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Giant magnetoconductivity in non-centrosymmetric superconductors

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We discuss a novel physical mechanism which gives rise to a giant magnetoconductivity in non-centrosymmetric superconducting films. This mechanism is caused by a combination of spin-orbit interaction and inversion symmetry breaking in the system, and arises in the presence of an in-plane magnetic field $\mathbf{H}_\parallel$. It produces a contribution to the conductivity, which displays a strong dependence on the angle between the electric field $\mathbf{E}$ and $\mathbf{H}_\parallel$, and is proportional to the inelastic relaxation time of quasiparticles. Since in typical situations the latter is much larger than the elastic one this contribution can be much larger than the conventional Mattis-Bardeen conductivity thus leading to giant microwave absorption.

Non-centrosymmetric conductors with spin-orbit interaction exhibit interesting phenomena which are forbidden in centrosymmetric materials. Examples include spin magnetization induced by a current in metals [1–4], the quantized anomalous Hall effect on the surface of topological superconductors [5], linear coupling between supercurrent and an in-plane magnetic field in superconducting films [6–10], and the superconducting diode effect [11–18]. In this article we consider a new effect of this kind; we show that non-centrosymmetric superconducting films subjected to an in-plane magnetic field $\mathbf{H}_\parallel$ exhibit a giant anisotropic dissipative *ac* conductivity σ, which can be measured in microwave absorption experiments.

The *ac* conductivity σ relates the current density $\boldsymbol{j}$ to the applied electric field $\boldsymbol{E}(t) = \boldsymbol{E}_0 \cos \omega t$. At sufficiently low frequency ω this relation takes the form

$$\boldsymbol{j} = \boldsymbol{j}_{\rm eq}(\boldsymbol{p}_{\rm s}) + \hat{\sigma}\boldsymbol{E}(t), \tag{1}$$

where $\hat{\sigma}$ is a conductivity tensor, and $\boldsymbol{j}_{\rm eq}(\boldsymbol{p}_s)$ is the equilibrium current evaluated at the instantaneous value of the superfluid momentum $\boldsymbol{p}_{\rm s}$. The latter is defined as $\boldsymbol{p}_{\rm s} = \frac{1}{2}\left(\boldsymbol{\nabla}\chi - \frac{2e}{c}\boldsymbol{A}\right)$, where χ is the order parameter phase, $\boldsymbol{A}$ is the vector potential, and c is the speed of light. In the presence of microwave radiation $\boldsymbol{p}_{\rm s}$ evolves in time according to

$$\frac{d\boldsymbol{p}_s}{dt} = e\mathbf{E}(t). \tag{2}$$

The first term in the right hand side of Eq. (1) represents the dissipationless supercurrent. In the presence of superfluid momentum and in-plane magnetic field $\boldsymbol{H}_\parallel$ it can be written in the form

$$\boldsymbol{j}(\boldsymbol{p}_s) = \frac{e}{m}N_s\boldsymbol{p}_s + a[\mathbf{n}\times\mathbf{h}] + b\mathbf{h}. \tag{3}$$

Here e and m denote the electron charge and mass, N_s is the superfluid density, and we introduced the notation $\mathbf{h} = g\mu_{\rm B}\boldsymbol{H}_\parallel/2$, with g being the electron g-factor, and $\mu_{\rm B}$ the Bohr magneton. The last two terms in the right hand side of Eq. (3) describe the magnetoelectric effect [9] and are allowed by symmetry only in non-centrosymmetric superconductors.

We will assume that the electric field $\mathbf{E}$ is applied in the x-direction. In this case the component $\sigma_{xx} = \sigma$ of the conductivity tensor $\hat{\sigma}$ is responsible for the microwave absorption. The dissipative part of the *ac* conductivity of centrosymmetric superconductors has been extensively studied starting with the classic work of Bardeen and Mattis [19]. In the absence of a *dc* supercurrent and at sufficiently low frequencies, the value of the conductivity σ is controlled by elastic scattering of quasiparticles off impurities and is proportional to the quasiparticle elastic momentum relaxation time. In particular, in the vicinity of the critical temperature σ is nearly equal to the normal state conductivity $\sigma_{\rm n} = e^2\nu_{\rm n}D$, where $\nu_{\rm n}$ is the density of states at the Fermi level, and $D = v_{\rm F}^2\tau_{\rm el}/3$ is the diffusion coefficient, with v_F being the Fermi velocity and $\tau_{\rm el}$ the elastic relaxation time in the normal state.

