Quantum thermal transport in the charged Sachdev-Ye-Kitaev model: Thermoelectric Coulomb blockade

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Quantum thermal transport in the Schwarzian regime of the charged Sachdev-Ye-Kitaev model

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We present a microscopic theory for quantum thermoelectric and heat transport in the Schwarzian regime of the Sachdev-Ye-Kitaev (SYK) model. As a charged fermion realization of the SYK model in nanostructures we assume a setup based on Quantum Dot connected to the charge reservoirs through weak tunnel barriers. We analyze particle-hole symmetry breaking effects crucial for both Seebeck and Peltier coefficients. We show that the quantum charge and heat transport at low temperatures is defined by the interplay between elastic and inelastic processes such that the inelastic processes provide a leading contribution to the transport coefficients at the temperatures smaller compared to charging energy. We demonstrate that both electric and thermal conductance obey power law in temperature behavior while thermoelectric, Seebeck and Peltier coefficients are exponentially suppressed. This selective suppression of only non-diagonal transport coefficients have not been previously reported. We discuss validity of Kelvin formula in the presence of strong Coulomb blockade.

Introduction. – In recent years, the Sachdev-Ye-Kitaev (SYK) model [1, 2] became a hot spot for numerous studies due to its unique features. This model, which can be formulated in terms of Majorana fermions or conventional complex fermions (cSYK) in $(0+1)$ dimensions, has solvable non-trivial limits with absent quasiparticles, saturates the bound for quantum chaos [3, 4] and is connected holographically to black holes with AdS_2 ($1+1$ dimensional anti-de-Sitter) horizons [5]. Extensions of the model to the coupled cSYK clusters reproduce the benchmark properties of strange metals [6, 7], such as linear in temperate resistivity [8] and thermal diffusivity [9], observed in cuprates [10], pnictides [11] and twisted bilayer graphene [12]. The cSYK model allows a non-trivial analytic saddle-point solution if the dynamics of the model can be neglected. This solution possesses both conformal $SL(2,\mathbb{R})$ and gauge $U(1)$ symmetries. The low-energy limit of the SYK model is governed by the symmetry breaking mechanism of Goldstone reparametrization modes [13, 14], known as the Schwarzian regime in theories of gravity in a nearly AdS_2 space-time [4, 5, 15]. The cSYK model with charge density $\mathcal{Q}$ is shown to possess the residual zero-temperature entropy S per particle proportional to the parameter controlling particle-hole asymmetry $\mathcal{E}$ in the system, $\frac{\partial S}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$, ([16]) and identified as the Bekenstein-Hawking entropy of the charged black hole [17–19]. The finite Bekenstein-Hawking entropy is present in both conformal and Schwarzian regimes of the theory [20]. Various experimental realizations of the SYK model are proposed in quantum gases [21], Majorana wires [22] and topological superconductor [23] devices. Possible experimental realization of the cSYK model in irregularly shaped graphene flake quantum dots [24, 25] opens possibilities for direct studies of thermoelectric transport properties of the model.

The thermoelectric transport through quantum dots is a subject of extensive theoretical [26–29] and experimental [30–33] studies, as it opens broad possibilities for technological advancements in thermoelectric and microelectronic industries [34–36] and provides tools for better understanding of strongly correlated systems [37, 38]. Among all thermoelectric coefficients, the thermopower $\mathcal{S}$ is a subject of a particular interest due to its high sensitivity to the particle-hole asymmetry of the system. The thermopower measurements allow probing of particle-hole asymmetry related effects and provide information about low-energy excitations in the system [28, 39, 40]. These properties of the thermopower make it a useful tool for capturing Fermi liquid - non-Fermi liquid transitions (FL-NFL) by accessing NFL regime [41, 42].

Substantial progress was recently made in entropy measurement of mesoscopic quantum systems [43] of size up to a few particles [44, 45]. One of the most interesting and promising developments in these fields is studying the entropy via the thermoelectric properties of the system [46–49]. This approach employs the fact that under certain conditions the thermopower can be regarded as the entropy per particle [50]. The thermoelectric transport coefficients relate the charge and heat current, I_e and I_h, to applied voltage and temperature differences, ΔV and ΔT, in the linear response regime as

$$\begin{aligned} I_e &= G\Delta V + G_T\Delta T, \\ I_h &= G_T\Delta V + K\Delta T. \end{aligned} \quad (1)$$

G is the electric conductance, G_T is the thermoelectric coefficient, $\mathcal{S} = G_T/G$ is the thermopower, $\kappa = K - G\mathcal{S}^2/T$ is the thermal conductance [51–53].

