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Experimental Realization of Classical \mathbb{Z}_2 Spin Liquids in a Programmable Quantum Device

Shiyu Zhou,^{1,*} Dmitry Green,^{2,†} Edward D. Dahl,^{3,‡} and Claudio Chamon^{1,§}

¹*Physics Department, Boston University, Boston, MA, 02215, USA*

²*AppliedTQC.com, ResearchPULSE LLC, New York, NY, 10065, USA*

³*ColdQuanta, Inc., Boulder, CO 80301, USA*

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We build and probe a \mathbb{Z}_2 spin liquid in a programmable quantum device, the D-Wave DW-2000Q. Specifically, we observe the classical 8-vertex and 6-vertex (spin ice) states and transitions between them. To realize this state of matter, we design a Hamiltonian with combinatorial gauge symmetry using only pairwise-qubit interactions and a transverse field, i.e., interactions which are accessible in this quantum device. The combinatorial gauge symmetry remains exact along the full quantum annealing path, landing the system onto the classical 8-vertex model at the endpoint of the path. The output configurations from the device allows us to directly observe the loop structure of the classical model. Moreover, we deform the Hamiltonian so as to vary the weights of the 8 vertices and show that we can selectively attain the classical 6-vertex (ice) model, or drive the system into a ferromagnetic state. We present studies of the classical phase diagram of the system as function of the 8-vertex deformations and effective temperature, which we control by varying the relative strengths of the programmable couplings, and we show that the experimental results are consistent with theoretical analysis. Finally, we identify additional capabilities that, if added to these devices, would allow us to realize \mathbb{Z}_2 quantum spin liquids on which to build topological qubits.

Introduction – Quantum spin liquids (QSLs) have a long history. They were first proposed in the 1970s by Anderson¹ as an alternative to the spin 1/2 Néel antiferromagnetic state, and later as candidates for the insulating parent state of the high-temperature superconductors². QSLs do not display magnetic symmetry-breaking order, but instead display topological order³. They are also closely related to lattice gauge models in particle physics, also dating to the 1970s^{4–6}. Today, a QSL model known as the toric code is a potential platform for topological computing⁷. There have been many proposed materials, but to-date gapped QSLs have not been unambiguously observed in nature^{8–12}. Since they are so hard to find in materials, a more recent idea is to build a QSL synthetically out of superconducting circuits^{13–15}. In parallel, another recent idea has emerged to simulate familiar quantum phase transitions on programmable devices^{16,17}. In this paper we take these ideas one step further and show that, in principle, a programmable device can be used to emulate QSL phases so far unreachable by other means, a step towards realizing logical topological qubits in these same devices. While we cannot observe the full quantum regime due to the limitations of the current device, we do observe unmistakable signatures of the phase in its classical limit at the endpoint of the quantum annealing protocol. In the process of doing so, we identify additional features that a programmable device of this sort would need in order to realize QSLs. It is a testament to technological progress that a handful of theorists can observe and experiment with new physics while being equipped only with remote access to a commercial device.

From a theoretical perspective, our framework requires a new general construct: “Combinatorial Gauge Symmetry”¹⁸. This is an *exact and non-perturbative* symmetry

that stabilizes the topological phase for a wide range of parameters. From a practical perspective, it enables the programming of QSLs in D-Wave because it requires at most two-body Ising spin interactions. The vast majority of theoretical QSL models rely on pure multi-spin interactions, which heretofore are not attainable. The notable exception is the Kitaev hexagonal model¹⁹, but it requires XX , YY and ZZ interactions simultaneously, which have also been unattainable. In our case the model that we program is the \mathbb{Z}_2 lattice gauge theory. The toric code⁷, which is a central model for topological quantum computing, is a special limit of the \mathbb{Z}_2 model.

The D-Wave DW-2000Q quantum annealer comprises a superconducting circuit which implements programmable ZZ -couplings between pairs of spins (qubits): $J_{ij}\sigma_i^z\sigma_j^z$. Programmability means that the J_{ij} coefficients can be specified, although the available couplings are constrained by the connectivity of the device. A transverse field Γ is applied to each spin, coupling to the x-component of the spins, σ_i^x , inducing spin flips. The machine can be operated with the forward annealing protocol described in the device documentation²⁰, in which all active spins are initialized in the ground state of a Hamiltonian with a high transverse field Γ that is gradually decreased to zero, while the J_{ij} couplings are slowly increased from zero to their specified final values. At the endpoint of the protocol the system should land in a ground state of the classical Hamiltonian associated with the J_{ij} ’s if a dynamical obstruction (such as a glass transition) is not encountered.

