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Can bilayer graphene become a fractional metal?

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In a metal, electron interactions can cause a perfect spin polarization of the Fermi surface. For such a phase, only half of the non-interacting Fermi surface is available, and thus this state is commonly referred to as a 'half-metal'. Here we argue that, in multi-band electronic systems with nesting, further 'fractionalization' of the Fermi surface is possible. Taking the AA bilayer graphene as a convenient test case, we demonstrate that, under suitable conditions imposed on the electron interactions, doped AA bilayer graphene can host a 'quarter-metal' phase. Its Fermi surface (Fermi contour) is simultaneously polarized relative to spin-related and valley-related operators. The resultant state is nematic and possesses peculiar transport properties: the electric current carries both valley-related and spin-related quanta. In addition, since the two polarizations can be controlled independent of each other, the quarter-metal is a promising candidate for spintronics and valleytronics applications. Other types of Fermi surface fractionalization are also discussed.

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I. INTRODUCTION

In usual metals, the total spin polarization of the charge carriers at the Fermi surface is zero. A strong electron-electron interaction can lift the spin degeneracy, and induce spin polarization of the states at the Fermi surface. In the extreme case of the so-called half-metals[1–3], this polarization is perfect: all states at the Fermi energy have identical spin projection. Indeed, various rather different systems with transition-metal atoms are found to be half-metals[4–7]. The existence of spin-polarized currents in these half-metals makes them promising materials for applications in spintronics[3,8]. Several papers[9–13] predict half-metallicity in carbon-based systems. The half-metals free of heavy atoms could be of interest for bio-compatible applications and carbon-based electronics[14–19].

In previous works[20,21] we have proposed a mechanism for half-metallicity suitable for transition-metals-free systems. The mechanism requires the existence of two nested Fermi surface sheets (these sheets are distinguished below by their electron or hole 'flavor'[22]). The ground state of the undoped parent system is the spin- or charge-density wave state. Upon doping, the insulating density wave is replaced by a kind of half-metallic phase. In a multi-band system with nesting, besides spin, the valley quantum number ξ emerges, enumerating pairs (valleys) of nested Fermi surface sheets. One may wonder if also half-metals with a nested Fermi surface can be polarized in valley space. The stability of this peculiar conducting phase, which we call below "fractional metal" (FraM), is the main topic of this paper.

If the valleys are connected by a rotation, the polarization with respect to ξ implies that the resultant state is nematic. The electronic nematicity is one of the central issues in recent fundamental condensed matter research. Several actively studied materials demonstrate[23–30] this curios feature. From the viewpoint of possible applications, generalizing the celebrated concept of the ferromagnetic memory cell, one can envision a spin-valleytronic counterpart, which stores data using both valley-related and spin-related degrees of freedom coexisting on a single chip.

It follows from the FraM definition that only a material with a complex multi-sheet nested Fermi surface might host a FraM. This requirement makes AA bilayer graphene (AA-BLG) a promising candidate. The AA-BLG is less studied than the AB bilayer graphene. Yet, AA-BLG samples have been manufactured[31–34]. Moreover, progress in van der Waals heterostructures fabrication[35] allows one to hope that more efforts will be undertaken in the direction of producing high-quality AA-BLG samples.

As in other graphene structures, the low-energy states of the AA-BLG can be classified by their proximity to either the $\mathbf{K}_1$ or $\mathbf{K}_2$ Dirac point. A given Dirac point is encircled by an electron Fermi surface sheet and a hole sheet; altogether there are four Fermi surface sheets in the whole Brillouin zone. We argue that, for such a degenerate Fermi surface structure and under rather common assumptions about the electron-electron coupling, doped AA-BLG could enter the FraM phase. We investigate the stability of this phase and also briefly discuss its most immediate properties, such as its nematicity, transport of spin and valley quanta, and peculiar features of superconductivity.

II. MODEL

The electronic properties of AA-BLG are described by the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_{\rm int}$, where $\hat{H}_0$ is the single-

electron part and $\hat{H}_{\rm int}$ corresponds to the interaction between quasiparticles. For AA-BLG[19]:

$$\hat{H}_0 = -t\sum_{\langle \mathbf{mn}\rangle l\sigma} d^\dagger_{\mathbf{m}l0\sigma} d_{\mathbf{n}l1\sigma} - t_0 \sum_{\mathbf{n}a\sigma} d^\dagger_{\mathbf{n}0a\sigma} d_{\mathbf{n}1a\sigma} + \text{H.c.} - \mu n\,. \quad (1)$$

