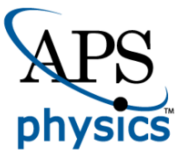

Space- and time-crystallization effects in multicomponent superfluids

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Space- and time-crystallization effects in multicomponent superfluids

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We observe that space- and time-crystallization effects in multicomponent superfluids—while having the same physical origin and mathematical description as in the single-component case—are conceptually much more straightforward. Specifically, the values of the temporal and spatial periods are absolute rather than relative, and the broken translation symmetry in space and/or time can be revealed with experiments involving only one equilibrium sample. We discuss two realistic setups—one with cold atoms and another one with bilayer superconductors—for observation of space and time crystallization in two-component counterflow superfluids.

The superfluid long-range order—either genuine or, in lower dimensions, topological/algebraic—is associated with the emergence of a well-defined (modulo 2π), defect-free field of a coarse-grained phase, $\Phi(\mathbf{r},t)$; see, e.g., Ref. [1]. In what follows, we discuss the genuine long-range order only. The generalization to the case of algebraic order is readily achieved along the lines described in Ref. [2]. Also, we use the classical-field (matter-wave) language, which, on one hand, captures the essence of superfluid phenomena and, on the other hand, is straightforwardly generalized to the case of quantum bosonic fields [1].

Long-range order in the coarse-grained matter field $\psi = \exp[i\Phi(\mathbf{r},t)]$ means that we are dealing with the broken global U(1) symmetry state. The very nature of this state implies the existence of a *space crystal* when the phase is a linear function of distance (and ψ is periodic in space with the period $2\pi/k$):

$$\psi(\mathbf{r},t) \;=\; \psi(\mathbf{0},t)\, e^{i\mathbf{k}\cdot\mathbf{r}}. \tag{1}$$

This is a state with finite superflow velocity proportional to the wavevector $\mathbf{k}$. In the literature on superfluidity, the term "space crystal" is almost never (if at all) used in the context of the state Eq. (1), because the matter density $n(\mathbf{r},t) = |\psi(\mathbf{r},t)|^2$ remains homogeneous in space. [It should not be confused with a supersolid—the superfluid state with spontaneously broken translation symmetry in the particle density.] However, it is now conventional to call various states of mater "solids" and/or "crystals" if there is some observable revealing broken translation invariance, and this observable need not be the particle density. One familiar example is the valance-bond crystal state of lattice bosons at half-integer filling factor. In superfluids, the phase field plays the role of such an observable. The interference fringes produced by superimposing two matter waves with opposite wavevectors [3] nicely visualize the fact that superflows break translation symmetry and thus qualify to be called space crystals. Supercurrent states in three-dimensional superfluids do not form naturally by cooling the system across the transition temperature [4]. Nevertheless, supercurrent states can be prepared by cooling the system in a rotating vessel which is stopped once the system is in the superfluid phase. In this sense, the period of the space crystal in the phase field depends on the experimental conditions used to prepare the sample, but otherwise we are dealing with a stable thermodynamic equilibrium described by the Gibbs distribution with an emergent quantized topological constant of motion (phase winding number) [1]. Academically speaking, such a persistent current state is metastable. However, its relaxation time due to rare quantum-tunneling or thermal-activation events is exponentially large in the inverse k, and easily exceeds the time of the Universe unless the period of the space crystal is microscopically small.

While the existence of plane-wave states (1) is a generic property of any statistical model with broken U(1) symmetry, the period depends on the reference frame. The Galilean transformation of the field Φ when going to the reference frame moving with the velocity $\mathbf{v}_0$ with respect to the original one,

$$\Phi(\mathbf{r},t) \;\to\; \Phi(\mathbf{r},t) \;-\; \frac{\mathbf{v}_0\cdot\mathbf{r}}{\gamma}, \tag{2}$$

implies that the state wavevector changes to:

$$\mathbf{k} \;\to\; \mathbf{k} - \mathbf{v}_0/\gamma. \tag{3}$$

Here γ is the system-specific parameter relating the wavevector of the matter wave to the flow velocity. In the quantum case, $\gamma = \hbar/m$, where $\hbar$ is the Planck's constant (in what follows we set it to unity) and m is the particle mass. This relativity of the period is quite unique for non-relativistic crystals!

