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Symmetry constraints on superconductivity in twisted bilayer graphene: Fractional vortices, $4e$ condensates or non-unitary pairing

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When two graphene sheets are twisted relative to each other by a small angle, enhanced correlations lead to superconductivity whose origin remains under debate. Here, we derive some general constraints on superconductivity in twisted bilayer graphene (TBG), independent of its underlying mechanism. Neglecting weak coupling between valleys, the global symmetry group of TBG consists of independent spin rotations in each valley in addition to valley charge rotations, $SU(2) \times SU(2) \times U(1)$. This symmetry is further enhanced to a full $SU(4)$ in the idealized chiral limit. In both cases, we show that any charge $2e$ pairing must break the global symmetry. Additionally, if the pairing is unitary the resulting superconductor admits *fractional* vortices. This leads to the following general statement: Any superconducting condensate in either symmetry class has to satisfy one of three possibilities: (i) the superconducting pairing is non-unitary, (ii) the superconducting condensate has charge $2e$ but admits at least half quantum vortices which carry a flux of $h/4e$, or (iii) the superconducting condensate has charge $2me$, $m > 1$, with vortices carrying $h/2me$ flux. The latter possibility can be realized by a symmetric charge $4e$ superconductor ($m = 2$). Non-unitary pairing (i) is expected for superconductors observed in the vicinity of flavor polarized states. On the other hand, in the absence of flavor polarization, e.g. in the vicinity of charge neutrality, one of the two exotic possibilities (ii) and (iii) is expected. We sketch how all three scenarios can be realized in different limits within a strong coupling theory of superconductivity based on skyrmions. Finally we discuss the effect of symmetry lowering anisotropies and experimental implications of these scenarios.

I. INTRODUCTION

The discovery of superconductivity in magic angle twisted bilayer graphene (TBG)¹ has inspired immense research activity to determine its origin²⁻¹⁹. Several mechanisms for the superconductivity were proposed including phonon-induced pairing^{7,8}, bosonic order parameter fluctuations⁹⁻¹⁷, and topological skyrmion superconductivity¹⁸⁻²⁰. At the same time, a few works have attempted to connect band topology to superfluid density^{21,22} or categorized different paired states on the basis of symmetry²³. TBG at small twist angles has a rather large symmetry, particularly when we ignore weak Hunds coupling that couples spins in opposite valleys. Here, we exploit this large symmetry group to expose strong constraints on the nature of superconductivity regardless of the mechanism responsible for it. Remarkably, these constraints imply that the superconductivity has to be exotic in one of three possible ways: either (i) the pairing is non-unitary^{24,25}, i.e. the pairing matrix is not proportional to a unitary matrix, which means that the different flavors pair independently, (ii) the pairing breaks the global symmetries leading to an order parameter manifold which admits fractional vortices, (iii) the superconducting condensate has charge $2me$ with $m > 1$ admitting fractional vortices with flux $h/2me$. Each of these scenarios leads to measurable physical consequences.

A notable feature of magic angle TBG is that electrons appear with both spin and valley flavors. The absence of spin-orbit coupling in graphene makes $SU(2)_S$ spin rotation an excellent symmetry. Similarly, the small angle leads to an additional $U(1)_V$ symmetry, implying conservation of the valley quantum number, in addition to charge $U(1)_C$ conservation. Finally, again in the small angle limit, the independent spin rotation symmetry in the two valleys is only weakly broken by a Hunds coupling, which can be ignored for many purposes. Thus while the low energy symmetry

group is $SU(2)_S \times U(1)_C \times U(1)_V$, it is meaningful to also consider the symmetry group $U(2)_K \times U(2)_{K'}$ denoting independent $U(2)$ symmetry groups in the opposite valleys. Finally, several works²⁶⁻²⁹ have suggested that this symmetry is enhanced to a full $U(4)$ flavor symmetry to a good approximation. This approximation becomes exact in the so-called chiral limit³⁰. In the following, we will omit the overall charge conservation $U(1)_C$ which will always be assumed and denote the three possible symmetry settings by $G_{\text{low}} = U_V(1) \times SU(2)_S$, $G = U_V(1) \times SU(2)_K \times SU(2)_{K'}$, and $G_{\text{high}} = SU(4)$. A hierarchy of energy scales lowers the symmetry in steps $G_{\text{high}} \rightarrow G \rightarrow G_{\text{low}}$ ²⁶. For the most part, we will ignore the weak intervalley Hunds coupling and concern ourselves with either G or the enlarged symmetry G_{high} . At the end we will discuss how the conclusions are modified on including the weak Hunds coupling, which can be either ferromagnetic or antiferromagnetic, and close with a discussion about the relevance to experiment.

Summary of the results— Before presenting the details of our analysis, let us summarize our main results. First, we note that the pairing function is a matrix in the space of flat bands labelled by band, spin, and valley indices. When considering pairing in a multi-orbital superconductor, it is important to distinguish the cases of unitary and non-unitary pairing first discussed in the context of Helium 3^{24,25}. Unitary pairing corresponds to the case where the pairing matrix is proportional to a unitary matrix which guarantees that its eigenvalues has the same magnitude leading to a single pairing gap. In non-unitary pairing, this is not the case and we can get different pairing gaps for different flavors e.g. different gaps for the two spin species. Usually, unitary pairing is energetically favored when the normal state neither breaks flavor symmetry^{31,32} nor is close to a flavor-symmetry breaking phase transition^{33,34}.

Our result can be phrased as a symmetry imposed con-

straint on the type of unitary pairing possible in TBG. This follows from the simple observation that there is no symmetric charge $2e$ superconductor either under $G_{\text{high}} = \text{SU}(4)$ or $G = \text{U}_V(1) \times \text{SU}(2)_K \times \text{SU}(2)_{K'}$ leaving one of two possibilities. First, a charge $2e$ superconductor which spontaneously breaks the global symmetry (G or G_{high}) by transforming as a non-trivial representation. As we will show, such superconductors always admit exotic vortices which carry a fractional flux. These arise from the non-trivial flavor structure of the symmetry breaking pairing function which allows us to compensate for a rotation in the phase of the condensate with an internal rotation in the flavor space. This is in complete analogy with the emergence of half-vortices in liquid Helium 3 or spin-triplet superconductors^{25,35,36} where a π phase rotation of the overall phase is combined with an internal rotation in spin space. The second possibility is a charge $2me$ superconductor with $m > 1$. Such higher charge superconductor is allowed to be symmetric under the global symmetry and it will automatically admit fractional vortices with flux $h/2me$. Higher charge superconductivity can be easily detected experimentally through the periodicity of the Fraunhofer pattern in Josephson junctions. In this regard, we note an important distinction between the fractional vortices of the $2e$ superconductor and the fractional vortices in higher charge symmetric superconductors. While the former have logarithmically divergent energies, even in the presence of a magnetic field, and can only be observed in certain mesoscopic geometries³⁷⁻³⁹, the latter has a finite energy and can be stabilized in the infinite system. This difference can be understood by recalling the difference between vortices in a superconductor whose logarithmic divergence is cancelled by the coupling to the electromagnetic gauge field and a neutral superfluid where the logarithmic divergence persists. The logarithmically divergent energy of the $2e$ fractional vortices arise precisely because they combine a rotation of the superconducting phase, which can be cancelled by an external flux, with an internal flavor rotation which cannot. On the other hand, fractional vortices in symmetric higher charge superconductors arise from pure phase winding which can be completely cancelled with an external flux.

We discuss the implications of these results for finite temperature physics and point out how, in certain cases, $2e$ -superconductors with fractional vortices are reduced by thermal fluctuations to higher charge superconductors. We also demonstrate how these different scenarios for superconductivity are explicitly realized in the recently proposed theory of skyrmion superconductivity^{18,19}, by constructing both $2e$ and $4e$ charged skyrmions near charge neutrality. We note here a recent work²⁰ which also investigated topological terms in TBG within a Dirac approximation and found level 1 WZW terms for half-filling and level 2 WZW term for neutrality. These correspond to cases (i) and (iii) in our analysis (see Fig. 1). Finally, we discuss the effects of further symmetry lowering perturbations such as the intervalley Hund coupling.