Here $d^\dagger_{\mathbf{m}la\sigma}$ ($d_{\mathbf{m}la\sigma}$) is the creation (annihilation) operator of an electron with spin projection σ in layer l [$l=0$ ($l=1$) corresponds to upper (lower) layer] on sublattice a [$a=0$ ($a=1$) represents sublattice A (B)] at the position $\mathbf{m}$. Also, $n=\sum_{\mathbf{n}la\sigma} d^\dagger_{\mathbf{m}la\sigma} d_{\mathbf{m}la\sigma}$ is the total charge density, μ is the chemical potential, and $\langle ...\rangle$ denotes nearest-neighbor pairs. The amplitude $t=2.57\,$eV ($t_0=0.36\,$eV) describes the in-plane (inter-plane) nearest-neighbor hopping. $\hat{H}_0$ can be readily diagonalized in a new basis $\gamma_{\mathbf{k}\ell\sigma}$: $\hat{H}_0=\sum_{\mathbf{k}\ell\sigma}\left(\varepsilon^{(\ell)}_{0\mathbf{k}}-\mu\right)\gamma^\dagger_{\mathbf{k}\ell\sigma}\gamma_{\mathbf{k}\ell\sigma}$, where $\ell=1,\ldots,4$ is the band index, $\mathbf{k}$ is the momentum; the eigenenergies and eigenoperators are

$$\begin{aligned} \varepsilon^{(1)}_{0\mathbf{k}} &= -t_0 - t\zeta_{\mathbf{k}}, \quad \varepsilon^{(2)}_{0\mathbf{k}} = -t_0 + t\zeta_{\mathbf{k}}, \\ \varepsilon^{(3)}_{0\mathbf{k}} &= +t_0 - t\zeta_{\mathbf{k}}, \quad \varepsilon^{(4)}_{0\mathbf{k}} = +t_0 + t\zeta_{\mathbf{k}}, \end{aligned} \quad (2)$$

$$\begin{aligned} d_{\mathbf{k}la\sigma} = \exp\left(-ai\varphi_{\mathbf{k}}\right)[\gamma_{\mathbf{k}1\sigma} + (-1)^a\gamma_{\mathbf{k}2\sigma} + \\ (-1)^l\gamma_{\mathbf{k}3\sigma} + (-1)^{a+l}\gamma_{\mathbf{k}4\sigma}]\,/2. \end{aligned} \quad (3)$$

In Eq. (3), $\varphi_{\mathbf{k}}=\arg(f_{\mathbf{k}})$, $\zeta_{\mathbf{k}}=|f_{\mathbf{k}}|$, where

$$f_{\mathbf{k}} = 1 + 2\exp\left(3ik_xa_0/2\right)\cos\left(\sqrt{3}k_ya_0/2\right), \quad (4)$$

and $a_0=1.42$ Å is the in-plane carbon-carbon distance. The band $\ell=2$ (band $\ell=3$) crosses the Fermi level and forms two electron (two hole) Fermi surface sheets, one centered at the Dirac point $\mathbf{K}_1=2\pi(\sqrt{3},1)/3\sqrt{3}a_0$, and another at $\mathbf{K}_2=2\pi(\sqrt{3},-1)/3\sqrt{3}a_0$. To distinguish electron and hole Fermi surface sheets, we introduce the charge flavor index $\nu=(-1)^\ell$: it equals $\nu=1$ ($\nu=-1$) for electrons (holes). If we label the graphene valley $\mathbf{K}_1$ (valley $\mathbf{K}_2$) by $\xi=+1$ (by $\xi=-1$), any sheet can be uniquely identified by values of ν and ξ (see Ref. 22 on the usage of term 'valley'). Since all sheets are circles of identical radius $k_{F0}=2t_0/3ta_0$, we have two nesting vectors: $\mathbf{0}$ and $\mathbf{Q}_0=\mathbf{K}_1-\mathbf{K}_2$.

The Coulomb interaction between electrons is

$$\hat{H}_{\rm int} = \frac{1}{2N_c}\sum_{\substack{\mathbf{kk'q}la\\ l'a'\sigma\sigma'}} V^{ll'}_{aa'}(\mathbf{q}) d^\dagger_{\mathbf{k}la\sigma} d_{\mathbf{k}+\mathbf{q}la\sigma} d^\dagger_{\mathbf{k'}l'a'\sigma'} d_{\mathbf{k'}-\mathbf{q}l'a'\sigma'}, \quad (5)$$

where N_c is the number of elementary cells in the sample and $V^{ll'}_{aa'}(\mathbf{q})$ is the Fourier transform of

$$V^{ll'}_{aa'}(\mathbf{r}) = V_{\rm C}\Big(\sqrt{[\mathbf{r}+(a-a')\boldsymbol{\delta}_1]^2+(l-l')^2D^2}\Big)\,. \quad (6)$$

Here, $V_{\rm C}(|\mathbf{r}|)$ is the screened Coulomb potential, $\boldsymbol{\delta}_1=(a_0,\,0)$, and $D=3.3$ Å is the inter-layer distance. The dependence of the interaction on various indices accounts for different distances between electrons at different sublattices and/or layers.

