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Optically-modulated tunneling current of dressed electrons in graphene and a dice lattice

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Based on the transmission coefficient of tunneling electrons, we have presented tunneling current and conductivity across a square-potential barrier for both graphene and α - \mathcal{T}_3 lattices under a linearly-polarized off-resonant dressing field. The presence of such a dressing field introduces an anisotropy factor in the energy dispersion of tunneling electrons so that the cross section of a Dirac-cone appears as elliptical. Consequently, the field-polarization controlled major axis of the ellipse will be misaligned with the normal direction of a barrier layer in the tunneling system, which exhibits an asymmetric Klein-paradox for an off-normal-direction tunneling. The resulting tunneling current in this system is calculated by using a transmission coefficient and a longitudinal group velocity (different from a longitudinal momentum) of electrons. By presenting numerically calculated tunneling conductivity modified by a laser dressing field, we demonstrate a significant enhancement of electrical conductivity by external laser-field intensity, which is expected to be crucial in application of ultrafast optical modulation of opto-electronic devices for photo-detection and fiber-optic communication.

I. INTRODUCTION

The α - \mathcal{T}_3 model represents an innovative and unusual type of Dirac materials in which their atomic structure leads to a pseudospin-1 Dirac-Weyl model Hamiltonian for electrons and an additional energy band which is completely flat and symmetric with respect to the valence and conduction bands corresponding to the upper and lower parts of the Dirac cone in graphene. [1–5] This type of energy bandstructure results in a variety of truly unique and previously unknown electronic, [6] topological, [7] collective, [8, 9] optical [10, 11] and magnetic properties [12–16] of α - \mathcal{T}_3 model.

The atomic composition of an α - \mathcal{T}_3 lattice consists of a regular graphene-like honeycomb lattice with additional fermionic atoms at the center of each hexagon, which are referred to as hub atoms in contrast to the rim atoms located at each of six hexagonal vertices. Here, the ratio between the hub-rim and rim-rim electron hopping coefficients defines a parameter α with its limiting values $\alpha = 0$ for graphene and $\alpha = 1$ for a dice lattice [17, 18]. One of the first realizations of a crystal lattice with a non-dispersive flat energy band in derivations of the bandstructure dates back to nearly a decade ago [19, 20]. At the present time, there has already been copious and convincing evidence for experimental confirmation and successful-fabrication efforts in producing different types of α - \mathcal{T}_3 materials, in which the most well-known cases include a three-layer structure of SrTiO₃/SrIrO₃/SrTiO₃ lattices with a cubic crystal symmetry, [4] Lieb and Kagome lattices with additional atoms at the edges [21–26], and a number of other planar materials [27–29] as well.

On the other hand, Floquet engineering [30–33] appears as a crucial tool for the modification of principal electronic properties of a planar structure [34, 35] or the surface states in a three-dimensional material [36–39] by employing an off-resonance dressing field with its frequency substantially higher than the characteristic energies of a system, such as the Fermi energy. Encouraged by rapid technological advancements in intense lasers and microwave field sources, lots of new field-induced phenomena in recently discovered low-dimensional structures, especially Dirac materials, have been demonstrated both theoretically and experimentally. [40, 41] Physically, a strong off-resonance field applied to a two-dimensional material enables the appearance of so-called dressed states, i.e., a quantum-mechanically composite involving both irradiated electrons and the applied field. Such a quantum composite acquires tunable electronic

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properties controlled by the strength of an external laser field. [42] Moreover, the laser-dressing field can also induce new or significantly modify existing fundamental quantum effects in the target system. [43–46]

The type as well as major dynamical properties of these dressed states are mainly manipulated by the polarization of external dressing field, e.g., circular-polarized light is known for opening or varying an existing energy gap, [47–50] while linearly-polarized field can induce an anisotropy [51] in optical response of irradiated materials. The theoretical study of laser-induced anisotropy in opto-electronic properties of graphene and a dice lattice is the focus of current research. Specifically, we will concentrate on the tunneling-transport properties of a system with light-induced tunable anisotropy of its electronic states. [52, 53]

The electronic properties of highly anisotropic Dirac fermions was first predicted theoretically by using the first-principles computations and then confirmed experimentally [54] based on angle-resolved photoemission spectroscopy of epitaxial graphene modulated by an island superlattice, [55] a Bi-square net of SrMnBi_2 , [56] an organic conductor, [57] and a B_2S honeycomb monolayer [58]. Such kind of materials has revealed unique features of an exceptional Dirac-material family as long as engineering a local directional asymmetry [55] can be introduced into the system. Since these materials with anisotropic dispersions are available and implemented within an electronic device, most of their electronic and collective properties have been addressed thoroughly, including plasmons [59, 60] whose frequency depends on the direction of wave vector, as well as angle-dependent screening and transport. [61–64] Meanwhile, specific attention has also been put onto materials with a tilted Dirac cone and having either a zero or finite bandgap, which exhibit a unique out-of-plane anisotropy [61, 65–68] and even include recently fabricated 8-Pmmn borophene as well as single-element materials presenting two ionic sublattices. [69–74]

Historically, Klein paradox, a complete and unimpeded tunneling of incoming charged carriers through a square-potential barrier with arbitrary height and width, has become one subject unique to all gapless Dirac cone materials. [75] This phenomena is attributed to the relativistic type of graphene Hamiltonian and the existence of chirality in its electronic wave functions. Interestingly, Klein tunneling was demonstrated extensively in graphene, a dice lattice and all types of interpolating $\alpha - \mathcal{T}_3$ even under a tilted potential barrier. [6, 76–79] Furthermore, Klein paradox is found persisting for an anisotropic Dirac cone. However, in this case, it occurs only at a finite angle of incidence and is referred to as an asymmetric Klein tunneling. [80, 81] Technically, however, various types of potential barriers, e.g., inhomogeneous and non-uniform spatial profiles of a potential and junctions, could be easily realized in graphene or a nanoscale-width nanoribbon by introducing a spatially-distributed gate voltage. [82–84] As a part of graphene-based optoelectronic device, [85–87] studying electron conductance through these different barrier arrangements, as well as revealing involved physics mechanism for ballistic transport, [88–91] or trapped electronic states [92] are paramount for quantitatively predicting the current level and characterizing the performance of an electric switch.

The remaining part of this paper is organized as follows. In Sec. II, we review some of important properties of optically modulated Dirac electrons with tunable-elliptical energy dispersions of electrons irradiated by linearly-polarized light, including the calculation of transmission of electrons and demonstrating the so-called asymmetric Klein tunneling associated with a non-head-on electron incidence. Based on computed transmission results, we arrive at an expression for the tunneling conductivity of electrons over a square potential barrier in Sec. III and provide a detailed investigation on how this conductivity relies on the crucial material and external irradiation parameters, e.g., the irradiation-induced anisotropy factor $a_\alpha(\lambda_0)$ in the energy dispersion of electrons and the misalignment angle between the direction of light polarization and the normal direction of a barrier layer. These new features are not known from previously considered cases for isotropic particles. The final conclusions are drawn in Sec. IV.

II. ASYMMETRIC KLEIN TUNNELING RESULTING FROM ELLIPTICAL DIRAC DISPERSIONS

The energy dispersion of electrons in a crystal usually appears spherical around a highly-symmetric valley such as the Γ point, within the first Brillouin zone. However, in the presence of strong laser-electron interaction, this energy dispersion is modified and becomes anisotropic, e.g., an elliptical one with a long-axis parallel to the polarization direction of an incident laser field. If the long-axis of the elliptical energy dispersion appears misaligned with the normal direction of the square-barrier layer in a tunneling structure, we expect to find that the tunneling conductivity will vary with this misalignment angle. Therefore, the first step for computing tunneling and conductivity in our system is to derive anisotropic dressed states of electrons under a linearly-polarized dressing field and to full understand how this anisotropy depends on the intensity of an applied optical field in different Dirac materials. Even though we only consider in this paper graphene ($\alpha \rightarrow 0$) and a dice lattice ($\alpha \rightarrow 1$) as two extreme limiting cases for the $\alpha - \mathcal{T}_3$ model, it would be useful to present a comprehensive description for the induced anisotropy by a linearly-polarized dressing field in all $\alpha - \mathcal{T}_3$ materials.

For a two-dimensional (2D) $\alpha - \mathcal{T}_3$ material, the low-energy Hamiltonian takes the form

$$\hat{\mathcal{H}}_\alpha(\mathbf{k} | \tau, \phi) = \hbar v_F \begin{bmatrix} 0 & k_-^\tau \cos \phi & 0 \\ 0 & 0 & k_-^\tau \sin \phi \\ 0 & 0 & 0 \end{bmatrix} + H.c. , \quad (1)$$

where the geometry phase $\phi = \tan^{-1} \alpha$ is directly related to the relative hopping parameter α , and $k_\pm^\tau = k_x^\tau \pm i k_y^\tau$ are obtained from components of the electron wave vector $\mathbf{k} = \{k_x, k_y\}$. Once an optical dressing field *with a linear polarization* is applied, the components of \mathbf{k} introduced in the Hamiltonian in Eq.(1) are modified according to a canonical substitution, i.e. $k_i \rightarrow k_i - eA_i^{(L)}(t)$, where the transient vector potential $\mathbf{A}^{(L)}(t)$ of this linearly-polarized light is given by

$$\mathbf{A}^{(L)}(t) = \begin{bmatrix} A_x^{(L)}(t) \\ A_y^{(L)}(t) \end{bmatrix} = \frac{E_0}{\omega} \begin{bmatrix} \cos \beta \\ \sin \beta \end{bmatrix} \cos(\omega t) , \quad (2)$$

with E_0 representing the electric field amplitude, ω is the frequency of light, and β stands for the polarization angle of light made with the x axis. Moreover, in the presence of both the dressing field and an electrostatic finite-width square barrier $V(x) = V_0 \Theta(x) \Theta(W_B - x)$, the non-interacting Hamiltonian in Eq.(1) becomes

$$\hat{\mathbb{H}}_0^\tau(\phi | x, y) = -v_F \hat{\mathbf{S}}(\phi) \cdot [i\hbar \nabla_\tau + e\mathbf{A}^{(L)}(t)] + V(x) , \quad (3)$$

where $\Theta(x)$ is the Heaviside step function, V_0 and W_B are barrier height and thickness respectively, $\nabla(\tau) \equiv \{\tau \partial / \partial x, \partial / \partial y\}$, and the two ϕ -dependent matrices $\hat{\mathbf{S}}(\phi) = \{\hat{S}_x(\phi), \hat{S}_y(\phi)\}$ are defined as

$$\hat{S}_x(\phi) = \begin{bmatrix} 0 & \cos \phi & 0 \\ \cos \phi & 0 & \sin \phi \\ 0 & \sin \phi & 0 \end{bmatrix} , \quad (4)$$

$$\hat{S}_y(\phi) = i \begin{bmatrix} 0 & -\cos \phi & 0 \\ \cos \phi & 0 & -\sin \phi \\ 0 & \sin \phi & 0 \end{bmatrix} . \quad (5)$$