To discuss the implications of these results for TBG, we need to distinguish the cases of pairing in the presence or absence of flavor polarization. We define a flavor polar-

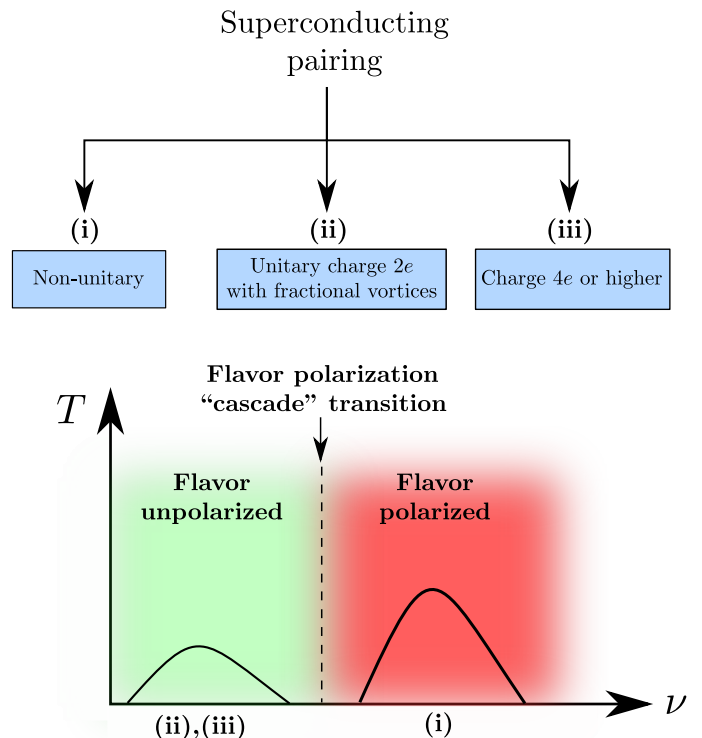


FIG. 1: Schematic illustration of the three possibilities for superconducting pairing with the global symmetry $G = \text{U}(1)_V \times \text{SU}(2)_K \times \text{SU}(2)_{K'}$ or $G_{\text{high}} = \text{SU}(4)$.

Non-unitary pairing (i) is expected in TBG for superconductivity in the vicinity of flavor symmetry breaking whereas unitary charge $2e$ superconductivity with fractional vortices (ii) or higher charge superconductivity (iii) is expected in the *absence* of flavor symmetry breaking. The unitary charge $2e$ superconductor always breaks the global (G or G_{high}) symmetry and is thus only possible at $T = 0$.

The possible order parameter manifolds for the unitary charge $2e$ superconductor and its associated vortices are summarized in Table I.

ized state as any state whose symmetry breaking reduces the Fermi surface fourfold spin-valley degeneracy. This implies that, at least within a weak coupling perspective, we have split Fermi surfaces where pairing takes place independently. For instance, spin polarization reduces the Fermi surface degeneracy from 4 to 2 generically leading to a spin-split Fermi surface where pairing takes place in each spin species separately. In the extreme case where one spin species is completely empty or full, pairing takes place in only one species. Another more subtle example is the K-IVC state²⁶ which spontaneously breaks $\text{U}_V(1)$ valley symmetry leading to a spin-split Fermi surface at finite doping¹⁵ and independent pairing in each Fermi surface⁴⁰ (note that at infinitesimal doping, the Fermi surface is not split due to an emergent Kramers time-reversal symmetry). On the other hand, a spin-unpolarized valley Hall states^{26,41} breaks C_2 symmetry but retain $\text{U}_V(1)$ valley symmetry as well as time-reversal symmetry \mathcal{T} which interchanges the valleys, thus it does not break the fourfold

degeneracy i.e. it is not flavor polarized.

This leads to the following conjectures about the nature of superconducting pairing in TBG, summarized in Fig. 1. First, the superconductors obtained in regions with flavor polarization – which can be identified experimentally through the evolution of the Hall number or quantum oscillations – likely correspond to non-unitary pairing, which is not constrained by our theory. This corresponds to the majority of the superconductivity observed in experiments close to half filling^{1,2,5,42,43} or to neutrality in the presence of symmetry breaking³. However, in certain isolated cases, superconductivity in the vicinity of charge neutrality⁴, or reached from charge neutrality in the absence of flavor polarization^{4,43}, has also been observed. We conjecture that these cases present a fertile hunting ground for exotic forms of superconductivity, either paired superconductors featuring fractional vortices or even higher charge condensates such as $4e$ superconductors. We hope future experiments will search for signatures of such novel condensates.

Symmetry transformation of charge $2e$ superconductors— Let us begin by considering the transformation properties of an arbitrary charge $2e$ superconductor under a general unitary transformation. A charge $2e$ superconductor is generally described by an order parameter Δ which transforms as $\Delta \mapsto \Delta e^{2i\phi}$ under global $U_C(1)$ charge conjugation which acts on the electrons as $c \mapsto e^{i\phi}c$. This breaks $U_C(1)$ down to \mathbb{Z}_2 . We take the electron operators c_a to be labelled by an index a . For example, in a translationally symmetric system, this index $a = (\alpha, \mathbf{k})$ includes both orbital α and momentum \mathbf{k} degrees of freedom. The pairing function is thus taken to be a matrix of the form $\Delta_{ab} = \langle c_a c_b \rangle$. Under a general unitary transformation $c \mapsto U c$, Δ transforms the same way as cc^T leading to

$$\Delta \mapsto U \Delta U^T \quad (1)$$

Note that this relation holds as long as Δ transforms the same way as cc^T and does not make any additional assumption on its nature. In particular, it holds well beyond the weak coupling BCS limit.

Throughout most of this paper, we will focus on global symmetries which only act on the orbital index α of the electron operator. This orbital index will be taken to label the 8 flat bands of TBG. We employ the Chern-sublattice-spin basis defined in Refs.^{18,26} with Pauli matrices $\gamma_z = \pm$ distinguishing the \pm Chern sectors, $\eta_z = \pm = K/K'$ distinguishing the two valleys and $s_z = \pm = \uparrow / \downarrow$ distinguishing the two spins. The generators of $U(4) = U(1)_C \times G_{\text{high}}$ are $\gamma_0 \eta_\mu s_\nu$ where $\mu, \nu = 0, x, y, z$ whereas the generators of $U(2) \times U(2) = U(1)_C \times G$ are $\gamma_0 \eta_{0,z} s_\nu$. We note that our analysis can be equally performed in the band basis which is more convenient for weak coupling. In that case, we can use the valley $\tau = K/K'$ and band $n = C/V$ (conductance/valence) indices instead of Chern and pseudospin.

We first note that the generators for G_{high} (and as a result also the generators of $G \subset G_{\text{high}}$) are all proportional to γ_0 . This means that the pairing function can be decomposed into

different pairing channels in the Chern space which can be considered independently

$$\Delta = \sum_{\mu=0,x,y,z} \Delta_\mu \gamma_\mu, \quad (2)$$

Here, Δ_μ satisfies

$$\Delta_\mu^T = \begin{cases} -\Delta_\mu & : \mu = 0, x, z \\ \Delta_\mu & : \mu = y \end{cases} \quad (3)$$

Although the results in the remainder of this paper hold for arbitrary pairing functions Δ_μ , it is instructive to consider the case of zero-momentum pairing between time-reversal related momenta \mathbf{k} and $\mathbf{k}' = -\mathbf{k}$ given by $\Delta(\mathbf{k}, \mathbf{k}') = \Delta(\mathbf{k}) \delta_{\mathbf{k}, -\mathbf{k}'}$. This allows us to classify the pairing function according to their transformation properties under spatial rotations into s , p , d , etc. We then see from Eq. 3 that $\Delta_{0,x,z}(-\mathbf{k})^T = -\Delta_{0,x,z}(\mathbf{k})$ whereas $\Delta_y(-\mathbf{k})^T = \Delta_y(\mathbf{k})$. Thus, antisymmetric pairing in orbital space $\Delta_\mu^T(\mathbf{k}) = -\Delta_\mu(\mathbf{k})$ is associated with even momentum pairing (s , d , etc) for $\Delta_{0,x,z}(\mathbf{k})$ and with odd momentum pairing (p , f , etc) for $\Delta_y(\mathbf{k})$. For symmetric pairing, the opposite is true. Physically, we expect the pairing between Chern sectors $\Delta_{x,y}$ to be favored compared to pairing within a Chern sector $\Delta_{0,z}$. This can also be seen in the band basis where inter-Chern pairing corresponds to pairing within the conductance or valence bands (among different flavors) whereas intra-Chern pairing corresponds to pairing between conductance and valence bands which is unlikely in the weak coupling limit. However, in the following, we will keep our discussion as general as possible.