What distinguishes superfluids from purely statistical models with broken U(1) symmetry is that broken U(1)

symmetry automatically entails breaking of the time-translation symmetry, and links superfluidity to yet another fundamental phenomenon of *time crystallization* [5, 6]. Indeed, the phase Φ evolves in time in accordance with the universal Beliaev–Josephson–Anderson relation (in the reference frame of the normal component)

$$\dot{\Phi} = -\mu. \tag{4}$$

Here $\mu \equiv \mu_k$ is the chemical potential that depends on the wavevector of the superflow. Equation (4) readily follows from the generalised Gibbs distribution for a superfluid [1]. Its remarkable simplicity and universality is rooted in the fact that the phase Φ is canonically conjugated to the total amount of matter

$$N = \int |\psi|^2 d^d r, \tag{5}$$

(the U(1) symmetry in question is the Noether's symmetry responsible for the conservation of N). With the relation (4), the expression (1) can be upgraded to the formula

$$\psi(\mathbf{r}, t) \;=\; \psi(\mathbf{0}, 0)\, e^{i\mathbf{k}\cdot\mathbf{r} - i\mu_k t}, \tag{6}$$

showing that, in the long-wave limit, the superfluid order parameter has the form of a running plane wave. Hence, the superfluid state with a superflow is a space-time crystal, or, a time crystal in the of absence of the superflow.

It is important to keep in mind that the value of μ_k is relative. This is formally reminiscent of (and even partially connected to) the relative nature of the wavevector $\mathbf{k}$: changing the reference frame we change $\mathbf{k}$ and μ_k. Furthermore, the chemical potential is defined only up to a global constant prescribed by the convention about the ground state energy per particle. In the non-relativistic physics, the rest energy of a free particle is typically set to zero. The merely conventional character of this choice results in a certain constraint on the protocols of measuring μ and the time-crystallization effect in superfluids, but does not exclude the effect itself.

In their paper on the no-go theorem for equilibrium time crystals [7], Watanabe and Oshikawa argued that it is the above-discussed relativity of the chemical potential that reconciles their theorem with Eq. (6). However, the proof of the no-go theorem in Ref. [7] is based on the implicit assumption that energy is the only additive constant of motion in a system (cf. Ref. [8]), which is certainly not true for superfluids where (5) is also a key constant of motion. The actual restriction implied by the no-go theorem for equilibrium time crystals is two-fold: (i) An equilibrium time crystal is supposed to have at least one additive constant of motion besides the energy. (ii) The observable revealing the time crystallization has to violate the conservation of this constant.

In the light of the above discussion, it is instructive to identify a class of superfluid systems that feature the effect of equilibrium space-/time-crystallization in the form of Eq. (6) while being free of the subtleties originating from the relative nature of $\mathbf{k}$ and μ_k. We observe that multicomponent (counterflow) superfluids belong to such a class. Here the quantity of interest, $\Phi_{\rm ab}(\mathbf{r}, t)$, is the coarse-grained field of the phase difference between the components "a" and "b" (the description stays exactly the same for arbitrary number of components, so we restrict ourselves to the two-component case for simplicity). We further limit ourselves with counterflow superfluid states (see, e.g., [1]) where the superfluid order exists exclusively in the field $\Phi_{\rm ab}(\mathbf{r}, t)$ but not in the individual phases of the components. The long-wave equilibrium statistics of the two-component counterflow superfluid is isomorphic to that of a single-component superfluid, rendering the system particularly simple and relevant for our purposes. To exclude irrelevant long-wave degrees of freedom, we also assume that the normal component is pinned by either disorder, walls, or an external periodic potential. The Beliaev–Josephson–Anderson relation for $\Phi_{\rm ab}$ (its derivation from the Gibbs distribution is directly analogous to that in the single-component case),

