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# Thermoelectric properties and low-temperature transport anomalies in p-type full-Heusler compounds $\text{Fe}_{2-x}\text{Cr}_x\text{VAI}$

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Elemental substitutions are successfully used for the optimization of thermoelectric properties of a specific material; it requires, however a deep understanding of its impact. For this purpose, based on the full-Heusler material  $\text{Fe}_2\text{VAI}$ , various compounds ( $\text{Fe}_{2-x}\text{Cr}_x\text{VAI}$ ) were synthesized by substituting Cr for Fe in a wide range from  $x = 0.005$  up to  $x = 0.4$ . X-ray diffraction analysis revealed full solubility of Cr for all concentrations. Bulk thermoelectric properties such as electrical resistivity, Seebeck coefficient, thermal conductivity and Hall resistivity were measured from 2 K up to 780 K and all results were discussed in the context of the outcome of density functional calculations. Transport anomalies, resembling Kondo scattering, were observed for all samples below 30 K. Finally, an increased effective number of valence electrons of 7 for Cr was phenomenologically determined, which revealed good agreement with other p-type doping studies of  $\text{Fe}_2\text{VAI}$ .

Keywords:  $\text{Fe}_2\text{VAI}$ , thermoelectric transport, Seebeck coefficient, Kondo effect, DFT calculation

## I. Introduction

The research on thermoelectric devices, whether on waste heat harvesting, cooling or other applications has seen great advancements in the last decades.[1–3] For such devices a reasonably large thermoelectric figure of merit  $ZT = S^2\sigma T/\kappa$  is the determining factor for most practical applications.  $S$  is the Seebeck coefficient,  $\sigma$  the electrical conductivity,  $T$  the absolute temperature and  $\kappa$  the total thermal conductivity. Consequently, a solid understanding of the underlying thermoelectric transport mechanisms and how to manipulate them is paramount for the development of thermoelectric devices. Full-Heusler compounds based on  $\text{Fe}_2\text{VAI}$  have recently shown to offer a rich playground for studying and developing new enhancement principles.[4, 5]

Pristine, ternary  $\text{Fe}_2\text{VAI}$  consists of cheap and abundant elements, is easy to synthesize and has been extensively studied in recent years.[6–13] Both experimental data and density functional theory (DFT) based calculations show  $\text{Fe}_2\text{VAI}$  as a semi-metal with a small negative indirect band gap, as can be seen in Figure 1. This results in a very low DOS (density of states) at the Fermi energy  $E_F$  and a high sensitivity of the transport parameters to doping or elemental substitution. Even though its high semiconductor-like electrical resistivity, large thermal conductivity and only moderate Seebeck coefficient are not suitable for thermoelectric application,  $\text{Fe}_2\text{VAI}$ -based compounds with moderate amounts of other substituted elements have shown substantial improvements of all parameters, achieving high power factors  $S^2\sigma$  up to  $10.3 \text{ mWm}^{-1}\text{K}^{-2}$  for Ta- and Si-based

substitutions by band engineering.[12] While a wide range of elements for n-type doping has been studied, research into p-type doping is sparse. Additionally, nearly all of these elemental substitutions are based on substituting either V or Al.

Disorder, such as Fe/V anti-site occupations, plays a significant role in undoped  $\text{Fe}_2\text{VAI}$ , influencing thermoelectric properties as well as magnetic features.[14, 15]. For example, both super-paramagnetic cluster glass behavior (see Ref. [16]) with an average magnetic moment  $\mu \approx 5000 \mu_B$  and isolated magnetic impurities arising from Fe/V anti-site defects with  $\mu = 3.7 \mu_B$  have been observed in the otherwise nonmagnetic host matrix of fully ordered  $\text{Fe}_2\text{VAI}$ . [17, 18]

The present study aims to provide deeper understanding of p-type transport in  $\text{Fe}_2\text{VAI}$  and how it can be utilized to create enhanced thermoelectric materials through band structure engineering. This will be achieved by quantifying the effect of Fe/Cr substitution on the thermoelectric properties. Moreover, low-temperature transport anomalies are observed, resembling Kondo-like carrier scattering, which is corroborated by electronic structure calculations, indicating that Cr atoms located at the Fe site lead to magnetic impurities.

## II. Experimental and Computational Methods

### *Synthesis*

For the synthesis of the bulk material, elements of high purity (Fe 99.98%, Cr 99.99%, V 99.93%, Al 99.99%, Si 99.999%) were weighed in and then melted in a high-frequency induction furnace. This resulted in polycrystalline ingots of around 4 to 5 g with only a marginal mass loss of less than 0.1%. As-cast samples were then annealed at 1073 K for 168 h inside an evacuated ( $p \approx$

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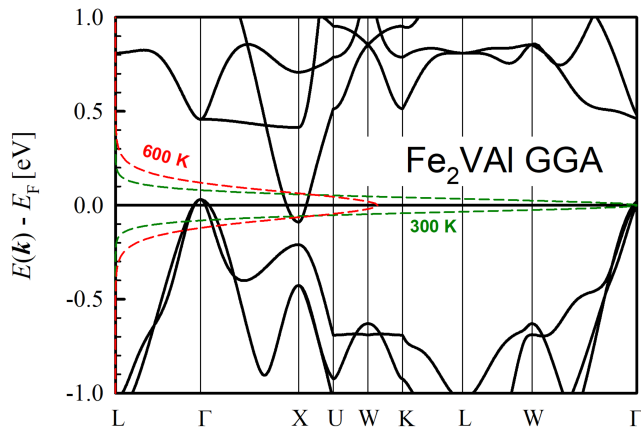


FIG. 1. Electronic band structure of  $\text{Fe}_2\text{VAI}$  around the Fermi energy  $E_F$ . Energy derivatives of the Fermi-Dirac distribution at  $T = 300$  and  $600$  K are indicated as colored dashed lines. Reproduced from Ref. [12].