In this work, we program the J_{ij} coefficients so that the spectrum represents a system with exact \mathbb{Z}_2 gauge symmetry. We observe and study the Hamiltonian in the classical regime at the endpoint of the annealing path, and map out its phases, which include both 8- and 6-vertex

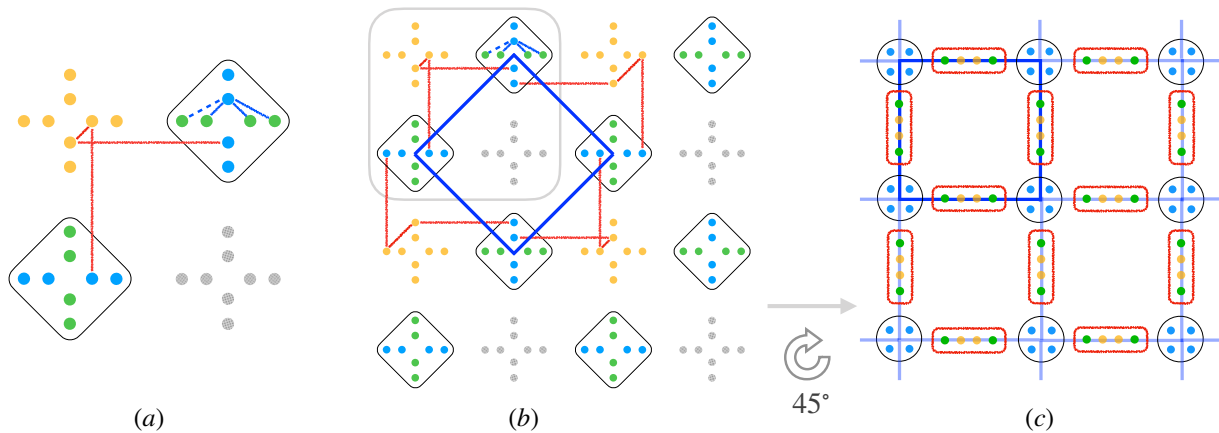


FIG. 1. (a) Embedding details for two adjacent star operators in Eq. (1). Each boxed cell represents one star operator consisting of four gauge spins σ_i (green dots) and four matter spins μ_a (blue dots). In a star, gauge spins only couple to matter spins according to the W matrix in Eq. (2), modulated by the strength J (blue lines). Only one set of J connections is shown for clarity. Note that one of the couplings (shown by a dashed blue line) is different from the other three (solid blue lines) and corresponds to the diagonal entries in the W matrix. Ghost spins (orange dots) are the necessary bridges to connect adjacent stars together. Each σ_i is copied to two ghost spins in the nearby unit cell with strong ferromagnetic coupling K (shown by a red line). There are unused spins in this embedding (grey dots). (b) The zoomed out view of the embedding, showing more stars connected through ghost spins. A plaquette of the effective lattice is indicated by thick blue lines. (c) The construction rotated through 45° , is equivalent to a lattice model where the effective gauge spins σ_i are represented by the four strongly coupled qubits that act as one. These four spins (two blue and two orange) reside on the bonds. The matter spins μ_a reside on the vertices (blue dots) of the effective lattice.

models. The 8-vertex model is the classical version of the toric code, while the 6-vertex phase is also known as planar spin ice²¹. Both planar²² and three-dimensional²³ spin ice have been observed; in our construction, we observe planar ice as a special case. In contrast, to our knowledge, neither the classical nor quantum 8-vertex models have been observed. We synthesize them here and probe the classical version.

Model and Embedding – The model is based on a square lattice, where we place “gauge” qubits on each link and four “matter” qubits at each vertex. The embedding of this geometry within the available D-Wave architecture is shown in Fig. 1. First, each gauge qubit is coupled to its neighboring matter qubits with strength proportional to J within the unit (Chimera) cell of the device. Second, we utilize four strongly-coupled qubits, with strength K , to effectively act as one gauge qubit, which is required because of the constrained coordination number of the Chimera architecture²⁰. To flip an effective gauge qubit, a transverse field must flip a total of four device qubits, so the effective transverse field on the gauge qubits is of the order $\tilde{\Gamma} \sim \Gamma^4/K^3$.

