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# Melting phase diagram of bubble phases in high Landau levels

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A low-disorder, two-dimensional electron system (2DES) subjected to a large perpendicular magnetic field and cooled to very low temperatures provides a rich platform for studies of many-body quantum phases. The magnetic field quenches the electrons' kinetic energy and quantizes the energy into a set of Landau levels, allowing the Coulomb interaction to dominate. In excited Landau levels, the fine interplay between short- and long-range interactions stabilizes bubble phases, Wigner crystals with more than one electron per unit cell. Here we present the screening properties of bubble phases, probed via a simple capacitance technique where the 2DES is placed between a top and a bottom gate and the electric field penetrating through the 2DES is measured. The bubbles formed at very low temperatures screen the electric field poorly as they are pinned by the residual disorder potential, allowing a large electric field to reach the top gate. As the temperature is increased, the penetrating electric field decreases and, surprisingly, exhibits a pronounced minimum at a temperature that appears to coincide with the melting temperature of the bubble phase. We deduce a quantitative phase diagram for the transition from bubble to liquid phases for Landau level filling factors  $4 \leq \nu \leq 5$ .

Two-dimensional electron systems (2DESs) in a perpendicular magnetic field reveal a fascinating set of many-body quantum phases [1–3]. In the lowest orbital Landau level (LL), there is a plethora of fractional quantum Hall states. In excited LLs, however, the nodes in the wavefunction lead to a weakened short-range interaction and a preferred long-range order, manifested by the formation of charge-density-wave and stripe/nematic phases [4–9]. Consistent with this expectation, magnetotransport data for very low disorder 2DESs at very low temperatures have revealed anisotropic phases at half-filled, high-index LLs [10, 11] which are interpreted as stripe (or nematic) phases. Moreover, away from exact half-fillings, e.g., at LL filling factors  $\nu \simeq i + 1/4$  and  $\simeq i + 3/4$ , there are unusual phases with a vanishing longitudinal resistance and a Hall resistance that is quantized at a value corresponding to the nearest integer quantum Hall state (IQHS), namely, at  $ih/e^2$  and  $(i + 1)h/e^2$ , respectively ( $i$  is an integer  $\geq 2$ ) [12–15]. These are believed to be bubble phases, Wigner crystal states with more than one electron per unit cell, as shown in Fig. 1(a). Because they are pinned by the small but ubiquitous disorder, they are insulating but phenomenologically appear as reentrant IQHSs (RIQHSs). Even though the bulk of the 2DES is insulating, the longitudinal resistance vanishes because of the conducting edge states of the underlying LLs (see Fig. 1(b)). Besides magnetotransport, numerous other experimental techniques have been employed to study the bubble phases; these include measurements of non-linear  $I$ - $V$  [12, 16, 17], microwave resonance [18, 19], and surface acoustic waves [20, 21]. The results of all these measurements are consistent with the presence of bubble phases.

Here we present experimental data, probing the bubble phases and their melting into a liquid state, using a technique that measures their screening efficiency. As

highlighted in Fig. 1(c), this is a simple capacitance technique [22–25] where the application of an AC voltage between the top and back gates induces an electric field  $E_P$  that penetrates through the 2DES. The magnitude of  $E_P$  depends on the screening efficiency of the 2DES bulk, and the size of the penetrating current  $I_P$  is then probed in response to  $E_P$ . At the lowest temperatures, we find that the pinned bubble phases screen the electric field between the gates poorly and a large  $I_P$  is observed. With increasing temperature, the screening efficiency of the 2DES improves and  $I_P$  drops, as expected. However,  $I_P$  shows a distinct minimum at a temperature that corresponds to the melting temperature of the bubble phases. This observation suggests that the bubble phases becomes particularly efficient at screening near their melting. We use the data to construct a bubble-liquid melting phase diagram as a function of filling factor.