Let us briefly comment on the role of spatial symmetries. First, they place constraints on the \mathbf{k} -dependence of the pairing function $\Delta_\mu(\mathbf{k})$. In addition, as shown in the supplemental material, symmetries which exchange valleys, such as mirror or two-fold rotation, relate the two intra-Chern pairing channels $\gamma_\pm = \frac{1 \pm \gamma_z}{2}$ which will then form a higher-dimensional representation of the full symmetry group. These considerations, which are analyzed in detail in the supplemental material, do not affect any of the conclusions we will derive below which only rely on the transformation properties under global symmetries. For notational simplicity, we will usually drop the \mathbf{k} dependence of the pairing function since we are mostly concerned with the symmetry transformation properties under global symmetries which only act on the internal (orbital) indices. We will also drop the μ subscript since the action of the global symmetries do not mix the different Chern channels.

Absence of symmetric charge $2e$ superconductors— Let us first consider the high-symmetry limit with $G_{\text{high}} = \text{SU}(4)$ whose action on Δ is given by Eq. 1. One fundamental observation is that it is not possible to choose a non-zero pairing matrix Δ which transforms as a singlet under $\text{SU}(4)$. Instead, the action of $\text{SU}(4)$ on Δ decomposes into a sum of a 6-dimensional and a 10-dimensional representations. This can be understood by noting that the action of $\text{SU}(4)$ corresponds to the tensor product of two copies of the fundamental

representation **4** which decomposes as $4 \otimes 4 = \mathbf{6} \oplus \mathbf{10}$. The **6** and **10** correspond to the antisymmetric and symmetric tensor representations of $SU(4)$, respectively.

Notice that $SU(4)$ symmetry mixes inter-valley and intra-valley pairing and there is no sense in distinguishing the two. This may seem at odds with our physical expectation that pairing takes places between time-reversal related states which live in opposite valleys. However, note that $SU(4)$ allows for unitary rotations which mix the two valleys. As a result, we can define a modified time-reversal symmetry \mathcal{T}' that acts within the same valley by combining such rotation with the standard time-reversal \mathcal{T} . The existence of intra-valley time-reversal was noted in Ref.²⁶ and more recently studied in Ref.⁴⁴.

Next, we can consider what happens in the more realistic limit where G_{high} is broken down to $G = U(1) \times SU(2) \times SU(2)$. Let us first consider the $SU(2) \times SU(2)$ part. The antisymmetric representation **6** splits into $4 \oplus 1 \oplus 1$ under $SU(2) \times SU(2)$ whereas the symmetric representation splits into $\mathbf{10} = 4 \oplus \mathbf{3} \oplus \mathbf{3}$. These can be understood as follows: first recall the representations of $SU(2)$ are labelled by a half-integer S with dimension $2S + 1$. Thus, the 1D irrep of $SU(2) \times SU(2)$ is simply the singlet representation in both valleys $(S_K, S_{K'}) = (0, 0)$. The 3D irreps above correspond to a singlet in one valley and triplet in the other $(S_K, S_{K'}) = (0, 1)$ or $(1, 0)$ whereas the 4D irreps correspond to $S = 1/2$ in both valleys. Alternatively, this 4D irrep can be understood as the fundamental of $Spin(4)$ which is isomorphic to $SU(2) \times SU(2)$. Note that, unlike the $SU(4)$ case, it is possible to have a $SU(2) \times SU(2)$ singlet. These correspond to the intravalley pairing channels $\propto \eta_{0,z}$. However, these pairing channels carry a charge under $U_V(1)$. Thus, similar to the G_{high} case, there is no symmetric pairing channel under the full G symmetry. We note that our statement for the symmetry group G is consistent with the symmetry analysis of Ref.²³.

Although the above argument applies to all pairing channels, the most natural expectation in the case of the symmetry group G is pairing between time-reversal related electrons in opposite valleys^{9–12} (Otherwise, the Cooper pairs would have a finite momentum on the graphene scale). Restriction to intervalley pairing is equivalent to the condition $\{\Delta, \eta_z\} = 0$. This selects 8 of the 16 generators of $U(4)$ given explicitly by $\eta_{x,y} s_\nu$, $\nu = 0, x, y, z$. Among these generators, four are symmetric and four are antisymmetric. Thus, the action of $SU(2) \times SU(2)$ on the inter-valley pairing (which is invariant under $U_V(1)$) splits into $4 \oplus 4$. For the remainder of this work, we will focus on the physically relevant case of intervalley pairing. The case of intravalley pairing is discussed in the supplemental material for completeness.

Charge $4e$ superconductors— The previous discussion leaves open the more intriguing possibility of a symmetric charge $4e$ condensate. The existence of such condensate can be seen by first writing the charge $4e$ pairing function

$$\Delta_{\alpha\beta\gamma\delta} = \langle c_\alpha c_\beta c_\gamma c_\delta \rangle \quad (4)$$

$\Delta_{\alpha\beta\gamma\delta}$ transforms as

$$\Delta_{\alpha\beta\gamma\delta} \mapsto U_{\alpha\alpha'} U_{\beta\beta'} U_{\gamma\gamma'} U_{\delta\delta'} \Delta_{\alpha'\beta'\gamma'\delta'} \quad (5)$$

A singlet pairing channel is obtained by taking Δ to be proportional to the antisymmetric tensor $\Delta_{\alpha\beta\gamma\delta} \propto \epsilon_{\alpha\beta\gamma\delta}$ since

$$U_{\alpha\alpha'} U_{\beta\beta'} U_{\gamma\gamma'} U_{\delta\delta'} \epsilon_{\alpha'\beta'\gamma'\delta'} = \det U \epsilon_{\alpha\beta\gamma\delta} = \epsilon_{\alpha\beta\gamma\delta} \quad (6)$$

Thus, in contrast to the charge $2e$ superconductor, an $SU(4) = G_{\text{high}}$ symmetric charge $4e$ superconductor is possible. This conclusion also holds for the symmetry group $G \subset G_{\text{high}}$.

Unitary pairing and fractional vortices— We start by discussing the case of unitary pairing²⁴ which is usually energetically favored if the pairing amplitude is the same in all flavors. For unitary pairing, $\Delta \Delta^\dagger$ is proportional to the identity matrix so that we can rescale Δ to be unitary (away from the points where it vanishes).

To study different types of vortices, we start by recalling the action of $U_C(1)$ charge conjugation $c \mapsto e^{i\theta} c$ on the superconducting order parameter: $\Delta \mapsto e^{2i\theta} \Delta$. Thus, for a unitary superconductor described by an $n \times n$ matrix order parameter Δ , the superconducting phase is given by

$$\varphi = \frac{1}{n} \arg \det \Delta \quad (7)$$

which guarantees that the phase changes as $\varphi \mapsto \varphi + 2\theta$ under the action of $U_C(1)$. The phase φ couples to the background gauge field with charge $2e$ according to the Ginzburg-Landau (GL) functional

$$\mathcal{L} \propto (\hbar \partial_\mu \varphi - 2e A_\mu)^2 + \dots \quad (8)$$

Variations with respect to A leads to

$$\int dl_\mu A_\mu = \frac{\hbar}{2e} \int dl_\mu \partial_\mu \varphi \quad (9)$$

Let us see the implications of this for the different symmetry settings.

Let us first discuss the more physical G symmetry and focus on the physically relevant intervalley pairing. This can be decomposed into a symmetric and an antisymmetric component given by

$$\Delta^{S/A} = \begin{pmatrix} 0 & \tilde{\Delta} \\ \pm \tilde{\Delta}^T & 0 \end{pmatrix}_n \quad (10)$$

where $\tilde{\Delta}$ is unitary. Notice that due to the $SU(2) \times SU(2)$ symmetry, the $SU(2)$ spin singlet and triplet components are joined in a single 4-dimensional irrep. As a consequence, we cannot split $\Delta^{S/A}$ further into a valley-singlet spin-triplet channel and a valley-triplet spin-singlet channel^{23,45}. This is only possible in the presence of Hund's coupling which breaks $SU(2)_K \times SU(2)_{K'}$ down to $SU(2)_S$ as we will discuss later. For either the symmetric or antisymmetric representation, we can write $\varphi = \frac{1}{4} \arg \det \Delta = \frac{1}{2} \arg \det \tilde{\Delta}$. Using (9), we see

that if only one of the two eigenvalues of $\tilde{\Delta}$ winds by 2π , the phase φ winds by π leading to a vortex with a flux of $h/4e$.