Below, we show that for non-centrosymmetric superconductors placed in an in-plane magnetic field $\boldsymbol{H}_\parallel$ the linear conductivity σ acquires an additional contribution $\sigma_{\rm DB}$, which is proportional to the inelastic relaxation time of quasiparticles, $\tau_{\rm in}$, and has a pronounced dependence on the angle between the microwave field $\boldsymbol{E}$ and $\boldsymbol{H}_\parallel$. Since in typical superconductors $\tau_{\rm in}$ exceeds $\tau_{\rm el}$ by several orders of magnitude this leads to a giant anisotropic magnetoconductivity. The frequency dispersion of $\sigma_{\rm DB}(\omega)$ takes place at relatively low frequency $\omega \sim 1/\tau_{\rm in}$, which is much smaller than the characteristic dispersion frequency $1/\tau_{\rm el}$ of the normal state conductivity $\sigma_{\rm n}$.

The existence of this phenomenon can be traced to the fact that the quasiparticle density of states in superconductors $\nu(\epsilon, \mathbf{p}_{\rm s})$ depends on the superfluid momentum $\boldsymbol{p}_{\rm s}$. In the presence of microwave radiation the latter changes with time according to Eq. (2) producing a time-dependent density of states. At small frequencies this may be described in terms of the spectral flow, i.e. motion of individual quasiparticle energy levels in energy space. The quasiparticles which occupy these levels are entrained by the spectral flow. As a result, the quasiparticle distribution acquires a non-equilibrium component.

Its relaxation causes energy dissipation at a rate that is proportional to the inelastic relaxation time $\tau_{\rm in}$. This dissipation mechanism is similar to the Debye mechanism in centrosymmetric superconductors [20–22]. We therefore refer to it as the Debye contribution to conductivity and denote it by $\sigma_{\rm DB}$. The total dissipative part of the conductivity is given by the sum of $\sigma_{\rm DB}$ and the Mattis-Bardeen contribution proportional to $\tau_{\rm el}$.

Due to the scalar character of the density of states $\nu(\epsilon, \mathbf{p}_{\rm s})$ its linear coupling to $\mathbf{p}_{\rm s}$ induced by the microwave field $\boldsymbol{E}$ is possible only if the symmetry of the system allows for a presence of a polar vector. Therefore, in centrosymmetric superconductors the contribution proportional to $\tau_{\rm in}$ is possible only in a current-carrying state [21, 23]. In superconductors with broken inversion symmetry a polar vector enabling linear coupling of the density of states to microwave radiation may be formed from the pseudovector $\boldsymbol{H}_{\|}$. As a result, in films of non-centrosymmetric superconductors placed in an in-plane magnetic field, the linear conductivity acquires a contribution proportional to $\tau_{\rm in}$ even in the absence of dc supercurrent.

We note that the presence of such an additive contribution to the conductivity is in drastic contrast with the phenomenological Matthiessen's rule, which states that the resistivity (including the microwave resistivity of superconductors) is proportional to the sum of partial momentum relaxation rates due to each type of relaxation process. The latter implies that the resistivity is controlled by the shortest relaxation time.

