Isotropically conducting (hidden) quantum Hall stripe phases in a two-dimensional electron gas

Yi Huang (黄), M. Sammon, M. A. Zudov, and B. I. Shklovskii

Phys. Rev. B 101, 161302 — Published 20 April 2020

DOI: 10.1103/PhysRevB.101.161302
Isotropically conducting (hidden) quantum Hall stripe phases in a two-dimensional electron gas

Yi Huang (黄奕)* M. Sammon, M. A. Zudov, and B. I. Shklovskii

School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA

(Received March 30, 2020)

Quantum Hall stripe (QHS) phases, predicted by the Hartree-Fock theory, are manifested in GaAs-based two-dimensional electron gases as giant resistance anisotropies. Here, we predict a “hidden” QHS phase which exhibits isotropic resistivity whose value, determined by the density of states of QHS, is independent of the Landau index \( N \) and is inversely proportional to the Drude conductivity at zero magnetic field. At high enough \( N \), this phase yields to an Ando-Unemura/Coleridge-Zawadski-Sachrajda phases in which the resistivity is proportional to \( 1/N \) and to the ratio of quantum and transport lifetimes. Experimental observation of this border can provide a new way to obtain quantum relaxation time.

Quantum Hall stripe (QHS) phases in spin-resolved Landau levels (LLs) near half-integer filling factors \( \nu = 9/2, 11/2, 13/2, \ldots \), were predicted by the Hartree-Fock (HF) theory [1–3]. These phases consist of alternating stripes with filling factors \( \nu \pm 1/2 \), which, at exactly half-filling, both have the width \( \Lambda/2 \approx 1.42 R_c \) [1, 2, 4, 5], where \( R_c \) is the cyclotron radius (see Figure 1). QHSs are formed due to a repulsive box-like interaction of electrons having ring-like wave functions. Such an unusual interaction leads to an energy gain when electrons occupy the nearest states within the same stripe and avoid interacting with electrons in neighboring stripes. The self-consistent HF theory is valid at LL indices \( N \gg 1 \), where \( R_c = l_B(2N + 1)^{1/2} \gg l_B \), where \( l_B = (\hbar eB)^{1/2} \) is the magnetic length, a measure of quantum fluctuations of an electron’s cyclotron orbit center, and \( B \) is the magnetic field. These fluctuations play a minor role even at \( N = 2 \), and QHSs determine the ground state for all \( \nu \geq 9/2 \) [2, 4, 5].

QHSs were confirmed by the discovery of dramatic resistance anisotropies in two-dimensional electron gases in GaAs/AlGaAs heterostructures [6, 7]. These anisotropies emerge because the diffusion mechanisms along and perpendicular to the stripes are different [8]. In the stripe direction \((\hat{y})\) electrons drift along the stripe edge in the internal electric field \( E \) until they are scattered to an adjacent stripe edge by impurities. If such scattering is weak, this mechanism leads to a large diffusion coefficient in the \( \hat{y} \) direction (large conductivity \( \sigma_{yy} \), large resistivity \( \rho_{xx} \)) and a small diffusion coefficient in the orthogonal \((\hat{x})\) direction (small \( \sigma_{xx} \), small \( \rho_{yy} \)). As a result [9], if \( N \) is not too large,

\[
\frac{\rho_{xx}}{\rho_{yy}} \approx \left( \frac{\tilde{\sigma}_0}{8\gamma\alpha^2N^2} \right)^2 \gg 1, \tag{1}
\]

where \( \tilde{\sigma}_0 = n_e \hbar \tau / m^* \) is the Drude conductivity at \( B = 0 \) in units of \( e^2/h \), \( n_e \) is the electron density, \( \tau \) is the momentum relaxation time, \( m^* \) is the electron effective mass, and \( \gamma \) is a discussed below numerical factor depending on the nature of scattering. To derive Eq. (1) we used the HF potential, shown in Figure 1(a), with the amplitude \( \Gamma_s \approx 0.4 \hbar \omega_c / \alpha \), where \( \omega_c \) is the cyclotron frequency, and \( \alpha \approx 18 \) is the ratio of the density of states (DOS) in the middle of a spin split LL to that without magnetic field, but per spin [10]. In Ref. 9 we showed that Eq. (1) agrees well with the data from high mobility samples.