The case of $SU(4)$ symmetry can be analyzed similarly. For the antisymmetric representation with $\Delta^T = -\Delta$, the eigenvalues come in pairs $\{\lambda, -\lambda\}$ which means that the winding of φ is necessarily a multiple of π leading also to half-vortices. For the symmetric representation, there is no constraint on the winding of individual eigenvalues of Δ , which means that the winding of φ is a multiple of $\pi/2$ leading to *quarter* vortices with flux $h/8e$ in this case. The physically less relevant case of intravally pairing for the symmetry group G is discussed in supplemental material and similarly leads to $h/4e$ vortices for the antisymmetric representation and $h/8e$ vortices for the symmetric one.

Order parameter manifolds for unitary pairing— We will now present an alternative and more abstract argument for the existence of fractional vortices in the different unitary charge $2e$ superconductors. This argument will not rely on any GL functional. Let us again start with the case of interval pairing with G symmetry. The symmetric/antisymmetric pairing channels can be generated starting with an arbitrary symmetric/antisymmetric valley off-diagonal matrix and applying $U(2) \times U(2)$ symmetry according to (1). The resulting pairing matrix is

$$\Delta^{S,A} = \begin{pmatrix} 0 & U_1 U_2^T \\ \pm U_2 U_1^T & 0 \end{pmatrix}_\eta, \implies \tilde{\Delta} = U_1 U_2^T \quad (11)$$

Thus, $\tilde{\Delta}$ is parametrized by a pair of 2×2 unitary matrices U_1 and U_2 modulo the operation $U_1 \mapsto U_1 V$, $U_2 \mapsto U_2 V^*$ for any 2×2 unitary matrix V . This means that $\tilde{\Delta}$ parametrizes the coset space $U(2) \times U(2)/U(2) \simeq U(2)$. The existence of half-vortices can be understood from the decomposition the decomposition $U(2) = \frac{U_C(1) \times SU(2)}{\mathbb{Z}_2}$. This can be written explicitly as $\tilde{\Delta} = e^{i\varphi} V$ with $V \in SU(2)$ where the identification of (φ, V) and $(\varphi + \pi, -V)$ allows for a π vortex in the superconducting phase φ combined with a rotation which sends V to $-V$. On the other hand, a full vortex in $\tilde{\Delta}$ corresponds to a loop in the subgroup $U_C(1)$ generated by $\tilde{\Delta} = e^{i\phi} \mathbb{1}_2$ (where $\mathbb{1}_n$ is $n \times n$ identity matrix) with $\phi : 0 \rightarrow 2\pi$. Thus, the full vortex corresponds to a phase winding which is twice that of a minimal vortex. This can be written as $\pi_1(U(2)) = \mathbb{Z}/2$, where we identify the element $1 \in \mathbb{Z}$ with the fundamental loop in $U_C(1)$ generated by $\tilde{\Delta} = e^{i\phi} \mathbb{1}_2$, $\phi : 0 \rightarrow 2\pi$. The minimal loop in this space $1/2 \in \mathbb{Z}/2$ thus corresponds to a half quantum vortex with $h/4e$ flux.

The existence of half (quarter) vortices for the antisymmetric (symmetric) representations of $SU(4)$ can also be understood by analyzing the order parameter manifolds. For the antisymmetric representation Δ_μ^A , the manifold is generated by acting with $U(4)$ on any given unitary antisymmetric matrix according to (1) leading to

$$\Delta^A = U \eta_y s_0 U^T \quad (12)$$

where $U \in U(4)$. We notice that Δ_μ^A is invariant under multi-

plying U from the right by any unitary matrix K satisfying

$$K \eta_y s_0 K^T = \eta_y s_0 \quad (13)$$

which is the definition of a 4×4 symplectic matrix $K \in Sp(4)$. Thus, Δ_μ^A parametrizes the coset space $U(4)/Sp(4)$. Writing $U = e^{\frac{i}{2}\theta} V$ where $V \in SU(4)$ yields $\Delta = e^{i\theta} V \eta_y s_0 V^T$. We see that we can form a closed path in $U(4)$ by taking $V \mapsto -iV$ and $\theta \mapsto \theta + \pi$. We note however that doing this twice yields a regular vortex since the the path $V \mapsto -V$ can be deformed into a path $\mathbb{1}_4 \mapsto -\mathbb{1}_4$ lying completely in the symplectic group which is identified with the identity. Thus, $\pi_1\left(\frac{U(4)}{Sp(4)}\right) = \mathbb{Z}/2$. A similar argument applies to the symmetric 10-dimensional representation. In this case, the order parameter manifold $\frac{U(4)}{O(4)}$ which admits half-vortices similar to the case of $\frac{U(4)}{Sp(4)}$. However, due to the structure of the $O(4)$ group, we can also form *quarter* vortices. These are obtained by taking a trajectory in $U(4)$ which starts at an element $K_+ \in O(4)$ with $\det K_+ = 1$ and ends at an element K_- with $\det K_- = -1$. Since $O(4) \subset U(4)$ and $U(4)$ is path connected, we can always find such trajectory. This trajectory is a loop in the coset space $\frac{U(4)}{O(4)}$ which involves a winding of the phase of θ by $(2l+1)\frac{\pi}{2}$ yielding a quarter vortex for $l = 0$. An explicit example of such trajectory is given by the diagonal matrix $U = \text{diag}(1, 1, 1, e^{i\theta})$, $\theta : 0 \mapsto \pi$ which connects the identity to the matrix $U = \text{diag}(1, 1, 1, -1)$ which is contained in $O(4)$.

In general, we can derive the flux quantization condition without a GL function by employing the following two axioms: (a) the flux is identified as a homomorphism from the first homotopy group of the order parameter manifold, which we will denote by \mathcal{M}_Δ , to the real numbers $F : \pi_1(\mathcal{M}_\Delta) \mapsto \mathbb{R}$ and (b) the element of $\pi_1(\mathcal{M}_\Delta)$ corresponding to the fundamental loop in $U_C(1)$ generated by $\Delta = e^{i\phi} \mathbb{1}_n$, $\phi : 0 \rightarrow 2\pi$, is assigned a flux of $h/2e$. Using this definition, we can reproduce the half and quarter vortices identified above as follows. We identify the minimal L_{\min} loop in \mathcal{M}_Δ and then find how many times we need to add it to itself to get the fundamental loop L_F in $U_C(1)$. If $mL_{\min} \simeq L_C$, then the flux satisfies $F(L_C) = F(mL_{\min}) = mF(L_{\min})$ since F is a homomorphism. Thus, by identifying the fundamental loop with the element $1 \in \pi_1(\mathcal{M}_\Delta)$ whose flux is $F(1) = h/2e$, we find that the minimal loop is $1/m$ with flux $F(1/m) = F(1)/m = h/(2me)$. For example, the full vortices of the manifold $U(2)$ were identified with the element $1 \in \pi_1(U(2))$ and the half vortices were identified with the element $1/2$. Since the flux F is a homomorphism, we have $2F(1/2) = F(1) = h/2e \implies F(1/2) = h/4e$.

Non-unitary pairing— Let us now see what happens if we lift the restriction of unitary pairing. Non-unitary pairing is expected if the parent state has broken flavor symmetry such that pairing takes place in each flavor separately. Although not very common, there are a few known exam-

ples for non-unitary pairing, particularly in ferromagnetic superconductors^{31,32,46,47} where pairing develops on top of a spin-split Fermi surface and has different amplitude for the different spin species. Another example is the A_1 phase of He_3 ²⁵. In the following, we will show that for non-unitary pairing, it is possible to have no fractional vortices. For this purpose, it suffices to focus on the most physically relevant case of intervalley pairing with $G = \text{U}(1) \times \text{SU}(2) \times \text{SU}(2)$ symmetry. In general, we can specify the pairing function $\tilde{\Delta}$ using the eigenvalues of $\tilde{\Delta}^\dagger \tilde{\Delta}$ which are invariant under the symmetry transformation (1). For inter-valley pairing, the pairing function has the form given in Eq. 10. Thus, the most general form of the pairing functions $\tilde{\Delta}$ is

$$\tilde{\Delta} = U_1 \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} U_2^T \quad (14)$$

For $|\lambda_1| \neq |\lambda_2|$, this corresponds to so-called two gap superconductors with different amplitudes of the gap function for the different electron species³⁷. The GL theory in this case is given by

$$\mathcal{L} = \rho_1 (\partial_\mu \varphi_1 - \frac{2e}{\hbar} A_\mu)^2 + \rho_2 (\partial_\mu \varphi_2 - \frac{2e}{\hbar} A_\mu)^2 + \dots \quad (15)$$

where the two $\text{U}(1)$ phases $\varphi_{1,2} = \arg \lambda_{1,2}$ correspond to the gap functions of the two different spin species. The vortex quantization condition is

$$\int dl_\mu A_\mu = \frac{\hbar}{2e} \int dl_\mu \frac{\rho_1 \partial_\mu \varphi_1 + \rho_2 \partial_\mu \varphi_2}{\rho_1 + \rho_2} \quad (16)$$

We see that this generically leads to fractional vortices which depend on the relative stiffness of the two superconductors when both λ_1 and λ_2 are non-zero³⁷.