Here, even if the potential $V(x)$ only remains piecewise-constant, the translational symmetry of the system is still maintained and, therefore, we can simplify our eigenvalue equation using $\{\partial / \partial x, \partial / \partial y\} \rightarrow i \{k_x, k_y\}$ and assuming the wave function in the form of $\Psi(x, y) \sim e^{ik_x x} e^{ik_y y}$. Consequently, the energy dispersions obtained from the Hamiltonian in Eq.(3) for the dressed states keep the flat band $\mathcal{E}_\alpha^{\gamma=0}(\lambda_0, \mathbf{k}) = 0$ under the dressing field, while two anisotropic Dirac-cone dispersions are given by

$$\mathcal{E}_\alpha^{\gamma=\pm 1}(\lambda_0, \mathbf{k}) = \pm \hbar v_F \sqrt{k_x^2 + [a_\alpha(\lambda_0) k_y]^2} \equiv \pm \hbar v_F k f_\alpha(\theta_{\mathbf{k}}, \lambda_0) , \quad (6)$$

which appear as an ellipse in the (x, y) -plane. Here, the angular factor in Eq.(6) is calculated as

$$f_\alpha(\theta_{\mathbf{k}}, \lambda_0) = \cos^2 \theta_{\mathbf{k}} + [J_0^2(2\lambda_0) \cos^2(2\phi) + J_0^2(\lambda_0) \sin^2(2\phi)] \sin^2 \theta_{\mathbf{k}} , \quad (7)$$

where $J_0(x)$ stands for the zeroth-order Bessel function of the first kind. In addition, we know that, for fixed $\theta_{\mathbf{k}}$, the anisotropic factor $f(\theta_{\mathbf{k}}, \lambda_0)$ in Eq.(7) depends on α or phase ϕ and it reaches a maximum for $\phi = 0$ (graphene) but a minimum for $\phi = \pi/4$ (dice lattice). In particular, the anisotropy employed in Eq.(6) takes the form

$$a_\alpha(\lambda_0) = 1 - \frac{\lambda_0^2}{8} [5 + 3 \cos(4\phi)] , \quad (8)$$

which is always less than one (isotropy with $f_\alpha(\theta_{\mathbf{k}}, \lambda_0) \equiv 1$) if the irradiation intensity $\lambda_0 \propto E_0^2 \neq 0$ and reveals how the strength of a dressing field modifies the interaction property of $\alpha - \mathcal{T}_3$ materials. For example, we find from Eq.(8)

a relatively larger anisotropy with $a_0(\lambda_0) = 1 - \lambda_0^2$ for graphene but a smaller anisotropy with $a_1(\lambda_0) = 1 - \lambda_0^2/4$ for dice lattice.

The anisotropic dispersions $\mathcal{E}_\alpha^{\gamma=\pm 1}(\lambda_0, \mathbf{k}) = \pm \hbar v_F \sqrt{k_x^2 + [a_\alpha(\lambda_0) k_y]^2} = \pm \hbar v_F k f_\alpha(\theta_{\mathbf{k}}, \lambda_0)$ for electron dressed states in the presence of a linearly-polarized off-resonant field is obtained using the Floquet-Magnus expansion technique. Mathematically, it is a high-frequency perturbation expansion with respect to the interaction Hamiltonian in powers of $(1/\hbar\omega)$, which works particularly well in the off-resonance limit $\hbar\omega \gg ev_F E_0/\omega$. Eventually, this allows for an approximate solution whenever the exact diagonalization of an interaction matrix becomes impossible or at least it greatly simplifies the calculation.

The dressed states, corresponding to linearly-polarized irradiation, was first presented for graphene in Ref. [53] and for a dice lattice and $\alpha - \mathcal{T}_3$ materials in Ref. [52]. The most important features in the obtained dispersions is their anisotropy and the fact that they are totally independent of time. This is an exact result under the Floquet-Magnus expansion, which is not based on any approximation.

The corresponding wave function for graphene was derived in Ref. [53]. Except for a general $e^{i\mathcal{E}_0 t/\hbar}$ factor, the calculated wave function for a linearly-polarized light field acquires additional $\exp[\pm i\lambda_0 \sin(\omega t)]$ terms. However, it is straightforward to show that the phase of the oscillations in these terms never changes substantially, and therefore could be neglected. Consequently, the considered electromagnetic wave can be treated as a purely dressing field which can be neither absorbed nor emitted by conduction electrons.

Our model for the electron dressed states is based on the off-resonant limit condition $\hbar\omega_0 \gg E_F^*$, i.e., the photon energy is much larger than any characteristic energies of our material (both graphene and a dice lattice), and we use perturbation theory to calculate the energy dispersions of these states. The off-resonant limit could be achieved solely by the high frequency of the dressing field for a wide range of the light intensity. Moreover, we would like to indicate that the multi-photon transitions considered in Ref. [93] is a clearly resonant phenomena which is incompatible with our off-resonant field limit. Since the single-photon energy $\hbar\omega_0$ of a dressing field is much larger than the Fermi energy of electrons, the assumption that multiple photons with energy $n\hbar\omega_0$ could participate in the process of creating dress states becomes much less realistic. Furthermore, the condition $\hbar\omega_0 \gg E_F^*$ for the off-resonant field limit is also the mathematical condition for excluding the multi-photon contributions to the dressed states.

For measuring charge transport through potential barriers, a feasible experimental realization with a gated square barrier has demonstrated strong evidence for Klein tunneling in a Dirac-cone material [94–96] (specifically, it was done for graphene but could now be repeated for a dice lattice as well). In this experiment, the density inside the top-gated region is set by both a back gate and top gate voltages without a substantial restriction on the doping level of electrons. In the experimental setup, an array of tapered electrodes were mounted on the upper side of the graphene sheet, while the interface side of the arrangement consists of several silicon-based substrates. Furthermore, we should mention that the vertical electric field associated with a depletion gate for the potential barrier cannot interfere with a horizontally-polarized light electric field for dressing electronic states. Therefore, the potential barrier and off-resonant dressing field could be implemented completely independently from each other.

The dimensionless coupling parameter $\lambda_0 = c_0/\hbar\omega$ is directly related to the electron-photon interaction strength c_0 which is given by $c_0 = ev_F E_0/\omega \sim 10^{-22} \text{ J} \sim 0.1 - 1 \text{ meV}$, where $\hbar\omega$ is the energy of the incident photons, $v_F = 10^6 \text{ m/s}$ is the Fermi velocity for all $\alpha - \mathcal{T}_3$ materials including graphene as the limiting case when $\alpha \rightarrow 0$, and E_0 represents the amplitude of an incident light field. This estimate based on the values for $\mathcal{I}_0 = 1 \text{ kW/cm}^2 = 10^7 \text{ W/m}^2$ for the light intensity, while the light frequency $\omega = 2\pi f$ falls in the terahertz range $f \sim 10^{12} - 10^{13} \text{ Hz}$ (or $\omega \sim 10^{13} - 10^{14} \text{ rad}$) pertaining to a realistic experimental setup. The corresponding electric-field amplitude is $E_0 = 10^5 \text{ V/m}$ since $\mathcal{I}_0 = \epsilon_0 c E_0^2/2$, where ϵ_0 and c are the permittivity and speed of light in vacuum. Therefore, it is clear that the coupling parameter λ_0 will vary from 0.01 to almost 0.5. Meanwhile, the off-resonant limit can also be achieved by assuming a high frequency $\omega \sim 10^{13} - 10^{14} \text{ rad}$ for the dressing field rather than employing its intensity \mathcal{I}_0 so that the requirement for low light intensity is no longer needed. Consequently, both \mathcal{I}_0 and ω could still be varied independently by up to one order of magnitude. All parameter values utilized in our plots are realistic and could be accessed in an actual experiment. In fact, the predicted phenomena in our paper would still be valid qualitatively even if the value of λ_0 is increased by five times.

Here, we would like to emphasize that our initial model for the dressed states of electrons is obtained within the off-resonant limit in which the photon energy is actually much higher than any of the characteristic energies of the material, such as the electronic Fermi energy. In contrast, resonant multi-photon states and transitions between them are related to a nonlinearly driven oscillator with strong electron-photon coupling and photon-assisted tunneling taken into account, as seen in Ref. [93].

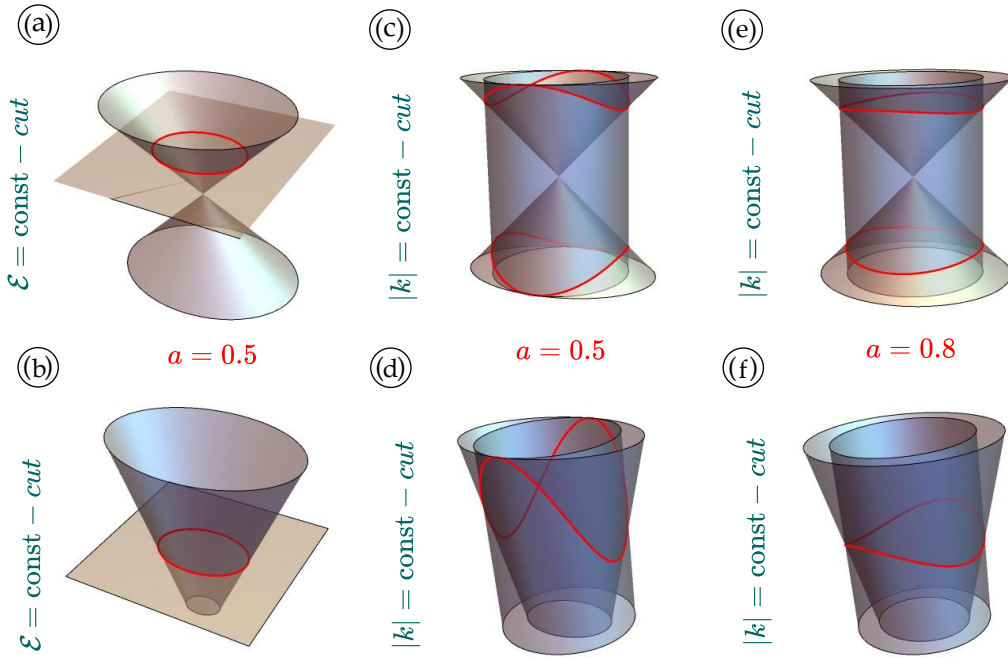


FIG. 1. (Color online) A horizontal (constant energy \mathcal{E}_0) cut, as well as a vertical (constant wave-vector magnitude $|\mathbf{k}|$) cut, of anisotropic Dirac-cone dispersions for both electron and hole states ($\gamma = \pm 1$), which are represented by intersection curves of the dispersion cones with either a horizontal plane or an upright cylinder. Panels (a)-(d) describe the situations with $a_\lambda = 0.5$ for a larger anisotropy, while plots (e) and (f) correspond to $a_\lambda = 0.8$ for a smaller anisotropy. Here, the lower panels (b), (d), (f) are zoom-in views for the energy range $0.2 < \mathcal{E}_0/E_F^* < 1.0$ with Fermi energy E_F^* for a host material.