We studied a 2DES confined to a 30-nm-wide modulation-doped GaAs quantum well grown on a GaAs (001) substrate. The Si dopant atoms were placed in doping wells [26], leading to a very high-quality 2DES. The sample has density  $n = 2.96 \times 10^{11} \text{ cm}^{-2}$  and a low-temperature mobility  $\mu \simeq 30 \times 10^6 \text{ cm}^2/\text{Vs}$ . We measured a 4 mm  $\times$  4 mm van der Pauw geometry sample with alloyed (InSn) electrical contacts at the corners. The device is mounted on a header using In that serves as a back gate, and on top has a deposited, semi-transparent, 15-nm-thick Al film, top gate. We illuminated our sample before the measurements [27]. We used lock-in techniques for magnetoresistance ( $\simeq 17 \text{ Hz}$ ) and capacitance measurements. For the latter, the frequency ( $f$ ) range was  $2 \text{ Hz} \lesssim f \lesssim 1000 \text{ Hz}$ . The sample and a calibrated, RuO thermometer next to it were placed on the cold finger of a dry dilution refrigerator.

Figure 2(a) provides magnetoresistance data as a func-

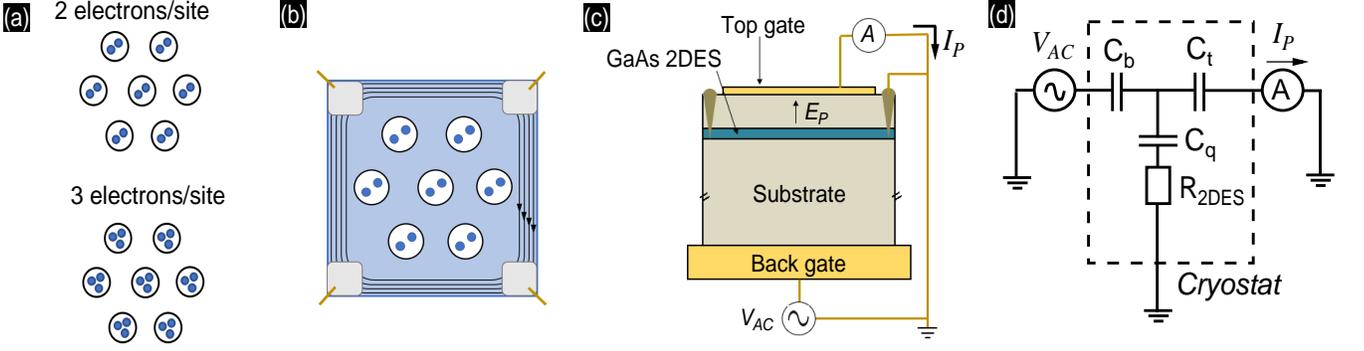


FIG. 1. (a) Schematic representations of bubble phases that have 2 or 3 electrons per lattice site. Bubble phases are formed in the excited Landau levels. (b) Measurement geometry (van der Pauw), where the grey squares represent the diffused electrical contacts to the GaAs 2DES. The edge currents flow near the perimeter of the sample, while the bubble phases occupy the bulk of the system. (c) Penetrating electric field measurement set-up. An AC excitation voltage ( $V_{AC}$ ) is applied between the top and back gates and the GaAs 2DES screens the established electric field, allowing only a penetrating electric field ( $E_P$ ) to pass through. From the top gate we collect the penetrating current ( $I_P$ ) induced by  $E_P$ . (d) A lumped-element circuit model of our device. The different components such as back-gate capacitance ( $C_b$ ), top-gate capacitance ( $C_t$ ), and the 2DES quantum capacitance ( $C_q$ ) and resistance ( $R_{2DES}$ ) are located inside of the cryostat (dashed box).