III. MEAN FIELD APPROACH

Theory predicts[36–39] that the electron repulsion converts the electronic "liquid" of the AA-BLG into a spin-density wave (SDW) insulator. The SDW order is characterized by non-zero values of $\langle\gamma^\dagger_{\mathbf{k}2\sigma}\gamma_{\mathbf{k}3\bar{\sigma}}\rangle$ and $\langle\gamma^\dagger_{\mathbf{k}1\sigma}\gamma_{\mathbf{k}4\bar{\sigma}}\rangle$, which describe excitonic pairs with vanishing total momentum. It is possible to define a different order parameter oscillating in space with the wave vector $\mathbf{Q}_0$, e.g., $\langle\gamma^\dagger_{\mathbf{k}+\mathbf{Q}_0 2\sigma}\gamma_{\mathbf{k}3\bar{\sigma}}\rangle$. However, the oscillating order parameter has lower coupling constant, because it cannot interact with another oscillating order parameter unless they have opposite momenta. This condition strongly reduces the effective coupling constant. As a result, such a phase has higher energy, and we will not consider it here. Switching to band operators γ and neglecting the terms irrelevant to the mean field approximation, we transform Eq. (5) and write $\hat{H}_{\rm int}=\hat{H}^{(1)}+\hat{H}^{(2)}+\hat{H}^{(3)}+\hat{H}^{(4)}$, where

$$\begin{aligned} \hat{H}^{(1)} &= -\frac{1}{N_c}\sum_{\mathbf{kp}\sigma} V^{(1)}_{\mathbf{kp}}\Big[(\gamma^\dagger_{\mathbf{k}1\sigma}\gamma_{\mathbf{k}4\bar{\sigma}})(\gamma^\dagger_{\mathbf{p}4\bar{\sigma}}\gamma_{\mathbf{p}1\sigma}) \\ &\quad +(\gamma^\dagger_{\mathbf{k}3\bar{\sigma}}\gamma_{\mathbf{k}2\sigma})(\gamma^\dagger_{\mathbf{p}2\sigma}\gamma_{\mathbf{p}3\bar{\sigma}})\Big], \quad (7) \\ \hat{H}^{(2)} &= -\frac{1}{2N_c}\sum_{\mathbf{kp}\sigma} V^{(2)}_{\mathbf{kp}}\Big[(\gamma^\dagger_{\mathbf{k}1\sigma}\gamma_{\mathbf{k}4\bar{\sigma}})(\gamma^\dagger_{\mathbf{p}1\bar{\sigma}}\gamma_{\mathbf{p}4\sigma}) \\ &\quad +(\gamma^\dagger_{\mathbf{k}2\sigma}\gamma_{\mathbf{k}3\bar{\sigma}})(\gamma^\dagger_{\mathbf{p}2\bar{\sigma}}\gamma_{\mathbf{p}3\sigma}) + \text{H.c.}\Big], \quad (8) \\ \hat{H}^{(3)} &= -\frac{1}{N_c}\sum_{\mathbf{kp}\sigma} V^{(3)}_{\mathbf{kp}}\Big[(\gamma^\dagger_{\mathbf{k}1\sigma}\gamma_{\mathbf{k}4\bar{\sigma}})(\gamma^\dagger_{\mathbf{p}3\bar{\sigma}}\gamma_{\mathbf{p}2\sigma}) \\ &\quad +(\gamma^\dagger_{\mathbf{k}2\sigma}\gamma_{\mathbf{k}3\bar{\sigma}})(\gamma^\dagger_{\mathbf{p}4\bar{\sigma}}\gamma_{\mathbf{p}1\sigma})\Big], \quad (9) \\ \hat{H}^{(4)} &= -\frac{1}{2N_c}\sum_{\mathbf{kp}\sigma} V^{(4)}_{\mathbf{kp}}\Big[(\gamma^\dagger_{\mathbf{k}1\sigma}\gamma_{\mathbf{k}4\bar{\sigma}})(\gamma^\dagger_{\mathbf{p}2\bar{\sigma}}\gamma_{\mathbf{p}3\sigma}) \\ &\quad +(\gamma^\dagger_{\mathbf{k}2\sigma}\gamma_{\mathbf{k}3\bar{\sigma}})(\gamma^\dagger_{\mathbf{p}1\bar{\sigma}}\gamma_{\mathbf{p}4\sigma}) + \text{H.c.}\Big], \quad (10) \end{aligned}$$