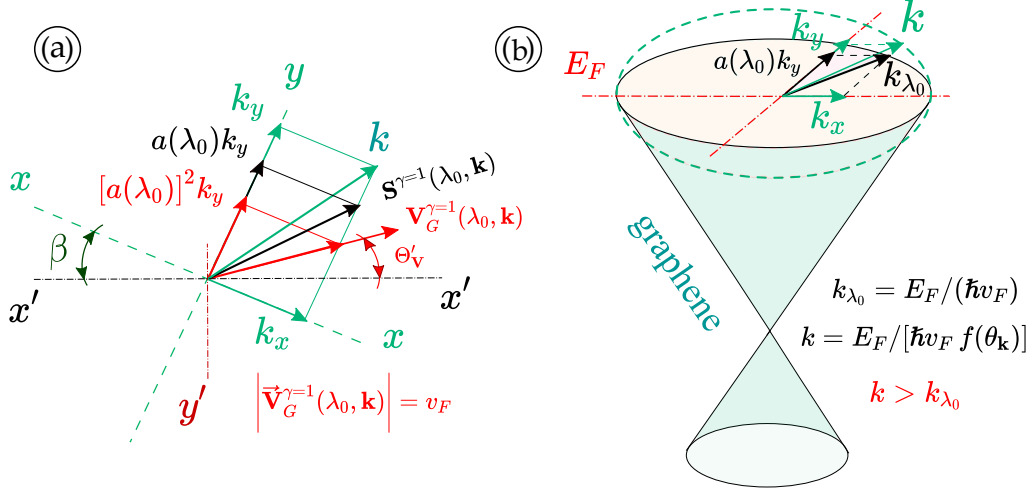


FIG. 2. (Color online) (a) The relationships between the electron wave vector \mathbf{k} , spinor $\mathbf{S}^{\gamma=1}(\lambda_0, \mathbf{k})$ and group-velocity vector $\mathbf{V}_G^{\gamma=1}(\lambda_0, \mathbf{k})$, i.e., a visualization of Eq. (14), outside the barrier layer. (b) Determining the components of wave vector \mathbf{k} within the barrier layer, corresponding to a given energy $\mathcal{E}_0 = E_F^*$ required for calculating tunneling conductivity in Eq. (29).

Figure 1 displays both horizontal and vertical cuts of an anisotropic Dirac-cone in different energy scales.⁰ As shown in Fig. 1, the constant-energy or horizontal cut of the anisotropic Dirac-cone presented in Eq. (6) turns into an ellipse with a shortened k_y semi-axis. This ellipse is inscribed by a $|\mathbf{k}|$ -circle which includes all \mathbf{k}_λ wave vector satisfying

⁰ A brief and informative description and a recipe for finding an intersection curve for two given surfaces using Wolfram Mathematica similar to what was used here could be found in <https://community.wolfram.com/groups/-/m/t/177994>.

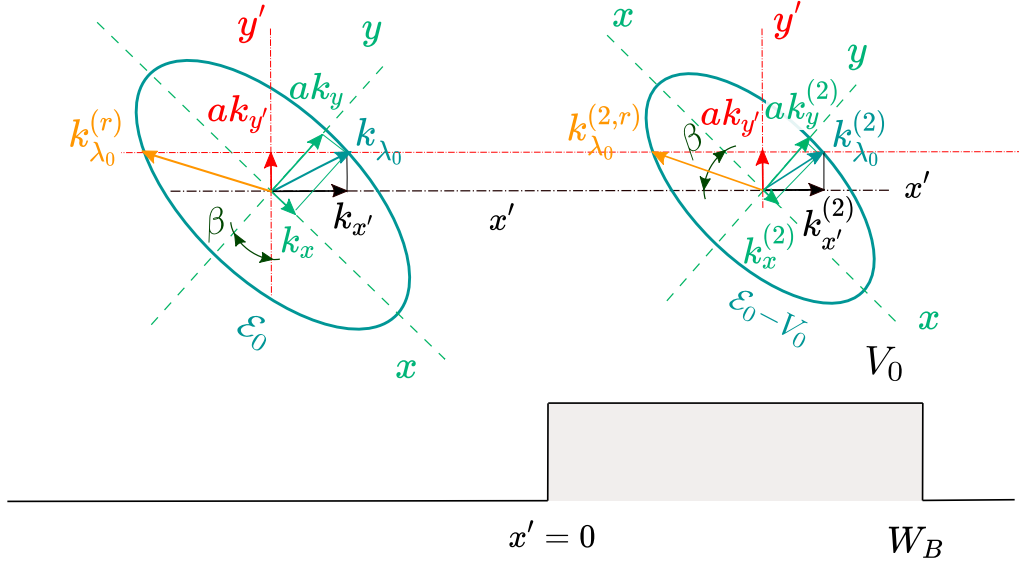


FIG. 3. (Color online) A schematic graph for determining the momentum of transmitted $\mathbf{k}^{(2)}$ and reflected $\mathbf{k}^{(r)}$ electrons, as well as the group-velocity vector $\mathbf{V}_G^{\gamma=1}(\lambda_0, \mathbf{k})$ in the barrier region characterized by a barrier height V_0 . These quantities could be computed by the known energy difference $\mathcal{E}_0 - V_0$ in the barrier region and the fact that the transverse electron momentum k'_y remains conserved in this system.

Eq. (6) and $|\mathbf{k}_\lambda| \leq |\mathbf{k}|$. The constant $|\mathbf{k}|$ -cut by an upright cylinder, on the other hand, appears as a much more sophisticated three-dimensional curve which acquires its maximum values on the k_y axis as $\theta_{\mathbf{k}} = \pi/2$.

Corresponding to energy dispersion in Eq. (6), the wave functions for the electron and hole states (i.e., conduction and valence bands with $\gamma = \pm 1$) in a dice lattice ($\alpha = 1$) are given as

$$\Psi_1^{\gamma=\pm 1}(\tau, \lambda_0, \mathbf{k}) = \frac{1}{4} \begin{bmatrix} e^{-i\Theta_S^{(1)}(\tau, \mathbf{k} | \lambda_0)} \\ \sqrt{2}\gamma \\ e^{+i\Theta_S^{(1)}(\tau, \mathbf{k} | \lambda_0)} \end{bmatrix}, \quad (9)$$

where the phase factor, as defined in Fig. 2, is calculated from

$$\Theta_S^{(1)}(\tau, \mathbf{k} | \lambda_0) = \tan^{-1} \left[\left(\frac{\tau k_y}{k_x} \right) a_1(\lambda_0) \right] = \tan^{-1} [\tau a_1(\lambda_0) \tan \theta_{\mathbf{k}}]. \quad (10)$$

Moreover, the wave function attributed to the flat band ($\gamma = 0$) is

$$\Psi_1^{\gamma=0}(\tau, \lambda_0, \mathbf{k}) = \begin{bmatrix} e^{-i\Theta_S^{(1)}(\tau, \mathbf{k} | \lambda_0)} \\ 0 \\ -e^{+i\Theta_S^{(1)}(\tau, \mathbf{k} | \lambda_0)} \end{bmatrix} = \frac{1}{k_\lambda} \begin{bmatrix} k_x - i\tau a_1(\lambda_0)k_y \\ 0 \\ -k_x - i\tau a_1(\lambda_0)k_y \end{bmatrix}, \quad (11)$$

where $k_\lambda = \sqrt{k_x^2 + a_1^2(\lambda_0)k_y^2}$ represents a measure for the kinetic energy of incident particles.

We further see from Fig. 3 that the wave vector \mathbf{k} , the spinor vector $\mathbf{S}^\gamma(\lambda_0, \mathbf{k})$ and the group-velocity vector $\mathbf{V}_G^\gamma(\lambda_0, \mathbf{k})$ are different and not aligned to each other in the existence of a finite anisotropy $a_1(\lambda_0) \neq 1$. Explicitly, $\mathbf{S}^\gamma(\lambda_0, \mathbf{k})$ and $\mathbf{V}_G^\gamma(\lambda_0, \mathbf{k})$ are defined as

$$\mathbf{S}^\gamma(\lambda_0, \mathbf{k}) = \frac{\gamma}{\sqrt{k_x^2 + [a_1(\lambda_0)k_y]^2}} \begin{bmatrix} k_x \\ a_1(\lambda_0)k_y \end{bmatrix}, \quad (12)$$

$$\mathbf{V}_G^\gamma(\lambda_0, \mathbf{k}) = \frac{1}{\hbar} \begin{bmatrix} \partial/\partial k_x \\ \partial/\partial k_y \end{bmatrix} \varepsilon_1^\gamma(\lambda_0, \mathbf{k}) = \frac{\gamma v_F}{\sqrt{k_x^2 + [a_1(\lambda_0)k_y]^2}} \begin{bmatrix} k_x \\ a_1^2(\lambda_0)k_y \end{bmatrix}. \quad (13)$$

Here, the spinor vector and its angle with the x -axis in Eq. (10) represent crucial components of the wave functions presented in Eqs. (9) and (11), while \mathbf{V}_G^γ specifies the actual incidence direction of incoming particles and is used to distinguish transmitted and reflected waves associated with the second interface of a barrier layer. From Eqs. (12) and (13), the angles of these two vectors relative to the x -axis are determined by

$$\begin{aligned}\tan \Theta_{\mathbf{S}}(\lambda_0) &= \left(\frac{k_y}{k_x} \right) a_1(\lambda_0) = a_1(\lambda_0) \tan \theta_{\mathbf{k}} , \\ \tan \Theta_{\mathbf{V}}(\lambda_0) &= \left(\frac{k_y}{k_x} \right) a_1^2(\lambda_0) = a_1^2(\lambda_0) \tan \theta_{\mathbf{k}} ,\end{aligned}\tag{14}$$

which will be utilized in our hereafter computations.