$$\dot{\Phi}_{\rm ab} = \mu_{\rm b} - \mu_{\rm a}, \tag{7}$$

has the form of the Josephson relation for the standard ac Josephson effect between two single-component superfluids (made of the same type of matter but having different chemical potentials). Similarly, the protocol of detecting the rotation of the phase $\Phi_{\rm ab}$ can be based on simply creating a "Josephson link" between components a and b. In this regard, note that any protocol of revealing the time crystallization effect in the field $\Phi_{\rm ab}$ has to deal with interactions explicitly violating the U(1)×U(1) symmetry of the original system, which implies a process converting components "a" and "b" into each other. The conceptual difference between this "internal" Josephson effect and its conventional counterpart is that now the frequency of the phase rotation—and thus the period of oscillations of the ac Josephson current—is independent of the choice for counting energy in a single equilibrium sample.

In the presence of disorder or external periodic potential, the system has a natural reference frame. The absence of Galilean invariance in this case does not yet mean that the period of the space crystal (1) is not relative. One can, in principle, design an experiment when this period is observed from a moving frame, in which case Eqs. (2)–(3) still apply. However, in a counterflow superfluid with two components having equal parameters γ, Galilean transformation (2) leaves the phase field intact,

$$\Phi_{\rm ab} \;\to\; \Phi_{\rm ab} \;-\; \frac{\mathbf{v}_0 \cdot \mathbf{r}}{\gamma_a} \;+\; \frac{\mathbf{v}_0 \cdot \mathbf{r}}{\gamma_b} \;\equiv\; \Phi_{\rm ab}, \tag{8}$$

and the period of the corresponding space crystal is the same in any reference frame.

The difference between the single-component and counterflow superfluids becomes even more dramatic and instructive in the case of toroidal geometry and rotating

frame. The fictitious vector potential $\mathbf{A}_{\rm fict}$ emerging in the rotating frame brings about the gauge freedom. In the single-component superfluid in the rotating frame, the gauge freedom renders the notion of spatial phase difference ambiguous and thus ill defined. The gauge-invariant equivalent of the phase difference between the points $\mathbf{r}_1$ and $\mathbf{r}_2$ has now the form of the line integral:

$$\int_{\mathbf{r}_1}^{\mathbf{r}_2} (\nabla\Phi - \mathbf{A}_{\rm fict}) \cdot d\mathbf{l}. \tag{9}$$

Even in the absence of topological defects in the field Φ, this integral depends on the form of the line because of the term with $\mathbf{A}_{\rm fict}$. This, in particular, means that there is no experimental way of unambiguously measuring the phase difference in the rotating frame. In the case of counterflow superfluid, the counterpart of the integral (9) reads [cf. Eq. (8)]

$$\int_{\mathbf{r}_1}^{\mathbf{r}_2} (\nabla\Phi_{\rm ab} - \mathbf{A}_{\rm fict} + \mathbf{A}_{\rm fict}) \cdot d\mathbf{l} \equiv \int_{\mathbf{r}_1}^{\mathbf{r}_2} \nabla\Phi_{\rm ab} \cdot d\mathbf{l}. \tag{10}$$

Now the phase difference is well defined and invariant with respect to the choice of the reference frame.

Experimental implementation 1: Cold atoms. One possible realization of the counterflow superfluidity is with multicomponent ultracold bosons in optical lattices [9]. A straightforward generalization of the protocol discussed in Ref. [2] (in the context of algebraic time crystallization in a single-component two-dimensional superfluid) allows one to simultaneously study both the space and time crystallization in such systems. The protocol is as follows.

• Consider a toroidal shape sample with close, but different, chemical potentials μ_a and μ_b, in a state with finite counterflow supercurrent. Introduce switchable internal Josephson links between the two components on two special sites of the optical lattice separated by a distance $\mathbf{r}$.

• At time zero, turn the first link on for the duration Δt, such that $(\Delta\mu = |\mu_a - \mu_b|)$

$$\Delta t\, \Delta\mu \ll 1. \tag{11}$$

• Keep both links switched off for a much longer time interval $t\,\Delta\mu > 1$ and then turn the second link on for the duration Δt.