At large enough \( N \), Eq. (1) predicts that the anisotropy of resistivity vanishes. In this Rapid Communication we theoretically study \( \rho(N, \tilde{\sigma}_0) \) at half-integer \( \nu \) in emerging at such \( N \) isotropic phase. Our results are summarized in the “phase diagram” of \( \tilde{\sigma} \equiv (e^2/h)\rho(N, \tilde{\sigma}_0) \) depicted in Figure 2. In the top-left corner it shows the anisotropic QHS phase, discussed above. The remaining three phases are isotropic. The Ando-Unemura (AU) phase [11] and the Coleridge-Zawadski...

![Figure 1](image-url)
(b). Although, generally speaking, we treat this phase as an isotropic one and call it the “hidden phases, has not been discussed in the literature. In this phase conductivity than the impurity scattering, which leads to hops. For a given $\sigma_0$ the “hard” $\tilde{\rho}_{xx}$ scales with $N^{-4}$ whereas the “easy” $\tilde{\rho}_{yy}$ is $N$-independent. The border between the QHS and hQHS phases in Figure 2 is determined by the condition $\tilde{\rho}_{xx} \approx \tilde{\rho}_{yy}$ or

$$\tilde{\sigma} \approx 4\alpha^2 N^2.$$  

**QHS phase.** Combining Eq. (1) with $\gamma \approx 0.5$ and Eq. (36) of Ref. 8, $(\rho_{xx}/\rho_{yy})^{1/2} \approx 1/8N^2$, we find

$$\tilde{\rho}_{xx} \approx \frac{\tilde{\sigma}_0}{32\alpha^2 N^4},$$  

$$\tilde{\rho}_{yy} \approx \frac{\alpha^2}{2\tilde{\sigma}_0}.$$  

For a given $\tilde{\sigma}_0$ the “hard” $\tilde{\rho}_{xx}$ scales with $N^{-4}$ whereas the “easy” $\tilde{\rho}_{yy}$ is $N$-independent. The border between the QHS and hQHS phases in Figure 2 is determined by the condition $\tilde{\rho}_{xx} \approx \tilde{\rho}_{yy}$ or

$$\tilde{\sigma} \approx 4\alpha^2 N^2.$$  

**hQHS phase.** We show below that the hQHS phase resides between its upper border, Eq. (4), and its lower border $\tilde{\sigma}_0 \approx 3.5 \alpha^2 N$, i.e., when

$$3.5 \alpha^2 N \lesssim \tilde{\sigma}_0 \lesssim 4\alpha^2 N^2.$$  

To find $\tilde{\rho}(N, \tilde{\sigma}_0)$, we start with Eqs. (38-39) of Ref. 15 [16],

$$\tilde{\sigma} = \frac{\hbar v_F^2 g_B \tau_B}{2(1 + \omega_c^2 \tau_B^2)} = \frac{\tilde{\sigma}_0}{2(1 + \omega_c^2 \tau_B^2)},$$  

where

$$\frac{1}{\tau_B} \approx \frac{g_B}{\gamma \tilde{g}_0}.$$  

Here, $\tau_B$ and $g_B$ are the scattering time and the DOS at the center of the LL at $B \neq 0$, while $v_F$ and $\tilde{g}_0 = m^*/2\hbar^2$ are the Fermi velocity and the DOS per spin at $B = 0$. [17]. Per Eq. (5), $\omega_c \tau_B = \omega_c \tau/\alpha = \tilde{\sigma}_0/2\alpha N \gg 1$ and Eq. (6) yields

$$\tilde{\sigma} \approx \frac{2\alpha^2 N^2}{\tilde{\sigma}_0}.$$  

**Equation (5)** also implies that $\tilde{\sigma} \ll \tilde{\sigma}_{xy} \approx 2N$ and we find

$$\tilde{\rho} \approx \frac{\tilde{\sigma}^2}{\tilde{\sigma}_{xy}} \approx \frac{\alpha^2}{2\tilde{\sigma}_0},$$  

which coincides with Eq. (3). The independence of $\tilde{\rho}$ on $N$ and its inverse proportionality to $\tilde{\sigma}_0$ are the hallmarks of the hQHS phase. At the end of this Rapid Communication we confirm this finding by calculating impurity scattering dominated $\sigma_{xx}$ and $\sigma_{yy}$ using a Kubo formula and being guided by Ref. 18.
AU phase. Using $\tilde{\sigma} = 2N/\pi$ calculated in Ref. 11 for low mobility samples with $\tau = \tau_0$ and $\tilde{\sigma}_{xy} \simeq 2N$ we find

$$\tilde{\rho} = \frac{\tilde{\sigma}}{\sigma^2 + \tilde{\sigma}_{xy}^2} \simeq \frac{0.14}{N^2}.$$  \hspace{1cm} (10)