The embedding as described above is a realization of the following Hamiltonian

$$H = - \sum_s \left[J \sum_{\substack{a \in s \\ i \in s}} W_{ai} \sigma_i^z \mu_a^z + \Gamma \sum_{a \in s} \mu_a^x \right] - \tilde{\Gamma} \sum_i \sigma_i^x, \quad (1)$$

where σ_i and μ_a are Pauli matrices representing the gauge and matter spins, respectively, around each vertex s . The 4×4 interaction matrix,

$$W = \begin{pmatrix} -1 + \eta & 1 & 1 & 1 \\ 1 & -1 + \eta & 1 & 1 \\ 1 & 1 & -1 + \eta & 1 \\ 1 & 1 & 1 & -1 + \eta \end{pmatrix}, \quad (2)$$

encodes the ferromagnetic/anti-ferromagnetic interactions between gauge and matter spins that are programmed into the device.

At the special point $\eta = 0$, W is a 4×4 Hadamard matrix, which has the property that $W^T W = 4\mathbb{I}$ and every entry is equal to ± 1 . This point is characterized by a local symmetry which is generated by a group of monomial matrices that represent flips of single spins as well as permutations among matter spins on each vertex. This local combinatorial symmetry ensures that the Hamiltonian obeys a \mathbb{Z}_2 gauge symmetry for *any value* of J , Γ and $\tilde{\Gamma}$.¹⁸

At zero-temperature, for Γ and/or $\tilde{\Gamma}$ small, the model is in its topological phase¹⁸, as schematically depicted in Fig. 2(a). The energy gap of this QSL phase is on the order of $10^{-3} - 10^{-2}J$ ²⁴, and the temperature of the device renders the gap unobservable. (We note that dynamics in this regime may still reveal coherent propagation of quasiparticles, see Ref. 25 for a theoretical discussion.) While the QSL is inaccessible in the current devices, the annealing cycle in D-Wave, designed to adiabatically transform the system from a region of finite to

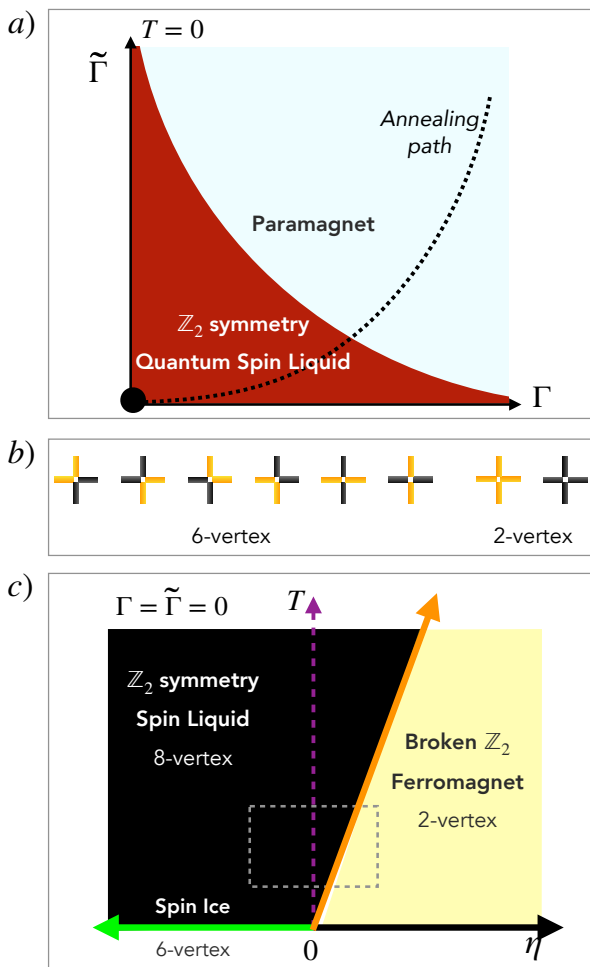


FIG. 2. (a) Schematic phase diagram of the Hamiltonian in Eq. (1) for $\eta = 0$. The annealing path goes from the paramagnetic to the quantum spin liquid with \mathbb{Z}_2 symmetry. The Hamiltonian obeys this symmetry along the entire annealing path. The system allows only one transverse field, however there are two effective ones, Γ and $\tilde{\Gamma}$, as a result of the embedding as described in the text. Measurements are taken at the classical limit where $\Gamma = \tilde{\Gamma} = 0$. (b) Vertex configurations for the gauge spins in the ground state. The 6-vertex and 2-vertex groupings are separated by an energy gap $\delta = 4\eta$. (c) Phase diagram for the classical limit ($\Gamma = \tilde{\Gamma} = 0$) of the 8-vertex model as a function of η and T . The dotted box depicts a window of η and T parameters where experimental measurements are accessible.

zero transverse field, lands the system at a classical \mathbb{Z}_2 spin liquid state at $\Gamma = \tilde{\Gamma} = 0$. Because the Hamiltonian obeys the combinatorial \mathbb{Z}_2 gauge symmetry *exactly* for any value of the couplings, the symmetry is respected throughout the entire quantum annealing path, minimizing the number of defects at the point of observation. The ground states at zero transverse field and $\eta = 0$ are the same as that of the 8-vertex model²⁶, in which the number of up and down gauge qubits around any vertex is an even number (even parity star), thus producing the loop

structure associated with the \mathbb{Z}_2 spin liquid state. In the Supplementary Material²⁷ we show the symmetries and ground state degeneracy explicitly.

