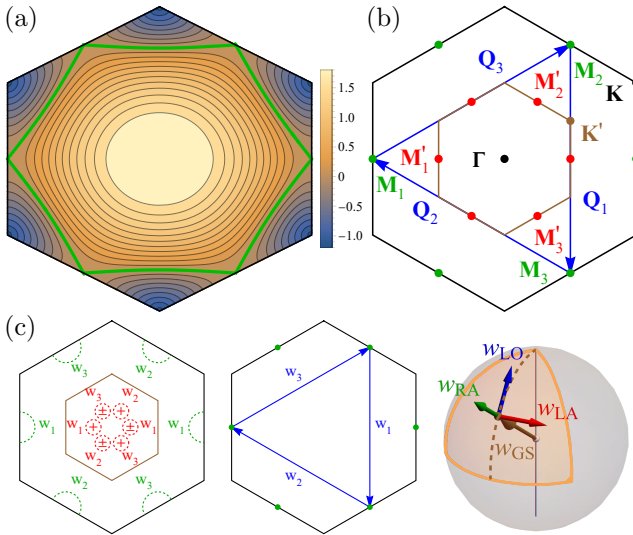


FIG. 2. Model setup. (a) The middle band of the kagome lattice with the (green) VH Fermi surface in the (black) BZ. In addition to the nearest-neighbor hopping $t_1 = 1$, we introduce an intra-sublattice hopping $t_2 = 0.05$ between next-nearest-neighbor unit cells to induce the nonperfect nesting. (b) High symmetry points and nesting momenta in the (black) original and (Brown) reduced BZs. (c) In (left) the patch models and at (middle) the nesting momenta, the symmetry channels can be mapped onto (right) a Bloch sphere.

Near the VHS, the CDW expands a strong $3Q+$ phase with high critical temperature T_{CDW} . These $3Q+$ orders develop the simultaneous ordering $|\Delta_1| = |\Delta_2| = |\Delta_3|$ under $\Delta_1\Delta_2\Delta_3 > 0$, which maximizes the gap structures by opening the gaps at the Fermi level [34, 45]. In the practical systems, the Fermi level usually lies away from the VHS. The nonzero doping $\mu \neq 0$ can draw the reconstructed Fermi surface and alter the energetic favor. While the $3Q+$ orders remain stable at negative doping $\mu < 0$, the $3Q-$ orders with $\Delta_1\Delta_2\Delta_3 < 0$ can emerge at positive doping $\mu > 0$. Near the phase transition, the CDW is pinned to the $1Q$ orders at single momenta. The nonzero doping suppresses the CDW phase and reduces the critical temperature T_{CDW} gradually. This is attributed to the reduction of density of states and Fermi surface nesting. When the CDW vanishes at the critical dopings, the ground state returns to the $0Q$ normal metal (NM). Note that different phases are separated by the first-order transitions, where the ground state switches abruptly between different orders. While the phase diagram has been presented previously [35], we further show that it contains several properties essential to the multi-dome superconductivity.

Remarkably, the CDW phase can be suppressed by different variations in the kagome metals AV_3Sb_5 . In addition to the nonzero doping [23], the suppression was also observed under pressure [6, 8, 24, 27] and uniaxial strain [29]. These effects may be related to the Fermi level shifting [6] or the interaction reduction under structural variation [18]. In the low-energy theory, the variations can be summarized by the tunings of doping, Fermi surface nesting, and interaction strength. We model these effects by a single parameter, which defines the shifting away from the perfectly-nested VHS at a fixed dimensionless interaction. The shifting further determines the reduction of the critical temperature T_{CDW} . Our analysis adopts the doping μ as the tunable parameter. This setup supports a tunable framework where the properties of the suppressed CDW phase can be examined.

We are interested in the reconstructed Fermi surface under nonzero doping. To obtain the reconstructed band structure, we adopt the ground state of the effective Hamiltonian (2) to a full four-band mean-field Hamiltonian in the reduced BZ [34]. The choice of an $(s+d)$ -wave form factor diminishes the CDW away from the Fermi level. Despite the mismatch in the model details, the computation captures the essential features of the reconstructed band structure. We monitor the evolution of the density of states $D(\mu)$ [Fig. 3(a)] and map out the reconstructed Fermi surface [Fig. 3(b)] at zero temperature $T = 0$. In the $3Q+$ phase, the small $\mathbf{M}'_{\alpha}$ pockets appear under the gaps at the Fermi level. The increasing doping enlarges the $\mathbf{M}'_{\alpha}$ pockets and raises the density of states. Importantly, the enlarged $\mathbf{M}'_{\alpha}$ pockets lose the energetic favor to the $\mathbf{K}'_{\alpha}$ pockets at sufficiently large positive doping $\mu > 0$. This corresponds to a transition to the $3Q-$ phase. A Lifshitz transition is naturally expected between the two phases, which hosts a reconstructed VHS.