As seen in Fig. 3, direction of the polarization for an imposed dressing field usually mismatches the normal direction of a barrier layer in tunneling structures. As a result, it is reasonable to introduce two coordinate frames of reference corresponding to these two directions, e.g., $\{x, y\}$ for the $\hat{\mathbf{x}}$ vector whereas $\{x', y'\}$ for the $\hat{\mathbf{x}}'$ vector. These two frames can be related to each other by an in-plane rotation angle β with a rotation matrix $\hat{\mathbb{R}}(\beta)$ given by

$$\hat{\mathbb{R}}(\beta) = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} ,\tag{15}$$

where β is the polarization angle of light made with the $\hat{\mathbf{x}}$ direction. From a physics perspective, what makes our transmission problem unique and a lot more complicated results from the fact that both $\{x, y\}$ and $\{x', y'\}$ frames are required to define the wave function of electrons. First, components of incident-electron momenta outside the barrier layer should be defined in the (x', y') frame with respect to the normal direction of a barrier layer, and meanwhile, both spinor and group-velocity angles must be calculated in the (x, y) frame within which the energy dispersions can be determined by Eq. (6). Similarly, for electrons within the barrier layer, we also deal with both frames simultaneously

as we decide the electron momentum components based on $\mathcal{E}_0 - V_0 = \hbar v_F \sqrt{\left[k_x^{(2)} \right]^2 + \left[a_1(\lambda_0) k_y^{(2)} \right]^2}$ in the (x, y) -frame, and the conservation of transverse k'_y component in the (x', y') -frame, as illustrated by Fig. 3.

Another intriguing feature for electron tunneling in a dice lattice with anisotropic dispersions comes from the requirement of a new type of boundary conditions which are different from simply matching the components of a wave function at two boundaries of a barrier region for graphene and isotropic dice lattice. [97] In our current system, the new boundary conditions possess an additional $k_{x'}$ -related term due to an extra discontinuity at two boundaries of a barrier layer in the (x', y') -frame and that the main Hamiltonian in Eq. (3) is defined in the (x, y) -frame. As a whole, after a lengthy calculation, we find the resulting boundary conditions for a dice lattice are given by $\varphi_2(-\delta x') = \varphi_2(\delta x')$ and

$$\aleph_\tau^+(\lambda_0, \beta) \varphi_1(-\delta x') + \aleph_\tau^-(\lambda_0, \beta) \varphi_3(-\delta x') = \aleph_\tau^+(\lambda_0, \beta) \varphi_1(\delta x') + \aleph_\tau^-(\lambda_0, \beta) \varphi_3(\delta x') ,\tag{16}$$

where $\varphi_j(x)$ for $j = 1, 2, 3$ represent three wave function components, and

$$\aleph_\tau^\pm(\lambda_0, \beta) = \tau \cos \beta [1 \pm i\tau a_1(\lambda_0) \tan \beta] .\tag{17}$$

It is known that a skew or non-symmetric type of Klein paradox is associated with a full transmission independent of barrier height V_0 , width W_B or the energy \mathcal{E}_0 of incoming particles. Such a phenomenon can be observed at a finite incidence angle $\Theta'_V \neq 0$, in contrast with the conventional Klein paradox with $\Theta'_V = 0$. In this paper, however, we are interested in investigating how this finite angle $\Theta'_V \neq 0$ depends on parameters of our system setup, e.g., anisotropy factor $a_1(\lambda_0)$ and light-polarization angle β made with x' -axis or the potential barrier normal direction.

Common Klein paradox could be observed only if the transverse component of electron momentum vanishes, or equivalently $k'_y = 0$. Obviously, this condition could be met as $\theta'_k = 0$ or $\theta_{\mathbf{k}} = \beta$ and $\mathbf{k} = \{k \cos \beta, k \sin \beta\}$. In fact, this critical incidence angle could be found from $\partial \mathcal{T}[E_{\text{dc}}, \mathcal{E}_1^{\gamma=1}(k), \Theta_{\mathbf{V}'}^{(1)} | \lambda_0, \beta] / \partial \Theta_{\mathbf{V}'}^{(1)} = 0$, which gives rise to [98]

$$\Theta_{\mathbf{V}'}^{(1)} = \Theta_{\mathbf{V}}^{(1)} - \beta = \tan^{-1} \left\{ \frac{[a_1^2(\lambda_0) - 1] \tan \beta}{1 + a_1^2(\lambda_0) \tan^2 \beta} \right\} .\tag{18}$$

Consequently, we immediately discern that for $|\beta| < \pi/2$, the largest asymmetry of the Klein paradox is achieved if $\beta_c = \cot^{-1}[a_1(\lambda_0)]$, yielding

$$\left[\Theta_{\mathbf{V}'}^{(1)}\right]_{\max} = \tan^{-1}[a_1(\lambda_0)] - \cot^{-1}[a_1(\lambda_0)]. \quad (19)$$

Therefore, the reach to $\left[\Theta_{\mathbf{V}'}^{(1)}\right]_{\max}$ could be fulfilled by adjusting either anisotropy factor $a_1(\lambda_0)$ or misalignment angle β .

Our calculated results for electron transmission in both graphene and dice lattice are presented in Figs. 4, 5 and 6, which clearly demonstrate asymmetric Klein paradox for a finite $\Theta_{\mathbf{V}'}^{(1)}$ angle of electron incidence as the anisotropy factor $a_1(\lambda_0) \neq 1$ and the misalignment angle $\beta \neq 0$ between the (x, y) and (x', y') frames. For the case of a dice lattice, however, we need consider the energy \mathcal{E}_0 of incoming particles separately in the ranges of $\mathcal{E}_0 < V_0$ and $\mathcal{E}_0 > V_0$ so as to exclude the possibility for particle scattering into a zero-energy state in the barrier region with an infinite degeneracy for its momentum. The observed transmission is generally larger for a dice lattice and very close to unity for a wide range of incident angles, which is not the case for graphene with a series of separated distinguished peaks. The dice lattice is also known for its “magic case”, i.e., a complete transmission for all angles of incidence if $\mathcal{E}_0 = 2V_0$ is satisfied. The angular dependence of transmission is not symmetric with respect to $\Theta_{\mathbf{V}'}^{(1)} = 0$ if both β and $a_1(\lambda_0)$ are finite [see panel (d) of Figs. 5 and 6]. In these cases, the Klein paradox line for full transmission is moved to negative values for $\Theta_{\mathbf{V}'}^{(1)}$ in accordance with Eq. (18).

III. SEQUENTIAL-TUNNELING CURRENT AND CONDUCTIVITY

Under a DC electric field E_{dc} along the x' direction for tunneling transport of electrons within monolayer graphene (with $\alpha = 0$), which includes a potential barrier distributed along the x' direction with a barrier width W_B and a barrier height V_0 , the sequential-tunneling sheet current $J_s(E_{\text{dc}} | E_F, \lambda_0, \beta)$ in this system is calculated as

$$J_s(E_{\text{dc}} | E_F, \lambda_0, \beta) = \frac{g_s g_v e}{(2\pi)^2} \int d^2 \mathbf{k} \mathcal{T}[\mathcal{E}_0^{\gamma=1}(\mathbf{k}), \Theta_{\mathbf{V}'}^{(1)} | \lambda_0, \beta] V_{G,x'}^{\gamma=1}(\lambda_0, \mathbf{k}) \times \left\{ f_0[\mathcal{E}_0^{\gamma=1}(\mathbf{k})] - f_0[\mathcal{E}_0^{\gamma=1}(\mathbf{k}) + eE_{\text{dc}}W_B] \right\}, \quad (20)$$

where $\mathcal{T}[\mathcal{E}_0^{\gamma=1}(\mathbf{k}), \Theta_{\mathbf{V}'}^{(1)} | \lambda_0, \beta]$, which is independent of E_{dc} for a thin barrier layer, represents the coefficient for sequential tunneling of electrons through the square-potential barrier, $g_s = 2$ and $g_v = 2$ are spin and valley degeneracies of graphene sheet, and \mathbf{k} is the wave vector of electrons within the 2D lattice. Additionally, $f_0(x) = \{1 + \exp[(x - \mu)/k_B T]\}^{-1}$ is the Fermi function for thermal-equilibrium electrons, μ and T are the graphene chemical potential and the system temperature, and the incident energy $\mathcal{E}_0^{\gamma=1}(\mathbf{k})$ is below the Fermi energy of electrons in the source electrode.

