Reentrant superconductivity in a quantum dot coupled to a Sachdev-Ye-Kitaev metal

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Planckian superconductor

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The Planckian relaxation rate $\hbar/t_{\mathrm{P}} = 2\pi k_{\mathrm{B}}T$ sets a characteristic time scale for both equilibration of quantum critical systems and maximal quantum chaos. In this note, we show that at the critical coupling between a superconducting dot and the complex Sachdev-Ye-Kitaev model, known to be maximally chaotic, the pairing gap Δ behaves as $\eta\,\hbar/t_{\mathrm{P}}$ at low temperatures, where η is an order one constant. The lower critical temperature emerges with a further increase of the coupling strength so that the finite Δ domain is settled between the two critical temperatures.

The Bardeen-Cooper-Schrieffer mechanism of conventional superconductivity [1] requires two species of fermions coupled by an attractive two-body interaction [2]. The mean-field analysis of such a model results in the gapped quasiparticle excitation spectrum below the critical temperature. Meanwhile, the absence of long-living quasiparticles in high-temperature superconducting materials *above* the critical temperature is an immutable characteristic of the so-called strange metal state [3, 4]. In contrast to the quasiparticle nature of superconductors, strange metals exhibit a power-law behavior in the spectral function [5], similarly to quantum critical systems [6]. Lack of quasiparticles manifests itself in fast equilibration at low temperature on a time scale set by the Planckian relaxation time $t_{\mathrm{P}} = \hbar/(2\pi k_{\mathrm{B}}T)$ [6, 7]. The same time scale appears as an upper bound on quantum chaos setting the maximal rate of information scrambling [8]. It is usually formulated [8–10] in terms of the out-of-time ordered correlator [11] (OTOC): In quantum many-body systems the OTOC grows no faster than exponentially $\mathrm{e}^{t/t_{\mathrm{L}}}$ with the Lyapunov time t_{L} bounded from below as $t_{\mathrm{L}} \geq t_{\mathrm{P}}$ [8].

The widely-known Sachdev-Ye-Kitaev (SYK) model [12, 13], describing strongly interacting Majorana zero modes in $0+1$ dimensions, saturates the chaos bound $t_{\mathrm{L}} = t_{\mathrm{P}}$ [13, 14]. It does not possess an underlying quasiparticle description while being solvable in the infrared, with a spectral function that scales as a power-law of frequency. These properties do not change upon replacing Majoranas with conventional fermions (complex SYK model) [15, 16]. The extensions of this model to the cSYK coupled clusters predict thermal diffusivity [17] $\propto t_{\mathrm{P}}$ and reproduce the linear in temperature resistivity [18], observed in strange metals [19, 20]. Recently a proposed theory of a Planckian metal [21], based on the destruction of a Fermi surface by the cSYK-like interactions, shows that the universal scattering time equals to the Planckian time t_{P}. The latter one characterizes the linear in temperature resistivity property [22] and was detected in cuprates [23], pnictides [24], and twisted bilayer graphene [25], regardless of their different microscopic nature.

The successes in applying the SYK model to the qualitative studies of strange metals and the minimalistic structure of the model itself fostered the effort to find a mechanism by which the superconducting state is formed out of an incoherent SYK metal [26–29]. Driven by the same curiosity, we consider a $0+1$ dimensional toy model which consists of a superconducting quantum dot [30] coupled to the complex-valued SYK model [15]. At the critical coupling the pairing gap turns out to be proportional to the Planckian relaxation rate at low temperatures:

$$\Delta \approx \eta\,\frac{\hbar}{t_{\mathrm{P}}}, \tag{1}$$

where η is a number close to one. This theoretical finding that we refer to as Planckian superconductor draws parallels to the phenomenon of reentrant superconductivity [31, 32] in Kondo superconductors [33–35] and physics of the Andreev billiards [36–40].