Although this transition is covered by an intermediate $1Q$ phase in our model, the proximity to the reconstructed VHS still enhances the density of states significantly. On the other hand, the reconstructed Fermi surface recovers its noninteracting form at the critical dopings. With the closeness to the VHS, the density of states is again elevated. Note that the reconstructed Fermi surface changes abruptly at the first-order transitions. Correspondingly, the density of states experiences the sharp jumps at the regime boundaries.

Due to the presence of multiple transitions, the reconstructed metallic phase is divided into multiple regimes. Whether each regime appears is system-dependent. Interestingly, the transitions between different CDW phases were observed under pressure [6, 8, 24, 27]. Furthermore, the reconstructed VHS was probed close to the

Fermi level for $A = \text{Rb}$ [21]. The multi-regime nature of the reconstructed metal is essential to the low-energy phenomena.

Multi-dome superconductivity.—As a central spirit of our work, we propose that the superconductivity develops as a weak-coupling instability on the reconstructed Fermi surface. This establishes a ‘parent-child relationship’ between the CDW and the superconductivity, where the large separation of energy scales is naturally explained. Moreover, the multi-regime reconstructed metal naturally hosts the intriguing multi-dome superconductivity. Deep in the CDW phase, the small density of states under the $3Q+$ orders supports the superconductivity at low critical temperature T_{SC} . As the doping increases, the increasing density of states enhances the superconductivity and raises its critical temperature T_{SC} . This is opposite to the suppression of the parent CDW phase, consistent with the experimental observations under pressure [6, 8, 24, 27], doping [23], and uniaxial-strain [29]. Importantly, the reconstructed VHS can support significant enhancement and create a peak in the critical temperature T_{SC} . Although it is inaccessible in our model, the enhancement still manifests adequately across the $1Q$ regime [46, 57]. The elevated critical temperature T_{SC} depicts a superconducting dome in the parent CDW phase. When the CDW is driven across a critical doping, the superconductivity experiences another enhancement from the closeness to the VHS. This marks another superconducting dome which extends beyond the parent CDW phase. Thus, our model predicts a ‘multi-dome’ child superconductivity from the parent CDW (Fig. 1). The double-dome structure under positive doping $\mu > 0$ is close to the observations under pressure [6, 24, 27]. Interestingly, the observed peaks are reminiscent of those from the (reconstructed) VHS. On the other hand, the double-dome structure under negative doping $\mu < 0$ was also observed [23], where the domes are separated by a sharp transition at the critical doping.

Patch model and symmetry analysis.—To further understand the superconductivity, we adopt a low-energy patch model in the reconstructed metal. The patch model takes a six-patch form, where the small patches $\mathbf{p}_\alpha$ capture the relevant portions of the reconstructed Fermi surface in the $\Gamma\text{-}\mathbf{K}'\text{-}\mathbf{K}'$ $1/6$ subzones [Fig. 2(c)]. The low-energy theory manifests the six-component fermion $(\psi_+^T, \psi_-^T)^T$ with $\psi_\pm = (\psi_{\pm 1}, \psi_{\pm 2}, \psi_{\pm 3})^T$. Correspondingly, the density of states reads $(D^T, D^T)^T$ with $D = (D_1, D_2, D_3)^T$. The patch model offers a natural basis for the symmetry analysis. There exist six orthonormal representations $(w_+^T, w_-^T)^T$, which correspond to the form factors $w_{\mathbf{k}}$ in six symmetry channels. Three of the channels are even/odd under the inversion $w_+ = \pm w_- = w = w_{w_1 w_2 w_3}$, where $w_{w_1 w_2 w_3}$ denotes the normalized form of (w_1, w_2, w_3) . The symmetry channels are clearly illustrated by a mapping onto the Bloch sphere [Fig. 2(c)]. At the ground state (GS) w_{GS} , the characteristic orthonormal basis defines the channels as radial (RA) w_{RA} , longitudinal (LO) w_{LO} , and latitudinal (LA) w_{LA} . Under