The entropy and thermopower are connected by the Kelvin formula $\mathcal{S}{=}\left(\frac{\partial S}{\partial N}\right)_{T,V}$, where N is the number of particles [54] (we use units $e{=}\hbar{=}k_B{=}1$ for the electron charge, Planck's and Boltzmann constants). This relation is an empirical approximation of thermopower given in terms of the transport coefficients Eq.(1). It is considered to be applicable in the thermodynamic limit of the transport [55], but does not necessarily hold in a general case [56, 57]. While approximate in most cases, the Kelvin formula is shown to be exact in systems with the $SL(2,\mathbb{R})$ symmetry [9]. One of the most well-studied examples of the systems realizing this symmetry is the SYK model in the conformal regime [19].

The transport properties of the cSYK dots are extensively studied in the conformal regime [19, 25, 58, 59]. It is shown that the system exhibits FL-NFL transitions governed by the parameters of the dot. In a recent study [60], its authors examine the Schwarzian regime of the SYK model realized in a quantum dot with addition of the finite charging energy, E_C, and argue that the relation between the entropy and the thermopower holds there as well, which opens possibilities for direct measurements of the residual Bekenstein-Hawking entropy in proposed experimental realization of the cSYK model [24, 25]. However, these studies account for only direct tunneling (aka "elastic tunneling" [61]) in quantum transport, while the inelastic co-tunneling transport can significantly affect transport properties of quantum systems [27, 28], including the transport through the cSYK quantum dot [62].

In the present *Letter*, we examine the low temperature limit of quantum transport through the cSYK quantum dot and address the violation of the relation between thermopower and entropy due to the dominant role of inelastic processes in presence of the Coulomb blockade.

Model. – We consider a cSYK quantum dot (QD) coupled to two identical metallic leads, in a set-up similar to [58, 60]. Fermions of the cSYK dot and the lead are coupled via the tunneling term.

The Hamiltonian of the cSYK dot with i=1...N electronic orbitals represented by N complex spinless fermions [63] is given by

$$H_{\mathrm{SYK}} = \frac{1}{(2N)^{3/2}} \sum_{ijkl=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_l - \mu \sum_{i=1}^{N} c_i^\dagger c_i, \quad (2)$$

where $J_{i,j;k,l}$ is the random Gaussian interaction constant with zero mean value $\langle J_{ij;kl}\rangle = 0$ and nonzero variance $\langle |J_{ij;kl}|^2\rangle = J^2$, μ is the chemical potential of fermions inside the dot. Since the leads are identical, we consider QD coupled to symmetric superposition of the electronic states (one "effective" contact). The full Hamiltonian, which takes the lead into account, reads

$$H = H_{\mathrm{SYK}} + E_C^{(0)}\hat{n}^2 + \sum_q \varepsilon_q a_q^\dagger a_q + \sum_i \left(\lambda_i c_i^\dagger a_q + H.c.\right), \quad (3)$$

λ_i is the random tunneling constant, we assume that it is Gaussian with zero mean $\langle \lambda_i \rangle = 0$ and non-zero variance $\langle |\lambda_i|^2\rangle = \lambda^2$. Operators a, $a^\dagger$ represent a symmetric combination of fermions in the leads with dispersion relation ε_q. $E_C^{(0)}$ is the charging energy and $\hat{n} = \sum_i^N c_i^\dagger c_i$ is the charge on the dot [64].

The average tunneling term $\langle \lambda_i \rangle = 0$, so the direct tunneling does not contribute to the system's currents. Since both the elastic and inelastic co-tunnelings are present, this situation resembles tunneling through a barrier randomly fluctuating in time [65]. For an arbitrary tunneling constant variance λ^2, the fermion Green's function (GF) of the dot is effectively renormalized in the presence of the lead. In the conformal limit of the cSYK model [2, 19], this renormalization of the Green's function was discussed in [58, 60]. Physics of an isolated SYK dot in the low temperature regime $Je^{-N/2} \ll T \ll J/[N \log N]$ (we denote further $T^* \equiv J/[N \log N]$) is defined by the Schwarzian action (derived in [13, 14]), which appears due to breaking of conformal symmetry $SL(2,\mathbb{R})$ group with an additional contribution to the the cSYK model from breaking $U(1)$ symmetry (discussed in [62]). The inequality $Je^{-N/2} \ll T$ in the limit of large N ensures that effects of mean-level spacing can be neglected [13] , all further results are discussed within this assumption. The appearing Goldstone modes renormalize the saddle point solution for the conformal GF [66]. We assume that the dot-lead coupling λ is the smallest energy scale of the system ($\lambda \ll min\{T, T^*)\}$). It allows consideration of the system in the vicinity of the original saddle point of the isolated cSYK dot, similar to [25, 62]. This assumption is not related to the effects of breaking particle-hole symmetry in the conformal and Schwarzian limits of the model, that we are interested in the scope of this *Letter*.