We begin our quantitative treatment by writing down the Hamiltonian of a normal metal,

$$H_n(\mathbf{p}) = \xi_{\mathbf{p}} - \boldsymbol{b}(\boldsymbol{p})\cdot\boldsymbol{\sigma} - \mathbf{h}\cdot\boldsymbol{\sigma}, \tag{4}$$

where $\xi_{\mathbf{p}} = E(\boldsymbol{p}) - E_F$ is the quasiparticle energy relative to the Fremi energy E_F. Below we assume isotropic dispersion, $E(\boldsymbol{p}) = p^2/2m$, and the spin-orbit coupling of the form

$$\boldsymbol{b}(\boldsymbol{p}) = \alpha\boldsymbol{p}\times\boldsymbol{n} + \beta\boldsymbol{p}, \tag{5}$$

which includes both Rashba ($\beta = 0$) and Deresselhaus ($\alpha = 0$) cases. Here $\boldsymbol{\sigma}$ are spin Pauli matrices, and $\boldsymbol{n}$ is one of the two non-equivalent normals to the sample.

The eigenstates of the Hamiltonian (4) can be characterized by helicity (projection of spin on the direction of $\boldsymbol{b}(\boldsymbol{p})+\boldsymbol{h}$). The dispersion of the bands with positive and negative helicity has the form

$$\epsilon_{\mathbf{p}\pm} = \xi_{\mathbf{p}} \pm |\boldsymbol{b}(\boldsymbol{p}) + \boldsymbol{h}|. \tag{6}$$

Although the spectrum Eq. (6) contains an odd in $\mathbf{p}$ component, the equilibrium current in the normal state vanishes; $\mathbf{j}_N = \sum_{\pm}\int n_{\rm F}(\epsilon_{\mathbf{p}\pm})d\mathbf{p} = 0$. Here $n_{\rm F}$ is the Fermi distribution function.

In non-centrosymmetric superconductors in the presence of an in-plane magnetic field the equilibrium current may be written in the form of Eq. (3). To evaluate the coefficients $a, b, N_{\rm s}$ one can use the BCS Hamiltonian for superconductors with spin-orbit coupling. We write our Hamiltonian as $H = H_0 + U(\boldsymbol{r})\tau_3$, where

$$\begin{aligned} H_0(\boldsymbol{p}) &= \begin{pmatrix} H_n(\boldsymbol{p}) & \hat{\Delta} \\ \hat{\Delta} & -H_n^{\mathcal{TR}}(-\boldsymbol{p}) \end{pmatrix} \\ &= (\xi_{\mathbf{p}} - \boldsymbol{b}(\boldsymbol{p})\cdot\boldsymbol{\sigma})\,\tau_3 + \hat{\Delta}\tau_1 - \boldsymbol{h}\cdot\boldsymbol{\sigma}. \end{aligned} \tag{7}$$

Here $\mathcal{TR}$ denotes time-reversal conjugation, $\boldsymbol{\tau}$ are the Pauli matrices in the Gorkov-Nambu space, and $U(\boldsymbol{r})$ is the random impurity potential, whose strength will be characterised by the value of the electron elastic mean free time $\tau_{\rm el}$. For simplicity, we assume local interactions. In this case the order parameter is a singlet, and $\hat{\Delta} = \Delta\hat{I}$ is proportional to the identity matrix in spin space as we have chosen our basis vector in Nambu-Gorkov space to be $\Psi = \left(\psi_{\mathbf{k}\uparrow}, \psi_{\mathbf{k}\downarrow}, \psi^\dagger_{-\mathbf{k}\downarrow}, -\psi^\dagger_{-\mathbf{k}\uparrow}\right)^T$, where $\psi^\dagger_{\mathbf{k}\sigma}$ creates an electron with momentum $\boldsymbol{k}$ and spin σ. The superfluid momentum is included by making the gauge transformation $\boldsymbol{p} \to \boldsymbol{p} + \boldsymbol{p}_{\rm s}\tau_3$.

We focus on the temperature interval near the critical temperature, $T_c - T \ll T_c$. In this case the coefficients a and b in Eq. (3) can be evaluated in the second order in Δ using the conventional diagram technique,

$$a = e\alpha m\frac{\Delta^2}{T_c^2}g(T_c\tau_{\rm el}), \quad b = e\beta m\frac{\Delta^2}{T_c^2}g(T_c\tau_{\rm el}), \tag{8}$$

In the clean and dirty cases the function $g(T_c\tau_{\rm el})$ is given by

$$g(T_c\tau_{\rm el}) = \begin{cases} \frac{7}{8\pi^2}\zeta(3), & T_c\tau_{\rm el} \gg 1, \\ \frac{3}{2\pi^2}\zeta(2)\,T_c\tau_{\rm el}, & T_c\tau_{\rm el} \ll 1, \end{cases} \tag{9}$$

where $\zeta(n)$ is the Reimann zeta function, and we have assumed $\alpha p_F, \beta p_F \gg \max\left\{T, \sqrt{T/\tau_{\rm el}}\right\}$.