$10^{-5}$  mbar) and sealed quartz tube. The annealed ingots were then cut into the appropriate geometry for the respective X-ray and transport measurements using an Aluminum oxide cutting wheel.

#### *X-ray diffraction*

To examine the phase purity and the lattice parameters of the polycrystalline  $\text{Fe}_{2-x}\text{Cr}_x\text{VAI}$  samples, X-ray diffraction (XRD) measurements were performed on ground samples and the lattice parameter was determined from Rietveld refinements.  $\text{Cu-K}\alpha$  radiation and a Bragg-Brentano geometry was used. (PANalytical XPert Pro MPD)

#### *Electronic and Thermal Transport*

The electrical resistivity and Seebeck coefficient above 300 K were simultaneously obtained using a ZEM3-ULVAC device. Seebeck measurements below 300 K were performed separately but on the same sample, utilizing a so-called ‘seesaw heating’ method.[19] For the low-temperature resistivity measurement the Van-der-Pauw method was used. The low-temperature data were adjusted by a small constant multiplicative factor to fit the standardized high-temperature measurements.

Thermal conductivity was measured in a steady state heat flow experimental setup. The measured data were then corrected for radiation loss by fitting the Callaway model [20] with an additional  $T^3$  term, as has been successfully done in previous studies.[8, 21, 22] To acquire the lattice thermal conductivity the electronic portion was subtracted using the Wiedemann-Franz law with a Lorenz number of  $2.44 \times 10^{-8} \text{ W}\Omega \text{ K}^{-2}$ .

For the calculation of the Hall mobility and charge carrier concentration, Hall resistivity at room temperature was measured. The Hall mobility and charge carrier concentration were then evaluated within a single-band model. While this seems to be justified by the linear Hall resistivity, it has to be mentioned that, due to the

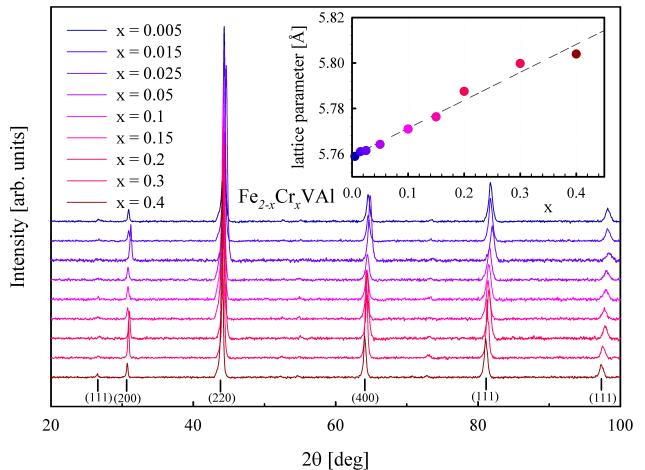


FIG. 2. Powder XRD patterns of  $\text{Fe}_{2-x}\text{Cr}_x\text{VAI}$  taken at room temperature for various concentrations of  $x$ . The inset shows the lattice parameter of each sample calculated by Rietveld refinement from the corresponding XRD graph, assuming a full-Heusler  $L_{21}$  crystal structure.

semi-metal-like band structure, contributions from both hole-like and electron-like may be relevant. For details of the evaluation of the Hall measurements see the Supplemental Materials.[23]

#### *Density Functional Theory Calculations*

Density functional theory (DFT) calculations of the electronic density of states of  $\text{Fe}_{2-x}\text{Cr}_x\text{VAI}$  were carried out. Performing supercell calculations on a  $3\times 3\times 3$  rhombohedral supercell with 108 atoms allowed to investigate the effect of partial Cr substitution, where a single Cr atom was replaced for one of the 54 Fe atoms, and compared to results of the quaternary  $\text{FeCrVAI}$  and pristine  $\text{Fe}_2\text{VAI}$ .

### III. Results

#### A. Crystal structure

XRD patterns and the composition-dependent lattice parameters are presented in Figure 2. It can be seen that all samples, even up to high Cr concentrations of  $x = 0.4$ , exhibit the same characteristic full-Heusler structure ( $L_{21}$ ) pattern. The linear dependence of the lattice parameter on the Cr concentration also indicates high solubility of Cr and homogeneity of the Heusler phase. Additionally, our DFT based calculations show the defect-formation enthalpy for a Fe/Cr substitution to be  $H_{\text{Fe}/\text{Cr}} = -7.83$  eV compared to  $H_{\text{V}/\text{Cr}} = +0.84$  eV, indicating that Cr occupying the Fe sublattice is more energetically favorable than Cr occupying the V sublattice and V the Fe sublattice. ( $\text{V}_{\text{Fe}}$  antisite:  $+1.513$  eV) [24]