In current study, we calculate the conduction (dark) current of doped graphene or $\alpha - \mathcal{T}_3$ materials under a weak bias field E_{dc} and a strong dressing field as well. In this way, a bias-independent conductance of this system can be found within the linear-response regime. Meanwhile, we also know that the optical absorption in the same system can occur and will further lead to a photo-current if $\hbar\omega_0 \geq 2E_F^*$. However, such an induced electron-hole photo-current becomes insignificant in comparison with the high conduction current resulted from doped electrons in the system, and then can be neglected safely in our model. This implies that the conductance of system will not be changed by the appearance of very small electron-hole photo-current. Physically, for graphene or $\alpha - \mathcal{T}_3$ materials, either a diffusion-based (undoped p - n junction) or a sequential-tunneling-based (doped) photo-current can be generated and observed experimentally if the thermally-excited electron-hole or doped-electron dark current is kept small enough. Refs. [99 and 100]

For a thin barrier with $eE_{\text{dc}}W_B \ll \mathcal{E}_0^{\gamma=1}(\mathbf{k})$, we find from Eq. (20) that

$$f_0[\mathcal{E}_0^{\gamma=1}(\mathbf{k})] - f_0[\mathcal{E}_0^{\gamma=1}(\mathbf{k}) + eE_{\text{dc}}W_B] = eE_{\text{dc}}W_B \left[-\frac{\partial}{\partial \mathcal{E}_0^{\gamma=1}(\mathbf{k})} f_0[\mathcal{E}_0^{\gamma=1}(\mathbf{k})] \right] \simeq eE_{\text{dc}}W_B \delta[\mathcal{E}_0^{\gamma=1}(\mathbf{k}) - E_F^*], \quad (21)$$

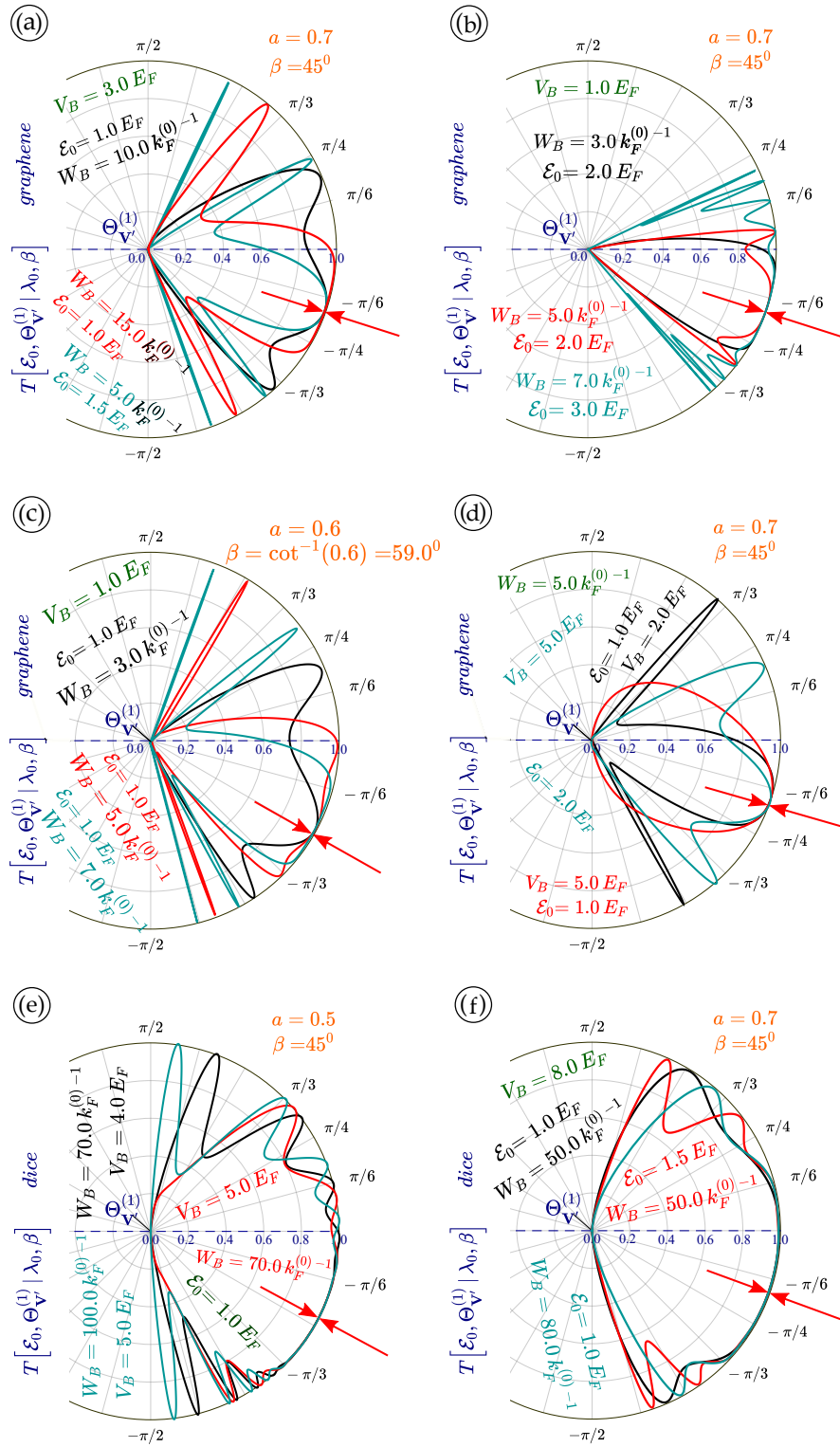


FIG. 4. (Color online) Angular plots for electron transmission $T[\mathcal{E}_0, \Theta_{\mathbf{V}}^{(1)} | \lambda_0, \beta]$ as a function of incidence angle $\Theta_{\mathbf{V}}^{(1)}$ of incoming-particle group-velocity vector in graphene and a dice lattice. Each panel corresponds to a different value of the anisotropy factor $a_\alpha(\lambda_0)$ of irradiated dressed states as well as the misalignment angle β and other parameters, as labeled. The Klein paradox is detected as full transmission is observed for different selected values of barrier height V_0 , incidence energy \mathcal{E}_0 and barrier width W_B , e.g. $V_0/E_F = 3.0$, $\mathcal{E}_0/E_F = 1.0, 1.5$ and $k_F^{(0)} W_B = 10.0, 15.0, 5.0$ for black, red and blue curves, respectively, in panel (a).

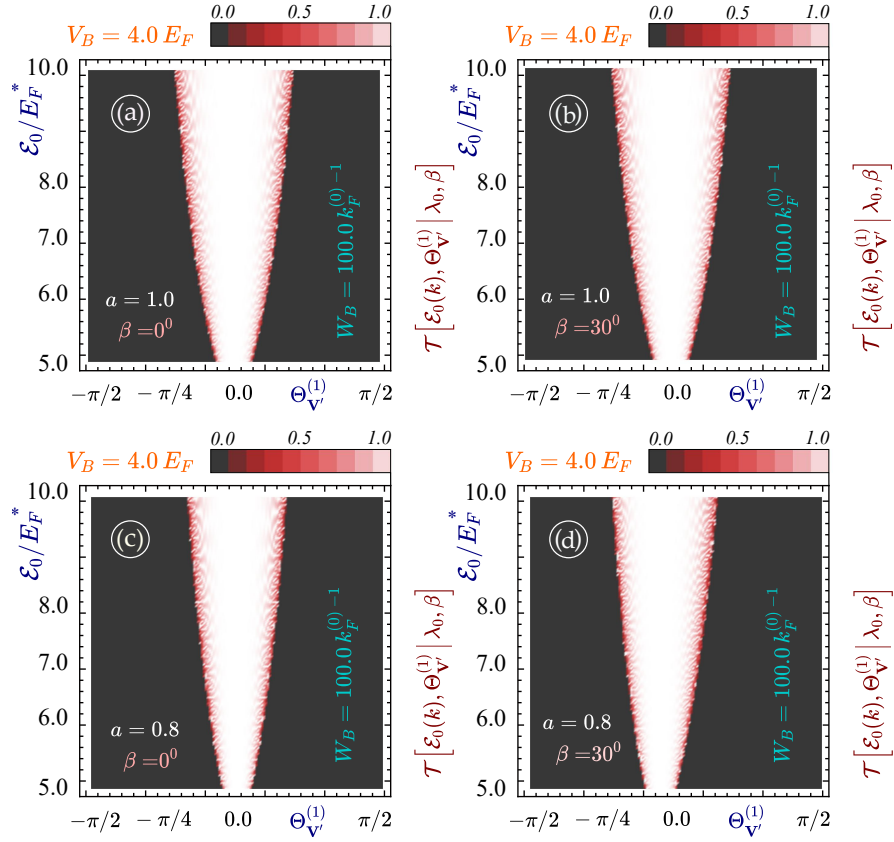


FIG. 5. (Color online) Density plots for $\mathcal{T}[\mathcal{E}_0, \Theta_{\mathbf{V}'}^{(1)} | \lambda_0, \beta]$ as functions of $\Theta_{\mathbf{V}'}^{(1)}$ and \mathcal{E}_0 in a dice lattice. Each panel corresponds to a specific value of $a_1(\lambda_0)$ and β , as labeled. Here $k_F^{(0)} W_B = 100.0$ for all panels. The values of \mathcal{E}_0/E_F are chosen above the barrier height $V_0/E_F = 4.0$ such that e - h (electron-to-hole) transition never occurs in the barrier layer.

where $\mathcal{E}_0^{\gamma=1}(\mathbf{k}) = \hbar v_F \sqrt{k_x^2 + a_0^2(\lambda_0) k_y^2}$, E_F^* stands for the Fermi energy of graphene layer at low temperatures $k_B T \ll E_F^*$, and v_F is the Fermi velocity. In our computations of Eq. (20), we have used the following relations:

$$\int d^2 \mathbf{k} = \int_0^\infty k dk \int_0^{2\pi} d\theta_{\mathbf{k}}, \quad (22)$$

$$\theta_{\mathbf{k}} = \tan^{-1} \left\{ \frac{1}{a_0^2(\lambda_0)} \tan \left(\Theta_{\mathbf{V}'}^{(1)} + \beta \right) \right\}, \quad (23)$$

$$d\theta_{\mathbf{k}} = a_0^2(\lambda_0) \left\{ \cos^2 \left(\Theta_{\mathbf{V}'}^{(1)} + \beta \right) \left[a_0^2(\lambda_0) + \tan^2 \left(\Theta_{\mathbf{V}'}^{(1)} + \beta \right) \right]^{-1} \right\} d\Theta_{\mathbf{V}'}^{(1)}. \quad (24)$$

Meanwhile, we also acquire

$$\begin{bmatrix} V_{G,x'}^\gamma(\lambda_0, \mathbf{k}) \\ V_{G,y'}^\gamma(\lambda_0, \mathbf{k}) \end{bmatrix} = \frac{\gamma v_F}{\sqrt{k_{x'}^2 + a_0^2(\lambda_0) k_{y'}^2}} \begin{bmatrix} k_{x'} \\ a_0^2(\lambda_0) k_{y'} \end{bmatrix} = \frac{\gamma v_F}{\sqrt{k_x^2 + a_0^2(\lambda_0) k_y^2}} \hat{\mathbb{R}}(-\beta) \begin{bmatrix} k_x \\ a_0^2(\lambda_0) k_y \end{bmatrix}, \quad (25)$$

or simply $V_{G,x'}^{\gamma=1}(\lambda_0, \mathbf{k}) \equiv v_F \cos \left(\Theta_{\mathbf{V}'}^{(1)} \right)$.