The existence of fractional vortices can also be understood from our more abstract definition in the previous section. The order parameter manifold can be understood as follows. Allowing λ_1 and λ_2 to be complex, we can absorb the overall phases of U_1 and U_2 as well as the effect of the transformations $U_{1,2} \mapsto e^{i\phi_{1,2}\sigma_z} U_{1,2}$ in the phases of λ_1 and λ_2 . This means that $\tilde{\Delta}$ parametrizes the manifold $\text{U}(1) \times \text{U}(1) \times S^2 \times S^2$ which can be understood as follows. Due to the independent $\text{SU}(2)$ spin rotation in each valley, we can choose the spin quantization axis independently in the two valleys leading to two unit vectors parametrizing $S^2 \times S^2$. The two $\text{U}(1)$ phases describe the phase of the pairing function for the two spin species. Note that $\pi_1(\text{U}(1) \times \text{U}(1) \times S^2 \times S^2) = \mathbb{Z} \oplus \mathbb{Z}$ where the fundamental vortex in $\text{U}(1)$ corresponds to the element $(1, 1) \in \mathbb{Z} \oplus \mathbb{Z}$ which represents 2π phase winding in both $\text{U}(1)$ phases. The flux quantization condition can then be written as $F[(1, 1)] = h/2e = F[(0, 1)] + F[(1, 0)]$. In the absence of any symmetry relating $(0, 1)$ and $(1, 0)$, i.e. the two spin species, we can write $F[(0, 1)] = xh/2e$ for some arbitrary $0 \leq x \leq 1$ which implies $F[(1, 0)] = (1 - x)h/2e$. Comparing with (16), we can identify x with $\rho_2/(\rho_1 + \rho_2)$. Note that whenever the two spin species are related by an unbroken symmetry, $x = 1/2$ and we recover half vortices as in the unitary case.

In the extreme non-unitary limit where pairing takes place in only one species, i.e. one of $\lambda_{1,2}$ (and consequently $\rho_{1,2}$) is zero, there are only regular $h/2e$ vortices. This can be seen by noting that the order parameter manifold for this case reduces to $\text{U}(1) \times S^2 \times S^2$ because phase rotations on the vanishing eigenvalue have no effect. This manifold has a single $\text{U}(1)$ with the GL functional (8) leading to the same quantization condition (9) where the winding of φ has to be a multiple of 2π leading to standard $h/2e$ flux. This serves to illustrate that the non-unitary pairing allows for a conventional scenario with a charge $2e$ superconductor that does not host fractional vortices.

No-go theorem— In summary, we have proved the following general statement: *For any pairing in the ideal $\text{SU}(4)$ symmetric model or the more realistic $\text{U}(1) \times \text{SU}(2) \times \text{SU}(2)$ model, one of three possibilities should occur: (i) the superconducting pairing is non-unitary, (ii) the superconducting condensate has charge $2e$ but breaks the global symmetry and admits at least half quantum vortices with flux of $h/4e$, or (iii) the superconducting condensate has charge of at least $2me$ with $m > 1$ with vortices carrying $h/2me$ flux.*

Note an important difference between the cases (ii) and (iii). The fractional vortices of the charge $2e$ superconductor combine a winding in the overall phase with a winding in internal orbital indices. Whereas the former is compensated by external magnetic flux, the latter is not, leading to logarithmically divergent energy at long distances. This means that these vortices are only relevant at sufficiently short distances e.g. in mesoscopic geometries. Such type of vortices were already observed in mesoscopic samples in liquid Helium 3³⁵ and spin-triplet condensates⁴⁸. On the other hand, vortices of a charge $2me$ superconductor with $m > 1$ which carry a fractional flux of $h/2me$ have finite energy and can be observed in the infinite system.

Nonzero temperature and BKT transitions— Here we discuss how the different scenarios we have discussed manifest at nonzero temperatures. At nonzero temperature in two spatial dimensions fractional vortices in charge $2e$ superconductors manifest as pure phase windings of a higher charge superconductor once a non-zero temperature is included; above a critical temperature T_c the superconductivity is lost due to a BKT transition driven by the condensation of these fractional vortices^{6,49–52}.

We now describe this picture for the candidate superconductors allowed by our statement, starting with the case of $\text{SU}(4)$ symmetry. Mermin-Wagner theorem forbids the existence of the charge $2e$ superconductors at $T > 0$ since, as we discussed earlier, such superconductor necessarily breaks $\text{SU}(4)$. On the other hand, algebraic order in the $\text{U}(1)$ phase may still persist. In particular, the unitary charge $2e$ order parameters $\Delta^{\text{S,A}}$ that parameterize the manifolds $\text{U}(4)/\text{Sp}(4)$ and $\text{U}(4)/\text{O}(4)$ respectively must be disordered, but the respective submanifolds $\text{U}(1)/\mathbb{Z}_4$ and $\text{U}(1)/\mathbb{Z}_8$ can retain XY order that melts for sufficiently high temperatures through a BKT transition. The BKT transition is driven by the condensation of half quantum vortices and quarter-quantum vortices

Symmetry	Irreps	generators	SC Manifold (unitary pairing)	Vortices
SU(4)	6	$\eta_y s_{0,x,z}, \eta_{0,x,z} s_y$	$\frac{U(4)}{Sp(4)}$	$\mathbb{Z}/2$
	10	$\eta_{0,x,z} s_{0,x,z}, \eta_y s_y$	$\frac{U(4)}{O(4)}$	$\mathbb{Z}/4$
SU(2) \times SU(2) \simeq Spin(4)	4	$\eta_y s_{0,x,z}, \eta_x s_y$	U(2)	$\mathbb{Z}/2$
Inter-valley pairing	4	$\eta_y s_y, \eta_x s_{0,x,z}$	U(2)	$\mathbb{Z}/2$

TABLE I: Summary of the order parameter manifolds and the irreducible representations for the unitary pairing charge $2e$ superconductors. For the ideal SU(4) case, the symmetry action splits into a 6-dimensional antisymmetric and a 10-dimensional symmetric irrep whose corresponding manifolds admit half and quarter quantum vortices respectively. For the more realistic $U(1)_V \times SU(2) \times SU(2)$, we focus on the physically relevant inter-valley pairing which is $U(1)_V$ singlet and splits into two 4-dimensional irreps under $SU(2) \times SU(2)$ whose manifolds admit half vortices.

for the SU(4) antisymmetric and symmetric superconductors, respectively. We may interpret this in terms of higher charge superconductors as follows. For $T > 0$ the charge $2e$ superconductors are unstable: $\Delta^{A,S}$ is disordered. However, for the antisymmetric case,

$$\Delta_{4e} = \varepsilon^{\alpha\beta\gamma\delta} \Delta_{\alpha\beta}^A \Delta_{\gamma\delta}^A \quad (17)$$

is an SU(4) singlet and parameterizes $U(1)/\mathbb{Z}_4$. It retains algebraic order and gives rise to a charge $4e$ superconductor for $T > 0$. The $h/4e$ vortices in Δ_{4e} are the finite temperature remnants of the half quantum vortices in Δ_{2e}^A . Similarly the operator

$$\Delta_{8e} = \varepsilon^{\alpha\beta\gamma\delta} \varepsilon^{\alpha'\beta'\gamma'\delta'} \Delta_{\alpha\alpha'}^S \Delta_{\beta\beta'}^S \Delta_{\gamma\gamma'}^S \Delta_{\delta\delta'}^S \quad (18)$$

is an SU(4) singlet built out of Δ^S that parameterizes $U(1)/\mathbb{Z}_8$. It can have algebraic order for $T > 0$ and host $h/8e$ vortices.

For intervalley pairing in the $U(1) \times SU(2) \times SU(2)$ symmetric case, the operator

$$\tilde{\Delta}_{4e} = \varepsilon^{ab} \varepsilon^{a'b'} \tilde{\Delta}_{aa'} \tilde{\Delta}_{bb'} \quad (19)$$

is a singlet with charge $4e$. Here we used (10) to write the order parameter in terms of an arbitrary unitary matrix $\tilde{\Delta}$. The unprimed and primed indices transform under the first and second copy of SU(2), rotations in valley K and K' , respectively. For the antisymmetric representation (17) and (19) coincide up to a constant.