For $\eta \neq 0$, the W matrix is deformed away from the point of combinatorial gauge symmetry, allowing us to access other states. Non-zero η splits the 8 vertices into groups of 2 and 6 vertices, as shown in Fig. 2(b), where up and down gauge qubits are pictured as black and gold links, respectively (the configurations of the matter qubits are tethered to those of the gauge qubits in the ground state). The energy separation is $\delta = 4J\eta$; $\eta < 0$ favors the 6-vertex model and $\eta > 0$ favors the ferromagnetic states, where all spins in a star are up or down. The topological phase $\eta = 0$ sits at a zero-temperature critical point.

At non-zero temperature, vertices with odd parity are allowed but exponentially suppressed at low temperatures. This enables us to study the phases of the classical 8-vertex model, whose theoretical phase diagram is shown in Fig. 2(c). In our programmed Hamiltonian we vary η and J , but since the operating temperature of the device is fixed, we are effectively varying η and J/T . This corresponds to varying the relative Boltzmann weight (fugacity) between the 6- and 2-vertex states, $d = e^{\delta/T} = e^{4J\eta/T}$. The 8-vertex spin liquid phase is stable in the range $0 < d < 3$ ^{28,29}. In this region there is a continuously varying imbalance between the 6- and 2-vertex types. The system transitions to an ordered ferromagnet at $d = 3$, and for $d > 3$ it remains magnetized, settling to one out of the two 2-vertex states. Strictly speaking, the $d = 0$ limit exists at zero temperature only, where $\eta < 0$ and we have the 6-vertex, or spin ice, phase. In the experiments, however, there is a small range of temperatures where it occurs because the system size is finite.

Results – The experiments were run on the D-Wave DW-2000Q machine at Los Alamos National Laboratory (LANL). The J and K couplings are programmed into the D-Wave’s Chimera architecture according to the embedding in Fig. 1. The ratio J/K is fixed to 1/2 in the experiments, which allows the ghost qubits to be coupled strongly while allowing sufficiently broad range for varying J . The operating temperature of the device is approximately 12 mK, but by scaling J and K (by the same factor) we effectively control the ratio T/J . The lowest temperature in our measurements corresponds to choosing the value $J_{\max}^{\text{API}} = 1/2$ in the Application Programming Interface (API) for the D-Wave DW-2000Q²⁰; by decreasing the input variable to $J^{\text{API}} = J_{\max}^{\text{API}}/\tau$, where $\tau \geq 1$, we increase the effective temperature by a factor τ . The factor can be expressed as $\tau = \alpha T/J$, where α is the dimensionless ratio between the maximum physical coupling J_{\max} (in units of mK) to the device temperature when $J_{\max}^{\text{API}} = 1/2$. We calibrate α by fitting the spectrum of the Hamiltonian for independent stars, and find it to be approximately 15 (see Supplementary Material²⁷).

The total annealing time in the experiments is chosen to be 2000 μs , with a pause from 20 μs to 1000 μs