In the ground state the current density must vanish. Substituting Eqs. (8), (9) into Eq. (3) we obtain the value $\mathbf{p}_{\rm s}^{\rm (gs)}$ of ground state superfluid momentum in the form

$$\mathbf{p}_{\rm s}^{\rm (gs)} = \tilde{a}\boldsymbol{n}\times\boldsymbol{h} + \tilde{b}\boldsymbol{h}, \tag{10}$$

where the coefficients $\tilde{a}$ and $\tilde{b}$ are given by

$$\tilde{a} = \frac{m\alpha}{2E_F}\tilde{g}(T_c\tau_{\rm el}), \quad \tilde{b} = \frac{m\beta}{2E_F}\tilde{g}(T_c\tau_{\rm el}), \tag{11a}$$

$$\tilde{g}(T_c\tau_{\rm el}) = \begin{cases} 1, & T_c\tau_{\rm el} \gg 1, \\ 2 & T_c\tau_{\rm el} \ll 1. \end{cases} \tag{11b}$$

In deriving Eq. (11) we have used the standard results [24]

$$N_s = \begin{cases} \frac{7p_F^2}{8\pi^3}\zeta(3)\frac{\Delta^2}{T^2}, & T_c\tau_{\rm el} \gg 1 \\ \frac{p_F^2}{8}\frac{\Delta^2\tau_{\rm el}}{T}, & T_c\tau_{\rm el} \ll 1 \end{cases} \tag{12}$$

for the superfluid density. Equations (8) - (11) were obtained by Edelstein [7, 9] for the case of Rashba spin-orbit coupling. For a more general spin-orbit coupling the derivation is presented in Appendix A [25].

We now turn to the consideration of the dissipative part of the conductivity. A general expression for the Debye contribution to the linear microwave conductivity, $\sigma_{\rm DB}(\omega)$ was obtained in Ref. 21. For finite microwave frequencies ω it has the form

$$\frac{\sigma_{\rm DB}}{\sigma_{\rm n}}=\frac{3}{4}\frac{\tau_{\rm in}}{\tau_{\rm el}}\frac{1}{1+(\omega\tau_{\rm in})^2}\int\frac{d\epsilon}{T}\frac{\nu(\epsilon)}{\nu_{\rm n}}\frac{V^2(\epsilon)}{v_{\rm F}^2\cosh\left(\epsilon/(2T)\right)}, \tag{13}$$

where

$$\boldsymbol{V}(\epsilon)=-\frac{1}{\nu(\epsilon)}\int_0^{\epsilon}d\tilde{\epsilon}\frac{\partial\nu(\tilde{\epsilon})}{\partial\boldsymbol{p}_{\rm s}} \tag{14}$$

characterizes the sensitivity of quasiparticle energy levels to changes in $\boldsymbol{p}_{\rm s}$. The derivation of Eqs. (13) and (14) is based on the concept of spectral flow. Accordingly, the value of σ_{DB} is completely determined by the dependence of the quasiparticle density of states in a superconductor, $\nu(\epsilon,\mathbf{p}_s)$, on the superfluid momentum.

In s-wave superconductors $\sigma_{\rm DB}$ given by the integral in Eq. (13) is controlled by the energy interval close to the gap $\epsilon-\Delta\ll\Delta$. The derivative in Eq. (14) must be evaluated at zero current, that is at $\mathbf{p}_{\rm s}=\mathbf{p}_{\rm s}^{\rm (gs)}$, where $\mathbf{p}_{\rm s}^{\rm (gs)}$ is given by Eq. (10). In the absence of time-reversal symmetry breaking, $h=p_{\rm s}^{(gs)}=0$, the density of states has the BCS square-root singularity at $\epsilon=\Delta$. At $h\neq 0$ this singularity is broadened. The Debye conductivity $\sigma_{\rm DB}$ is dominated by the motion of quasiparticle energy levels inside this energy interval. Its character depends on the magnitude of the magnetic field h and the elastic relaxation rate in the normal state, $1/\tau_{\rm el}$.