At temperatures above the charging energy, $E_C^{(0)} \ll T \ll J$, the effects of the Coulomb blockade are reduced and the direct tunneling dominates the transport properties. This case was studied in details in [60]. In the present *Letter*, we focus on transport properties at low energy scales, where inelastic co-tunneling processes give crucial contribution to the currents. So, we are interested in two energy scales, $T^* \ll T \ll min\{E_C^{(0)}, J\}$ and $T \ll T^* \ll min\{E_C, J\}$. The former corresponds to the conformal regime, while the latter is described by the Schwarzian physics. The relevance of the latter case was additionally addressed in [62], as the charging energy is effectively renormalized by the Goldstone modes, $E_C = E_C^{(0)} + \mathcal{K}$ with the additional contribution to the charging energy $\mathcal{K} \sim T^*$. It ensures that the effective charging energy is always $E_C > T^*$. We suppose further that E_C includes this renormalization of the charging energy.

The thermoelectric properties of the system are expressed via the electric I_e and heat I_h currents, given in the weak tunneling limit by the Fermi golden rule [35]

$$I_e = -2\pi \int_{-\infty}^{\infty} d\varepsilon \rho_a(\epsilon)\rho_c(\epsilon)\Delta f(\epsilon, T),$$
$$I_h = -2\pi \int_{-\infty}^{\infty} d\varepsilon \varepsilon \rho_a(\epsilon)\rho_c(\epsilon)\Delta f(\epsilon, T), \quad (4)$$

where ρ_a and ρ_c are density of states (DoS) in the lead and the dot correspondingly, $f(\epsilon, T)$ is the Fermi distribution function at temperature T, ΔV is the applied voltage and $\Delta f(\epsilon, T) = f(\epsilon + \Delta V, T + \Delta T) - f(\epsilon, T)$.

Free fermion DoS in metal has weak energy dependence around the Fermi level, so it can be put to a constant $\rho_a = (2\pi v_F)^{-1}$ [25, 58, 60], v_F is the Fermi velocity.

Eqs.(1) and (4) allow us to find the system's electric conductance G, thermoelectric coefficient G_T and thermal conductance κ [67]. Following the approach of [27, 28], we express these entities through the T-matrix in the Matsubara representation $\mathcal{T}$ [38] (the T-matrix is supposed to be momentum independent, which is the case for short-range interactions).

$$G = \frac{1}{2v_F}\int_{-\infty}^{\infty} dt \frac{1}{\cosh(\pi T t)} \mathcal{T}\left(\frac{1}{2T} + it\right), \tag{5}$$

$$G_T = -\frac{i\pi}{2v_F}\int_{-\infty}^{\infty} dt \frac{\sinh(\pi T t)}{\cosh^2(\pi T t)} \mathcal{T}\left(\frac{1}{2T} + it\right), \tag{6}$$

$$K = \frac{\pi^2 T}{v_F}\int_{-\infty}^{\infty} dt \frac{1}{\cosh^3(\pi T t)} \mathcal{T}\left(\frac{1}{2T} + it\right) - T\pi^2 G. \tag{7}$$

As follows from Eqs. (5)-(7), the thermoelectric transport properties of the considered system are completely defined by the T-matrix $\mathcal{T}$. For direct tunneling (dt), the leading term contributing to the T-matrix is proportional to the two-point finite temperature Matsubara GF $\mathcal{G}_T(\tau)$, $\mathcal{T}_{dt}(\tau) \simeq \lambda^2 \mathcal{G}_T(\tau)$. In the inelastic co-tunneling case (in), it is expressed via the four-point finite temperature cSYK correlator $\mathcal{F}_T(\tau)$, $\mathcal{T}_{in}(\tau) \simeq \lambda^4 \frac{T}{\sin(\pi T \tau)} \mathcal{F}_T(\tau)$ [68].