## B. Electronic structure

Figure 3 shows the spin-polarized electronic densities of states of  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}$  in the dilute limit of Cr substitution ( $x \approx 0.02$ ) and for the full-substitution of one Fe sublattice by Cr, i.e. the quaternary Heusler compound  $\text{FeCrVAL}$ . The electronic structure of  $\text{Fe}_2\text{VAL}$  was calculated as well and is plotted for comparison (dashed grey lines). The first immediate feature which can be noticed upon substituting with Cr is the appearance of narrow impurity states inside the pseudogap. Moreover, the spin degeneracy of these states is removed, meaning that Cr on the Fe site forms a magnetic impurity. Upon increasing Cr substitution the degree of spin-polarization increases substantially and leads to a half-metallic-like ground state in  $\text{FeCrVAL}$ , i.e., one spin channel shows a metallic solution whereas the minority spin has a band gap at the Fermi energy. By analyzing the partial atom-projected DOS, we find that the majority contribution of electronic states in the impurity band arises from Fe and Cr atoms, followed by V, with very little Al contribution. Interestingly, for the dilute case ( $x \approx 0.02$ ) the main contribution arises from the Fe atoms near the Cr impurity whereas for the quaternary Heusler compound, in-gap states have predominant Cr character.

Such a behavior, where the contribution of the impurity states mostly stems from the atoms of the host material surrounding the impurity and not the impurity itself has for example also been found in other semiconductors such as thallium-doped  $\text{PbTe}$ .<sup>[25]</sup> Thus, it is possible that small amounts of Cr impurities in  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}$  act in a similar manner as a catalyst for the formation of resonant-like states. However, as the amount of Cr impurities increases, the overlap of wave functions will lead to the formation of a real impurity band, which has predominant Cr character. We further point out that the DOS of the minority spin retains its integrity and shows similar features even for the  $\text{FeCrVAL}$  quaternary Heusler compound.

In summary, DFT calculations suggest the formation of magnetic impurity states due to Fe/Cr substitution and a half-metallic-like groundstate for the quaternary  $\text{FeCrVAL}$  compound. With increasing Cr substitution the in-gap states become broader around the Fermi level and eventually fill out the entire pseudogap.

## C. Electrical Resistivity

As shown in Figure 4, the temperature-dependent electrical resistivity  $\rho(T)$  of  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}$  ( $0 < x \leq 0.4$ ) shows for all  $x$  a semiconductor-like behavior above room temperature due to the activation of charge carriers across the pseudogap. Presumably, this is also the case in undoped  $\text{Fe}_2\text{VAL}$ , which exhibits this behavior across the whole temperature range. For higher concentrations of Cr, the semiconductor-like behavior diminishes as the samples become more and more metal-like below room

temperature. Additionally, at very low temperatures, below 30 K, a logarithmic increase of the resistivity was obtained for all samples.

These low-temperature anomalies might result from a single-impurity-type Kondo effect due to the interaction of conduction electrons with the almost localized Cr ions and will be further discussed in subsection IV B. The overall change of the temperature-dependent resistivity for different Cr concentrations can be understood as arising from two competing factors: Cr contributes less electrons and therefore lowers the Fermi-energy further into the valence band. This increases the charge carrier concentration and consequently decreases the electrical resistivity. The other competing factor is the random potential fluctuation induced by the Cr atoms on the Fe sublattices. This leads to significant additional impurity scattering, since the majority of electronic states in the valence band has Fe orbital character. <sup>[26]</sup> Moreover, electronic structure calculations suggest that the partial substitution of Cr leads to narrow, less dispersive or localized impurity states at the valence band edge. For small concentrations such as  $x = 0.05$  the additional impurity scattering is dominant while for high concentrations such as  $x = 0.4$  the additional charge carriers outweigh the effect of impurity scattering leading to an significantly reduced low-temperature resistivity.

## D. Seebeck Effect

Measurements of the temperature-dependent Seebeck coefficient  $S(T)$  from 4 K up to more than 700 K are shown in Figure 5. All samples exhibit a maximum between 200 and 400 K, with the sample  $x = 0.025$  showing the highest value of  $S = 75 \mu\text{V/K}$ .

Qualitatively, the composition-dependent behavior of the maximum Seebeck coefficient for  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}$  with increasing Cr concentration can be explained as follows: The Seebeck coefficient is the sum of all contributions of all bands near the Fermi level with hole-like valence bands contributing with a positive sign and electron-like conduction bands, contributing a negative signed thermopower. In the case of  $\text{Fe}_2\text{VAL}$  or at very low concentrations (i.e.  $x = 0.005$ ), both hole-like and electron-like carriers contribute significantly, leading to only moderately large values of the Seebeck coefficient. For slightly higher Cr concentrations, the Fermi level drops and the negative contribution of the conduction band diminishes, which increases the Seebeck coefficient up to Cr concentrations of  $x \approx 0.025$ . A further decrease of the Fermi level into the valence band increases the density of states (DOS) at  $E = E_F$  and therefore decreases the Seebeck coefficient as described by the Mott formula

$$S \propto \frac{\partial N(E_F)/\partial E}{N(E_F)},$$

with  $N(E_F)$  being the DOS at the Fermi energy. This is dominant for high Cr concentrations and diminishes