One can distinguish between ballistic and diffusive regimes of microwave absorption. In the former, the broadening exceeds the rate of elastic scattering of quasiparticles, whose energies lie inside the broadened BCS singularity. In the latter the opposite inequality takes place. The value of the magnetic field h separating the ballistic and diffusive regimes may be estimated by equating h with the elastic relaxation rate for quasiparticles in the energy interval $|\epsilon-\Delta|\sim h$. Recalling that elastic relaxation rate for quasiparticles depends on the energy ϵ as (see, for example, Ref. 26)

$$\frac{1}{\tau_{\rm el}^{\rm s}(\epsilon)}\sim\frac{1}{\tau_{\rm el}}\sqrt{\frac{\epsilon-\Delta}{\Delta}}$$

one finds that the crossover between the ballistic and the diffusive regimes occurs at $h\tau_{\rm el}^2\Delta\sim 1$.

Ballistic regime, $h\tau_{\rm el}^2\Delta\gg 1$: In the superconducting state, the excitation spectrum of quasiparticles with positive and negative helicity in the presence of an in-plane Zeeman field $\mathbf{H}_{\parallel}$ and $\boldsymbol{p}_{\rm s}$ takes the form (up to first order in $h\ll|\mathbf{b}|$ and $p_{\rm s}$)

$$\tilde{\epsilon}_{\pm}(\boldsymbol{p})=\sqrt{(\xi_{\mathbf{p}}\pm|\boldsymbol{b}(\boldsymbol{p})|)^2+\Delta^2}+\tilde{\boldsymbol{v}}_{\pm}(\boldsymbol{p})\cdot\boldsymbol{p}_{\rm s}\mp\boldsymbol{h}\cdot\hat{\boldsymbol{b}}(\boldsymbol{p}).$$

Here $\hat{\boldsymbol{b}}=\boldsymbol{b}(\boldsymbol{p})/|\boldsymbol{b}(\boldsymbol{p})|$, and $\tilde{\boldsymbol{v}}_{\pm}(\boldsymbol{p})=\boldsymbol{p}/m\pm\boldsymbol{b}(\boldsymbol{p})/|\boldsymbol{p}|$ is the normal state velocity in the band with helicity $\pm$. The resulting density of states is given by

$$\nu(\epsilon)=\sum_{\pm}\int\frac{d\phi}{4\pi}\frac{\nu_{\pm}\left(\epsilon-\tilde{\boldsymbol{v}}_{\rm F\pm}\cdot\boldsymbol{p}_{\rm s}\pm\boldsymbol{h}\cdot\hat{\boldsymbol{b}}(\boldsymbol{p})\right)}{\sqrt{[\epsilon-\tilde{\boldsymbol{v}}_{\rm F\pm}\cdot\boldsymbol{p}_{\rm s}\pm\boldsymbol{h}\cdot\hat{\boldsymbol{b}}(\boldsymbol{p})]^2-\Delta^2}}, \tag{15}$$

where $\tilde{\boldsymbol{v}}_{\rm F\pm}$ is the band velocity, $\tilde{\boldsymbol{v}}_{\pm}(\boldsymbol{p})$ evaluated on the Fermi circle with the corresponding helicity ($\pm$), ϕ is the azimuthal angle of $\boldsymbol{p}$, and $\nu_{\pm}=\nu_{\rm n}\left(1\pm b/E_F\right)$ is the density of states on the corresponding Fermi circle.