with the coupling constants $V^{(1,2,3,4)}_{\mathbf{kp}}$ defined by

$$V^{(1,3)}_{\mathbf{kp}} = \frac{1}{8}\left[V^{00}_{AA}+V^{10}_{AA} \pm \left(V^{00}_{AB}+V^{10}_{AB}\right)e^{-i\Delta\varphi} + \text{C.c.}\right], \quad (11)$$

$$V^{(2,4)}_{\mathbf{kp}} = \frac{1}{8}\left[V^{00}_{AA}-V^{10}_{AA} \mp \left(V^{00}_{AB}-V^{10}_{AB}\right)e^{-i\Delta\varphi} + \text{C.c.}\right]. \quad (12)$$

Here $V^{ll'}_{aa'}=V^{ll'}_{aa'}(\mathbf{k}-\mathbf{p})=V^{ll'}_{a'a}(\mathbf{p}-\mathbf{k})$, and $\Delta\varphi=\Delta\varphi_{\mathbf{kp}}=\varphi_{\mathbf{k}}-\varphi_{\mathbf{p}}$. One can assume[40,41] that intra-layer and inter-layer interactions in a graphene bilayer are approximately equal (at small momentum): $V^{00}_{aa'}\approx V^{10}_{aa'}$. In such a limit, we have in the first approximation

$$V^{(1,3)}_{\mathbf{kp}} \approx \frac{1}{2}V_{\rm C}(\mathbf{k}-\mathbf{p})\left[1\pm\cos(\Delta\varphi_{\mathbf{kp}})\right], \quad V^{(2,4)}_{\mathbf{kp}}\approx 0. \quad (13)$$

Thus, the interaction can be approximated as $\hat{H}_{\rm int}\approx\hat{H}^{(1)}+\hat{H}^{(3)}$. We analyze this Hamiltonian using mean field theory, and the terms $\hat{H}^{(2,4)}$ will be taken into account perturbatively. The mean field version of $\hat{H}_{\rm int}$ is

$$\hat{H}^{\rm MF}_{\rm int} = -\sum_{\mathbf{p}\sigma}\tilde{\Delta}_{\mathbf{p}\sigma}\gamma^\dagger_{\mathbf{p}4\bar{\sigma}}\gamma_{\mathbf{p}1\sigma} + \Delta_{\mathbf{p}\sigma}\gamma^\dagger_{\mathbf{p}3\bar{\sigma}}\gamma_{\mathbf{p}2\sigma} + \text{H.c.} + B, \quad (14)$$

where

$$\Delta_{\mathbf{k}\sigma} = \frac{1}{N_c}\sum_{\mathbf{p}}\left[V^{(1)*}_{\mathbf{pk}}\langle\gamma^\dagger_{\mathbf{p}2\sigma}\gamma_{\mathbf{p}3\bar{\sigma}}\rangle + V^{(3)}_{\mathbf{pk}}\langle\gamma^\dagger_{\mathbf{p}1\sigma}\gamma_{\mathbf{p}4\bar{\sigma}}\rangle\right],$$

$$\tilde{\Delta}_{\mathbf{k}\sigma} = \frac{1}{N_c}\sum_{\mathbf{p}}\left[V^{(1)}_{\mathbf{pk}}\langle\gamma^\dagger_{\mathbf{p}1\sigma}\gamma_{\mathbf{p}4\bar{\sigma}}\rangle + V^{(3)}_{\mathbf{pk}}\langle\gamma^\dagger_{\mathbf{p}2\sigma}\gamma_{\mathbf{p}3\bar{\sigma}}\rangle\right],$$

$$B = \sum_{\mathbf{p}\sigma}\left[\Delta_{\mathbf{p}\sigma}\langle\gamma^\dagger_{\mathbf{p}3\bar{\sigma}}\gamma_{\mathbf{p}2\sigma}\rangle + \tilde{\Delta}_{\mathbf{p}\sigma}\langle\gamma^\dagger_{\mathbf{p}4\bar{\sigma}}\gamma_{\mathbf{p}1\sigma}\rangle\right]. \quad (15)$$