At a finite but low temperature (i.e., $k_B T \ll E_F^*$), the negative derivative of the Fermi-Dirac distribution function under a weak bias field E_{dc} becomes

$$f_0[\mathcal{E}_0^{\gamma=1}(\mathbf{k})] - f_0[\mathcal{E}_0^{\gamma=1}(\mathbf{k}) + eE_{dc}W_B] \approx \frac{eE_{dc}W_B}{2k_B T} \left\{ 1 + \cosh \left[\frac{\mathcal{E}_0^{\gamma=1}(\mathbf{k}) - E_F^*}{k_B T} \right] \right\}^{-1}, \quad (26)$$

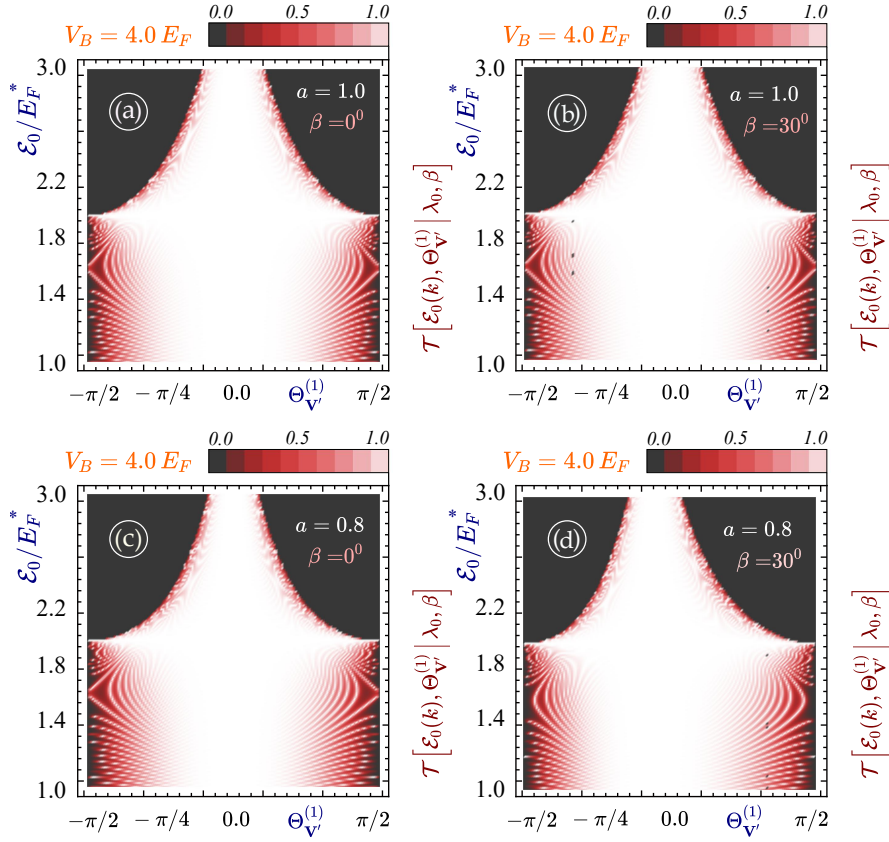


FIG. 6. (Color online) Density plots for $\mathcal{T}[\mathcal{E}_0, \Theta_{\mathbf{V}}^{(1)} | \lambda_0, \beta]$ as functions of $\Theta_{\mathbf{V}}^{(1)}$ and \mathcal{E}_0 in a dice lattice. Each panel corresponds to a specific value of $a_1(\lambda_0)$ and β , as labeled. Here, The $k_F^{(0)}W_B = 100.0$ for all panels. The values of \mathcal{E}_0/E_F are chosen below the barrier height $V_0/E_F = 4.0$ such that e - h transition always occurs in the barrier region.

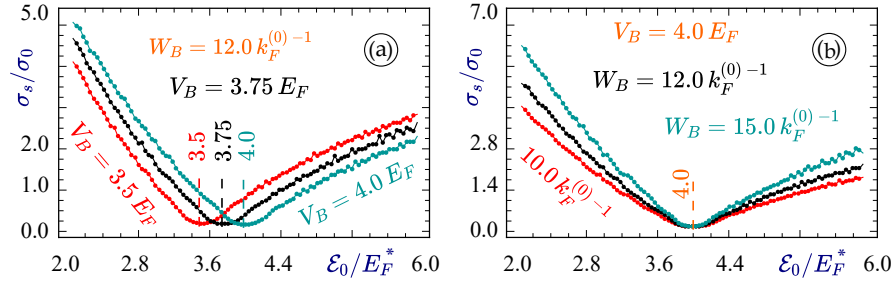


FIG. 7. (Color online) Tunneling conductivity $\sigma_s(E_F^*, \lambda_0)$ for $\lambda_0 = 0$ in the units of $\sigma_0 = 2e^2/h$ for isotropic graphene electronic states in the absence of a dressing field as a function of incoming-particle energy $\mathcal{E}_0 = E_F^*$ in the units of E_F . Panel (a) is plotted for three different values of barrier height V_0 and the fixed barrier width $k_F^{(0)}W_B = 12.0$, while panel (b) displays $\sigma_s(E_F^*, \lambda_0)$ for three different values of W_B , as labeled, for fixed V_0 and W_B .

where its low-temperature asymptotic delta-function representation gives rise to a peak with an “energy width” $k_B T$ around $\mathcal{E}_0^{\gamma=1}(\mathbf{k}) = E_F^*$. Consequently, the above result could be replaced by any other δ -function representation, e.g., a Gaussian-type function. As a result, we immediately conclude that the temperature-induced broadening of our k -integral in Eq. (20) will be on the order of $\mathcal{O}(k_B T/E_F^*) \ll 1$. On the other hand, for estimate of the temperature-induced modification of our calculated tunneling conductance, we would indicate that there exist two competing factors, i.e. monotonic dependence of $\theta_{\mathbf{k}}$ on the temperature-independent group velocity angle $\Theta_{\mathbf{V}}^{(1)}$ and a decrease of electron occupations around the Fermi energy E_F^* due to the presence of an elliptical cross-section.

For the quantities, proportional to the Fermi energy E_F^* , such as the polarization function, the largest temperature-induced effect comes from the decrease of the chemical potential $\mu(T)$ due to $\mu(T) \neq E_F^*$. In fact, the Boltzmann

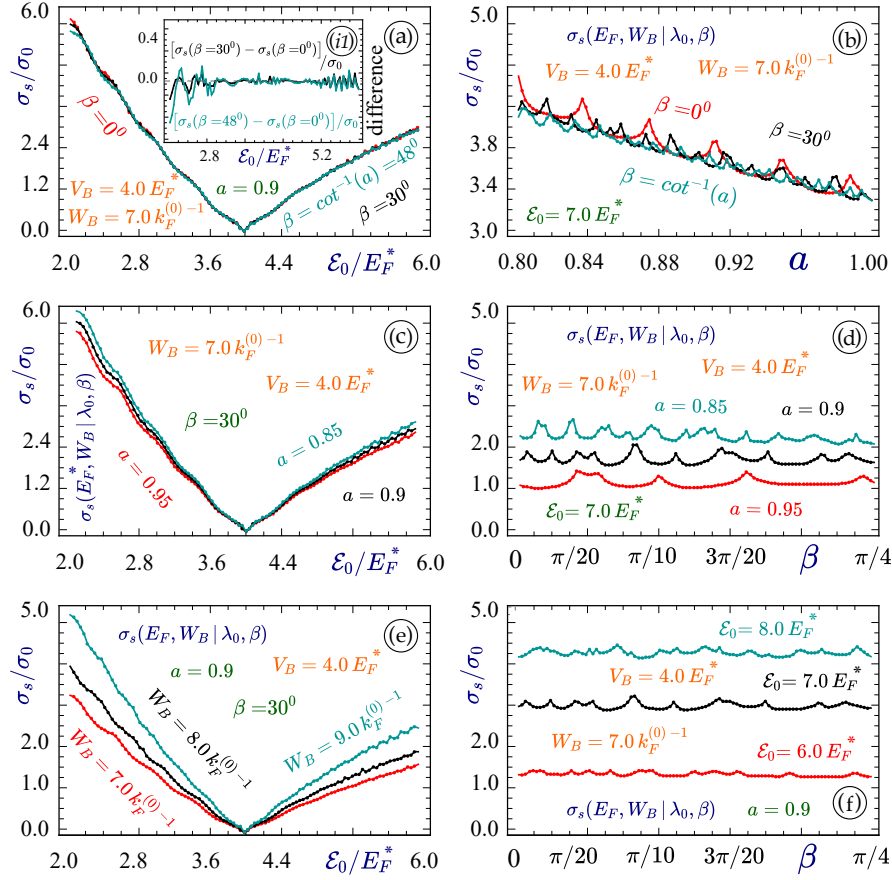


FIG. 8. (Color online) $\sigma_s(E_F^*, \lambda_0, \beta)$ with $\lambda_0 = 0$ in the units of $\sigma_0 = 2e^2/h$ for graphene anisotropic dressed electronic states. On the left panels (a), (c) and (e), $\sigma_s(E_F^*, \lambda_0, \beta)$ is plotted as a function of $\mathcal{E}_0 = E_F^*$ in the units of E_F , as a function of anisotropic factor $a_0(\lambda_0)$ in panel (b) and a function of misalignment angle β in panels (d) and (f). Inset (i1) in panel (a) displays the difference of tunneling conductivities with $\beta = 30^\circ$ and $\beta = 0^\circ$.

conductivity of graphene remains nearly independent of the temperature for unscreened electrons, but shows a steady decrease as a function of temperature if the screening is included.[101] Our study in this paper corresponds to the former case. Furthermore, we should emphasize that both the transmission $\mathcal{T}[\mathcal{E}_0, \Theta_{\mathbf{V}'}^{(1)} | \lambda_0]$ and group velocity $\mathbf{V}_G^\gamma(\lambda_0, \mathbf{k})$ do not exhibit a monotonic dependence on wave number k or the particle energy $\mathcal{E}_0^{\gamma=1}(\mathbf{k})$, as can be verified from Figs. 5 and 6. Therefore, the averaging over a broadened k -range will not lead to significant changes of the calculated conductance or its dependence on temperature.