The superfluid momentum in Eq. (15) can be written as $\boldsymbol{p}_{\rm s}=\mathbf{p}_{\rm s}^{\rm (gs)}+\delta\mathbf{p}_{\rm s}$, where $\delta\mathbf{p}_{\rm s}(t)$ is the superfluid momentum related to the electric field by Eq. (2). For the case where the spin-orbit coupling has the form of Eq. (5), to linear order in $\delta\mathbf{p}_{\rm s}$, the density of states in Eq. (15) depends only on the component of $\delta\mathbf{p}_{\rm s}$ that is parallel to $\mathbf{p}_{\rm s}^{\rm (gs)}$. This follows from the fact that, according to Eqs. (5), (10), and (11a) for $\boldsymbol{p}_{\rm s}=\mathbf{p}_{\rm s}^{\rm (gs)}$ the anisotropic terms $\tilde{\boldsymbol{v}}_{\rm F\pm}\cdot\boldsymbol{p}_{\rm s}$ and $\boldsymbol{h}\cdot\hat{\boldsymbol{b}}(\boldsymbol{p})$ in Eq. (15) have identical dependence on the azimuthal angle ϕ. Assuming the longitudinal polarization of the electric field, $\boldsymbol{p}_{\rm s}\parallel\mathbf{p}_{\rm s}^{\rm (gs)}$, and performing the angular integration in Eq. (15) we get the density of states in the form

$$\nu(\epsilon)=\frac{1}{2\pi}\sum_{\pm}\nu_{\pm}\gamma_{\pm}^{-1/2}\,\theta\left(w+\gamma_{\pm}\right)\mathrm{Re}\;K\left(\frac{w+\gamma_{\pm}}{2\gamma_{\pm}}\right). \tag{16}$$

Here $K(x)=\int_0^{\pi/2}d\phi(1-x\sin^2\phi)^{-1/2}$ is the complete elliptic integral of the first kind, and we introduced the following variables: $w=(\epsilon-\Delta)/\Delta\ll 1$, and $\gamma_{\pm}=\left(h\pm v_{\rm F}p_{\rm s}^{(gs)}\right)/\Delta$.

Next, we evaluate the level sensitivity, Eq. (14). There are two energy intervals to consider. For energies $-\gamma_-\leq w$ the bands with opposite helicities give almost equal but opposite contributions to V, and the level sensitivity is $V\sim v_F b/E_F$. Therefore, the contribution of this interval to the conductivity in Eq. (13) is quadratic in spin-orbit coupling strength, $\propto(b/E_F)^2$. Inside the second energy interval, $-\gamma_+\leq w\leq-\gamma_-$, only the band with $+$ helicity contributes to the density of states in Eq. (16). As a result, the level sensitivity in this energy interval is given by

$$V(\epsilon)=-v_{\rm F}. \tag{17}$$

Therefore, although the width of this interval is relatively small, $2v_{\rm F}p_{\rm s}^{\rm (gs)}\propto b/E_{\rm F}$, it provides the main contribution to the conductivity in Eq. (13). Substituting Eq. (17) into Eq. (13), and performing integration over ϵ in the interval $-\gamma_+\leq w\leq-\gamma_-$ we obtain the Debye contribution to the longitudinal conductivity, ($\boldsymbol{E}_0\parallel\mathbf{p}_{\rm s}^{\rm (gs)}$). The angular dependence of the Debye conductivity relative to the orientation of magnetic field is then restored with the aid of Eqs. (10) and (11a),

$$\frac{\sigma_{\rm DB}}{\sigma_{\rm n}}=\frac{3}{16\left(1+(\omega\tau_{\rm in})^2\right)}\frac{\tau_{\rm in}}{\tau_{\rm el}}\frac{p_F\sqrt{\Delta h}}{TE_F}\frac{(\beta\cos\theta-\alpha\sin\theta)^2}{\sqrt{\alpha^2+\beta^2}}. \tag{18}$$

Here θ is the angle between the electric field $\boldsymbol{E}$ and the in-plane magnetic field $\boldsymbol{h}$. Notably, as $\tau_{\rm in}/\tau_{\rm el}$ can be as large as 10^6-10^8, the Debye magnetoconductivity can be much larger than σ_n and thus Eq. (18) can be much larger than 1.