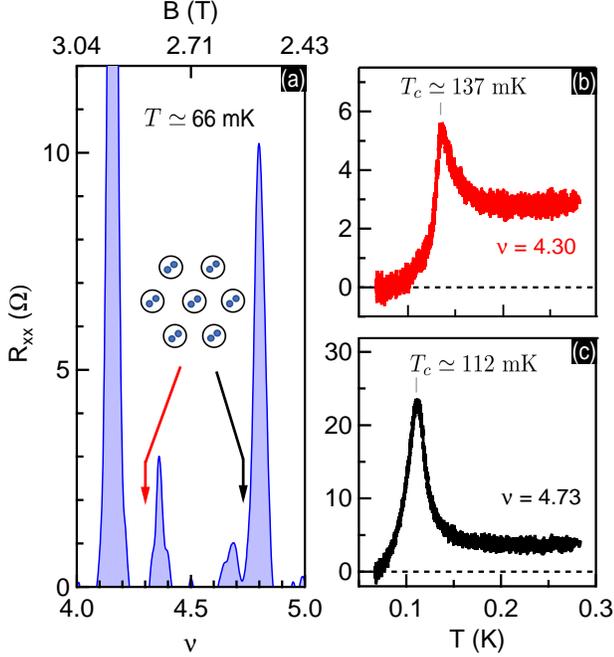


FIG. 2. (a) Longitudinal resistance ( $R_{xx}$ ) vs. Landau level filling factor ( $\nu$ ) for our 2DES with density  $n = 2.96 \times 10^{11} \text{ cm}^{-2}$ , at temperature  $T \approx 66$  mK. The trace represents the resistance along the “easy-axis” direction. The red and black arrows point to the regions where the bubble phases, exhibiting a RIQHS behavior, are observed. The inset shows a bubble phase with 2 electrons/site that forms at the lowest temperatures. (b)  $R_{xx}$  vs.  $T$ , measured at  $\nu = 4.30$  (red) and 4.73 (black). As the temperature is raised, the resistance peaks at  $T_c$ .

tion of  $\nu$  at the temperature  $T \approx 66$  mK. Near  $\nu = 4.3$

and 4.7 (marked by arrows), the longitudinal resistance ( $R_{xx}$ ) has vanishing values and the Hall trace is quantized to the resistance value of the nearest IQHS, signaling a RIQHS behavior [10–14, 16, 18]. See the Supplementary Material SM [28] for  $R_{yy}$  and  $R_{xy}$  traces. These characteristics have been associated with disorder-pinned bubble phases, which are observed in the highest-quality 2DESs. Pinned bubble phases have vanishingly small conductivity, but in transport measurements, the insulating behavior is shunted by the underlying edge currents of the filled LLs (see, e.g., Fig. 1(b)). We will return to the temperature dependence of the  $R_{xx}$  shown in Figs. 2(b,c) later in the manuscript.

By probing the screening efficiency of the 2DES, we derive information regarding its bulk. Figure 3(a) shows the sample’s screening efficiency for  $\nu$  ranging from 4 to 5, at  $T \approx 61$  mK [33]. At the  $\nu = 4$  and 5 IQHSs,  $I_P$  is a maximum, indicating minimal screening, i.e., an insulating bulk as the quasiparticles are localized by the disorder potential. As  $\nu$  increases from  $\nu = 4$ ,  $I_P$  decreases and reaches a minimum at  $\nu \approx 4.15$ , hinting at the delocalization of quasiparticles that are able to screen better the penetrating electric field. For larger  $\nu$ ,  $I_P$  rises and peaks at  $\nu = 4.30$ ; the local maximum in  $I_P$  coincides with the bubble phase region in transport experiments, namely the vanishing of  $R_{xx}$  (see Fig. 2(a)). In pinned electron solids, the electrons are fixed to their positions and are unable to screen effectively the penetrating electric field, thus leading to signatures of minimal screening (maximal  $I_P$ ) [22, 23], as seen in Fig. 3(a) at  $\nu \approx 4.30$ . When  $\nu$  starts at 5 and decreases,  $I_P$  follows a similar trend, reaching a local maximum at  $\nu \approx 4.73$ .