This parameter-free result matches Eq. (9) at the upper border of the AU phase, $\tilde{\rho}_0 = 3.5\alpha^2N$, mentioned above and shown in Figure 2. This border can also be estimated by equating $\Gamma_s$ and $\Gamma_1 = h\sqrt{2\omega_c/\pi\tau}$. CZS phase. Using Eq. (6) with $g_B = g_0\sqrt{\omega_c/\tau} \gg 1$ [12, 19, 20], we find $\omega_c\tau_B = \sqrt{\omega_c\tau^2/\tau_q} \gg 1$ and Eq. (6) gives

$$\tilde{\sigma} \simeq \frac{\tau_q}{\pi} N,$$  \hspace{1cm} (11)

which has an extra factor of $\pi\tau_q/2\tau$ compared to $\tilde{\sigma} = 2N/\pi$ in the AU phase [11]. For $\tau_q/\tau \ll 1$, we have $\tilde{\sigma} \ll \tilde{\sigma}_{xy}$ and

$$\tilde{\rho} \simeq \frac{\tilde{\sigma}}{\tilde{\sigma}_{xy}^2} = \frac{\tau_q}{4\pi\tau N},$$  \hspace{1cm} (12)

which agrees with Eq. (6) of Ref. 12. Equation (12) matches $\tilde{\rho}$ in the AU phase, Eq. (10), at $\tau \approx 1.7\tau_0$ or at

$$\tilde{\sigma}_0 \approx 1.7\tilde{\sigma}_q, \quad \tilde{\sigma}_q = \frac{\hbar n_e\tau_q}{m^*}.$$  \hspace{1cm} (13)

Equation (12) also matches $\tilde{\rho}$ in the hQHS phase, Eq. (9), at

$$N \approx N_q \equiv \frac{\tilde{\sigma}_q}{2\alpha^2} = \frac{\hbar n_e\tau_q}{2\alpha^2 m^*}.$$  \hspace{1cm} (14)

Let us now discuss predictions of our phase diagram (Figure 2) for $\tilde{\rho}(N)$ of three hypothetical samples with $\tilde{\sigma}_0 = 2 \times 10^3, 4 \times 10^3$ and $1 \times 10^3$, shown in Figure 3. The first sample resides in the A U phase at all $N$ and obeys Eq. (10). The second one is in the hQHS phase at $N < 4$, and therefore, its $\tilde{\rho}(N)$ is given by Eq. (9) and is independent on $N$. This plateau ends at $N \geq 4$, at which $\tilde{\rho}(N)$ starts declining as $1/N$ as prescribed by Eq. (10). Finally, the third sample shows anisotropic $\tilde{\rho}(N)$ at $N = 2$, then a plateau between $N = 3$ and $N = N_q = 7$, and eventually follows Eq. (12) of the CZS phase at $N > 7$. If such a plateau is observed, one should be able to find $\tau_q$ from experimental $N_q$.

Above we considered a smectic phase pinned by disorder [8, 9]. If the disorder is able to create dislocations leading to a nematic phase, the conductivity of the hQHS phase will not be affected as it is sensitive only to the density of states. In the highest mobility sample of Figure 3, the anisotropic QHS phase happens at $\Gamma_1 \ll \Gamma_s$ when dislocations are sparse and, as a result, their effect on the electron drift along the stripe edges is negligible.

So far we dealt with very low temperatures $T$ at which long-range stripe order is preserved. It is known, however, that at $T \gtrsim 0.1$ K the resistivity anisotropy vanishes while the local stripe order is believed to persist [21]. One can imagine this emerging phase as an ensemble of randomly oriented stripe domains [22–24]. The resistivity of such a 2D polycrystal is known [25] to be $\rho = (\rho_{xx}\rho_{yy})^{1/2}$. This means that the hQHS phase should not be affected, while in the QHS phase one expects isotropic $\tilde{\rho} \propto N^{-2}$ (see dashed line in Figure 3).