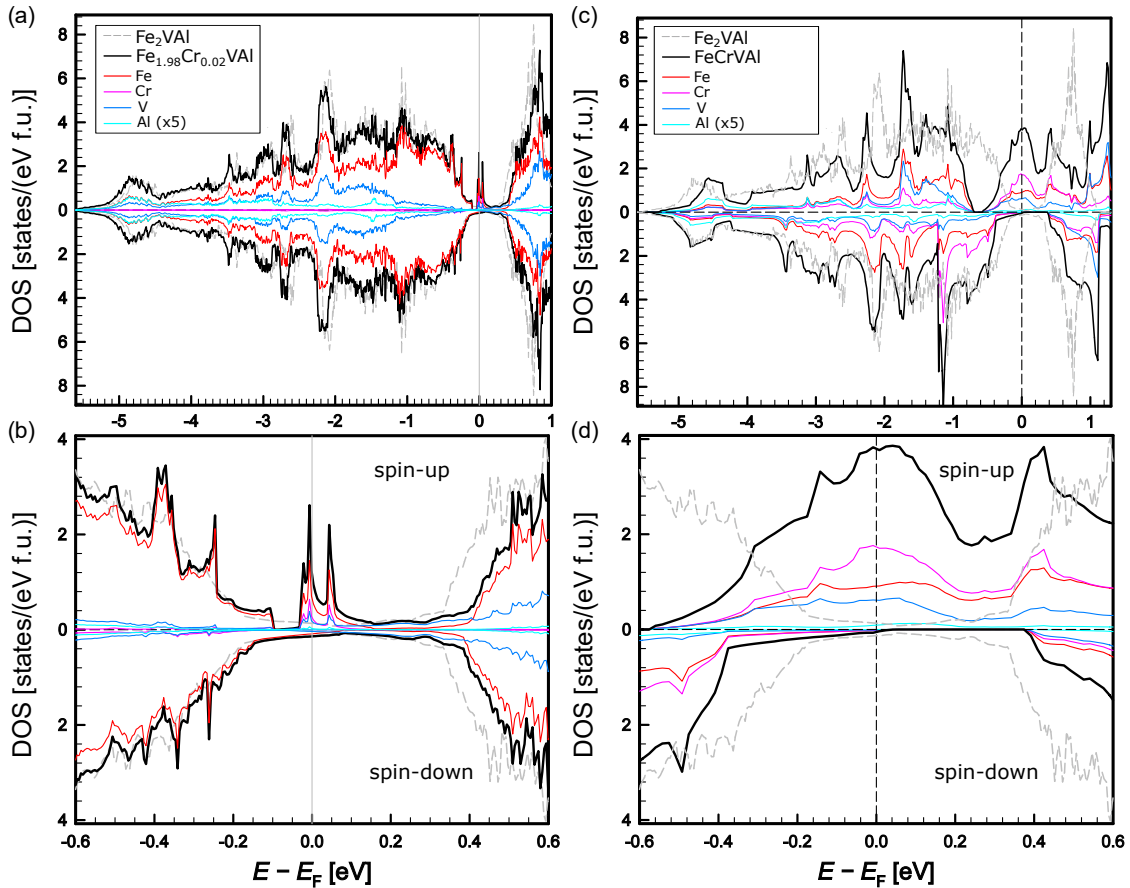


FIG. 3. Energy-dependent density of states (DOS) of a) dilute Cr doping of  $\text{Fe}_2\text{VAI}$  ( $\text{Fe}_{1.98}\text{Cr}_{0.02}\text{VAI}$ ) and b) the fully substituted quaternary compound  $\text{FeCrVAI}$ . The DOS around the Fermi energy  $E_F$  is shown in b) and d) respectively.

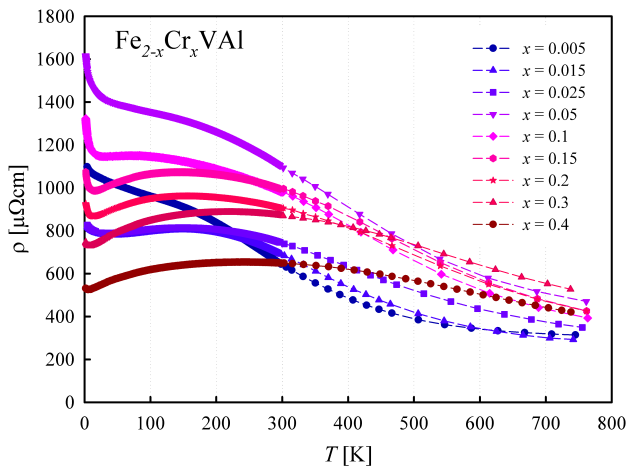


FIG. 4. Temperature-dependent electrical resistivity,  $\rho$ , of  $\text{Fe}_{2-x}\text{Cr}_x\text{VAI}$  for various concentrations  $x$ . Dashed lines are added as a visual guide.

the resulting Seebeck coefficient, while shifting the maximum to higher temperatures, due to a loss of

bipolar conduction.

Even though these arguments (i.e. a rigid-band-like shift) neglect other effects such as the formation of a Cr-impurity band or other changes in the band-structure, they can reasonably well explain the observed behavior. These simple arguments are further supported by the dependence of  $S$  on the valence electron concentration as shown in Figure 8 (discussed in subsection IV A).