It is possible that at $T = 0$ these higher charge superconductors are the ground state anyway and there is no charge $2e$ order at $T = 0$. The $T > 0$ phenomena of higher charge superconductivity are similar in both cases, though the correlation length for charge $2e$ order will diverge exponentially as $T \rightarrow 0$ if a charge $2e$ superconductor is the quantum ground state. For Josephson junctions with linear extent smaller than the charge $2e$ correlation length the system will behave like a charge $2e$ superconductor.

Symmetry Lowering by Intervalley Hund's Coupling— Ultimately $SU(2) \times SU(2)$ symmetry is broken down to its diagonal subgroup of overall spin and charge rotations $SU(2)$ by intervalley Hund's terms^{23,45,53}. These terms are suppressed

by the lattice to moiré scale $a/L \ll 1$, and have magnitude $|J_H| \approx 0.2 - 0.5 \text{meV}$ ⁵⁴. Nonzero J_H splits the **4** representation of $SU(2) \times SU(2)$ to $\mathbf{3} \oplus \mathbf{1}$ of $SU(2)$, spin triplet and spin singlet respectively²³. The sign of J_H is unknown and determines whether spin triplet or spin singlet states are favored.

If the singlet state is favored, then at zero temperature there will only be full quantum vortices. The critical temperature for the spin-singlet pairing will be smaller than the critical temperature for charge $4e$ order for sufficiently small J_H . By analogy with the Heisenberg model with a small easy plane bias⁵⁵⁻⁵⁸, we expect the critical temperature to have the following form as $J_H \rightarrow 0$

$$T_{C \text{ singlet}, 2e} = \frac{c_{1\rho}}{\log \frac{c_{2\rho}}{J_H}}, \quad (20)$$

where $c_{1,2}$ are numbers and ρ is the phase stiffness. Thus even with small but finite J_H due to the slow variation of the logarithm, there may not be a significant separation of scales between $T_{C \text{ singlet}, 2e}$ and $T_{C 4e} \propto \rho$. On the other hand, if the triplet state is favored and the pairing is unitary, half quantum vortices persist. An extreme non-unitary triplet state is worth mentioning as well, and was considered in Ref. 59. Here the order parameter actually parameterizes $SO(3)$ which has fundamental group \mathbb{Z}_2 – a double vortex is no longer a metastable configuration. Here there is no BKT transition²³, however; the \mathbb{Z}_2 nature of the vortices implies that there is no composite operator built from the zero temperature order parameter that retains XY order for nonzero temperatures. As we discuss later, this scenario seems incompatible with experiments which observe signatures of BKT physics.

Spin orbit coupling at very small scales may also break the overall SU(2) spin rotation and stabilize a charge $2e$ state with no half quantum vortices at nonzero temperature. However, spin orbit coupling is extremely weak and so even with a logarithmic critical temperature dependence similar to (20) we expect that there will be a large window where the charge $2e$ order has melted but the charge $4e$ order is still present with observable half quantum vortices.

Skyrmions with charge $2e$ and $4e$ and superconductivity— Let us now discuss how the different pairing scenarios discussed in this work arise very naturally from the topological mechanism of superconductivity proposed in Ref. 18 which

we review briefly below. In that work, the authors proposed a topological mechanism for superconductivity based on pairing topological skyrmion textures. They considered a simplified problem where the spin degree of freedom is neglected. This is likely relevant to the extreme non-unitary limit where superconductivity takes place in only one spin species. In this limit, the system consists of two bands in each Chern sector which are labelled by the pseudospin index η . The low energy states of the system with vanishing Chern number correspond to ferromagnetic order of the pseudospin vector in each Chern sector which are coupled antiferromagnetically between the opposite Chern sectors. A pseudo-spin skyrmion texture in the $C = \pm$ sector carries charge $\pm e$. The antiferromagnetic coupling between the opposite Chern sectors leads to an attractive interaction between a skyrmion in one sector and an antiskyrmion in the opposite sector despite having the same charge. This attraction leads to the formation of a charge $2e$ bound state which leads to superconductivity upon condensation.

Let us now see how this picture changes when we consider the fully spinful model in the absence of flavor symmetry breaking where unitary pairing is expected. In this case, each Chern sector contains four bands labelled by spin and pseudospin. The low energy states with zero total Chern number are specified by filling 2 out of the 4 bands within each sector²⁶. Such state is parametrized by the 4×4 matrices Q_{\pm} in the spin-pseudospin space satisfying $Q_{\pm}^2 = 1$ and $\text{tr} Q_{\pm} = 0$. Within each Chern sector, Q_{\pm} parametrizes the manifold $\frac{U(4)}{U(2) \times U(2)}$ which, similar to the 2-sphere $S^2 = \frac{U(2)}{U(1) \times U(1)}$, admits skyrmion textures due to $\pi_2 \left(\frac{U(4)}{U(2) \times U(2)} \right) = \mathbb{Z}$. A general skyrmion texture in this manifold intertwines the pseudospin and spin degrees of freedom. We can understand the different types of possible skyrmions by restricting ourselves to some simple limiting cases. One possibility is to consider a pseudospin skyrmion in only one of the spin species. Such a skyrmion can be written as

$$Q_+(\mathbf{r}) = \begin{pmatrix} \mathbf{n}_0 \cdot \boldsymbol{\eta} & 0 \\ 0 & \mathbf{n}_{\text{sk}}(\mathbf{r}) \cdot \boldsymbol{\eta} \end{pmatrix}_{\uparrow/\downarrow} \quad (21)$$

where \mathbf{n}_0 is some fixed vector whereas $\mathbf{n}_{\text{sk}}(\mathbf{r})$ describes a skyrmion texture. This skyrmion is the same as the spinless pseudospin skyrmion and will consequently carry an electric charge of $\pm e$. Another more symmetric possibility is to take the same skyrmion in both spin directions so that

$$Q_+(\mathbf{r}) = s_0 \mathbf{n}_{\text{sk}}(\mathbf{r}) \cdot \boldsymbol{\eta} \quad (22)$$

This skyrmion consists of two copies of the spinless charge e skyrmion and will thus carry a total charge of $2e$. Notice that these Chern skyrmions cannot condense since they feel a net magnus force (effective magnetic field). However, as in the spinless case, the two Chern sectors are antiferromagnetically decoupled by a term of the form $J \text{tr} Q_+ Q_-$ which favors $Q_+ = -Q_-$ leading to an attractive interaction between skyrmions in one Chern sector and antiskyrmions with the same charge in the opposite sector. The resulting bound

state feels no net magnetic field and can condense leading to a charge $2e$ or charge $4e$ superconductor. The former is realized by pairing the single spin skyrmions in Eq. 21 which is expected to break the $SU(4)$ symmetry while the latter is realized by pairing the spin-singlet skyrmions in Eq. 22 which is expected to yield an $SU(4)$ -singlet. A detailed analysis of the symmetry transformation properties of the different skyrmion superconductors will be provided in an upcoming work⁶⁰.

The discussion above can be substantiated by writing the field theory derived in Ref.¹⁸ (although that work focused on the spinless limit, the field theory derived was valid for any number of flavors)

$$\mathcal{L}[Q_+, Q_-] = \mathcal{L}_+[Q_+] + \mathcal{L}_-[Q_-] + J \text{tr} Q_+ Q_- \quad (23)$$

with $\mathcal{L}[Q]$ given by

$$\begin{aligned} \mathcal{L}_{\pm}[Q] &= \frac{\rho}{2} \text{tr}(\partial_i Q)^2 + ie A_{\mu} J_{\pm, \mu}[Q] + \mathcal{L}_B[Q] \pm \mathcal{L}_{\text{WZW}}[Q], \\ J_{\pm, \mu}[Q] &= \pm \frac{i\epsilon_{\mu\nu\lambda}}{16\pi} \text{tr} Q \partial_{\nu} Q \partial_{\lambda} Q \end{aligned} \quad (24)$$

Here, \mathcal{L}_B is the Berry phase term⁶¹ whereas \mathcal{L}_{WZW} is a Wess-Zumino-Witten term which ensures that charge e skyrmions are fermions as expected. This term, which was not included in Ref.¹⁸, will not play a role in our discussion but is generally important to obtain the right symmetry properties as we will show in a subsequent paper⁶⁰. What is important to our current discussion is the skyrmion charge given by integrating $J_{\pm, 0}[Q]$. We see that for the single spin skyrmion defined in (21), the expression in (24) simplifies to $\frac{1}{4\pi} \mathbf{n}_{\text{sk}} \cdot (\partial_x \mathbf{n}_{\text{sk}} \times \partial_y \mathbf{n}_{\text{sk}})$ which corresponds to the charge density of a standard spin skyrmion that integrates to 1, thus indicating it is indeed a charge e skyrmion. On the other hand, the spin singlet skyrmion defined in (22) has twice the topological density leading a total electrical charge of $2e$. An effective field theory describing the different insulators and skyrmion superconductors analogous to the CP^1 theory of Ref.¹⁸ will be presented in Ref.⁶⁰.