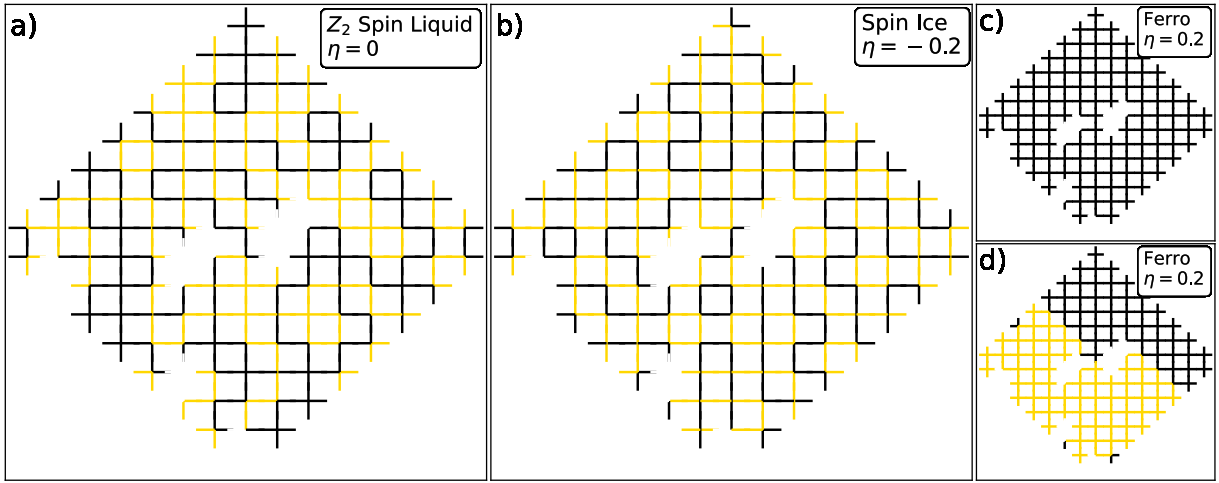


FIG. 3. Four typical configurations of final states returned by the device at the endpoint of the annealing cycle, obtained at base temperature ($\tau = 1$) for different values of the parameter η . The color of the bonds represent the state of the gauge spin [associated to that of the four strongly coupled qubits on the bonds, see Fig. 1(c)]: up gauge spins are colored in gold, and down spins are colored in black. The matter spins, which are tethered to the gauge spins at low energies, are omitted in the picture. The blank areas indicate punctures on the lattice due to the inactive qubits in the machine. (a) at $\eta = 0$, a \mathbb{Z}_2 spin liquid state is observed where closed loops or connected strings of either spin up or down are formed, and can intersect; (b) at $\eta = -0.2$, a spin ice state (or 6-vertex model) is obtained where similar loops and strings are formed, but do not intersect. (c) at $\eta = 0.2$, system fully magnetizes; and (d) at $\eta = 0.2$, two ferromagnetic domains with opposite magnetization are separated by a domain wall.

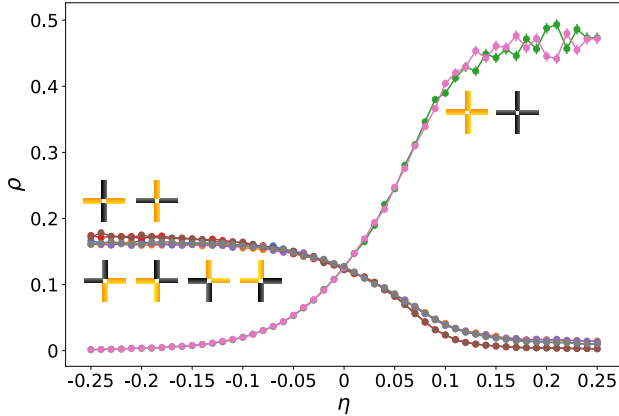


FIG. 4. Densities of all 8-vertex types for η varying from -0.25 to 0.25 , at base temperature ($\tau = 1$). The data clusters according to three different families of stars, indicated next to the curves. At $\eta = 0$ all 8-vertex types appear with equal densities, consistent with the presence of the combinatorial gauge symmetry. The 6-vertex types are favored for $\eta < 0$, and the 2-vertex types for $\eta > 0$.

in which Γ is held at approximately half its maximum value. A wait time of $100 \mu\text{s}$ is allowed between consecutive annealing cycles. Each run returns 1000 different configurations of final states, projected onto the z -basis. Typical configurations for different parameters η and at base temperature ($\tau = 1$), are shown in Fig. 3. The total number of active stars in the system is $N_{\text{stars}} = 114$ (if all qubits were active, there would be 128 stars.)

We observe the \mathbb{Z}_2 spin liquid in the expected range of parameters, with an example at $\eta = 0$ in Fig. 3(a). At each vertex, or star, the parity of the qubits at the links is even, and each of the allowed 8-vertex configurations occurs with equal proportion (see Fig. 4). Strings are visible in Fig. 3(a) by following a gold or black line. These strings either form closed loops in the bulk, or are open but terminate either at the external boundary or at the internal holes formed by the cluster of inactive qubits. We also observe the spin ice, or 6-vertex model, when $\eta < 0$, as shown in Fig. 3(b). The parity constraints are satisfied but the 2-vertex configurations are suppressed resulting in non-intersecting strings. Figs. 3(c) and (d) show ferromagnetic configurations, where the up/down symmetry is broken and there is an imbalance between the two 2-vertex types. In (c) the fully magnetized state is reached, while in (d) the system breaks into two domains with opposite magnetization and separated by a domain wall.