Moreover, a three-parabolic-band model, as has been employed successfully in similar compounds [8, 10, 12], was used to fit the Seebeck data (depicted as dashed lines in Figure 5). Obviously, experimental data above about 50 K are well accounted for by this model, revealing reasonable facts and figures regarding the respective valence and conduction bands (details are collected in the Supplemental Materials[23]). Below about 50 K, the three-parabolic-band model predicts an almost linear dependence of  $S(T)$ ; experimental data, however, deviate from this linearity because of the presence of Kondo interactions. (details on the fit parameters can be found in Supplemental Materials[23])

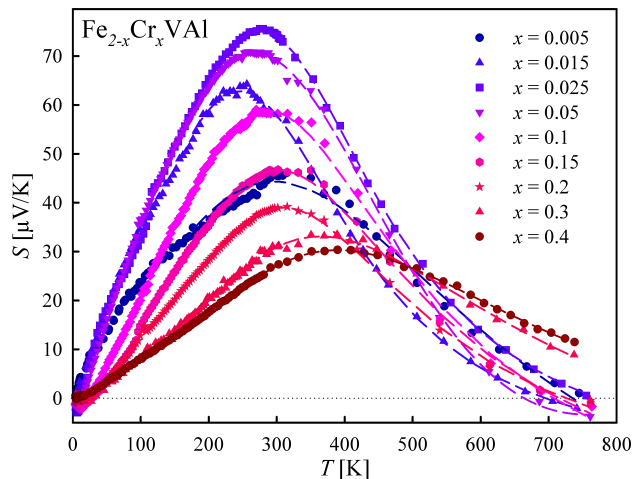


FIG. 5. Temperature dependent Seebeck coefficient measurements of  $\text{Fe}_{2-x}\text{Cr}_x\text{VAl}$  between 4 and 800 K. The dashed lines indicate a parabolic three-band model fit in the temperature range of 100 to 750 K.

### E. Thermal Conductivity

The temperature-dependent thermal conductivity  $\kappa$  was measured for representative samples with Cr concentrations  $x = 0.1$ ,  $x = 0.15$  and  $x = 0.2$ . Results are as depicted in Figure 6 a) and b). The lattice thermal conductivity (solid lines in Figure 6) was calculated by fitting a Callaway model [20] with an additional  $T^3$  radiation loss term and subtracting the electronic contribution, which was calculated using the Wiedemann-Franz law.

In Figure 6 b), the Cr concentration dependence of the lattice contribution  $\kappa_{\text{lat}}$  shows a decrease due to point defect scattering of phonons by mass and strain field fluctuations. The decrease of  $\kappa_{\text{lat}}$  with composition is found to be comparable to n-type  $\text{Fe}_2\text{VAl}_{1-x}\text{Si}_x$ -based Heusler compounds.[9] Qualitatively, the composition-dependent behavior can be well described employing the Klemens model[27], as shown in Figure 6 b). Within this model the ratio of the thermal conductivity of a defective solid  $\kappa_d$  to that of the pure solid  $\kappa_0$  is given by

$$\frac{\kappa_d}{\kappa_0} = \frac{\tan^{-1}u}{u} \text{ and } u^2 = \frac{(6\pi^5 V_0^2)^{1/3}}{2k_B v_s} \kappa_0 \Gamma$$

where  $V_0$  is the volume per atom,  $v_s$  the speed of sound and  $\Gamma$  the point defect scattering parameter which is related to the defect concentration by  $\Gamma \propto x(1-x)$ . [28]

### F. Mobility and Charge Carrier Concentration

The calculated charge carrier mobility and charge carrier concentrations from Hall measurements (see Supplemental Materials[23]) at  $T = 300$  K can be seen in

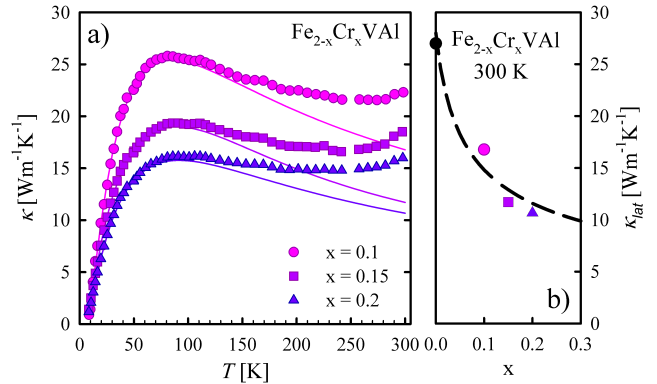


FIG. 6. (a) Temperature dependent thermal conductivity of a selection of samples of  $\text{Fe}_{2-x}\text{Cr}_x\text{VAl}$  from 4 K to room temperature. Data points in a) represent the total measured thermal conductivity. The solid lines are least squares fits as explained in the text. b) Lattice thermal conductivity for various concentrations of Cr at  $T = 300$  K. The dashed line in b) indicates a Klemens model fit. [27] For comparison an additional data point for undoped  $\text{Fe}_2\text{VAl}$  was added.

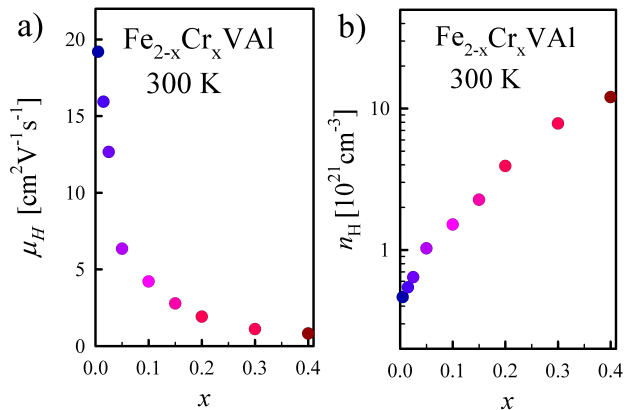


FIG. 7. Cr concentration dependence of the charge carrier mobility and density calculated from Hall-resistivity measurements at room temperature of  $\text{Fe}_{2-x}\text{Cr}_x\text{VAl}$ .