We next justify our use of the above semi-quantitative approach of Eq. (6). For a $\sigma_{xx}$ due to short range impurity scattering the Kubo formula yields

$$\sigma_{xx} = \frac{\pi\hbar e^2}{L_xL_y} \sum_{i,j} \left| \langle \chi' \rangle \right|^2 \delta(E_F - \epsilon_i)\delta(E_F - \epsilon_j),$$  \hspace{1cm} (15)

where $L_x$ and $L_y$ are sample dimensions, $E_F$ is the Fermi energy, $i$ and $j$ run over states with energies $\epsilon_i$ and $\epsilon_j$, $X = -l_B^2\rho_{yy}/\hbar$ and $U(r) = U_0\alpha^3 \sum_l \delta^{(3)}(r - r_l)$ is the impurity potential with range of the lattice constant $a$. The electron wavefunction in the Landau gauge is given by

$$\psi_i(r) = \phi(z) \exp\left(-i\frac{X_i}{l_B} \right) \frac{\chi_N(x - X_i)}{\sqrt{L_y}},$$  \hspace{1cm} (16)

$$\chi_N(x) = \exp\left(-x^2/2l_B^2\right) H_N(x/l_B),$$  \hspace{1cm} (17)

$$\phi(z) = (2/w)^{1/2} \sin(\pi z/w),$$  \hspace{1cm} (18)

where $w$ is the width of the quantum well. The velocity matrix element can be written as $X_{i,j} = (i/h)(X_i - X_j)U_{ij}$ and

$$\sigma_{xx} = \frac{\pi e^2}{\hbar L_xL_y} \sum_{i,j} \left| \langle U_{ij} \rangle \right|^2 (X_i - X_j)^2 \times \delta(E_F - \epsilon_i)\delta(E_F - \epsilon_j).$$  \hspace{1cm} (19)

For short range impurities with the 3D concentration $N_3$ and the correlator $\langle U(r)U(r') \rangle = N_3(U_0\alpha^3)^2 \delta^{(3)}(r - r')$, we have

$$\left| \langle U_{ij} \rangle \right|^2 = \int \frac{d^2r}{2\pi} \psi^*_i(r)\psi_j(r) \psi^*_j(r') \psi_i(r') \langle U(r)U(r') \rangle = \frac{3N_3(U_0\alpha^3)^2}{2wL_y} \int dx \chi_N^2(x - X_i) \chi_N^2(x - X_j).$$  \hspace{1cm} (20)
Using \( \delta(E_F - \epsilon_i) = \sum_m \delta(x_i - x^{(m)})/\epsilon E \) where \( x^{(m)} \) is the \( m \)-th solution of \( \epsilon(x) = V(x) = E_F \), \( eE = |d\epsilon/dx|_{x=x^{(m)}} \), and \( \sum_i = (L_y/2\pi l_B^2) \int dX_i \), we arrive at

\[
\sigma_{xx} = \frac{e^2g_B^2R^2}{2g_0\tau} \eta_x.
\]  

(21)

Here we have ignored all terms in the summation with \( |X_i - X_j| > \Lambda/2 \), as these terms are exponentially suppressed by the overlap of the wave functions in \( U_{ij} \). Additionally, we have introduced the transport relaxation rate in zero magnetic field

\[
\frac{1}{\tau} = \frac{2\pi}{\hbar} \frac{3N_3(U_0a^3)^2}{2w},
\]  

(22)

and a dimensionless coefficient

\[
\eta_x = \left( \frac{\Lambda}{2R_c} \right)^2 \Lambda \int dx \chi_N^2(x) \beta_N(x - \Lambda/2),
\]  

(23)

which oscillates with \( N \) and tends to 1.07 at \( N \to \infty \). For \( \Lambda = 2.84R_c \) and \( N > 2 \), \( \eta_x(N) = 1.07 \pm 0.15 \). By equating \( \sigma_{xx}/g_Be^2 \) in Eq. (21) and \( D_{xx} \) given by Eqs. (2) and (7) of Ref.9, we can now conclude that the coefficient \( \gamma \) entering Eq. (1), and used in Eqs. (2), (3), (4), is \( \gamma \approx \eta_x/2 \approx 0.53 \).

Similar to \( \sigma_{xx} \), we can compute \( \sigma_{yy} \) as

\[
\sigma_{yy} = \frac{\pi e^2}{L_xL_y} \sum_{i,j} \left\langle \psi^*_{ij}(r) \psi_{ij}(r) \right\rangle \delta(E_F - \epsilon_i)\delta(E_F - \epsilon_j).
\]  

(24)

The velocity operator along \( \hat{y} \) can be written as

\[
\hat{Y} = \frac{i}{\hbar}[\hat{H}, \hat{Y}] = -\frac{l_B^2}{\hbar} \frac{\partial U}{\partial x},
\]  

(25)

where we have ignored the drift in internal electric field \( \mathbf{E} \) and used \( \hat{Y} = l_B^2/\hbar i \). The matrix element \( \hat{Y}_{ij} \) can then be evaluated via integration by parts,

\[
\hat{Y}_{ij} = \frac{l_B^2}{\hbar} \int dr \frac{\partial}{\partial x} \left[ \psi^*_i(r) \psi_j(r) \right].
\]  