We gather 1000 sample configurations for each of 51 values of η equally spaced in the window $-0.25 \leq \eta \leq 0.25$ and 31 values of τ equally spaced in the range $1 \leq \tau \leq 2.5$. The data accumulated over these 51×31 bins can be aggregated to yield evidence that the 8-vertex constraint is satisfied in this window of the $\eta - \tau$ plane. Violations of the 8-vertex constraints take two forms in this embedding: (a) an odd parity star, or (b) a broken gauge link where the four physical qubits representing it are not all aligned. Out of the $51 \times 31 \times 1000 = 1581000$ samples gathered, 1007751 samples (or 63.7%) had no defects in any of the stars or links comprising the sys-

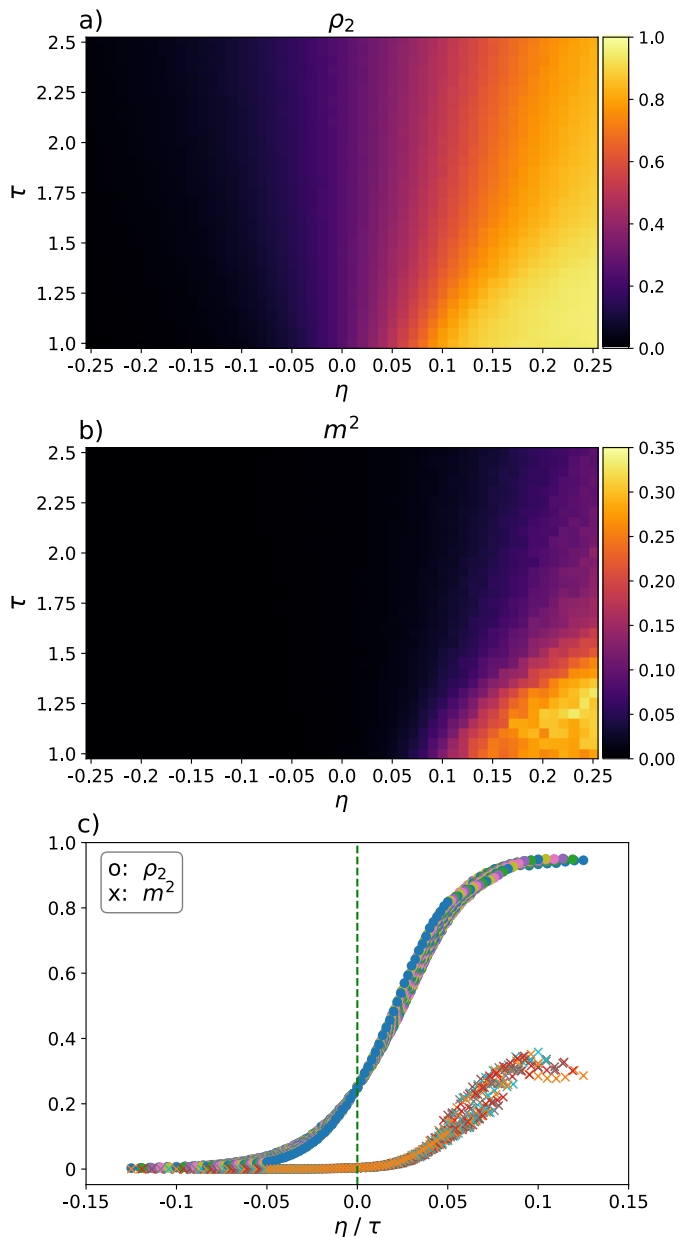


FIG. 5. Heat plots (a) and (b) show the observables ρ_2 and m^2 for η varying from -0.25 to 0.25 in intervals of 0.01 , and τ varying from 1 to 2.5 in intervals of 0.05 . The dependence of ρ_2 on the slopes τ/η of lines on the $\eta - \tau$ plane can be observed in (a). Map (b) shows that the magnetization vanishes in the spin liquid region (black region), and continuously grows past the transition to the ferromagnetic state (purple and yellow regions). (c) shows ρ_2 and m^2 as a function of η/τ . The collapse of the data shows that ρ_2 and m^2 are controlled by a single parameter, the ratio η/τ .

tem, while 374217 (or 23.7%) had only a single defect, 146865 (or 9.3%) had two defects, and 38857 (or 2.5%) had three defects. We note that most defects occur at higher temperatures, as expected, and in the ferromagnetic region because of domain walls. At base tempera-

ture and in the region of negative η where the topological phases of the 8-vertex model occur, 86.0% of the samples had zero defects, 11.7% had one defects, 2.0% had two defects, and 0.2% had three defects. We note that the fraction of single stars with no defects is 99.9% at base temperature and in the $\eta \leq 0$ region. We present in the Supplementary Material²⁷ a detailed study of the temperature dependence of the defect densities, which we also use to calibrate the constant α relating τ to T/J . The low number of defects is evidence that the 8-vertex model is realized within the experimental window.