Figure 7. The concentration dependence of the mobility shows an dramatic decrease and is more than one order of magnitude smaller for  $x = 0.4$ . This arises from increased carrier scattering on the randomly distributed Cr ions on the Fe sublattices. The increase of the charge carrier concentration is consistent with the experimental results of the Seebeck coefficient and indicates a shift of the Fermi energy deeper into the valence band.



## IV. Discussion

### A. Prospects of band structure engineering by Fe/Cr substitution

The concept of band structure engineering follows the intentional modification of both the shape of the band structure near the Fermi level and the position of the Fermi level itself, to achieve intended changes of the physical properties. This is usually accomplished by elemental substitution or changes in the synthesis method. For this purpose, it is necessary to understand and be able to predict the effect of specific elemental substitutions on the pristine material.

To quantify and compare the results of different elemental substitutions, the Seebeck coefficient of various  $\text{Fe}_2\text{VAL}$ -based compounds is plotted as a function of the valence electron concentration. Results are shown in Figure 8. Assuming a rigid band structure, a change in the valence electron concentration (VEC) is phenomenologically comparable with a change of the Fermi level and is remarkably consistent for various  $\text{Fe}_2\text{VAL}$ -based compounds (see examples from literature in Figure 8). The VEC is commonly calculated by averaging over the valence electron count  $N$  of all constituent elements. In case of  $\text{Fe}_2\text{VAL}$  ( $N_{\text{Fe}}$ : 8,  $N_{\text{V}}$ : 5,  $N_{\text{Al}}$ : 3), this leads to  $\text{VEC} = 6$ . Although this simple counting of valence electrons works for many elemental substitutions in  $\text{Fe}_2\text{VAL}$ , it seems to be different for Cr, with a formal valence electron count  $N_{\text{Cr}} = 6$  when substituted on the Fe-site in the  $\text{Fe}_2\text{VAL}$ -structure (see Figure 8) which indicates significant changes in the electronic structure.

When, however, considering  $N_{\text{Cr}} = 7$ , the overall behavior of the elemental substitution is in very good agreement with all other substitutions and is reminiscent of a rigid-band-like shift of the Fermi energy. While this observation is of pure phenomenological nature and does not have any direct physical meaning, it is crucial when tuning the Fermi level for optimal thermoelectric performance. To corroborate this finding, several  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}_{1-y}\text{Si}_y$ -based samples were synthesized in a second batch and their respective Seebeck coefficient at  $T = 300$  K was measured. Here, co-substitution of Al/Si is used as an n-type doping agent to increase both the total valence electron concentration and the position of the Fermi level, see Figure 8. It can be seen that, in accordance with previous arguments, these samples also show better consistency with previous doping studies when assuming  $N_{\text{Cr}} = 7$  instead of 6. Again, it has to be reiterated that this observation does not imply a true rigid-band-like behavior as it is most likely the case in pure Al/Si substitution but is only of phenomenological nature. For further details on these substitutions see Supplemental Materials.[23]

Additionally, the high solubility of Cr is crucial because it facilitates the reduction of the lattice thermal conductivity due to increased crystallographic disorder.

In conclusion, Cr as an elemental substitution for Fe is

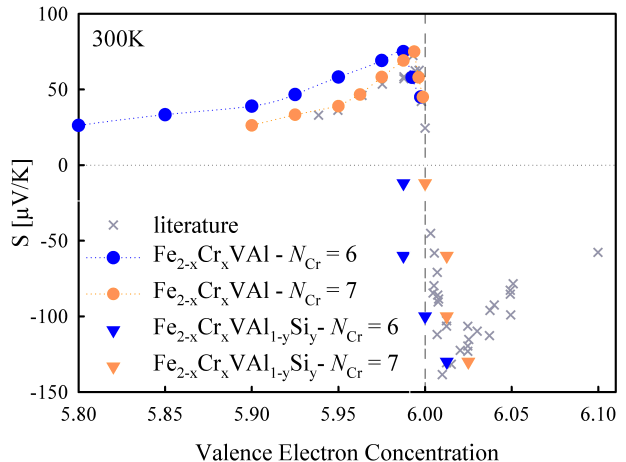


FIG. 8. Valence electron concentration dependency of the Seebeck coefficient shown for  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}$  and  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}_{1-y}\text{Si}_y$ . While an effective valence electron count  $N_{\text{Cr}}$  of 6 for Cr is conventionally expected, a count of 7 fits very well with data from other doping studies on  $\text{Fe}_2\text{VAL}$ . [9, 11, 29–40]

a useful and predictable tool in the careful crafting of  $\text{Fe}_2\text{VAL}$  based compounds optimized for thermoelectrical applications.