The spectrum of the mean-field Hamiltonian can be easily derived:

$$E^{(2,3)}_{\mathbf{k}\sigma} = \mp E^{\rm m}_{\mathbf{k}\sigma}, \qquad E^{(1,4)}_{\mathbf{k}\sigma} = \mp E^{\rm h}_{\mathbf{k}\sigma}, \quad (16)$$

$$E^{\rm m}_{\mathbf{k}\sigma}=\sqrt{|\Delta_{\mathbf{k}\sigma}|^2+(t_0-t\zeta_{\mathbf{k}})^2},\ E^{\rm h}_{\mathbf{k}\sigma}=\sqrt{|\tilde{\Delta}_{\mathbf{k}\sigma}|^2+(t_0+t\zeta_{\mathbf{k}})^2}.$$

The grand potential of the system is equal to

$$\Omega = \sum_{v=1}^{4}\sum_{\mathbf{k}\sigma}(E^{(v)}_{\mathbf{k}\sigma}-\mu)\,\Theta(\mu - E^{(v)}_{\mathbf{k}\sigma}) + B\,, \quad (17)$$

where $\Theta(E)$ is the step-function. Minimization of Ω with respect to $\langle\gamma^\dagger_{\mathbf{p}3\bar{\sigma}}\gamma_{\mathbf{p}2\sigma}\rangle$ and $\langle\gamma^\dagger_{\mathbf{p}4\bar{\sigma}}\gamma_{\mathbf{p}1\sigma}\rangle$ gives us the system of equations for $\tilde{\Delta}_{\mathbf{k}\sigma}$ and $\Delta_{\mathbf{k}\sigma}$:

$$\Delta_{\mathbf{k}\sigma} = \sum_{\mathbf{p}}\left\{\frac{V^{(1)*}_{\mathbf{pk}}\Delta_{\mathbf{p}\sigma}}{2N_cE^{\rm m}_{\mathbf{p}\sigma}}\left[\Theta(\mu+E^{\rm m}_{\mathbf{p}\sigma})-\Theta(\mu-E^{\rm m}_{\mathbf{p}\sigma})\right]\right.$$
$$\left.+\frac{V^{(3)}_{\mathbf{pk}}\tilde{\Delta}_{\mathbf{p}\sigma}}{2N_cE^{\rm h}_{\mathbf{p}\sigma}}\left[\Theta(\mu+E^{\rm h}_{\mathbf{p}\sigma})-\Theta(\mu-E^{\rm h}_{\mathbf{p}\sigma})\right]\right\}, \quad (18)$$

$$\tilde{\Delta}_{\mathbf{k}\sigma} = \sum_{\mathbf{p}}\left\{\frac{V^{(1)}_{\mathbf{pk}}\tilde{\Delta}_{\mathbf{p}\sigma}}{2N_cE^{\rm h}_{\mathbf{p}\sigma}}\left[\Theta(\mu+E^{\rm h}_{\mathbf{p}\sigma})-\Theta(\mu-E^{\rm h}_{\mathbf{p}\sigma})\right]\right.$$
$$\left.+\frac{V^{(3)}_{\mathbf{pk}}\Delta_{\mathbf{p}\sigma}}{2N_cE^{\rm m}_{\mathbf{p}\sigma}}\left[\Theta(\mu+E^{\rm m}_{\mathbf{p}\sigma})-\Theta(\mu-E^{\rm m}_{\mathbf{p}\sigma})\right]\right\}. \quad (19)$$

The summation in Eqs. (18,19) covers the whole Brillouin zone. However, the interaction $V^{(1,3)}_{\mathbf{pk}}$ is strongest when $\mathbf{p}\approx\mathbf{k}$, and decays for larger $|\mathbf{p}-\mathbf{k}|$. In the limit of vanishing backscattering

$$V^{(1,3)}_{\rm bs} \equiv V^{(1,3)}_{\mathbf{K}_1,\mathbf{K}_2} \approx 0, \quad (20)$$

it is possible to define order parameters localized near the specific Dirac point $\mathbf{K}_\xi$: $\Delta_{\mathbf{k}\sigma} = \Delta_{\mathbf{k}\xi\sigma}$, when $\mathbf{k}\approx\mathbf{K}_\xi$. We see that, within our approximations, the electronic states and the order parameters can be split into four independent sectors, labeled by the multi-index $s=(\sigma,\xi)$. A sector with label $s=(\sigma,\xi)$ contains electron states with spin σ from valley ξ, and hole states with spin $-\sigma$ from the same valley. This definition implies that all states within a sector have identical value of the product $\sigma\xi$. The sectors are weakly coupled by neglected contributions proportional to $V_{\rm bs}$ and $V^{(2,4)}$. These corrections will be studied perturbatively.