The samples enable us to study several observables, such as the densities of all 8-vertex types, shown in Fig. 4 for η from -0.25 to 0.25 , at base temperature ($\tau = 1$). At $\eta = 0$, the system is placed at the combinatorial gauge symmetry point, and all 8-vertex types appear with equal densities. To the left of the symmetry point, for $\eta < 0$, the 6-vertex configurations are favored for sufficiently negative η , where the densities of 2-vertex types become exponentially small and thus not observable in the finite lattice. (We note that the densities of 2-vertex types are strictly zero only at $T = 0$.) We observe a small splitting between the 6 configurations, with slightly different densities for the two stars that are inversion symmetric as compared to the four that are not. To the right at $\eta > 0$, the 2-vertex configurations are favored. In this region, spontaneous magnetization is detected via the order parameter $m^2 = (\rho_{2+} - \rho_{2-})^2$, which measures the imbalance between the 2-vertex types with positive (ρ_{2+}) and negative (ρ_{2-}) magnetization.

In Figs. 5(a) and (b) we show experimental data for $\rho_2 = \rho_{2+} + \rho_{2-}$ and m^2 in the $\eta - \tau$ plane. Fig. 5(c) is a plot of the data for ρ_2 and m^2 as a function of η/τ , for the 31 values of τ . The collapse of the data confirms that ρ_2 and m^2 are indeed controlled by a single parameter, the relative fugacity $d = e^{4\eta/\tau}$. Notice that at $\eta = 0$, in particular, $\rho_2 \approx 1/3$ and $m^2 \approx 0$, consistent with the 8 vertices having the same density. Because the system size is finite, the phase transitions are rounded to smooth crossovers. Altogether, the experimental data in Figs. 3, 4, and 5 are consistent with the theoretical phase diagram presented in Fig. 2(c)

Outlook – The results above indicate that toric-code-like phases can be programmed in quantum hardware with only one- and two-body interactions. With the currently available hardware, with ZZ -interactions and X -fields, we succeeded in observing the classical 8-vertex model. In the annealing trajectory we traverse the region of the quantum phase diagram where the \mathbb{Z}_2 quantum spin liquid state resides, but the size of the many-body gap is too small to measure. A \mathbb{Z}_2 quantum spin liquid with a sizable gap can be implemented if XX -interactions and Z -fields are available¹⁸. The current hardware already has the latter. Extending the capability of the device to include XX -interactions, together with the theoretical notion of combinatorial gauge symmetry, would enable a realistic platform to build programmable topological qubits.