### B. Low-temperature transport anomalies

Both the Seebeck coefficient and the electrical resistivity of  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}$  exhibit a peculiar temperature-dependent behavior below  $T \approx 30$  K. In the case of the electrical resistivity, a logarithmic temperature dependence  $\rho(T) \propto -\ln(T)$  arises. This can best be observed for  $\text{Fe}_{1.9}\text{Cr}_{0.1}\text{VAL}$  (Figure 9) but is present in each other sample as well.

A peculiar low-temperature anomaly is also observable for the temperature-dependent Seebeck coefficient below 30 K (see Figure 10). The emergent minimum in  $S(T)$  is most pronounced for moderately doped samples such as  $\text{Fe}_{1.9}\text{Cr}_{0.1}\text{VAL}$ . For lower concentrations, the  $S(T)$  minimum occurs at lower temperatures. For higher concentrations, on the other hand, the low-temperature anomaly becomes suppressed and even vanishes for  $x$  approaching 0.4. Together with the anomalous increase of the low-temperature resistivity, this hints to more complex energy-dependent scattering mechanisms.

A possible explanation for both these phenomena can be given by isolated magnetic impurity scattering, i.e. the Kondo effect.[41] Here, a logarithmic energy dependence of the relaxation time  $\tau(E) \propto \ln(1/E)$  arises from intermediate spin-flip scattering between the magnetic impurity and the conduction electron spins, resulting in a negative logarithmic  $\rho(T)$  dependence. (see Figure 9) Such Kondo-type scattering has previously also been suggested in other substituted  $\text{Fe}_2\text{VAL}$ -based compounds.[32]

With respect to the Seebeck effect,  $S$  (and in absence of any other interaction mechanisms), a universal behaviour [42] of  $S(T)$  can be expected, revealing a smooth maximum/minimum around the characteristic Kondo temperature  $T_K$ . In this scenario, large thermopower values can be obtained as a consequence of an asymmetric relaxation time  $\tau(E)$  due to the Kondo resonance around the Fermi energy  $E_F$ . Studies of 3d magnetic impurities dissolved in a non-magnetic host like Cu or Au revealed pronounced extrema (about  $|10-20| \mu\text{V}/\text{K}$ ) [43]. Diluted chromium in Cu was reported to exhibit a negative thermopower anomaly at low temperatures [44]. In fact, depending on the Cr concentration in  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}$ , more or less pronounced minima in  $S(T)$  are obvious at low temperatures from Figure 10.

To quantitatively account for this Kondo contribution to  $S(T)$ , a theory developed by Fischer [45], based on the Heisenberg Hamiltonian, can be utilized. In this model, the Kondo contribution to the Seebeck effect is given by

$$S_{\text{Kondo}}(T) = \frac{\pi^2 J(J+1)\sin(2\alpha)}{(\ln^2(T_K/T) + \pi^2 J(J+1))^{3/2}}$$

where  $J$  is the effective spin moment,  $\alpha$  the scattering phase shift and  $T_K$  the Kondo temperature.[46] Thus, the total Seebeck coefficient is then given by

$$S_{\text{total}}(T) = cS_{\text{Kondo}}(T) + \gamma T$$

where  $c$  is a parameter, depending on the electrical resistivity and  $\gamma$  is the conventional linear diffusion term at low temperatures. The value of  $\gamma$  was obtained by interpolating the linear temperature dependence significantly above the Kondo contribution. Least squares fits of  $S_{\text{total}}(T)$  to the experimental data reveal good agreement (solid lines, Figure 10), thus confirming to some extent the Kondo scenario proposed. Parameters and further details on the fitting procedure can be found in Supplemental Materials.[23]

To further investigate the low-temperature transport anomalies, magneto-transport measurements have been performed at low temperatures. In the presence of an external magnetic field Kondo scattering diminishes due to the degeneracy of the intermediate spin-flip state being lifted. As a result, the electrical resistivity is decreased and the Kondo effect becomes entirely suppressed if the magnetic field energy corresponds with the Kondo energy  $k_B T_K$ . Indeed, as can be seen in Figure 11, the upturn of  $\rho$  at low temperatures is significantly suppressed in an external magnetic field. Both insets of Figure 11 exhibit the magnetoresistance  $\Delta\rho/\rho$  at various low temperatures. In accordance to the results of the main panels of Figure 11,  $\Delta\rho/\rho$  is negative, and the field dependence matches a typical Kondo scenario.

It is of note that there exist many other physical mechanisms that can originate a low-temperature anomaly

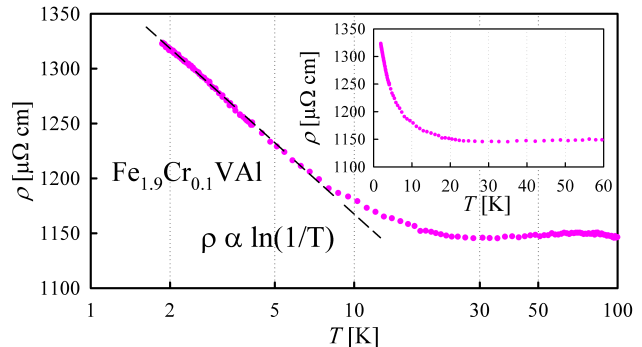


FIG. 9. Temperature dependent electrical conductivity measurement of  $\text{Fe}_{1.9}\text{Cr}_{0.1}\text{VAL}$ . The dashed line illustrates the logarithmic temperature dependency at low temperatures. The inset shows the same sample in a linear temperature dependence.