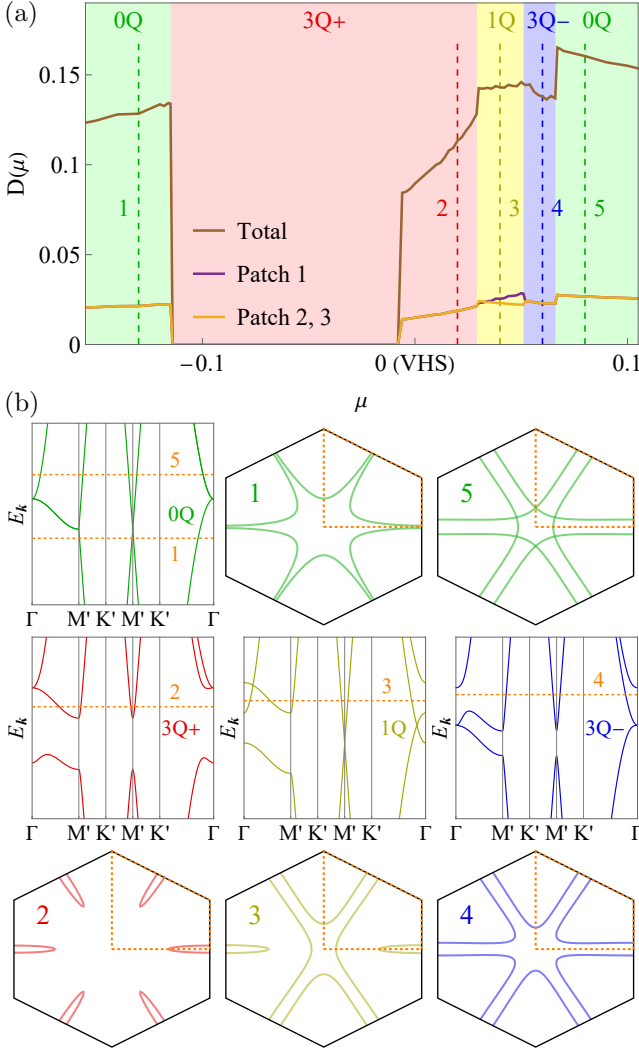


FIG. 3. Reconstructed Fermi surface under the parent CDW at $T = 0$. We compute (a) the density of states and obtain (b) the reconstructed band structure and Fermi surface. The density of states at each patch D_α is estimated by the total in each $\Gamma\text{-}\mathbf{K}'\text{-}\mathbf{K}'$ $1/6$ subzone.

the C_3 symmetry, the ground state $w_{\text{GS}} = w_{111}$ hosts $w_{\text{RA}} = w_{111}$ along with the degenerate $w_{\text{LO}} = w_{2-1-1}$ and $w_{\text{LA}} = w_{01-1}$.

Note that the ground state may break the C_3 symmetry, such as in the $1Q$ phase. Assume the C_3 symmetry breaking at the momentum $\mathbf{Q}_1$, which leads to a two-fold symmetry along the x direction. The reconstructed metal holds the (possibly weak) x and y reflection symmetries. In the patch model, the two-fold symmetry manifests in the anisotropic patch momenta $\mathbf{p}_1 \neq \mathbf{p}_2 = \mathbf{p}_3$ and density of states $D_1 \neq D_2 = D_3$. The ground state w_{GS} shifts away from w_{111} along the longitude on the Bloch sphere [Fig. 2(c)]. While the radial and longitudinal channels deform together in the reflection-even branch, the reflection-odd latitudinal channel remains invariant.