Finally, the optically-modulated sheet conductivity $\sigma_s(E_F^*, \lambda_0, \beta)$ of the system is given by

$$\begin{aligned} \sigma_s(E_F^*, \lambda_0, \beta) &\equiv \frac{J_s(E_{\text{dc}} | E_F^*, \lambda_0, \beta)}{E_{\text{dc}}} = \frac{e^2 W_B}{\pi^2} \int_0^\infty k dk \int_0^{2\pi} d\theta_{\mathbf{k}} \mathcal{T}[\mathcal{E}_0^{\gamma=1}(\mathbf{k}), \Theta_{\mathbf{V}'}^{(1)} | \lambda_0, \beta] \\ &\quad \times V_{G, x'}^{\gamma=1}(\lambda_0, \mathbf{k}) \delta[\mathcal{E}_0^{\gamma=1}(\mathbf{k}) - E_F^*]. \end{aligned} \quad (27)$$

In Eq. (27), the delta function $\delta[\mathcal{E}_0^{\gamma=1}(\mathbf{k}) - E_F^*]$ for the anisotropic dispersions $\mathcal{E}_0^{\gamma=1}(\mathbf{k}) = \hbar v_F k f_0(\theta_{\mathbf{k}})$ with $f_0(\theta_{\mathbf{k}}) = \sqrt{1 - [1 - a_0^2(\lambda_0)] \sin^2(\theta_{\mathbf{k}})}$ and $a_0(\lambda_0) < 1$ could be written as

$$\delta[\mathcal{E}_0^{\gamma=1}(\mathbf{k}) - E_F^*] = \left\{ \frac{\partial}{\partial k} [\mathcal{E}_0^{\gamma=1}(\mathbf{k}) - E_F^*] \right\}^{-1} \delta[k - k_F^*(\theta_{\mathbf{k}})] = \frac{1}{\hbar v_F f_0(\theta_{\mathbf{k}})} \delta[k - k_F^*(\theta_{\mathbf{k}})], \quad (28)$$

where $k_F^*(\theta_{\mathbf{k}}) = E_F^*/[\hbar v_F f_0(\theta_{\mathbf{k}})]$ is the $\theta_{\mathbf{k}}$ -dependent Fermi momentum for an elliptical Dirac-cone dispersion. By

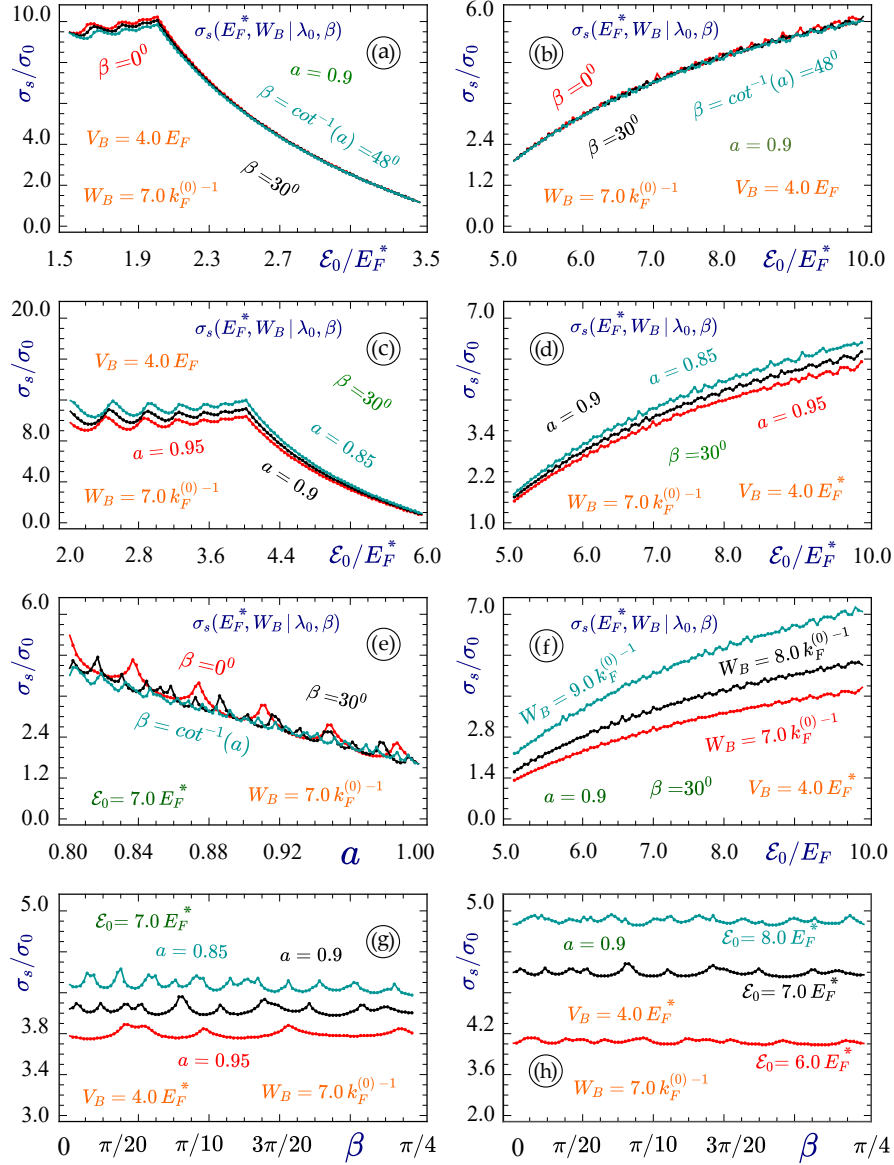


FIG. 9. (Color online) $\sigma_s(E_F^*, \lambda_0, \beta)$ in the units of $\sigma_0 = 2e^2/h$ for dice-lattice anisotropic dressed electron states. On the panels (a)-(d) and (f), $\sigma_s(E_F^*, \lambda_0, \beta)$ is plotted as a function of $\mathcal{E}_0 = E_F^*$ in the units of E_F , as a function of $a_1(\lambda_0)$ in panel (e) and as a function of β in panels (g) and (h). Here, for $\mathcal{E}_0 = E_F^*$ dependence in isotropic $\sigma_s(E_F^* | \lambda_0 = 0)$, we separate the cases for $\mathcal{E}_0 > V_0$ and $\mathcal{E}_0 < V_0$ in the barrier region.

using the result in Eq. (28) and considering a forward incidence of particles with $|\Theta_{\mathbf{V}}^{(1)}| \leq \pi/2$, the conductivity in Eq. (27) can be calculated explicitly as

$$\begin{aligned}
\sigma_s(E_F^*, \lambda_0, \beta) &= \frac{e^2 W_B E_F^*}{\pi^2 \hbar^2 v_F} \int_{-\pi/2}^{\pi/2} d\Theta_{\mathbf{V}'}^{(1)} \cos(\Theta_{\mathbf{V}'}^{(1)}) \mathcal{T}[E_F^*, \Theta_{\mathbf{V}'}^{(1)} | \lambda_0, \beta] \\
&\times \left[\frac{1 + \tan^2(\Theta_{\mathbf{V}'}^{(1)} + \beta)}{a_0^2(\lambda_0) + \tan^2(\Theta_{\mathbf{V}'}^{(1)} + \beta) / a_0^2(\lambda_0)} \right] \left[\frac{1 + \tan^2(\Theta_{\mathbf{V}'}^{(1)} + \beta) / a_0^4(\lambda_0)}{1 + \tan^2(\Theta_{\mathbf{V}'}^{(1)} + \beta) / a_0^2(\lambda_0)} \right] \\
&= \left(\frac{2e^2}{\hbar} \right) \frac{E_F^* W_B}{\hbar v_F} \int_{-\pi/2}^{\pi/2} \frac{d\Theta_{\mathbf{V}'}^{(1)}}{\pi} \cos(\Theta_{\mathbf{V}'}^{(1)}) \mathcal{T}[E_F^*, \Theta_{\mathbf{V}'}^{(1)} | \lambda_0, \beta] \left[\frac{1 + \tan^2(\Theta_{\mathbf{V}'}^{(1)} + \beta)}{a_0^2(\lambda_0) + \tan^2(\Theta_{\mathbf{V}'}^{(1)} + \beta)} \right], \quad (29)
\end{aligned}$$

where $\mathcal{E}_0^{\gamma=1}(\mathbf{k}) = E_F^*$ represents the kinetic energy of tunneling electrons at the Fermi level of graphene.

The only effect of finite temperature in our model is that we cannot use the δ -function approximation for the $\partial/\partial\mathcal{E}_0^{\gamma=1}(\mathbf{k})$ -derivative of the Fermi-Dirac distribution function. For a thin barrier with $eE_{\text{dc}}W_B \ll \mathcal{E}_0^{\gamma=1}(\mathbf{k})$, where $\mathcal{E}_0^{\gamma=1}(\mathbf{k}) = \hbar v_F \sqrt{k_x^2 + a_0^2(\lambda_0)k_y^2}$, we find

$$\begin{aligned}
f_0[\mathcal{E}_0^{\gamma=1}(\mathbf{k})] - f_0[\mathcal{E}_0^{\gamma=1}(\mathbf{k}) + eE_{\text{dc}}W_B] &\approx eE_{\text{dc}}W_B \left[-\frac{\partial}{\partial\mathcal{E}_0^{\gamma=1}(\mathbf{k})} f_0[\mathcal{E}_0^{\gamma=1}(\mathbf{k})] \right] \\
&= \frac{1}{k_B T} \exp\left[\frac{\mathcal{E}_0^{\gamma=1}(\mathbf{k}) - E_F^*}{k_B T}\right] \left\{ 1 + \exp\left[\frac{\mathcal{E}_0^{\gamma=1}(\mathbf{k}) - E_F^*}{k_B T}\right] \right\}^{-2} \neq eE_{\text{dc}}W_B \delta[\mathcal{E}_0^{\gamma=1}(\mathbf{k}) - E_F^*]. \quad (30)
\end{aligned}$$

For this case, neither of the two integrals $\int_0^\infty dk k \int_0^{2\pi} d\theta_{\mathbf{k}}$ can be eliminated by the delta function and both of them are retained in the final equation for the current. However, all the calculated quantities and their dependence on finite but low temperatures $k_B T \ll E_F^*$ remain qualitatively the same as the zero-temperature case. Even for elevated temperatures, we still expect to find two competing factors, i.e. monotonic dependence of $\theta_{\mathbf{k}}$ on the temperature-independent group velocity angle $\Theta_{\mathbf{V}'}^{(1)}$ and a decrease of electron occupations around the Fermi energy E_F^* due to the presence of an elliptical cross-section although the exact numerical value will vary. Also, the obtained tunneling conductivity will not be directly related to the density-of-states $\rho_S(\mathbb{E})$ in Eq. (31) below.