(26)

After averaging over impurities positions,

\[
\left\langle \hat{Y}_{ij} \right\rangle = \frac{l_B^2}{\hbar} \frac{3N_3(U_0a^3)^2}{2wL_y} \times \int dx \left\{ \frac{d}{dx} \left[ \chi_N(x - X_i)\chi_N(x - X_j) \right] \right\}^2,
\]  

(27)

we obtain

\[
\sigma_{yy} = \frac{\pi e^2}{L_xL_y} \frac{l_B^4}{(2\pi l_B^2 e)^2} \int dx_i dx_j \frac{3N_3(U_0a^3)^2}{2w} \times \int dx \left\{ \frac{d}{dx} \left[ \chi_N(x - X_i)\chi_N(x - X_j) \right] \right\}^2 \times \sum_{m,n} \delta(x_i - x^{(m)})\delta(x_j - x^{(n)}) = \frac{e^2g_B^2R^2}{2g_0\tau} \eta_y,
\]  

(28)

where the summation of the product of delta functions can be evaluated separately for \( m = n \) and for \( m \neq n \). If \( m = n \), then there are \( 2L_x/\Lambda \) terms with \( X_i = X_j \). On the other hand, if \( m \neq n \), there are \( 4L_x/\Lambda \) terms with \( |X_i - X_j| = \Lambda/2 \) (all other terms are negligible). This leads to \( \eta_y \) in Eq. (28),

\[
\eta_y = \frac{N_4}{2R_c} \int dx \left\{ \frac{d}{dx} \left( \chi_N(x) \right)^2 \right\}^2 + 2 \left\{ \frac{d}{dx} \left( \chi_N(x)\chi_N(x - \Lambda/2) \right)^2 \right\}.
\]  

(29)

For \( \Lambda = 2.84R_c \) and \( N > 2 \), \( \eta_y(N) = 1.06 \pm 0.01 \).

Using Eqs. (21), (28) we finally obtain dimensionless conductivities in the hQHS phase,

\[
\tilde{\sigma}_{xx} = \frac{2\eta_x\alpha^2 N^2}{\sigma_0}, \quad \tilde{\sigma}_{yy} = \frac{2\eta_y\alpha^2 N^2}{\sigma_0},
\]  

(30)

confirming that the conductivity agrees with Eq. (8) within 15%, thus justifying a semi-quantitative isotropic approach based on Eq. (6). Such a weak anisotropy is directly related to the period \( \Lambda = 2.84R_c \). We have found by varying \( \Lambda \) that at \( N \to \infty \) the anisotropy vanishes when \( \Lambda = 2.82R_c \). Thus, it is indeed expected to be small at \( \Lambda = 2.84R_c \).

In summary, we have predicted the existence of an isotropically conducting (hidden) quantum Hall stripe phase. This phase is expected to reside in a finite range of nearly half-filled LLs, between the anisotropic quantum Hall stripe and isotropic liquid phases. The hallmark of this new phase is the independence of the resistivity on the LL index and its inverse proportionality to the Drude resistivity at zero magnetic field. Although we focused on 2DEG in GaAs, the conjectured hQHS phase might be relevant to other realizations of 2DEG, such as, e.g., graphene, which might not exhibit conventional QHSs due to the lack of intrinsic orienting mechanism.

We thank I. Dmitriev, M. Fogler, and X. Fu for valuable discussions. Calculations by Y. H. and M. S. were supported primarily by the NSF through the University of Minnesota MRSEC under Award No. DMR-1420013. M. Z. acknowledges support from the U.S. Department of Energy, Office of Science, Basic Energy Sciences, under Award No. DE-SC0002567.

* Corresponding author: huan1756@umn.edu

We used Eqs. (13), (48), and (43) of Ref. 2 to find $\Gamma_s$ and $\alpha$ at $r_s \simeq 1$, typical of GaAs samples.

Note that $\tau_B$ in this Rapid Communication is four times smaller than the one in Ref. 9.

Our Eq. (6) has a factor $1/2$ compared to Eq. (39) of Ref. 15 because we deal with spin resolved LLs.

Here, we have chosen $N_q = 7$ which, for $n_e \simeq 3 \times 10^{11} \text{ cm}^{-2}$, $m^* \simeq 0.06 m_e$ [26–30], corresponds to $\tau_q \simeq 10^2 \text{ ps}$ [31].