Diffusive regime, $h\tau_{\rm el}^2\Delta \ll 1$: In this case we express the single particle density of states in terms of the retarded single-particle Green's function

$$\nu(\epsilon) = -\frac{1}{\pi}\,\mathrm{Im}\int \frac{d^2p}{(2\pi)^2}\,\mathrm{tr}\,\hat{G}^R(\boldsymbol{p},\epsilon) \tag{19}$$

where the trace is performed over both Gorkov-Nambu and spin spaces and $\hat{G}^R$ is the retarded Green function averaged over disorder. Using the standard diagram technique for averaging over the realizations of random potential [24] one gets

$$\hat{G}^R(\boldsymbol{p},\epsilon) = \left(\epsilon_+ - H_0(\boldsymbol{p}) - \hat{\Sigma}^R(\epsilon)\right)^{-1}, \tag{20}$$

where $\epsilon_+ = \epsilon + i0$, $H_0(\boldsymbol{p})$ is given by Eq. (7), and the disorder-induced self-energy is given by

$$\hat{\Sigma}^R(\epsilon) = \frac{-1}{2\pi\nu_{\rm n}\tau_{\rm el}}\int \frac{d^2p}{(2\pi)^2}\hat{G}^R(\epsilon,\boldsymbol{p}). \tag{21}$$

Integrating over momentum here, substituting the result into Eq. (20), and using Eq. (7) we write the retarded Green's function in the standard form

$$\hat{G}^R(\boldsymbol{p},\epsilon) = \Big(\tilde{\epsilon} - (\xi_{\mathbf{p}} - \mathbf{b}(\mathbf{p})\cdot\boldsymbol{\sigma})\,\tau_3 - \tilde{\Delta}\tau_1 + \boldsymbol{h}\cdot\boldsymbol{\sigma} + \boldsymbol{v}_\pm\cdot\boldsymbol{p}_{\rm s}\Big)^{-1}. \tag{22}$$

where the disorder-renormalized energy $\tilde{\epsilon}$ and gap function $\tilde{\Delta}$ are given by the solutions of the following equations:

$$\tilde{\epsilon} = \epsilon + \frac{i}{4\tau_{\rm el}}\sum_\pm\int\frac{d\phi}{2\pi}\frac{\tilde{\epsilon} - (v_F p_s \pm h)\cos\phi}{\sqrt{(\tilde{\epsilon} - (v_F p_s \pm h)\cos\phi)^2 - \tilde{\Delta}^2}}, \tag{23a}$$

$$\tilde{\Delta} = \Delta + \frac{i\tilde{\Delta}}{4\tau_{\rm el}}\sum_\pm\int\frac{d\phi}{2\pi}\frac{1}{\sqrt{(\tilde{\epsilon} - (v_F p_s \pm h)\cos\phi)^2 - \tilde{\Delta}^2}}. \tag{23b}$$

Here, as in the ballistic case, we assumed longitudinal polarization, $\boldsymbol{p}_{\rm s} \parallel \mathbf{p}_{\rm s}^{\rm (gs)}$. Equations 23 are similar to those arising in the theory of superconductors with magnetic impurities. It is easy to see that in the absence of perturbations breaking time-reversal symmetry, $\mathbf{h}, \mathbf{p}_{\rm s} = 0$, Eqs. (23) yield $\frac{\tilde{\epsilon}}{\epsilon} = \frac{\tilde{\Delta}}{\Delta}$, which reproduces the standard BCS result for the density of states. For weak time-reversal breaking perturbations, $\left|(h \pm v_{\rm F}p_{\rm s}^{\rm (gs)})/(\tilde{\epsilon} - \tilde{\Delta})\right| \ll 1$ the density of states Eq. (19) can be expressed in the form (see Appendix B for details [25])

$$\nu(\epsilon) = \frac{\nu_{\rm n}A^{-1/3}\theta\left(\tilde{w} + \frac{3}{2^{5/3}}\right)}{\sqrt{3}}\left[\frac{\tilde{\alpha}(\tilde{w})}{2^{4/3}} - \frac{2^{4/3}\tilde{w}^2}{\tilde{\alpha}(\tilde{w})}\right], \tag{24a}$$

$$\tilde{\alpha}(\tilde{w}) = \left(16\tilde{w}^3 + 27 + 3\sqrt{3}\sqrt{32\tilde{w}^3 + 27}\right)^{1/3}. \tag{24b}$$