Pairing formalism.—We now study the superconductivity in the low-energy theory. Consider the projection of general interaction on the Cooper channels [66]

$$H_{\text{CP}} = (P_{\text{CP}}^\nu)^\dagger V_{s/t} P_{\text{CP}}^\nu. \quad (3)$$

The spin-singlet and triplet Cooper pairings (CP) are $(P_{\text{CP}}^\nu)^\dagger_\alpha = \psi_{+\alpha}^\dagger(\sigma^\nu/\sqrt{2})[(i\sigma^2)(\psi_{-\alpha}^\dagger)^T]$ with the spin Pauli matrices $\sigma^0 = 1$ and $\sigma^{1,2,3}$, respectively. Under the Fermi statistics, these pairings are coupled by the symmetric/antisymmetric interactions $V_{s/t} = (V_{s/t,\alpha\beta})$ with $V_{s/t,\alpha\beta} = (V_{\mathbf{p}\alpha-\mathbf{p}\beta} \pm V_{\mathbf{p}\alpha+\mathbf{p}\beta})/2$. The interaction takes a general matrix form under the two-fold symmetry

$$V = \begin{pmatrix} V_1 & V_3 & V_3 \\ V_3^* & V_2 & V_4 \\ V_3^* & V_4 & V_2 \end{pmatrix}. \quad (4)$$

Here $V_{1,2,4} \in \mathbb{R}$ and $V_3 \in \mathbb{C}$ are assumed, which matches all candidates in our analysis. Note that $V_1 = V_2$ and $V_3 = V_4$ under the C_3 symmetry.

A diagonalization determines the symmetry channels of the pairing states. In each symmetry channel

$$H_{\text{SC}} = V_{\text{SC}}(P_{\text{SC}}^\nu)^\dagger P_{\text{SC}}^\nu, \quad (5)$$

the pairing $(P_{\text{SC}}^\nu)^\dagger = \psi_+^\dagger(\sigma^\nu/\sqrt{2})\hat{w}_{\text{SC}}[(i\sigma^2)(\psi_-^\dagger)^T]$ adopts an eigenstate $w_{\text{SC}} = \tilde{w}_{\text{SC}}/|\tilde{w}_{\text{SC}}|$ as the representation $\hat{w}_{\text{SC}} = \text{diag}(w_{\text{SC}})$. The projected interaction V_{SC} is given by the corresponding eigenvalue. We identify the pairing states in six symmetry channels

Radial/longitudinal:

$$V_{\text{SC}} = \frac{1}{2}(V_1 + V_\pm), \quad \tilde{w}_{\text{SC}} = \begin{pmatrix} V_1 - V_\mp \\ 2V_3^* \\ 2V_3^* \end{pmatrix}, \quad (6)$$

Latitudinal:

$$V_{\text{SC}} = V_2 - V_4, \quad \tilde{w}_{\text{SC}} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad (7)$$

with $V_\pm = V_2 + V_4 \pm \{[V_1 - (V_2 + V_4)]^2 + 8|V_3|^2\}^{1/2}$. Under the competition, the superconductivity develops from the

leading pairing state with the strongest dimensionless interaction $\lambda_{\text{SC}} = V_{\text{SC}}(w_{\text{SC}}^\dagger D w_{\text{SC}}) < 0$ [67]. The critical temperature is estimated as

$$T_{\text{SC}} = \Lambda_{\text{SC}} e^{-1/|\lambda_{\text{SC}}|}, \quad (8)$$

where Λ_{SC} is an energy cutoff in the low-energy theory.

A natural question arises as which pairing states are leading in the multi-dome superconductivity. Since the competition is interaction-dependent, the answer is strongly contingent on the pairing mechanism. Here we discuss some candidates and their resulting pairing states in the multi-dome superconductivity.

Phonon-mediated attraction.—The phonon-mediated attraction always serves as a feasible candidate. Assume the weak short-range attraction with $V_{1,2,3,4} \approx V_0 < 0$. Under the C_3 symmetry, the isotropic channel w_{111} is leading. This leading channel becomes anisotropic when the two-fold symmetry is manifest. Since the attraction is weakly momentum-dependent, the anisotropy is weak. This contradicts with the strong anisotropy in the experimental observations [52, 53]. Therefore, the phonon-mediated attraction is an unlikely pairing mechanism.

CDW/RPOM fluctuation.—The onset from the parent CDW suggests an intriguing pairing mechanism for the superconductivity. When the parent CDW manifests certain fluctuation, the Cooper pairs can form through its mediation at low energy. This scenario naturally matches the large separation of energy scales between the superconductivity and the CDW. The parent CDW manifests itself in the reconstruction of the band structure. At low energy, this reconstruction turns into the RPOM order on the Fermi surface. Therefore, the scenario is reminiscent of the pairing from the POM fluctuations [68–70].