In conclusion, it is possible to realize the three possibilities – non-unitary pairing, unitary charge $2e$ and charge $4e$ superconductivity – within the skyrmion mechanism. The former corresponds to the spinless limit where pairing takes place in a single spin species. The latter two possibilities are realized in the full spinful model by pairing charge e skyrmions (21) leading to an $SU(4)$ -breaking superconductor or charge $2e$ objects (22) leading to an $SU(4)$ -symmetric superconductor.

Experimental consequences— To discuss the experimental implications of the statement above for TBG, let us first review some experimental facts. In many experiments^{1,2,4,5,43,62}, superconductivity is observed in the vicinity of a flavor-symmetry-breaking insulator at $\nu = -2$, which suggests non-unitary pairing. However, there are a few samples⁴ where superconductivity is observed close to charge neutrality with no signature of flavor symmetry breaking nearby. This is likely to be unitary pairing where cases (ii) or (iii) of our theory should apply. In addition, several groups^{4,5,43} have observed super-

conductivity in the absence of an insulator close to $\nu = \pm 2$. The absence of insulator is not sufficient to rule out flavor symmetry breaking since it does not exclude the possibility of a flavor-polarized metal. A stronger evidence for flavor polarization is provided by the presence of a Landau fan emanating for $\nu = \pm 2$ indicating a Fermi surface reconstruction. Thus, our general criteria for unitary vs non-unitary pairing can be directly translated to the degeneracy of the Landau fan in the region where superconductivity is observed with the full four-fold degeneracy giving rise to unitary pairing and any reduced degeneracy yielding non-unitary pairing. We leave open the unusual scenario where non-unitary pairing develops on top of a flavor symmetric state. This was observed in UTe_2 ⁶³ and was argued to occur due to proximity to a ferromagnetic transition⁶⁴.

Having established the connection to the experimental setup in TBG, let us now investigate the implications of our results. In the vicinity of $\nu = \pm 2$, the most likely flavor polarization is spin which has two possibilities depending on the sign of Hund's coupling. For the ferromagnetic sign of Hund's coupling, the flavor polarized state is a spin ferromagnet which favors a spin-triplet superconductor where pairing takes place in a single spin species. This corresponds to the scenario discussed in Ref.⁵⁹ with only \mathbb{Z}_2 vortices. In principle, this implies the superconductor cannot exist at finite temperature and has no BKT transition. This appears hard to reconcile with the robust pairing state observed in experiments. On the other hand, antiferromagnetic Hund's coupling results a non-unitary pairing between anti-aligned spin directions in the two valleys. This state is a superposition of a spin singlet and the $L_z = 0$ component of spin-triplet. At finite temperature, $\text{SU}(2)$ rotation symmetry is restored and the spin-triplet component disorders leading to a spin-singlet superconductor. In the absence of spin anisotropies, this appears to be the likely candidate for the experimentally observed superconductivity.

For superconductors observed in the absence of flavor polarization, we expect unitary pairing with fractional vortices or higher charge superconductivity. Ordinary half quantum vortices are not present in large systems due to logarithmically divergent energies associated with their windings in flavor space. While the phase winding costs finite energy due to the presence of magnetic flux in the vortex core, there is no physical gauge flux that can cancel the flavor space winding. However, as discussed above, in two spatial dimensions fractional vortices in charge $2e$ superconductors manifest as pure phase windings of a higher charge superconductor once a non-zero temperature is included^{6,49,52}.

This has the following implications for the periodicity of the Fraunhofer patterns in magnetic Josephson junctions or SQUIDS. If the ground state is a charge $2me$ superconductor with $m > 1$, this will be observed as a modified periodicity of the Fraunhofer pattern with period $2\pi/m$ for any temperature below the KT transition. On the other hand, if the ground state is a charge $2e$ superconductor, we can only observe the modified periodicity for sufficiently large junctions

whose size exceed the charge $2e$ correlation length. The latter decreases with increasing temperature making it likely to observe the modified periodicity at sufficiently large temperatures provided that they are still below the KT temperature.

In mesoscopic annular geometries it may be possible to directly observe half quantum vortices as $h/4e$ jumps in the flux threaded through the hole of the annulus. Mesoscopic geometries are ideal to avoid the logarithmically divergent energy cost associated with the unscreened flavor space winding³⁷⁻³⁹ and were previously used to observe half quantum vortices^{35,48,65}.

As discussed above, a Hund's coupling that favors spin singlet pairing will stabilize a unitary charge $2e$ superconductor without half quantum vortices. Because the critical temperature (20) goes to zero very slowly for small J_H , it is likely that experiments done at cold temperatures would not observe such vortices if the Hund's coupling favors singlet pairing. However, the lack of a large separation of scales does not exclude a potentially large temperature window where the singlet Hund's pairing has melted but the charge $4e$ condensate has not. Within this window one would still observe fractional vortices.

Fractional vortices have previously received much attention due to the promise of Majorana zero modes⁶⁶⁻⁶⁸. However, these zero modes are specific to p -wave pairing where winding in a single spin mimics a full quantum vortex for a spinless $p + ip$ superconductor. Full quantum vortices are not expected to host Majorana zero modes in spinless TBG, and so half quantum vortices in spinful TBG also will not host them. However, both full and half quantum vortices are expected to host multiple protected fermionic zero modes if the pairing is between Chern sectors and valleys. We will discuss these zero modes in more depth in a subsequent paper.

Conclusion— In conclusion, we have proven a no-go theorem under mild assumptions which constraints the properties of superconducting condensates in TBG, particularly for regions in the phase diagram where flavor polarization is absent. The main outcome of our analysis is that such regions present ideal hunting grounds for unconventional and exotic types of superconductivity. Our results are independent of the underlying mechanism of superconductivity and are equally applicable to weak coupling BCS-type superconductor as well as strong coupling superconductors. We hope our study will inspire future experiments to look for signatures of fractional vortices and higher charge condensates in these superconducting regions.

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Appendix A: Transformation under spatial symmetries

In the main text, we only considered the transformation properties of the pairing function under global symmetries. Here, we consider the spatial symmetries. We follow the notations of Refs.^{18,26} by writing the microscopic symmetries acting on the microscopic annihilation operators $f_{\mathbf{k}}$ which is a vector in valley (τ), sublattice (σ), spin (s), and layer (μ):

$$C_3 : f_{\mathbf{k}} \mapsto e^{i\frac{\pi}{3}\sigma_z\tau_z} f_{O_3\mathbf{k}}, \quad C_2 : f_{\mathbf{k}} \mapsto \sigma_x\tau_x f_{-\mathbf{k}}, \quad M_y : f_{\mathbf{k}} \mapsto \sigma_x\mu_x f_{m_y\mathbf{k}} \quad (\text{A1})$$

where O_3 denotes the standard action of threefold rotation on a 2D vector and $m_y(k_x, k_y) = (k_x, -k_y)$. Note that all these symmetries acts trivially on the spin index due to the SU(2) rotation symmetry. Next, we project these symmetries onto the flat bands. Due to the finite sublattice polarization, we can label the wavefunctions with the sublattice σ their weight is concentrated on²⁶, which means that the flatband electron annihilation operators are labelled by valley (τ), sublattice (σ), and spin (s). A more convenient basis is obtained by replacing the valley-sublattice indices with the the Chern (γ) - pseudospin (η) indices defined via¹⁸:

$$\gamma = (\sigma_x, \sigma_y\tau_z, \sigma_z\tau_z), \quad \eta = (\sigma_x\tau_x, \sigma_x\tau_y, \tau_z) \quad (\text{A2})$$

Due to the non-trivial band topology of the flat bands, some symmetries acquire a \mathbf{k} -dependence when projected onto them. This dependence was derived in detail in Refs.^{18,26} leading to the results summarized in the table below

basis	C_2	C_3	M_y
microscopic	$\sigma_x\tau_x$	$e^{i\frac{\pi}{3}\sigma_z\tau_z}$	$\sigma_x\mu_x$
projected	$\sigma_x\tau_x e^{i\theta_2(\mathbf{k})}$	$e^{i\theta_3(\mathbf{k})\sigma_z\tau_z}$	$\sigma_x e^{i\theta_y(\mathbf{k})\sigma_z\tau_z}$
projected (γ, η)	$\eta_x e^{i\theta_2(\mathbf{k})}$	$e^{i\theta_3(\mathbf{k})\gamma_z}$	$\gamma_x e^{i\theta_y(\mathbf{k})\gamma_z}$