Reparametrization modes. – The Hamiltonian (2) is invariant under $U(1)$ and $SL(2,\mathbb{R})$ symmetry reparametrizations [19]. It does not change after the transformation $c_i(\tau) \to e^{-i\phi(\tau)}\left[\dot{h}(\tau)\right]^{1/4} c_i(h(\tau))$, where $h(\tau)$ is a monotonic time reparameterization with winding number 1, ϕ is a phase fluctuation with possibly arbitrary integer winding number [20, 62].

These symmetries are broken by both time derivative in the action and by additional terms in Eq.(3). This leads to an effective action associated with energy costs of $\phi(\tau)$ and $h(\tau)$ fluctuations. As is shown in [62], the effective action S_{eff} splits into two independent parts for the corresponding fluctuations up to $1/N$-corrections, $S_{\text{eff}} = S_h + S_\phi$, where $S_h = -m\int d\tau Sch(h(\tau),\tau))$; $S_\phi = 16m\int d\tau \left[\phi'(\tau)\right]^2$. $Sch(h,\tau) \equiv (h''/h')' - \frac{1}{2}(h''/h')^2$ is the Schwarzian operator, $m = \frac{N\log N}{64J}\sqrt{\frac{\cos 2\theta}{2\pi}}$ is the effective mass appearing during the renormalizaton procedure [62], θ is related to the average hole occupation Q as $Q \equiv \frac{\langle n\rangle}{N} = \frac{1}{2} - \frac{\theta}{\pi} - \frac{\sin(2\theta)}{4}$ [19, 69].

Conformal regime. – Here we consider tunneling in the conformal case of the cSYK model, valid at temperatures $T^* \ll T \ll J$. The tunneling in the system is a sum of the elastic and inelastic processes.

Direct tunneling dominates at high temperatures $T \gg E_C$ [25, 62]. The T-matrix for elastic processes in the leading order is $\mathcal{T}(\tau) = \lambda^2 G^c(\tau) D(\tau)$. The conformal cSYK GF $G^c(\tau)$ is

$$G^c(\tau) = -C\text{sgn}(\tau)\sin\left(\pi/4 - \text{sgn}(\tau)\theta\right)\left(\frac{TJ}{\sin(\pi T|\tau|)}\right)^{1/2}, \tag{8}$$

$C = \left[(8/\pi)\cos(2\theta)\right]^{-1/4}$, the spectral asymmetry parameter $\mathcal{E}$ defines θ as $e^{2\pi\mathcal{E}} = \tan(\theta + \pi/4)$ [20, 70]. The two-point Coulomb correlator $D(\tau)$ reads

$$D(\tau) = \frac{\theta_3\left(-iE_C\tau - i\mathcal{E}\pi, e^{-\frac{E_C}{T}}\right)}{\theta_3\left(-i\mathcal{E}\pi, e^{-\frac{E_C}{T}}\right)} e^{-E_C|\tau|}, \tag{9}$$

$\theta_3(\bullet, *)$ is the Jacobi theta function [67, 71].

In this regime, the electric conductance G is suppressed with T by the Arrhenius exponential factor $G \sim e^{-\frac{E_C}{T}}$ at $T \ll E_C$, while $G \sim \frac{1}{\sqrt{T}}$ at $T \gg E_C$ [62, 72–76]. Thermopower $\mathcal{S}$ in this regime grows at small temperatures, while it saturates to a constant proportional to the spectral asymmetry parameter $\mathcal{E}$ at large T [19]. In the pure SYK system without charging energy of the dot, the Lorenz ratio $L = \lim_{T\to 0}\frac{\kappa}{TG}$ at zero temperature is modified in accordance with [9]. The Wiedemann-Franz law breaks down at finite temperatures if the Coulomb blockade effect is present in the system ($T \ll E_C$), similar to [53].

Inelastic co-tunneling process dominates at low temperatures $T^* \ll T \ll E_C$, where the direct tunneling is exponentially suppressed by the Coulomb blockade. The leading term in inelastic component of the T-matrix $\mathcal{T}_{in}(\tau)$ consists of the two-point free fermionic correlator, four-point SYK, F_{SYK}, and Coulomb, F_C, correlators: $\mathcal{F}_T(\tau) \simeq F_{\text{SYK}}(\tau) F_C(\tau)$ [67]. As shown in [14], the leading term in the $1/N$ orders of the SYK four-point function in the conformal limit is a factorization of the two-point functions Eq. (8), $F_{\text{SYK}}(\tau) \simeq G^c(\tau)G^c(-\tau)$.