We add and subtract Eqs. (18) and (19), use Eqs. (13), and change the summation by integration over the momentum near the Dirac point $\mathbf{K}_\xi$. We also assume that both Δ and $\tilde{\Delta}$ depend on $|\mathbf{k}|$ only. Finally, using the symmetry of our theory with respect to the sign of μ, we derive for $0<\mu<t_0$

$$\Delta_{ks}+\tilde{\Delta}_{ks} = \int_p \overline{V}(k,p)\left[\frac{\Delta_{ps}}{2E^{\rm m}_{ps}}\Theta(E^{\rm m}_{ps}-\mu)+\frac{\tilde{\Delta}_{ps}}{2E^{\rm h}_{ps}}\right],$$

$$\Delta_{ks}-\tilde{\Delta}_{ks} = \int_p \overline{U}(k,p)\left[\frac{\Delta_{ps}}{2E^{\rm m}_{ps}}\Theta(E^{\rm m}_{ps}-\mu)-\frac{\tilde{\Delta}_{ps}}{2E^{\rm h}_{ps}}\right], \quad (21)$$

where $\int_p \ldots = (2\pi/v_{\rm BZ})\int pdp\ldots$, and the volume (area) of the Brillouin zone is $v_{\rm BZ} = 8\pi^2/(3\sqrt{3}a_0^2)$. In Eqs. (21), the averaged coupling constants are

$$\overline{V}(k,p) = \int\frac{d\phi}{2\pi}V_{\rm C}(\sqrt{k^2+p^2-2kp\cos\phi}), \quad (22)$$

$$\overline{U}(k,p) = \int\frac{d\phi}{2\pi}V_{\rm C}(\sqrt{k^2+p^2-2kp\cos\phi})\cos\phi,$$

and the spectrum (16) in sector $s=(\sigma,\xi)$ can be approximated as

$$E^{\rm m}_{ps} \cong \sqrt{|\Delta_s|^2+t_0^2(1-p/k_{F0})^2}, \quad (23)$$

$$E^{\rm h}_{ps} \cong \sqrt{|\tilde{\Delta}_s|^2+t_0^2(1+p/k_{F0})^2} \cong t_0(1+p/k_{F0}),$$

where $p=|\mathbf{p}-\mathbf{K}_\xi|$.

IV. BCS-LIKE APPROXIMATION

In general, we can choose some model for $V_{\rm C}(q)$ and solve Eqs. (21) numerically. However, modeling the effective Coulomb interaction in graphene bilayers is notoriously difficult, and no universal and compact answer is known. Indeed, to study the effects of the Coulomb interaction in graphene-based systems, the Hartree-Fock approximation[47], renormalization group[48,49], and the random phase approximation[38] were used. Unfortunately, a rigorous theoretical attempt to account for the Coulomb interaction may produce a non-universal and difficult-to-interpret results, see, for example, Fig. 3 in Ref.[49]. In this situation, finding an accurate numerical solution to the integral equations (21) is impractical. Instead, we use the simple BCS-like ansatz

$$\Delta_s(q) = \Delta_s\Theta(\Lambda-|q-k_{F0}|), \quad (24)$$

$$\tilde{\Delta}_s(q) = \tilde{\Delta}_s\Theta(\Lambda-|q-k_{F0}|) \quad (25)$$

for the order parameters (the cutoff momentum Λ satisfies $\Lambda<k_{F0}$), and assume that $\overline{V}$ and $\overline{U}$ are constants independent of k and p. We believe that this ansatz, despite its simplicity, captures all the necessary physics.

Now the integral equations become non-linear algebraic equations

$$\Delta_s + \tilde{\Delta}_s = g\Delta_s \ln\left(\frac{E^*}{\mu + \sqrt{\mu^2 - \Delta_s^2}}\right) + \tilde{g}\tilde{\Delta}_s,$$
$$\Delta_s - \tilde{\Delta}_s = \frac{g}{\alpha}\Delta_s \ln\left(\frac{E^*}{\mu + \sqrt{\mu^2 - \Delta_s^2}}\right) - \frac{\tilde{g}}{\alpha}\tilde{\Delta}_s, \quad (26)$$

where the energy scale is $E^* = 2t_0\Lambda/k_{F0}$ and the coupling constants are

$$g = \frac{t_0}{\sqrt{3}\pi t^2}\overline{V}, \quad \tilde{g} = \frac{\Lambda}{2k_{F0}}g, \quad \alpha = \overline{V}/\overline{U} > 1. \quad (27)$$