Here $A = \left(h^2 + (v_{\rm F}p_{\rm s}^{\rm (gs)})^2\right)\tau_{\rm el}\Delta$, and $\tilde{w} = wA^{-2/3}$. Using Eq. (24) we evaluate the level sensitivity Eq. (14), and substitute the result into (13) to obtain the Debye conductivity in the diffusive regime (see Appendix B [25])

$$\frac{\sigma_{\rm DB}}{\sigma_{\rm n}} = \frac{I_{\rm D}}{1+(\omega\tau_{\rm in})^2}\frac{\tau_{\rm in}}{\tau_{\rm el}}\frac{\Delta}{T}\left(\Delta\tau_{\rm el}^5h^4\right)^{1/3} \times\frac{(\beta\cos\theta - \alpha\sin\theta)^2p_F^2}{4E_F^2}\tilde{g}^2(T_c\tau_{\rm el}) \tag{25}$$

Here the numerical constant $I_{\rm D}\approx 0.38727$ is given by a definite integral in Eq. (B9) of the Appendix.

In summary, we have identified a new mechanism of magnetoconductivity of non-centrosymmetric superconductors, which arises from the quasiparticle spectral flow. It provides a contribution to the conductivity in the presence of an in-plane Zeeman field which is proportional to the inelastic quasiparticle relaxation time $\tau_{\rm in}$. In the ballistic, $h\tau_{\rm el}^2\Delta \gg 1$, and diffusive, $h\tau_{\rm el}^2\Delta \ll 1$, regimes this contribution is described by Eqs. (18) and (25), respectively. Since under typical conditions $\tau_{\rm in}$ exceeds the elastic relaxation time $\tau_{\rm el}$ by several orders of magnitude this contribution may exceed the Mattis-Bardeen contribution proportional to $\tau_{\rm el}$. Further, the Debye contribution to conductivity is strongly anisotropic; it exhibits a characteristic dependence on the angle between the direction of the external magnetic and the electric field of the microwave. This dependence is different in the ballistic, (18), and diffusive, (25), regimes. We also note that in typical situations, the positive magneto-conductance turns out to be much larger than the $H_\parallel$-dependence of the conductivity in normal metals, which may be estimated as $\frac{\sigma(H_\parallel)-\sigma(0)}{\sigma(0)} \sim \left(\frac{\mu_B H_\parallel}{E_F}\right)^2$.

Although we focused our consideration on the interval of temperatures near T_c, the mechanism of magnetoconductance discussed above is present in a much broader temperature interval. In particular, even at small temperatures, $T \ll \Delta$, where the quasiparticle concentration becomes exponentially small, $\propto \exp(-\Delta/T)$ it may give a large contribution to the low frequency magnetoconductivity. In this regime the quasiparticle relaxation is characterized by two time scales: *i)* quasiparticle scattering processes, which conserve the number of quasiparticles, occur on a time scale $\tau_{\rm in,sc}$, which is independent of the quasiparticle concentration, *ii)* quasiparticle recombination processes are characterized by a relaxation time, which is inversely proportional to the exponentially small concentration of quasiparticles, $\tau_{\rm in,r} \sim \tau_{\rm in,sc}\exp(\Delta/T)$. Since the Debye contribution to the conductivity is proportional to the longest relaxation time in the system, at sufficiently low frequencies the corresponding exponentially small factors cancel in the conductivity. Thus $\sigma_{\rm DB}$ at low temperatures is, roughly speaking, comparable to that near T_c. However, in this case the frequency dispersion of $\sigma_{\rm DB}(\omega)$ takes place at very low frequencies, $\omega \sim 1/\tau_{\rm in,r}$.

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