Evaluation of Eq. (5) in this case gives that the electric conductance G is linear in temperature $G \sim T$.

Electric conductance G of both elastic and inelastic processes were analyzed in [60, 62, 67]. In the conformal regime, G is linear in temperature in the $T \ll E_C$ limit, but the direct tunneling quickly becomes dominant with increase of temperature [77].

The thermoelectric coefficient G_T is exponentially small in the $T \ll E_C$ limit for both elastic and inelastic processes, the leading contribution is elastic, it gives $G_T \sim e^{-\frac{E_C}{T}}$.

Schwarzian regime. – In this section we study the transport properties away from the conformal regime. This case is realized at $T \ll T^*$. The physics of the direct tunneling is defined by the two-point Green's function. The two-point Coulomb correlator in this case is again given by Eq.(9). In this region of parameters, the SYK GF is strongly renormalized by the soft mode $h(\tau)$. The exact form of the temperature dependent GF in time representation was found in [78]. In the low-temperature limit, the renormalized GF is

$$G^r(\tau) \to -\mathrm{sgn}(\tau)\frac{\beta^{3/2}}{(4\pi)^{\frac{1}{4}}}\frac{\Gamma^4(\frac{1}{4})}{\pi}\frac{me^{-\frac{\pi^2}{\beta m}}}{|\tau|^{3/2}(\beta-|\tau|)^{3/2}}, \quad (10)$$

$\beta \equiv 1/T$. The contribution from the direct tunneling is strongly suppressed by the Arrhenius exponent $e^{-\frac{E_C}{T}}$ from the Coulomb correlator.

In the inelastic co-tunneling case, the four-point SYK correlator is renormalized by the $h(\tau)$-modes. Following [78, 79], one can express this correlator $F_{\mathrm{SYK}}(\tau) = \langle G_\tau[h]G_{-\tau}[h]\rangle_h$ in the limit of low temperature as

$$F^r(\tau) \to \frac{\beta^{\frac{3}{2}}\pi m^{\frac{1}{2}}e^{-\frac{\pi^2}{\beta m}}}{2^{\frac{3}{2}}|\tau|^{\frac{3}{2}}(\beta-|\tau|)^{\frac{3}{2}}}. \quad (11)$$

This correlator defines the scaling law of the the electric conductance as $G \sim T^{3/2}$. The four point Coulomb correlator does not depend on T and $\mathcal{E}$ in the leading order, so the electric conductance G retain the power-law scaling. In contrast, the thermoelectric coefficient G_T is always exponentially suppressed at $T \ll E_C$. It follows from the symmetries of Eq.(6), since the exponentially small corrections to $F_C(\tau)$ give the only non-vanishing contribution in this case. In the Schwarzian regime, the leading contribution is elastic, $G_T \sim T^{-2}e^{-\frac{E_C}{T}}$ (same as in the conformal regime). This fact changes the thermopower $\mathcal{S}$ below E_C both in conformal and Schwarzian regimes. The resulting thermopower is depicted in Fig.1. It saturates to $\mathcal{S} = \frac{4\pi}{3}\mathcal{E}$ at $T \gg E_C$, approaching this limit as $\sim \frac{1}{\sqrt{T}}$, but it is exponentially suppressed at $T \ll E_C$, so it reaches zero instead of having divergence, which were expected if one considered only the direct tunneling in the system.

$$\mathcal{S} \sim \begin{cases} (T^*/T)^3 e^{-\frac{E_C}{T}}, & T^* \ll T \ll E_C \\ (T^*/T)^{\frac{7}{2}} e^{-\frac{E_C}{T}}, & T \ll T^* \end{cases}. \quad (12)$$

The same analysis is applicable to the Peltier coefficient Π due to its connection to thermopower, $\Pi = T\mathcal{S}$ [35]. Both the thermopower and the Peltier coefficient are antisymmetric in the spectral asymmetry parameter $\mathcal{E}$ and exponentially suppressed at low temperature by the Arrhenius exponent. The exact dependence of $\mathcal{S}$ on $\mathcal{E}$ is shown in [67].