It trivially follows from Eqs. (26) that $\tilde{\Delta}_s = C\Delta_s$, where $C = (\alpha - 1)/(\alpha + 1 - 2\tilde{g})$. At zero doping, which corresponds to the case $\mu = \Delta_s$, one finds

$$\Delta_s = \Delta_0 = E^* \exp\left[-\frac{1}{g}\frac{2\alpha - \tilde{g}(1+\alpha)}{1 + \alpha - 2\tilde{g}}\right]. \quad (28)$$

This compact mean field solution is valid in the small coupling limit; that is, when g (and $\tilde{g}$) is small, and, consequently, Δ_0 and $\tilde{\Delta}_0$ are much less than t_0. The doped state is characterized by $\mu > \Delta_s$. To describe the solution of Eq. (26) in such a regime, let us define the partial doping x_s: the concentration of electrons residing in sector s, per single carbon atom. It is known[42–45] that a finite x_s acts to decrease the order parameter Δ_s:

$$\Delta_s(x_s) = \Delta_0\sqrt{1 - \frac{4x_s}{x_0}}, \quad \mu = \Delta_0\left(1 - \frac{2x_s}{x_0}\right). \quad (29)$$

where $x_0 = \Delta_0 t_0/(\pi\sqrt{3}t^2)$. It is easy to check that Eqs. (29) indeed guarantee that μ exceeds Δ_s, making the doping of sector s possible. At $T = 0$ the partial free energy (per unit cell) associated with doping is

$$\Delta F_s(x_s) = 4\int_0^{x_s}\mu(x)dx = 4\Delta_0\left(x_s - \frac{x_s^2}{x_0}\right). \quad (30)$$

Since a unit cell contains four carbon atoms, the factor 4 is required in this formula.

V. FRACTIONAL METAL STATE

The relations (29,30) describe a single sector. To determine the state of the whole system, we must understand how the total doping x is distributed between the sectors. One might expect that x is spread evenly: $x_s = x/4$. Yet such an assumption might not be most advantageous thermodynamically: we demonstrated[20,21], for a two-sector system, that placing all the extra charge x into a single sector optimizes the system free energy relative to the state with an even distribution of x. To settle this issue for our four-sector model, we must minimize the doping-related part of the free energy for the whole system

$$\Delta F = \sum_s \Delta F_s = 4\Delta_0 x - \frac{4\Delta_0}{x_0}\sum_{\xi\sigma} x_{\xi\sigma}^2 \quad (31)$$

at fixed doping $x = \sum_s x_s$. Simple calculations demonstrate that, for $x < x_0$, the term ΔF reaches its smallest value, $\Delta F_{\rm qm} = 4\Delta_0(x - x^2/x_0)$, when all extra electrons are placed into a specific sector s, while all other sectors are kept doping-free

$$x_s = x, \quad x_{s'} = 0 \quad \text{for} \quad s' \neq s. \quad (32)$$

For example, $\Delta F_{\rm qm}$ is smaller than $\Delta F_{\rm e} = 4\Delta_0 x - \Delta_0 x^2/x_0$, which is the free energy of the state with $x_s = x/4$ for all four s. For the distribution (32) the Fermi surface lies entirely in sector $s = (\sigma, \xi)$. Therefore, only states with spin σ near the Dirac point $\mathbf{K}_\xi$ reach the Fermi level. In other words, the Fermi surface is perfectly polarized in terms of both σ and ξ indices. Since the insulating gap persists in three other sectors, the state described by Eq. (32) may be called 'a quarter-metal', a first example of a series of 'fractional metals'. The quarter-metal is nematic. Indeed, valleys $\mathbf{K}_1$ and $\mathbf{K}_2$ are not equivalent, thus, the $\pi/3$-rotations of the underlying lattice are no longer symmetries of the ordered state.

As in the case of the half-metal in the system with nesting[20,21], the gap in the first sector closes when increasing doping. The doped electrons begin to enter the second sector, then to the third and fourth sectors. As a result, the system passes respectively through the states of a half-metal, 3/4-metal, and finally the gaps in all sectors close and the system occurs in the usual metallic phase. We can show that each transformation is a first-order phase transition. The analysis of the electronic states evolution with doping is quite similar to the half-metal case[20,21].