Let us now consider the transformation properties of the pairing function. We restrict ourselves to the physically relevant case of opposite momentum pairing:

$$\Delta_{\mathbf{k}} = c_{\mathbf{k}} c_{-\mathbf{k}}^T \quad (\text{A3})$$

which behaves under spatial symmetries as

$$c_{\mathbf{k}} \mapsto U_{\mathbf{k}} c_{O\mathbf{k}} \quad \Longrightarrow \quad \Delta_{\mathbf{k}} \mapsto U_{\mathbf{k}} \Delta_{O\mathbf{k}} U_{-\mathbf{k}}^T \quad (\text{A4})$$

As in the main text (Eq. 3), we can use the fact the global symmetries do not have structure in the Chern space γ to decompose the gap function into different channels

$$\Delta(\mathbf{k}) = \Delta_x(\mathbf{k})\gamma_x + \Delta_y(\mathbf{k})\gamma_y + \Delta_+(\mathbf{k})\frac{1+\gamma_z}{2} + \Delta_-(\mathbf{k})\frac{1-\gamma_z}{2} \quad (\text{A5})$$

The antisymmetry of the full pairing function $\Delta(\mathbf{k})^T = -\Delta(-\mathbf{k})$ means that

$$\Delta_{x,\pm}(\mathbf{k})^T = -\Delta_{x,\pm}(-\mathbf{k}), \quad \Delta_y(\mathbf{k})^T = \Delta_y(-\mathbf{k}) \quad (\text{A6})$$

The transformation properties under C_2 , C_3 and M_y can now be obtained:

$$C_2 : \Delta_{x,y,\pm}(\mathbf{k}) \mapsto \eta_x \Delta_{x,y,\pm}(-\mathbf{k}) \eta_x, \quad (\text{A7})$$

$$C_3 : \Delta_{x,y}(\mathbf{k}) \mapsto \Delta_{x,y}(O_3 \mathbf{k}), \quad \Delta_{\pm}(\mathbf{k}) \mapsto e^{\pm 2i\theta_3(\mathbf{k})} \Delta_{\pm}(O_3 \mathbf{k}) \quad (\text{A8})$$

$$M_y : \Delta_x(\mathbf{k}) \mapsto \Delta_x(m_y \mathbf{k}), \quad \Delta_y(\mathbf{k}) \mapsto -\Delta_y(m_y \mathbf{k}), \quad \Delta_{\pm}(\mathbf{k}) \mapsto e^{\pm 2i\theta_y(\mathbf{k})} \Delta_{\mp}(m_y \mathbf{k}). \quad (\text{A9})$$

The last two equations imply that the Δ_{\pm} pairing channels are tied together in a single irrep of the spatial symmetry group, whereas Δ_x and Δ_y transform as separate irreps. To understand the action of C_2 , we decompose each Δ_{μ} into symmetric and antisymmetric intervalley pairing and an intravalley pairing. For intervalley pairing considered in the main text

$$\Delta^{S/A}(\mathbf{k}) = \begin{pmatrix} 0 & \tilde{\Delta}^{S/A}(\mathbf{k}) \\ \pm \tilde{\Delta}^{S/A}(\mathbf{k})^T & 0 \end{pmatrix} \implies \tilde{\Delta}^{S/A}(\mathbf{k}) = \pm \tilde{\Delta}^{S/A}(-\mathbf{k})^T \quad (\text{A10})$$

Combining with Eq. A6 yields

$$\tilde{\Delta}_{x,\pm}^{S/A}(\mathbf{k})^T = \mp \tilde{\Delta}_{x,\pm}^{S/A}(\mathbf{k}), \quad \tilde{\Delta}_y^{S/A}(\mathbf{k})^T = \pm \tilde{\Delta}_y^{S/A}(\mathbf{k}) \quad (\text{A11})$$

This represents a restriction on the individual blocks $\tilde{\Delta}$ in the different pairing channels \pm, x, y for symmetric and antisymmetric pairing functions. Finally, we consider intravalley pairing

$$\Delta(\mathbf{k}) = \begin{pmatrix} \Delta_K(\mathbf{k}) & 0 \\ 0 & \Delta_{K'}(\mathbf{k}) \end{pmatrix}, \quad \implies \Delta_K(\mathbf{k}) = \Delta_{K'}(-\mathbf{k}) \quad (\text{A12})$$

which dictates that the pairing in both valleys has the same gap and symmetry properties (singlet or triplet).

Appendix B: Fractional vortices for intravalley pairing

Let us now consider the case of intra-valley pairing. This is not likely to be the relevant case physically but we include it here for completeness. Assuming unitary pairing, we can write the most general form for intra-valley pairing as

$$\Delta = \begin{pmatrix} \Delta_K & 0 \\ 0 & \Delta_{K'} \end{pmatrix}, \quad \Delta_K \Delta_K^\dagger = \Delta_{K'} \Delta_{K'}^\dagger \propto \mathbb{1} \quad (\text{B1})$$

If both Δ_K and $\Delta_{K'}$ are antisymmetric, which corresponds to the singlet channel of $SU(2)$ in each valley, then we can use the same considerations as in the main text to determine the type of vortices with the GL Lagrangian

$$\mathcal{L} = \rho_K (\partial_\mu \varphi_K - \frac{2e}{\hbar} A_\mu)^2 + \rho_{K'} (\partial_\mu \varphi_{K'} - \frac{2e}{\hbar} A_\mu)^2 + \dots \quad (\text{B2})$$

leading to the vortex quantization condition

$$\int dl_\mu A_\mu = \frac{\hbar}{2e} \int dl_\mu \frac{\rho_K \partial_\mu \varphi_K + \rho_{K'} \partial_\mu \varphi_{K'}}{\rho_K + \rho_{K'}} \quad (\text{B3})$$

Symmetries such as C_2 and time reversal exchange valleys and therefore relate Δ_K and $\Delta_{K'}$ as well as ρ_K and $\rho_{K'}$. As long as the superconductivity develops in a state that preserves one of C_2 , time reversal or any combination of one of them with an internal symmetry, we expect $\rho_K = \rho_{K'}$ for unitary pairing. We then obtain $h/4e$ vortices.

This conclusions can also be verified by considering the manifold of the superconducting order parameter by writing $\Delta = s_y e^{i(\phi\eta_0 + \phi_V \eta_z)}$. Thus, we can form a loop in the space of Δ where both ϕ and ϕ_V go from 0 to π . This will correspond to a half-vortex in the overall superconducting phase ϕ . This reflects the nature of the manifold of the superconducting order parameter $\frac{U_C(1) \times U_V(1)}{\mathbb{Z}_2}$ which admits half-vortices. This case is similar to the case of non-unitary pairing considered in the main text where π_1 of the superconducting manifold is $\mathbb{Z} \oplus \mathbb{Z}$ and vortices with arbitrary flux $xh/2e$ are in principle allowed. However, the presence of any symmetry relating the two valleys would force $x = 1/2$ leading to half quantum vortices.

If both C_2 and time reversal are broken in the parent state and there is no other unbroken symmetry that relates the two valleys

then ρ_K and $\rho_{K'}$ may be different. In such a state we would expect nonunitary pairing since there is also no symmetry relating Δ_K and $\Delta_{K'}$ which means unitary pairing should not be stable to RG flows; it must be fine tuned. If we nonetheless fine tune to obtain unitary pairing, and furthermore fine tune one of the stiffnesses to zero despite a nonzero pairing potential, we obtain conventional $h/2e$ vortices. We do not take this latter scenario into account for our main theorem due to the extreme amounts of unphysical fine tuning, but detailed its existence here for completeness.

Next, we consider the case where either Δ_K or $\Delta_{K'}$ is symmetric, i.e. SU(2) triplet. The existence of fractional vortices in this case follows immediately from the existence of half-vortices in the spin-triplet case whose manifold is $U(2)/O(2)$ with $\pi_1(U(2)/O(2)) = \mathbb{Z}/2$. When included in the bigger order parameter for both valleys, this yields fractional vortices. For instance, if pairing is triplet in both valleys, this will be a quarter vortex similar to the symmetric SU(4) case considered in the main text.