In between the bubble phases ( $4.40 \lesssim \nu \lesssim 4.65$ ),  $I_P$  is minimal. This range of  $\nu$  coincides with the region where

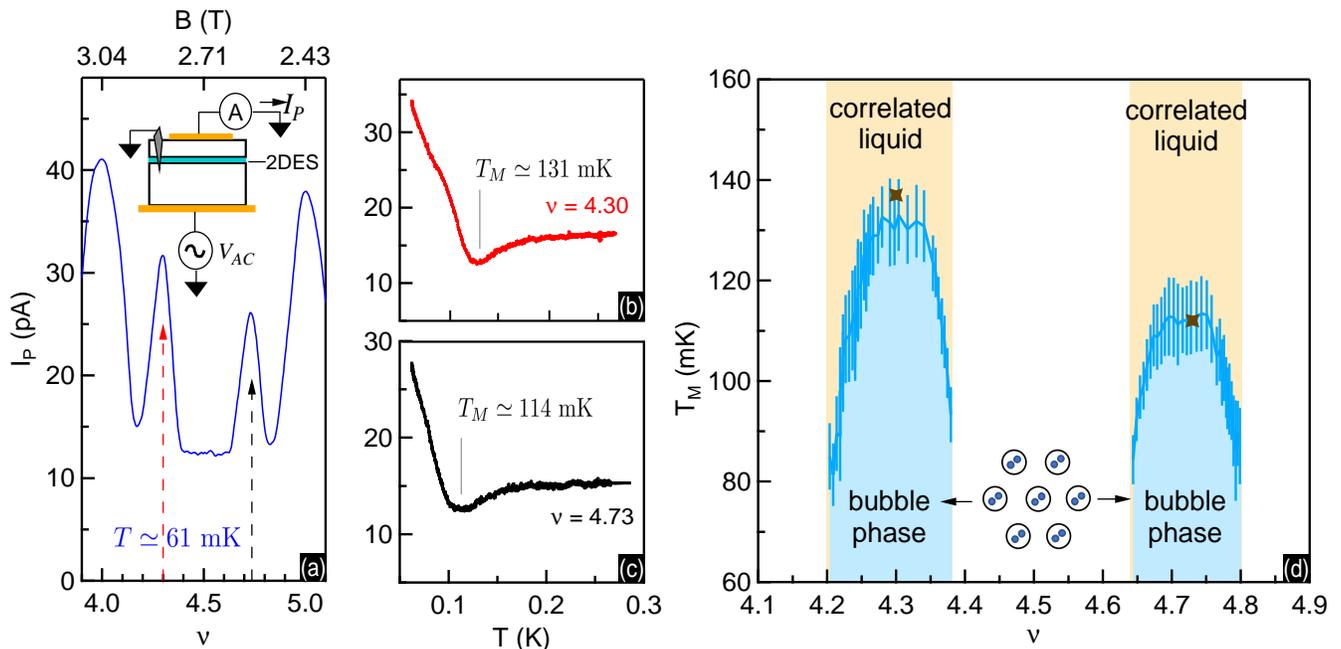


FIG. 3. (a) Penetration current ( $I_P$ ) vs.  $\nu$  at  $T \simeq 61$  mK. The bubble phases are marked by dashed lines. The inset shows the capacitance set-up. We apply an AC excitation voltage  $V_{AC} = 10$  mV between the gates at frequency  $f = 213$  Hz. (b) Temperature dependence of  $I_P$  for  $\nu = 4.30$  and  $4.73$ . A local minimum in  $I_P$  is observed at  $T_M$ , which we associate with the melting of the bubble phase. (c) Thermal melting phase diagram for the bubble phases for  $4 < \nu < 5$ . The blue regions are the bubble phases delimited by  $T_M$ . The yellow regions indicate the liquid states at high temperatures. The two brown crosses denote the maxima found in the non-monotonic temperature dependence of  $R_{xx}$  in Figs. 2(b,c). The cartoon shows a bubble phase with 2 electrons/site.

anisotropic stripe/nematic phases are detected in magnetotransport [4–6, 9–11] (see Fig. 2(a)). Stripe/nematic phases are unidirectional charge density waves that are highly conductive along the stripe/nematic direction at finite temperatures, thus an impinging electric field should be screened, in agreement with the local wide  $I_P$  minimum seen in Fig. 3(a). Note that we observe the same signatures of pinned bubble and stripe/nematic phases in  $I_P$  at higher LLs, up to  $\nu = 7$  [28].