Thermal conductance. – As follows from Eq.(7) and our analysis of the charge conductance and thermoelectric coefficient above, at temperatures below $T \ll E_C$, both in conformal and Schwarzian regimes, contribution to the thermal conductance κ proportional to G_T^2 is exponentially suppressed, the temperature dependence of κ has the same scaling in the leading order as TG and stems form the inelastic co-tunneling. The temperature dependence of κ is shown in Fig.2.

Lorenz ratio. – The authors of [9, 60] have demonstrated that the Wiedemann-Franz law and the Lorenz

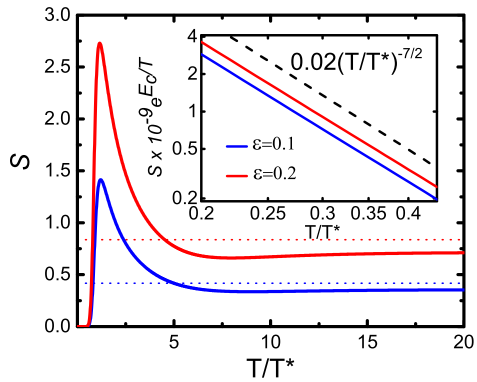

FIG. 1. Thermopower $\mathcal{S}$ accounting for direct tunneling and inelastic co-tunneling in the conformal regime for $\mathcal{E} = 0.1$ (blue line) and $\mathcal{E} = 0.2$ (red line), $N = 50$, $E_C/T^* = 10$, $(\lambda/T^*)^2 = 0.03$. Dotted lines are asymptotic limits $\frac{4\pi}{3}\mathcal{E}$ of thermopower for corresponding values of $\mathcal{E}$. Inset: *Log–log* plot for $\mathcal{S}$ multiplied by $e^{-\frac{E_C}{T}}$ in the Schwarzian regime for $\mathcal{E} = 0.1$ (blue line) and $\mathcal{E} = 0.2$ (red line). Black dotted line is $0.02(T/T^*)^{-7/2}$. Inset demonstrates the $(T/T^*)^{-7/2}e^{-\frac{E_C}{T}}$ law for thermopower [80].

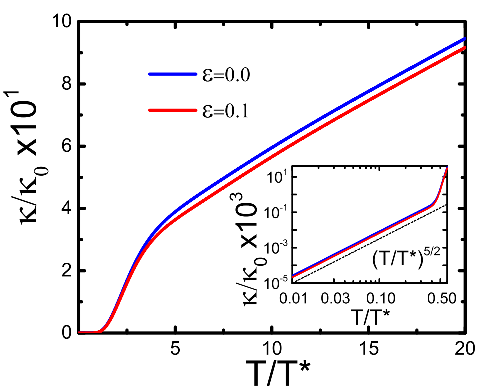

FIG. 2. Thermal conductance κ accounting for elastic and inelastic co-tunneling in the conformal regime, $N = 50$, $E_C/T^* = 10$, $(\lambda/T^*)^2 = 0.03$, $\kappa_0 = T^{*3}/v_F$. The lines correspond to $\mathcal{E} = 0$ (red line), and $\mathcal{E} = 0.1$ (red line). Inset: *Log-log* plot for thermal conductance κ in the Schwarzian regime for the same values of $\mathcal{E}$.

ratio [35, 36, 81] are violated in the conformal regime of cSYK model, $L_{\mathrm{SYK}} = \frac{\pi^2}{5}$. Accounting the inelastic co-tunneling contribution to the transport coefficients in this regime, we come to the Lorenz ratio

$$L_{in} = \lim_{T\to 0}\frac{\kappa(T, \mathcal{E}=0)}{TG(T, \mathcal{E}=0)} = \frac{\pi^2}{2}.$$

Considering the same entity in the Schwarzian case, we come to the Lorenz ratio $L_{in} \simeq 0.52\pi^2$.