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- * zhous@bu.edu
 † dmitry.green@aya.yale.edu
 ‡ denny.dahl@coldquanta.com
 § chamon@bu.edu
- ¹ P. Anderson, “Resonating valence bonds: A new kind of insulator?” *Materials Research Bulletin* **8**, 153–160 (1973).
 - ² P. W. Anderson, “The resonating valence bond state in La_2CuO_4 and superconductivity,” *Science* **235**, 1196–1198 (1987).
 - ³ X. G. WEN, “Topological orders in rigid states,” *International Journal of Modern Physics B* **04**, 239–271 (1990).
 - ⁴ F. J. Wegner, “Duality in generalized Ising models and phase transitions without local order parameters,” *J. Math. Phys.* **12**, 2259–2272 (1971).
 - ⁵ J. B. Kogut, “An introduction to lattice gauge theory and spin systems,” *Rev. Mod. Phys.* **51**, 659–713 (1979).
 - ⁶ E. Fradkin and L. Susskind, “Order and disorder in gauge systems and magnets,” *Phys. Rev. D* **17**, 2637–2658 (1978).
 - ⁷ A. Y. Kitaev, “Fault-tolerant quantum computation by anyons,” *Ann. Phys.* **303**, 2–30 (2003).
 - ⁸ L. Balents, “Spin liquids in frustrated magnets,” *Nature* **464**, 199–208 (2010).
 - ⁹ L. Savary and L. Balents, “Quantum spin liquids: a review,” *Reports on Progress in Physics* **80**, 016502 (2016).
 - ¹⁰ Y. Zhou, K. Kanoda, and T.-K. Ng, “Quantum spin liquid states,” *Rev. Mod. Phys.* **89** (2017), 10.1103/RevModPhys.89.025003.
 - ¹¹ Y. S. L. S. e. a. Wen, J., “Experimental identification of quantum spin liquids,” *npj Quantum Materials* **4** (2019), 10.1038/s41535-019-0151-6.
 - ¹² C. Broholm, R. J. Cava, S. A. Kivelson, D. G. Nocera, M. R. Norman, and T. Senthil, “Quantum spin liquids,” *Science* **367** (2020), 10.1126/science.aay0668.
 - ¹³ L. B. Ioffe, M. V. Feigel’man, A. Ioselevich, D. Ivanov, M. Troyer, and G. Blatter, “Topologically protected quantum bits using Josephson junction arrays,” *Nature* **415**, 503–506 (2002).
 - ¹⁴ L. B. Ioffe and M. V. Feigel’man, “Possible realization of an ideal quantum computer in Josephson junction array,” *Phys. Rev. B* **66**, 224503 (2002).
 - ¹⁵ C. Chamon and D. Green, “A superconducting circuit realization of combinatorial gauge symmetry,” (2020), arXiv:2006.10060.
 - ¹⁶ A. D. King, J. Carrasquilla, J. Raymond, I. Ozfidan, E. Andriyash, A. Berkley, M. Reis, T. Lanting, R. Harris, F. Altomare, K. Boothby, P. I. Bunyk, C. Enderud, A. Fréchet, E. Hoskinson, N. Ladizinsky, T. Oh, G. Poulin-Lamarre, C. Rich, Y. Sato, A. Y. Smirnov, L. J. Swenson, M. H. Volkmann, J. Whittaker, J. Yao, E. Ladizinsky, M. W. Johnson, J. Hilton, and M. H. Amin, “Observation of topological phenomena in a programmable lattice of 1,800 qubits,” *Nature* **560**, 456–460 (2018).
 - ¹⁷ R. Harris, Y. Sato, A. J. Berkley, M. Reis, F. Altomare, M. H. Amin, K. Boothby, P. Bunyk, C. Deng, C. Enderud, S. Huang, E. Hoskinson, M. W. Johnson, E. Ladizinsky, N. Ladizinsky, T. Lanting, R. Li, T. Medina, R. Molavi, R. Neufeld, T. Oh, I. Pavlov, I. Perminov, G. Poulin-Lamarre, C. Rich, A. Smirnov, L. Swenson, N. Tsai, M. Volkmann, J. Whittaker, and J. Yao, “Phase transitions in a programmable quantum spin glass simulator,” *Science* **361**, 162–165 (2018).
 - ¹⁸ C. Chamon, D. Green, and Z.-C. Yang, “Constructing quantum spin liquids using combinatorial gauge symmetry,” *Phys. Rev. Lett.* **125**, 067203 (2020).
 - ¹⁹ A. Kitaev, “Anyons in an exactly solved model and beyond,” *Ann. Phys.* **321**, 2–111 (2006).
 - ²⁰ D-wave Systems Inc, “D-wave system documentation,” <https://docs.dwavesys.com/docs/latest>.
 - ²¹ E. H. Lieb, “Residual entropy of square ice,” *Phys. Rev.* **162**, 162–172 (1967).
 - ²² E. Mengotti, L. J. Heyderman, A. F. Rodriguez, F. Noltling, R. V. Hügli, and H.-B. Braun, “Real-space observation of emergent magnetic monopoles and associated Dirac strings in artificial kagome spin ice,” *Nature Physics* **7**, 68–74 (2010).
 - ²³ A. P. Ramirez, A. Hayashi, R. J. Cava, R. Siddharthan, and B. S. Shastry, “Zero-point entropy in ‘spin ice,’” *Nature* **399**, 333–335 (1999).
 - ²⁴ K.-H. Wu, Z.-C. Yang, D. Green, A. Sandvik, and C. Chamon, “ \mathbb{Z}_2 topological order and first-order quantum phase transitions in systems with combinatorial gauge symmetry,” .
 - ²⁵ O. Hart, Y. Wan, and C. Castelnovo, “Coherent propagation of quasiparticles in topological spin liquids at finite temperature,” *Phys. Rev. B* **101**, 064428 (2020).
 - ²⁶ R. Baxter, *Exactly Solved Models in Statistical Mechanics* (Academic Press, 1982).
 - ²⁷ See Supplemental Material at [URL will be inserted by publisher] for the symmetry and ground state degeneracy of the Hamiltonian; detailed defect analysis of the data we present; and the calibration of D-Wave experimental temperature control parameter to the physical ratio T/J .
 - ²⁸ E. Ardonne, P. Fendley, and E. Fradkin, “Topological order and conformal quantum critical points,” *Annals of Physics* **310**, 493–551 (2004).
 - ²⁹ S. Papanikolaou, K. S. Raman, and E. Fradkin, “Topological phases and topological entropy of two-dimensional systems with finite correlation length,” *Phys. Rev. B* **76**, 224421 (2007).