We first discuss the fluctuations of the parent CDW. Diagonalizing the free-energy Hessian matrix $(\partial_\alpha \partial_\beta f[\vec{\Delta}])_{\vec{\Delta}=\vec{\Delta}_{\text{GS}}}$ at zero temperature $T = 0$, we identify the symmetry channels of the CDW fluctuations (Fig. 4)

$$\delta f[\vec{\Delta}_{\text{GS}}] = \frac{1}{2} m^2 |\delta \vec{\Delta}|^2 \quad (9)$$

with the representation $\delta \vec{\Delta} = \delta \Delta w$ and the effective mass m^2 . Interestingly, these channels match the characteristic orthonormal basis $\{w_{\text{RA}}, w_{\text{LO}}, w_{\text{LA}}\}$ at the ground state $\vec{\Delta}_{\text{GS}} = \Delta w_{\text{GS}}$ on the Bloch sphere [Fig. 2(c)]. While the ground state exhibits the long-range CDW orders, the fluctuations exhibit the short-range correlations according to the nonzero masses.

In the parent CDW, the reconstructed metal acquires an inversion-even RPOM order $\vec{\Delta}_{\text{RPOM}} = (\Delta_{\text{RPOM}}/\sqrt{2})(w^T, w^T)^T$. The RPOM order and its fluctuations correspond approximately to the CDW ones. Due to the positive semidefiniteness, the mapping onto the Bloch sphere now occurs in the first octant [Fig. 2(c)]. The fluctuation in each symmetry channel is described by an effective charge density (CD) coupling [65, 71]

$$H_{\delta\Delta_{\text{CD}}} = \sum_{\mathbf{q}} \chi_{\mathbf{q}}^{-1} \delta \Delta_{\text{CD},-\mathbf{q}} \delta \Delta_{\text{CD},\mathbf{q}}. \quad (10)$$

The susceptibility $\chi_{\mathbf{q}} > 0$ is a result of tracing out the high-energy modes beyond the patch model. While the model involves the fluctuations at general momentum $\mathbf{q}$, we assume the susceptibility peaks nondivergently $\chi_{\mathbf{q}=0} \approx 2/m^2$ at zero momentum. The fluctuation couples to the CD pairing $P_{\text{CD},\mathbf{q}} = \sum_{\mathbf{k}} \psi_{\mathbf{k}+\mathbf{q}}^\dagger (\sigma^0/\sqrt{2}) w_{\mathbf{k},-\mathbf{q}} \psi_{\mathbf{k}}$ as

$$H_{\psi\delta\Delta_{\text{CD}}} = g \sum_{\mathbf{q}} P_{\text{CD},\mathbf{q}} \delta\Delta_{\text{CD},-\mathbf{q}}, \quad (11)$$

where $w_{\mathbf{k},\mathbf{q}} \sim \delta\langle\psi_{\mathbf{k}+\mathbf{q}}^\dagger \psi_{\mathbf{k}}\rangle$ is the form factor and g is an effective coupling. Integrating out the fluctuation, we arrive at an effective attraction

$$H_{\text{CD}} = -g^2 \sum_{\mathbf{q}} \chi_{\mathbf{q}} P_{\text{CD},\mathbf{q}} P_{\text{CD},-\mathbf{q}}. \quad (12)$$

It is worth discussing the structure of the form factor $w_{\mathbf{k},\mathbf{q}}$. The diagonal components $w_{\mathbf{p}_\alpha,\mathbf{q}=0} = \hat{w}_{\alpha\alpha} = w_\alpha$ correspond to the intrapatch RPOM representations. Meanwhile, the off-diagonal components $w_{\mathbf{p}_\alpha,\mathbf{q}=\mathbf{p}_\beta-\mathbf{p}_\alpha} = \hat{w}_{\beta\alpha} = \hat{w}_{\alpha\beta}^*$ with $\alpha \neq \beta$ are interpatch. Although the incommensurate momenta $\mathbf{p}_\beta - \mathbf{p}_\alpha$ suggest the generally complex nature of the off-diagonal components $\hat{w}_{\beta\alpha}$, the symmetries can fix some of the complex phases. While the inversion evenness implies $\hat{w}_{\beta\alpha} = \hat{w}_{-\beta,-\alpha}$, the reflection evenness/oddness forces $\hat{w}_{\alpha\beta} = \pm \hat{w}_{\tilde{\alpha}\tilde{\beta}}$ for the reflection pairs $(\alpha, \tilde{\alpha})$ and $(\beta, \tilde{\beta})$. In the C_3 symmetric channel $w = w_{111}$, the off-diagonal components are $(\hat{w}_{2\pm 3}, \hat{w}_{3\pm 1}, \hat{w}_{1\pm 2}) = w_{111}$. On the other hand, the anisotropic channels may allow complex components due to the loss of certain reflection symmetries. Enforcing the reality, we find the components $(\hat{w}_{2\pm 3}, \hat{w}_{3\pm 1}, \hat{w}_{1\pm 2}) = w_{abb}$ with $a, b \in \mathbb{R}$ and w_{01-1} in the reflection-even and odd channels, respectively.