By measuring  $I_P$  at fixed  $\nu$  as the temperature is raised, we capture the screening behavior of the bubble phases as they melt. Figures 3(b,c) show  $I_P$  vs.  $T$  for  $\nu = 4.30$  and  $4.73$ . At the lowest temperatures,  $I_P$  has large values consistent with pinned bubble phases. At high temperatures,  $I_P$  saturates at a value that is lower than its maximum low-temperature value, consistent with a melted state, i.e., a liquid phase that has a higher screening efficiency than the pinned bubble phase. One would expect that as the bubbles melt,  $I_P$  would change monotonically between these two limits as the electrons become unpinned with increasing temperature. Surprisingly, however, in between these two limits,  $I_P$  reaches a local minimum at temperature  $T_M$ , implying maximal screening at this temperature [34].

We attempt to understand qualitatively the non-monotonic  $I_P$  response with a lumped-element circuit

model as shown in Fig. 1(d) [35]. The 2DES is characterized by its quantum capacitance ( $C_q$ ) and bulk resistance ( $R_{2DES}$ ). Since values of the top and bottom capacitances ( $C_t$  and  $C_b$ ) are fixed, changes in  $I_P$  are a direct consequence of variations in the 2DES impedance  $Z_{2DES} = R_{2DES} + \frac{1}{j\omega C_q}$ . The local minimum in  $I_P$ , at  $T_M$ , implies a dip in  $Z_{2DES}$ , which could mean that  $R_{2DES}$  decreases, or  $C_q$  increases, or both.

We associate  $T_M$  with the melting temperature of the bubble phases based on the following considerations. First, a qualitatively similar behavior in  $I_P$  vs.  $T$  traces was recently seen when studying the screening properties of the Wigner crystal at very small  $\nu$  (*lowest* LL) in low-density GaAs 2D electron and hole systems [22, 23]. Associating the temperature for maximum screening with the melting temperature of Wigner crystal, Ref. [22] found the measured dependence of this temperature on  $\nu$  to be consistent with the melting phase diagrams reported previously for the Wigner crystal in GaAs 2DESs [36–39]. Second, theories for the melting of a 2D solid [40–43] predict a divergence in the compressibility near the melting temperature. The compressibility is proportional to  $C_q$  in Fig. 1(d), and the 2DES screening ability is directly related to  $Z_{2DES}$ . A large compressibility would lead to increased screening. We surmise that the maximal screening at our measured  $T_M$  might be related

to an increase in  $C_q$  as the bubble phase melts.

A natural question that arises is whether the non-monotonic behavior of  $I_P$  and the value of  $T_M$  are intrinsic to the 2DES and not an artifact of our measurement circuitry. As discussed in the SM [28] we tested this by measuring  $I_P$  for frequencies ranging from 21.3 to 1000 Hz. The non-monotonic  $I_P$  behavior and the value of  $T_M$  are frequency independent over this range.

Associating  $T_M$  with the melting temperature, a plot of our measured  $T_M$  vs.  $\nu$ , as shown in Fig. 3(d), provides the bubbles' thermal melting phase diagram. The light-blue and yellow regions represent the bubble phases and correlated liquids, respectively. The error bars give an estimate of the uncertainty in determining the temperature of the local minimum in  $I_P$  vs.  $T$  traces. The left-side bubble-liquid boundary has a relatively broad maximum around  $\nu = 4.30$ , and on its flanks  $T_M$  decreases quickly reaching values as low as  $\simeq 80$  mK at  $\nu \simeq 4.20$  and  $\simeq 4.38$ . The right-side bubble-liquid boundary also has a broad maximum near  $\nu = 4.73$  and, away from this maximum,  $T_M$  falls rapidly to  $\simeq 80$  mK at  $\nu \simeq 4.65$  and 4.80. Note that 80 mK is our reliable lower temperature limit for detecting a minimum in  $I_P$ . The  $\nu$  range of the left-side bubble to liquid boundary in Fig. 3(d) falls within the available theoretical calculations for the bubble phase stability. Density-matrix-renormalization-group [8] and Hartree-Fock [44, 45] calculations predict  $4.23 \leq \nu \leq 4.39$  and  $4.21 \leq \nu \leq 4.44$ , respectively, for the  $\nu$  window where the bubbles are stable. In the simplest scenario the regions around  $\nu = 4.30$  and 4.73 should be particle-hole symmetric.