Discussion. – We considered effects of inelastic co-tunneling in thermoelectric transport of the cSYK model in conformal and Schwarzian limits. We demonstrated that even in the weak tunneling limit the related tunneling process gives the leading order contribution to electric conductance G, and thermal conductance κ. The thermoelectric coefficient G_T is exponentially suppressed in both elastic and inelastic co-tunneling regimes at temperatures below E_C, while the electric and thermal conductances retain the power law behavior. This selective suppression of the thermoelectric coefficient appears as a consequence of the particle-hole symmetry breaking. The diagonal transport coefficients are not sensitive to small particle-hole asymmetry and have finite values in the particle-hole symmetric point, while the off-diagonal coefficient, G_T, is non-zero only when this symmetry is broken. It is proportional to the asymmetry parameter $\mathcal{E}$ (at $\mathcal{E} \ll 1$). All the terms contributing to the two-point and four-point Coulomb correlators that contain $\mathcal{E}$ also contain some power of the Arrhenius exponent, which leads to the reported low-T suppression of G_T. In contrary, the diagonal transport coefficients stem at low temperatures predominantly from the $\mathcal{E}$-independent term of the four-point correlator, which does not have the exponential suppression at low temperatures, so they exhibit power law behavior in T. As a direct consequence of this thermoelectric Coulomb blockade, thermopower $\mathcal{S}$ is exponentially suppressed as well. In general, the importance of inelastic co-tunneling in the conductance peaks $\langle Q\rangle = N$ was underlined in [28, 82, 83] while in the conductance valleys $\langle Q\rangle = N + \frac{1}{2}$ the direct tunneling usually accounts for all relevant contributions [26]. However in the Schwarzian regime the effective charging energy always has non-zero value, so the inelastic contribution is crucial here, resulting in the exponential suppression of $\mathcal{S}$. The selective Coulomb blockade of different transport coefficients, namely electric and thermal conductances, were recently observed experimentally in context of the heat Coulomb blockade [84].

The Kelvin formula for thermopower, rigorous for the cSYK model at $T \gg E_C$, is not applicable when the Coulomb blockade effects cannot be neglected. This discrepancy arises due to the leading role of the inelastic processes, so the transport coefficients now have different energy-dependence from DoS [55].

Even when the transport coefficients of the cSYK model stem from the inelastic co-tunneling contribution, the Lorenz ratio of the cSYK model is not sensitive to the renormalization of the model by reparametrization modes and the Coulomb blockade effects. The similar finite temperature relation (the Wiedemann-Franz law) is violated due to the Coulomb blockade effect.

Conclusion. – In this *Letter* we analysed temperature behavior of the charge and heat transport coefficients in the Schwarzian regime of cSYK model. We showed that both electric and thermal conductance obey power law in temperature behavior characteristic for non-Fermi liquid regimes while Seebeck and Peltier coefficients are exponentially suppressed. The leading contribution to the transport coefficients in this regime is given by the inelastic processes. We suggest to test the theoretical predictions in quantum transport experiments in semiconductor nanostructures.

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[63] The neglection of spin degrees of freedom is justified in a strong magnetic field, which is the case for proposed experimental realizations of the model [24].
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[66] In case of the arbitrary large tunneling constant λ, the saddle point of the conformal Green's function is effectively shifted, so the whole reparametrization by Goldstone modes occurs around some new saddle point.
[67] See Supplemental Material at [URL will be inserted by publisher] for the details of transport coefficients derivation, calculation of the two- and four- point correlation functions, electric conductance behavior as a function of temperature and dependence of thermopower on spectral asymmetry parameter for arbitrary $\mathcal{E}$.
[68] In principle, there are higher order corrections in powers of λ to both elastic and inelastic parts of the T-matrix. In the considered weak-tunneling limit all the orders above λ^2 can be neglected for the elastic part. However, when the elastic contributions are exponentially suppressed, the inelastic processes become important, the corresponding lowest in λ inelastic contribution is proportional to λ^4.

[69] We average the fermion correlators of the quantum dot over the fluctuations following the idea of [13]. For instance, the two-point function is renormalized as $\mathcal{G}(\tau) \simeq \int DhD\phi \mathcal{G}_{T,\tau}[\phi, h]e^{S_h+S_\phi}$. Functional integrals are factorized (up to $1/N$-corrections), so the resulting two-point correlator can be written as $\mathcal{G}(\tau) = D(\tau)G(\tau)$, where $G(\tau)$ is the Green's function of the SYK model averaged over h-fluctuations, $D(\tau)$ is the Coulomb correlator obtained by integrating out the phase field ϕ. These entities were calculated in [13, 62].

[70] The $sgn(\tau)$ ensures that the fermionic GF in the Matsubara representation is antiperiodic $G^c(\frac{1}{2T}) = -G^c(-\frac{1}{2T})$.

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