Finalizing our calculations, we also obtain the density-of-states for asymmetric Dirac-cone electrons in graphene, as well as the dependence of their Fermi energy E_F^* on the electron density n_e [102]. In general, the density-of-states $\rho_S(\mathbb{E})$ is defined as

$$\rho_S(\mathbb{E}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \sum_{\gamma=\pm 1} \sum_{\xi, \sigma=\pm 1} \delta[\mathbb{E} - \mathcal{E}_0^\gamma(k)], \quad (31)$$

where σ and ξ refer to the spin and valley indices. As an example, we will only consider electron states with $\gamma = 1$, while the situation for hole states with $\gamma = -1$ will be exactly symmetric. Meanwhile, there exist valley and spin degeneracies so that $\sum_{\xi, \sigma=\pm 1} = 4$. Moreover, Eq. (31) looks identical to a part of the integrand in Eq. (29) for

conductivity, therefore, we can easily rewrite Eq. (31) as

$$\rho_S(\mathbb{E}) = \mathcal{C}_{\lambda_0} \mathbb{E} \equiv \frac{\mathbb{E}}{(\pi \hbar v_F)^2} \int_0^{2\pi} d\theta_{\mathbf{k}} \left[\frac{1 + \tan^2 \theta_{\mathbf{k}}}{1 + a_0^2(\lambda_0) \tan^2 \theta_{\mathbf{k}}} \right], \quad (32)$$

which is still proportional to energy \mathbb{E} although its coefficient depends on the anisotropy factor $a_0(\lambda_0) < 1$, and is reduced with increasing light intensity, in correspondence with the increase of tunneling current due to the anisotropic

dispersion. Furthermore, the Fermi energy E_F of the system is calculated as $E_F^* = (2n_e/\mathcal{C}_{\lambda_0})^{1/2}$ which reduces to a standard expression $E_F^* = \hbar v_F \sqrt{\pi n_e}$ after taking $a_0(\lambda_0) = 1$.

Figures 8 and 9 demonstrate the electron tunneling conductance $\sigma_s(E_F^*, \lambda_0, \beta)$ of irradiated graphene and a dice lattice for different values of dressing-field intensity ($\propto \lambda_0$) and misalignment angles β in comparison with non-irradiated materials displayed in Fig. 7. The calculated results for $\sigma_s(E_F^*, \lambda_0, \beta)$ in Figs. 8 and 9 are scaled with $\sigma_0 = 2e^2/h = 7.75 \times 10^{-5} S$, while the incoming-particle energy is scaled by the electron Fermi energy $E_F^* = \hbar k_F^* = \hbar \sqrt{\pi n_0} = 100 \text{ meV}$, corresponding to doping density $n_0 = 10^{10} - 10^{12} \text{ cm}^{-2}$ or Fermi wave number $k_F^* = 10^6 - 10^7 \text{ cm}^{-1}$. Meanwhile, the familiar V -shape dependence of $\sigma_s(E_F^*, \lambda_0, \beta)$ on the incident energy \mathcal{E}_0 shows up even under a dressing field. However, its correction $\Delta\sigma_s(E_F^*, \lambda_0, \beta)$ resulted from irradiation is limited by $\Delta\sigma_s \leq \sigma_0$.

The tunneling conductivity $\sigma_s(E_F^*, \lambda_0)$ for the standard case of isotropic dispersions ($\lambda_0 = 0$) and symmetric Klein paradox ($\Theta_{\mathbf{V}'}^{(1)} = 0$) is shown in Fig. 7. As it is well known, $\sigma_s(E_F^*, \lambda_0)$ reproduces a familiar V -shape dependence for $\mathcal{E}_0 = E_F^*$, where the transmission $\mathcal{T}[\mathcal{E}_0, \Theta_{\mathbf{V}'}^{(1)} | \lambda_0]$ is equal to zero for all non-zero $\Theta_{\mathbf{V}'}^{(1)}$ angles of incidence and reaches its minimal values at $\mathcal{E}_0 = V_0$, where the electron momentum in the barrier region is zero so that the particle still remains. However, the Klein paradox is still observed for the head-on collision. Similar \mathcal{E}_0 dependence in $\sigma_s(E_F^*, \lambda_0, \beta)$ is found for anisotropic dispersions with a finite angle β , as seen in Fig. 8, which is also reflected in $\mathcal{T}[\mathcal{E}_0, \Theta_{\mathbf{V}'}^{(1)} | \lambda_0, \beta]$. It is very interesting to find that the β dependence of $\sigma_s(E_F^*, \lambda_0, \beta)$ in Figs. 8(d) and 8(f) is weak, periodic and non-monotonic so that it plays no substantial role in $\sigma_s(E_F^*, \lambda_0, \beta)$. The fluctuations around a fixed value in Figs. 8(d) and 8(f) result from the fact that sub-resonances with $k_x^{(2)} W_B = n\pi$ with integer n [103] can equally contribute to tunneling current and the change of β corresponds to a rotation of these ‘‘transmission petals’’. In fact, as presented in Eq. (27), $\sigma_s(E_F^*, \lambda_0, \beta)$ is a convolution of $\mathcal{T}[\mathcal{E}_0, \Theta_{\mathbf{V}'}^{(1)} | \lambda_0, \beta]$ with $\cos\left[\Theta_{\mathbf{V}'}^{(1)}\right]$, which certainly relies on the misalignment angle β .

It is also crucial that $\sigma_s(E_F^*, \lambda_0, \beta)$ exhibits a stable decrease with increasing anisotropy factor $a_\alpha(\lambda_0)$, namely, $\sigma_s(E_F^*, \lambda_0, \beta)$ will reach its minimal value for the isotropic case with $a_\alpha(\lambda_0) = 1$ and demonstrate a steady growth with enhanced eccentricity $e_\alpha(\lambda_0) = \sqrt{1 - a_\alpha^2(\lambda_0)}$ of the electron dispersions. Therefore, the presence of an optical dressing field is enabled to enhance the tunneling conductivity of electrons with increasing field intensity, which demonstrates an attractive device application. This property remains true for both graphene and dice lattices, as can be verified from Fig. 9(e), and could be explained by a similar dependence of density-of-states in Eq. (32) as well as the dependence of E_F^* on n_e in Eq. (32) for $a_\alpha(\lambda_0) < 1$. As illustrated in the right panel of Fig. 3, the actual value of a transverse wave number k_y will be changed by the inclusion of anisotropy.

Mathematically, the dependence of $\sigma_s(E_F^*, \lambda_0, \beta)$ on the light-induced anisotropy factor $a_\alpha(\lambda_0)$ can be quantified by two competing factors, i.e., a reduced dependence of $\theta_{\mathbf{k}}$ on the group velocity angle $\Theta_{\mathbf{V}'}^{(1)}$ in Eq. (23) and a slight decrease of density-of-states $\rho_S(\mathbb{E})$ included in Eq. (29) due to the interchange of \mathbf{k} -integration defined in Eq. (24). On the other hand, the Fermi energy E_F^* is considerably reduced for an anisotropic Dirac cone under a fixed electron density n_e , and therefore, it should be regarded as a key parameter in our computations. Moreover, as $a_\alpha(\lambda_0) = 1$, the isotropic $\sigma_s(E_F^* | \lambda_0 = 0)$ also displays a steady enhancement for a wider barrier width W_B mainly due to a bigger voltage drop across the barrier layer, as seen in Eq. (29). Furthermore, the tunneling current in a dice lattice reveals qualitatively the same feature as in graphene by comparing Fig. 8 with Fig. 9, but the actual values of $\sigma_s(E_F^*, \lambda_0 = 0)$ are quite different due to an enhanced transmission coefficient for a dice lattice compared to that of graphene, as can be seen in Fig. 4 and from Ref. [104]. Meanwhile, we find additional saturation of current increasing as $\mathcal{E}_0 = E_F^*$ becomes lower than a critical value, as found from Figs. 9(a) and 9(c), due to unique features in the transmission coefficient of a dice lattice.

IV. SUMMARY AND REMARKS

The effort of our work has been directed towards carrying out a rigorous investigation of the tunneling conductance and the currents through a square potential barrier in graphene and a dice lattice in the presence of irradiation-induced anisotropy of the electronic states and, specifically, their energy dispersions. In order to do that, we have derived an analytical expression for the conductivity as a function of all the crucial tuning parameters: an incoming electron energy \mathcal{E}_0 , angle of incidence $\Theta_{\mathbf{V}'}^{(1)}$, anisotropy $a_\alpha(\lambda_0)$ and misalignment angle β between the directions of the light polarization and the direct incidence on the potential barrier.

For a finite angle β , the direction of motion of an incoming particle is determined by its group velocity $\vec{V}_G^\gamma(\lambda_0, \mathbf{k})$ and its angle $\Theta_{\mathbf{V}'}^{(1)}$ relative to the normal incidence on the potential barrier, which is also used to distinguish the

transmitted and reflected waves in the barrier region, but not the wavevector angle $\theta_{\mathbf{k}}$. In our case, however, both these angles enter the equations for conductance simultaneously, which brings in some new unexpected relations and physical behavior.

We have found that the tunneling conductance has only non-essential and periodic dependence on the misalignment angle β . Its dependence on anisotropy consists of two terms with competing effects: a monotonically increasing trend coming from the mismatch between angles $\Theta_{\mathbf{V}}^{(1)}$ and $\theta_{\mathbf{k}}$, and a separate decreasing dependence which exactly reflects the way how the density of states for such anisotropic Dirac electrons is related to their Fermi energy.

The external optical dressing field could modify the sequential-tunneling current of electrons with the help of polarization angle β and light-interaction strength λ_0 . In addition to the applied E_{dc} , the sequential tunneling of electrons can also be enhanced by the Fermi energy E_F in two electrodes and the barrier width W_B . A strong dependence of the tunneling conductance and the current on the electron doping and its fine-tuning by the applied irradiation leads to a possibility to engineer an electronic device with desirable characteristics. Therefore, our work is expected to be an important step in understanding the radiation-modulated properties and behavior of such optoelectronic devices.

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