• Quickly, on time scales $\ll 1/\Delta\mu$, apply a deep optical lattice to localize all atoms in the system and count atom numbers $N_{\rm a}$ and $N_{\rm b}$ using single-site microscopy [10, 11].

Repeating the protocol many times under identical conditions allows one to accumulate representative statistics and process the data with the help of an auxiliary experimental run that skips the next-to-last step of the above-described protocol. The outcome of the auxiliary run is the expectation value $\bar{N}_{\rm ab} = \langle N_{\rm a} - N_{\rm b} \rangle$ that averages typical particle number differences taking place *right before* the two samples are disconnected for a period of time t. The key statistical observable is then

$$K(t) \,=\, \langle\, [N_{\rm a}(t) - N_{\rm b}(t) - \bar{N}_{\rm ab}]^2 \,\rangle. \tag{12}$$

In this expression, random particle number differences characterizing irreproducibility of the initial state preparation cancel out and we are left with a signal reflecting spatial and temporal oscillations of the phase field

$$\langle\, \Phi_{\rm ab}(\mathbf{r}, t) - \Phi_{\rm ab}(\mathbf{0}, 0) \,\rangle. \tag{13}$$

State preparation fluctuations are independent of $\mathbf{r}$ and t and thus creates no problem except for that of a signal-to-noise ratio, which can be improved by collecting more statistics and optimizing setup parameters. To ensure that the space-/time-dependent contribution to dispersion is large, one needs to have $J_0/\Delta\mu \gg 1$, where J_0 is the Josephson constant (assumed to be the same for both links).

Experimental implementation 2: Bilayer superconductor. A different—and interesting on its own—realization of space-/time-crystallization effect in a counterflow superfluid is a bilayer superconducting annulus. When the thickness of the two layers is small enough to suppress finite-temperature bulk superconductivity and tunneling between the layers is negligibly small, the system still features (at appropriately low temperature) a two-dimensional neutral counterflow superfluid mode [12]. In this case, the coarse-grained phase field $\Phi_{\rm ab}(\mathbf{r}, t)$ describes the phase difference between the layers "a" and "b." Because of the long-range current-current interaction between the layers via the vector potential, the field $\Phi_{\rm ab}(\mathbf{r}, t)$ remains algebraically ordered while the individual phase fields $\Phi_{\rm a}(\mathbf{r}, t)$ and $\Phi_{\rm b}(\mathbf{r}, t)$ are destroyed at finite-temperature by the proliferation of vortices that cost finite energy [12]. The effect of surface superconductivity predicted recently by Samoilenka and Babaev [14] can be used to create an interesting modification of the bilayer superconducting setup [13], in which the two layers are formed by adjacent surfaces of two superconducting materials that remain normal in the bulk.

The setup with switchable Josephson link(s) and subsequent counting of the electrons in each of the two layers appears to be impractical. Instead, one can utilize the standard ac Josephson setup with one or two permanent links. One link would be sufficient for revealing the time crystallization through the current-current correlation function [2]. To reveal both the space and time crystallization, one needs the second link (assume that both have the same Josephson constant J_0) at a macroscopically large distance $\mathbf{r}$ from the first one. Operationally, the resulting device will behave as a hybrid of an ac Josephson junction and a SQUID. On the one hand, the it will be demonstrating the algebraic Josephson effect, see Ref. [2], with the frequency prescribed by the superconducting analog of relation (7), where the chemical potential difference $\mu_{\rm b} - \mu_{\rm a}$ is doubled because of the Cooper pairing. On the other hand, the net amplitude

of the Josephson current will depend on the phase shift $\mathbf{k}\cdot\mathbf{r}$ between the two junctions:

$$J_{\rm net} \propto J_0 \left|\cos\frac{\mathbf{k}\cdot\mathbf{r}}{2}\right|. \tag{14}$$

This way one can directly measure the projection of the wavevector $\mathbf{k}$ on the axis of $\mathbf{r}$.