VI. STABILITY OF FRACTIONAL METAL

Above we neglected interactions between electrons in different sectors. Then, treating individual sectors independently, we derived Eqs. (29,30). Now we want to assess the effects of the neglected terms. There are two types of interaction terms: (i) umklapp interaction $\hat{H}^{(2,4)}$, Eqs. (8,10), which couples sectors with the same ξ but different spins, and (ii) the backscattering amplitude $V_{\rm bs}^{(1,3)}$, which describes interactions between sectors with the same σ but different valley ξ, Eq. (20). In principle, $\hat{H}^{(2,4)}$ also contain the backscattering $V_{\rm bs}^{(2)}$, which is even weaker, and will be neglected. If the associated coupling constants are small, we can use perturbation theory. The lowest-order perturbative correction $F_{\rm um}$ to the free energy due to the umklapp term $\hat{H}^{(2)}$ equals $\langle\hat{H}^{(2)}\rangle$. Thus,

neglecting small contributions due to $\tilde{\Delta}_s$, we determine the umklapp correction to the free energy (per unit cell)

$$F_{\rm um} = -\frac{\mathcal{F}}{2}\sum_{\xi}\sqrt{\left(1-\frac{4x_{\uparrow\xi}}{x_0}\right)\left(1-\frac{4x_{\downarrow\xi}}{x_0}\right)}, \qquad (33)$$

where $\mathcal{F} = 8\alpha^2 g_{\rm um}\Delta_0 x_0/(1+\alpha)^2 g^2$, and the dimensionless Fermi-surface-averaged umklapp coupling constant is $g_{\rm um} = t_0\overline{V}_{\rm um}/\sqrt{3}\pi t^2$. We also used the fact that $V^{(1)}$, upon averaging over the Fermi surface, becomes equal to $g(1+\alpha)/2\alpha$. When x is low, one has

$$\frac{F_{\rm um}}{\mathcal{F}} \approx -1 + \frac{x}{x_0} + \sum_{\xi}\frac{(x_{\uparrow\xi}-x_{\downarrow\xi})^2}{x_0^2}, \qquad (34)$$

which is smallest at $x_s = x/4$. A similar result can be derived for the backscattering interaction. Thus, both the umklapp and the backscattering favor an even distribution of doping over the sectors. However, in the limit $g_{\rm um} \ll g^2$, $g_{\rm bs} \ll g^2$, their contributions are small, and cannot destroy the fractional metal phase. The perturbative derivation of the stability criterion is intuitively clear and transparent. Its primary purpose is to demonstrate that the fractional metal phase can survive weak deviations from the highly idealized model neglecting any couplings between the sectors. On the other hand, this criterion is very stringent, and one may wonder if it can be satisfied in a real material. Fortunately, a more complex non-perturbative approach, which accounts for the inter-sector couplings at the mean field level, allows to relax it: we demonstrated[46] that it is sufficient to have

$$g_{\rm bs} < g, \quad g_{\rm um} < g \qquad (35)$$

to maintain the stability of the FraM. More detailed stability analysis will be presented in future studies.

VII. DISCUSSION

Using AA bilayer graphene as a test example, we argue that in a system with a nested multi-sheet Fermi surface, a peculiar state (which we call fractional metal, or FraM) can be stabilized. In the FraM phase, part of the Fermi surface is gapped and charge carriers on the remaining gapless part of the Fermi surface belong to a specific sector of the low-energy electronic states. Similar to a half-metal, the states at the Fermi energy can be characterized in terms of polarization; but, unlike the usual half-metals, this is not spin polarization. Let us introduce the spin-flavor operator

$$\hat{S}_{\rm f} = \sum_{\sigma\xi\nu}\sigma\nu\hat{N}_{\sigma\xi\nu}, \qquad (36)$$

where $\hat{N}_{\sigma\xi\nu}$ is the number operator for fermions with spin σ, charge ν, in valley ξ. (In Refs. 20,21 an operator analogous to $\hat{S}_{\rm f}$ was called 'spin-valley' operator.) Since doping enters only in one sector, all states at the Fermi surface have the same value of $\sigma\nu$. Therefore, these states are eigenstates of $\hat{S}_{\rm f}$ with identical eigenvalue $\sigma\nu$. The same is true for the valley operator

$$\hat{S}_{\rm v} = \sum_{\sigma\xi\nu}\xi\hat{N}_{\sigma\xi\nu}, \qquad (37)$$

since a given sector is localized entirely in one valley.

Thus, *the FraM is the conducting nematic state whose Fermi surface is polarized in terms of two spin-like operators $S_{\rm f,v}$. The electric current though the FraM carries, in addition to the electric charge, spin-flavor and valley quanta.* Finally, note that, if superconductivity arises in a FraM phase, it should demonstrate rather peculiar properties. The superconducting order parameter might have a very unusual symmetry, classified according to a non-trivial spin and valley structure, and superconducting currents would be spin-flavor and valley polarized. However, the detailed analysis of this superconductivity requires the specification of the symmetric properties of the electron-phonon coupling.

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