We start with a superconducting Hamiltonian H_{SC} that contains $2M$ modes described by the Richardson Hamiltonian [41–43] without single-particle energies coupled to the SYK model H_{SYK} with N fermions through a random tunneling term H_{tun}:

$$H = H_{SC} + H_{SYK} + H_{tun}\,, \tag{2}$$

$$H_{SC} = -\frac{U}{M}\sum_{i,j=1}^{M}\psi^\dagger_{\uparrow i}\psi^\dagger_{\downarrow i}\psi_{\downarrow j}\psi_{\uparrow j} - \mu\sum_{i=1}^{M}\sum_{\sigma=\uparrow,\downarrow}\psi^\dagger_{\sigma i}\psi_{\sigma i}, \tag{3}$$

$$H_{SYK} = \frac{1}{(2N)^{3/2}}\sum_{i,j,k,l=1}^{N} J_{ij;kl}c^\dagger_i c^\dagger_j c_k c_l, \tag{4}$$

$$H_{tun} = \frac{1}{(MN)^{1/4}}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{\sigma=\uparrow,\downarrow}\Big(t^\sigma_{ij}c^\dagger_i\psi_{\sigma j} + \mathrm{h.c.}\Big). \tag{5}$$

The couplings t^{σ}_{ij} and $J_{ij;kl}$ are assumed to be independent Gaussian random variables with the finite variances $\overline{t^{\sigma *}_{ij} t^{\sigma'}_{ij}} = t^2 \delta_{\sigma\sigma'}$, $\overline{|J_{ij;kl}|^2} = J^2$ ($J_{ij;kl} = -J_{ji;kl} = -J_{ij;lk} = J^*_{kl;ij}$) and zero means.

The interaction terms in the Hamiltonian (2) are decoupled within the Hubbard-Stratonovich transformations [2, 15], so that in the large M, N limit the self-consistent saddle-point equations are [44]:

$$\Sigma_c(\tau) = J^2 G_c(\tau)^3 + 2\sqrt{p}\, t^2 G_+(\tau)\,, \tag{6}$$

$$G_c(\mathrm{i}\omega_n)^{-1} = \mathrm{i}\omega_n - \Sigma_c(\mathrm{i}\omega_n), \tag{7}$$

$$G_+(\mathrm{i}\omega_n) = \frac{\mathrm{i}\omega_n - \frac{t^2}{\sqrt{p}} G_c(\mathrm{i}\omega_n)}{\left(\mathrm{i}\omega_n - \frac{t^2}{\sqrt{p}} G_c(\mathrm{i}\omega_n)\right)^2 - |\Delta|^2}, \tag{8}$$

$$\frac{1}{U} = T \sum_{n=-\infty}^{+\infty} \frac{1}{\left(\omega_n + \frac{\mathrm{i}t^2}{\sqrt{p}} G_c(\mathrm{i}\omega_n)\right)^2 + |\Delta|^2}, \tag{9}$$

where $\omega_n = \pi T(2n+1)$ are Matsubara frequencies and $p = M/N$ controls the ratio between the "sites" [45–47] in the superconductor/SYK sector. The self-energy of the SYK fermions appears in the equations (6,7) as $\Sigma_c(\tau)$, while $G_c(\tau)$ denotes the corresponding Green's function $-N^{-1}\sum_{i=1}^{N} \langle \mathrm{T}_\tau c_i(\tau)\bar{c}_i(0)\rangle$. The Green's functions of the $\uparrow,\downarrow$–fermions in the superconductor $G_\sigma(\tau) = -M^{-1}\sum_{i=1}^{M} \langle \mathrm{T}_\tau \psi_{i\sigma}(\tau)\bar{\psi}_{i\sigma}(0)\rangle$ enter the equation (8) as a half trace of the Gor'kov's function [48] $G_+(\tau) = \frac{1}{2}(G_\uparrow + G_\downarrow)(\tau)$. Finally, relation (9) is a modified gap equation [2], which accounts for the amount of the SYK impurity in the superconductor through $G_c(\tau)$ under the assumption of frequency independent pairing Δ. The chemical potential μ can be accounted in the equations (6-9) by the shift $|\Delta|^2 \to |\Delta|^2 + \mu^2$. Below, we set $\mu = 0$.