The melting of bubble phases has also been studied in transport measurements. Deng *et al.* [14, 15] performed resistance measurements in a 2DES of very similar density and mobility to our sample. They showed that in the bubble phases' regime there is a sharp transition in  $R_{xy}$  as the temperature decreases. At high temperatures,  $R_{xy}$  has the classical Hall resistance value, and as the temperature decreases it quickly transitions to the resistance value of the nearest IQHS. Such a sharp transition was used to define the critical temperature ( $T_c$ ) for the formation/melting of bubble phases. The longitudinal resistance that accompanies the sharp transition in  $R_{xy}$ , has a non monotonic behavior and it peaks at the same  $T_c$ . The measured  $T_c$  in the  $R_{xx}$  temperature dependence can therefore be also used to pinpoint the formation/melting of bubble phases. Based on their measured  $T_c$ , Deng *et al.* [15] constructed thermal melting phase diagrams for bubble phases between  $2 \leq \nu \leq 4$ . Their deduced diagrams also exhibit a dome-shaped boundary, similar to those seen in Fig. 3(d).

It is instructive to compare our results for  $I_P$  with the temperature dependence of our  $R_{xx}$  data. Figures 2(b,c) feature  $R_{xx}$  vs.  $T$  for  $\nu = 4.30$  and 4.73. At the lowest temperatures  $R_{xx}$  is very close to zero, which is a manifestation of pinned bubble phases: the divergent bulk re-

sistance is shunted by the underlying edge currents of the filled LLs. For  $T \gtrsim 170$  mK, where the bubble phases have melted and the system is composed of unpinned quasiparticles,  $R_{xx}$  reaches a non-zero saturation value. The resistance reaches a maximum at an intermediate temperature  $T_c$  similar to what has been reported previously in Refs. [14, 15, 46, 47], as summarized in the preceding paragraph. In our measurements, we find that  $T_c$  is indeed very close to  $T_M$  deduced from screening efficiency data (see Figs. 3(b,c)). This can be best seen in the phase diagram in Fig. 3(d) where we have added our measured  $T_c$  as brown crosses. Note that our measured  $T_c$  values coincide (to within 5%) with those reported by Deng *et al.* [14].

Further contrasting of our data can be done with microwave resonance results from a very similar GaAs 2DES, again with comparable density and mobility to ours [18, 19]. At  $T \simeq 50$  mK, Lewis *et al.* [18] found resonances in the ranges  $4.20 \lesssim \nu \lesssim 4.37$  and  $4.62 \lesssim \nu \lesssim 4.82$ , providing support to the picture of pinned bubble phases. These filling factor ranges overlap with those where we see bubble phases in our experiments (Fig. 3(d)). Reference [18] also reported that the resonances are strongest at low temperatures and disappear when the temperature exceeds  $\simeq 110$  mK. This is somewhat smaller than our measured  $T_M \simeq 131$  mK. Given our measured  $T_c$  and  $T_M$  values, it appears that the melting temperatures derived from screening and transport measurements are slightly larger than those from microwave resonances.

In summary, we report the first screening efficiency measurements of pinned bubble phases in excited LLs. As the bubble phases melt, a minimum in  $I_P$  at temperature  $T_M$  is found, signaling maximal screening. We associate  $T_M$  with the bubble phase melting temperature and construct a melting phase diagram for the bubbles near  $\nu = 4.30$  and 4.70. We would like to highlight that the higher screening at an intermediate temperature appears to be a ubiquitous phenomenon for the melting of magnetic-field-induced electron solids formed in 2D systems, be it in the lowest [22, 23] or the excited LLs, thus it begs a rigorous theoretical explanation.

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