Conclusions and discussion. Multicomponent superfluids—most notably, counterflow superfluids—unquestionably feature the effects of space and time crystallization. In this context, the space crystallization is understood broadly as the broken translation symmetry, irrespective of its microscopic origin (including the role played by interactions) and relevant observables. The counterflow superfluidity was predicted theoretically some time ago but it has not been yet realized in the lab. Observation of the space-/time-crystallization effects can be used for detecting this superfluid state experimentally.

To comply with the no-go theorem [7], a system featuring the effect of time crystallization has to have an additive conserved quantity different from energy, and measurements have to violate the conservation of this quantity. The counterflow superfluids with potentially inter-convertible components satisfy these criteria; internal Josephson links between the two components probe the order in the phase difference field through temporal correlations and spacial interference of Josephson currents. The necessity for the time-crystallization probe to deal with the inter-conversion of the two components explicitly follows from the Beliaev–Josephson–Anderson relation (7). Otherwise, each of the two chemical potentials is defined up to its individual arbitrary additive constant reflecting the convention about the counting zero for energy.

We discussed two different experimental setups and, correspondingly, two different protocols for the observation of space-/time-crystallization in counterflow superfluids. The first setup, dealing with ultracold atoms in optical lattices, appears to be the most universal and conceptually transparent. Here the probes are local in space and time and unquestionably remove all concerns about non-equilibrium effects. Yet, an important aspect of realistic cold-atomic systems is that their sizes are rather moderate. This makes them especially suitable for studying the finite-size effects leading to phase decoherence and hence the finite linewidth of the Josephson effect power spectrum [15, 16]. This setup is equally good for detecting the genuine space and time crystallization in a three-dimensional system, as well as algebraic space and time crystallization in lower dimensions.

The second experimental setup is based on a bilayer supeconductor with two spatially separated Josephson links between the layers to study the effect of space crystallization via the interference of the two Josephson currents. The advantage of this setup is that it allows one to employ standard experimental techniques for detecting Josephson effect in electronic systems. In particular, one can use the emitted electromagnetic radiation to measure the frequency and the amplitude of the oscillating current. This setup also appears to be natural for utilizing the space-/time-crystallization effects to reveal and study some other superfluid phenomena and properties such as algebraic (as opposed to genuine) time crystallization [2], equilibrium statistics of supercurrent states [4], and surface superconductivity [14]. A minor shortcoming of this setup is that the system is two-dimensional so that it deals with the algebraic space and time crystallization.

It is instructive to put our results in a broader context of past and present activities addressing spontaneous breaking of time-translation symmetry in equilibrium, steady state, and periodically driven (Floquet) systems. At the moment, an exciting progress is being made—on both theoretical and experimental side—with Floquet time crystals (see, e.g., review [17] and references therein). By their very nature—the presence of a periodic drive—Floquet time crystals break discrete time-translation symmetry as opposed to breaking continuous time-translation symmetry. The discussed scenario for a macroscopic system to break continuous time-translation symmetry is most closely related to the Kuramoto synchronization mechanism (see review [18] and references therein), when, under appropriate conditions, local rotors get globally synchronized despite local fluctuations and disorder.

Our discussion was focused on the counterflow superfluid. Nevertheless, all the conclusions apply to any multicomponent superfluid because it inevitably has at least one counterflow mode. In particular, a simple two-component Bose-Einstein condensate would be a reasonable system for applying the above-mentioned protocol. In this regard, the manifestation of time crystallization in a two-component Bose-Einstein condensate has been already observed in Ref. [19]. As opposed to our protocol of detecting the time-translation symmetry breaking at equilibrium, the experiment of Ref. [19] starts by creating a coherent non-equilibrium initial state with well-defined relative phase between the two components (by producing the second component out of the condensed first one). The evidence for the broken time-translation symmetry then comes in the form of long-lived oscillations of the relative phase of the two condensates.

If the two-pulse protocol for equilibrium states is modified to render the weak interconversion interaction global (uniform) rather than local, then the experiment would demonstrate—by the very fact that a macroscopic equilibrium system features a finite response to such type of perturbation—a fundamental property of macroscopic time crystals: an inevitable presence of long-range spatial correlations along with the oscillations in the time domain. This property can be interpreted in terms of time-dependent order parameter, see, e.g., Ref. [20].

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