We now project the effective attraction on the Cooper channels

$$H_{\text{CP}} = -g^2 \sum_{\alpha\beta} \chi_{\alpha\beta} \hat{w}_{\beta\alpha} \hat{w}_{-\beta,-\alpha} \psi_{\alpha}^\dagger \psi_{-\alpha}^\dagger \psi_{-\beta} \psi_{\beta}. \quad (13)$$

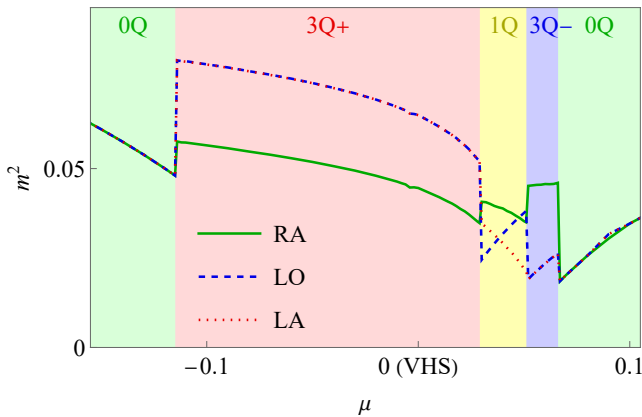


FIG. 4. The masses of the CDW fluctuations at $T = 0$.

Here $\chi_{\alpha\beta} = \chi_{\mathbf{q}=\mathbf{p}_\alpha-\mathbf{p}_\beta}$, and the four-fermion spin indices $(\sigma, \sigma', \sigma', \sigma)$ are suppressed. The symmetric/antisymmetric interactions read

$$V_{\alpha\beta} = -g^2 (\chi_{\alpha\beta} \hat{w}_{\beta\alpha}^2 \pm \chi_{\alpha-\beta} \hat{w}_{-\beta\alpha}^2). \quad (14)$$

Since the symmetric interactions are generally stronger, the spin-singlet pairings are energetically favorable. The diagonal RPOM components determine the symmetry channels of the pairing states. Meanwhile, the off-diagonal components perturb these channels and break the possible degeneracy. The leading pairing state is chosen by the strongest dimensionless interaction

$$|\lambda_{\text{SC}}| = C_{\text{SC}} m^{-2} V_{w,\text{SC}} (w_{\text{SC}}^\dagger D w_{\text{SC}}), \quad (15)$$

which does not necessarily originate from the leading fluctuation. Here C_{SC} is an effective parameter, and $V_{w,\text{SC}}$ is an eigenvalue of the dominant interaction representation $\text{diag}(w_1^2, w_2^2, w_3^2)$. Note that the CDW fluctuations can also trigger the superconductivity in the 0Q normal metal. With the unfolded patch model at $\mathbf{M}_\alpha$, the symmetry channels are dominated by the off-diagonal CDW components.

Remarkably, the pairing states with the strongly anisotropic two-fold structures $w_{\text{SC}} = w_{1\delta\delta,111}$ can be leading in the multi-dome superconductivity (Table I). Here $0 < \delta \ll 1$ (may be complex under the two-fold symmetry) represents the infinitesimal components from the off-diagonal perturbations. Since the CDW breaks the C_3 symmetry in the kagome metals AV_3Sb_5 , the two-fold structures are pinned along a single direction in the parent CDW phase. Interestingly, the two-fold structures were observed in the magnetoresistance measurement [52, 53]. Furthermore, the sign preservation matches the scanning tunneling microscopy and spectroscopy [54]. While a thermal conductivity measurement suggested a nodal gap [49], the measurements of magnetic penetration depth and specific heat drew an opposite conclusion [50]. This contradiction may originate from the infinitesimal component δ . Beyond the parent CDW phase, the degenerate pairing states $w_{011,101,110}$ may form the nodeless ground states with possible time-reversal symmetry breaking.