in the electrical resistivity with  $d\rho/dT < 0$ . For instance it is well known that in disordered systems, quantum interference phenomena can lead to a  $\sqrt{T}$ -increase in the electrical conductivity, i.e. weak localization. Moreover, as demonstrated recently for stoichiometric  $\text{Fe}_2\text{VAL}$  itself, the conduction mechanism within localized impurity bands arising from intrinsic antisite disorder, may follow a Mott-type variable-range-hopping behavior  $\rho(T) \propto \exp(T^{-1/4})$ , which will be modified in the presence of strong electronic correlations to  $\rho(T) \propto \exp(T^{-1/2})$  (called Efros-Shklovskii hopping). Supplementary Fig.S1 shows that, while Kondo-type scattering indeed fits the measured data best, the upturn at low temperatures can also qualitatively be described by several of the above-mentioned physical scenarios. Thus, an unambiguous and definite statement about the origin of low-temperature transport anomalies in  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}$  is not feasible.

However, the anomaly of the Seebeck coefficient, the negative magneto-resistance as well as the suggestion of magnetic impurities by DFT-based calculations are consistent with the Kondo-like magnetic scattering.

## V. Conclusion

XRD crystal structure studies of the compounds  $\text{Fe}_{2-x}\text{Cr}_x\text{VAL}$  revealed a high solubility of Cr in  $\text{Fe}_2\text{VAL}$ . The temperature and Cr concentration dependency of both the electrical resistivity and Seebeck coefficient can be well described taking into account a phenomenological valence electron count of 7 instead of 6. The resulting VEC dependence of the Seebeck is in very good agreement with other studies of p-type doping of  $\text{Fe}_2\text{VAL}$  and hints at the fact that the otherwise non-rigid-band-like Fe/Cr substitution can be phenomenologically compared to an almost rigid-band-like shift of the Fermi level with Cr acting as a p-type doping agent effectively donating one electron per Cr atom.



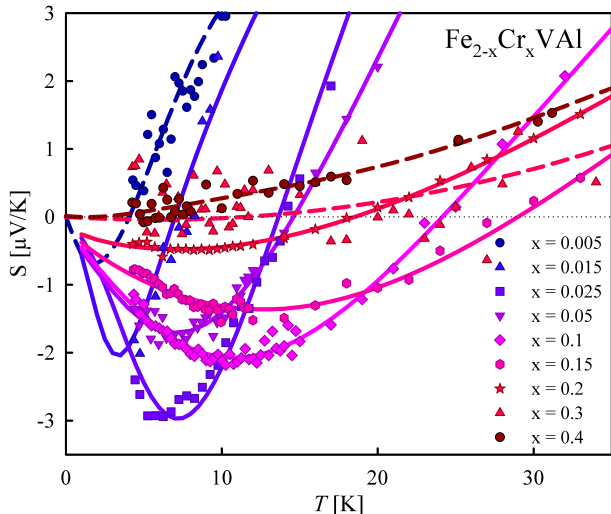


FIG. 10. Low-temperature Seebeck coefficient for  $\text{Fe}_{2-x}\text{Cr}_x\text{VAI}$ . Solid lines indicate a qualitative Kondo-model fit. Dashed lines indicate simple polynomial fits for visual clarity.

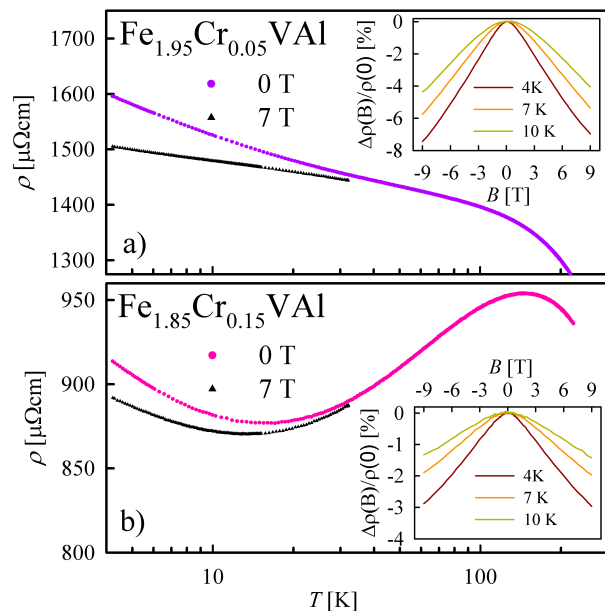


FIG. 11. Temperature dependent electrical resistivity  $\rho$  of  $\text{Fe}_{1.95}\text{Cr}_{0.05}\text{VAI}$  (a) and  $\text{Fe}_{1.85}\text{Cr}_{0.15}\text{VAI}$  (b) for external fields of 0 and 7 T. The insets show the field dependent magnetoresistance for various temperatures.

The low temperature ( $T < 30\text{K}$ ) properties of  $\text{Fe}_{2-x}\text{Cr}_x\text{VAI}$ , i.e., a negative logarithmic contribution to the electrical resistivity, the negative magnetoresistance, as well as a minimum in the Seebeck coefficient can be well described by incoherent Kondo scattering of conduction electrons.

The ability to predict changes of the effective VEC for maximizing the thermoelectric performance is an invaluable tool when designing thermoelectric materials for practical application and Cr substitution has shown to be a well understood and easy to utilize system and certainly will be effective in designing further  $\text{Fe}_2\text{VAI}$  based compounds.

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