Finally, we note some features of the critical temperature T_{SC} from the CDW/RPOM fluctuations. The competition between different channels can broaden the transition temperature window between $T_{\text{SC}}^{\text{onset}}$ and $T_{\text{SC}}^{\text{zero}}$ [6, 24, 27], where the resistivity drops and vanishes, respectively. Meanwhile, the enhanced fluctuations at the first-order transitions can strengthen the peaks of the critical temperature T_{SC} at the regime boundaries.

Discussion.—We show that the parent CDW naturally hosts the multi-dome child superconductivity in the kagome metals AV_3Sb_5 . The ‘parent-child relationship’ realizes the large separation of energy scales between the superconductivity and the CDW. Meanwhile, the multi-dome superconductivity originates from the multi-regime reconstruct metal with distinct (reconstructed) VHSs.

GS	w_{GS}	Fluctuation $\rightarrow$ Pairing State w_{SC}
In Parent CDW Phase		
$3Q+$	w_{111}	$w_{\text{LO}} \rightarrow w_{1\delta\delta} > w_{\text{LA}} \rightarrow \{w_{\delta 11} \gtrsim w_{01-1}\}$
$1Q$	w_{100}	$w_{\text{RA}} \rightarrow w_{1\delta\delta} > w_{\text{LO/LA}} \rightarrow \{w_{\delta 11} \gtrsim w_{01-1}\}$
$3Q-$	$w_{\mp 111}$	$w_{\text{LO}} \rightarrow w_{1\delta\delta} > w_{\text{LA}} \rightarrow \{w_{\delta 11} \gtrsim w_{01-1}\}$
Beyond Parent CDW Phase		
$0Q$	w_{000}	$w_{\text{RA/LO/LA}} = w_{100,010,001} \rightarrow w_{011,101,110}$

TABLE I. Leading pairing states from the CDW/RPOM fluctuations in the multi-dome superconductivity. Here $w_1 > w_2$ means that w_1 and w_2 are the leading and secondary channels, respectively, and $\{w_1 \gtrsim w_2\}$ represents the degeneracy with perturbative breakdown. The $\mp$ sign in the $3Q-$ phase corresponds to the CDW/RPOM description.

The pairing states with strong two-fold anisotropy can develop from the CDW/RPOM fluctuations. Our work sheds light on an unconventional pairing mechanism with strong evidences in the kagome metals AV_3Sb_5 .

Our analysis is exemplified with the real CDW on a two-dimensional single-orbital kagome lattice. The inclusion of three-dimensional multi-band orders may yield a more accurate description of the kagome metals AV_3Sb_5 , such as the general involvement of the two-fold symmetry [39]. A combination with the imaginary CDW [34] can further incorporate the time-reversal symmetry breaking. Meanwhile, the possible band topology [2, 3, 7, 34, 40, 47] can make the superconductivity geometrically enhanced [72–76] or topological [77]. Spin-orbit coupling may also lead to additional features. Note that the multi-dome structure follows solely from the evolution of the density of states, which will be shared by any weak-coupling pairing mechanism on the reconstructed Fermi surface. If the ferromagnetic fluctuation or the Kohn-Luttinger

renormalization [62, 66] is strong, the spin-triplet pairing states may develop. On the other hand, the first-order transitions may hinder the quantum critical behavior at the regime boundaries [78, 79]. The related discussion is beyond our mean-field framework and is an interesting topic for future work. Finally, since the VH Fermi surface is universal on the hexagonal lattices, our analysis is also applicable to the other hexagonal-lattice systems.

Note added. While we were finalizing the manuscript, we learned about an independent study of kagome superconductors from CDW fluctuations [80]. This work considered the unfolded unreconstructed theory, which is eligible outside the CDW. We adopt the folded reconstructed theory in the parent CDW phase. This captures more precisely the situation in the kagome metals AV_3Sb_5 and explains the double-dome superconductivity. Furthermore, our work conducts a systematic symmetry analysis of the multi- Q CDW orders and their fluctuations. This determines the leading pairing states with strong experimental relevance in a complete framework.

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