In the normal phase ($\Delta = 0$) the equations (6-8) can be written as

$$\Sigma(\tau) = J^2 G_c(\tau)^3, \tag{10}$$

$$\left(\mathrm{i}\omega_n - \Sigma(\mathrm{i}\omega_n)\right) G_c(\mathrm{i}\omega_n) = \frac{\mathrm{i}\omega_n - \frac{t^2(1-2p)}{\sqrt{p}} G_c(\mathrm{i}\omega_n)}{\mathrm{i}\omega_n - \frac{t^2}{\sqrt{p}} G_c(\mathrm{i}\omega_n)}, \tag{11}$$

mind a convenient self-energy translation $\Sigma \equiv \Sigma_c - 2\sqrt{p}\, t^2 G_+$. If $p \ll 1/2$ ($2M \ll N$), the bare SYK Green's function $G_{SYK}(\mathrm{i}\omega_n) = -\mathrm{i}\pi^{1/4}\mathrm{sgn}\left(\omega_n\right)/\sqrt{|J\omega_n|}$ solves the equations (10,11) in the infrared $\omega_n \ll J$. In this regime, the Green's function of the ψ-fermions $G_+(\mathrm{i}\omega_n) \propto \sqrt{\omega_n}$ for $\omega_n/J \ll p^{-1/3}(t/J)^{4/3}$. In the equal sites case $2M = N$, which corresponds to $p = 1/2$, the bare SYK Green's function survives for $(t/J)^{4/3} \ll \omega_n/J \ll 1$. Another solution appears at $p = 1/2$ if one supposes $\omega_n \ll \left\{t^2 |G_c|\,, |\Sigma|\right\}$. Then the equation (11) shortens to

$$\Sigma(\mathrm{i}\omega_n) = \frac{\mathrm{i}\omega_n}{\sqrt{2} t^2}\, G_c(\mathrm{i}\omega_n)^{-2}. \tag{12}$$

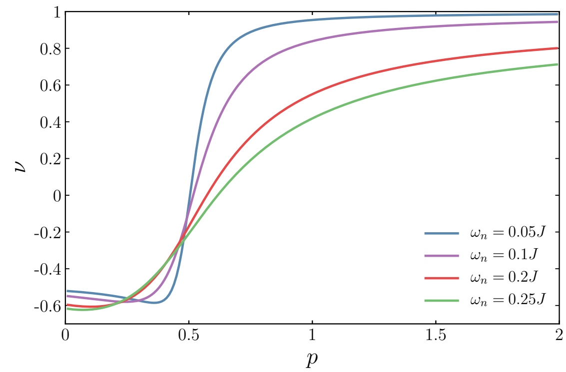


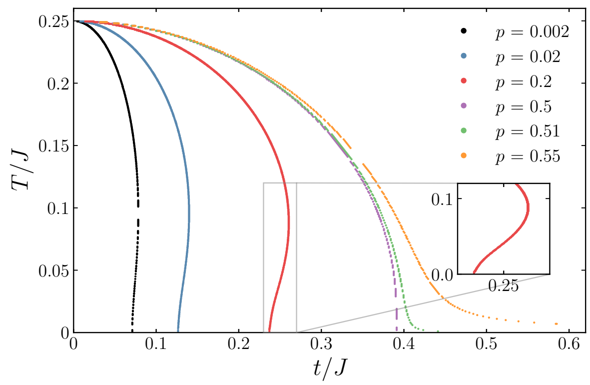
FIG. 1. **Scaling of the Green's function** G_c in the normal phase. We plot $\nu = \partial \ln G_c/\partial \ln \omega_n$ as a function of p at given frequencies and finite coupling $t = 0.475J$. At low frequencies, ν close to $-1/2$ is robust against p increase for $p < 1/2$. The frequency rise moves ν towards -1 (free fermion limit), while ν crosses over to 1 for large p. The temperature is $T = 10^{-4}J$.

The Green's function that satisfies the equations (10,12) is $G_c(\mathrm{i}\omega) \propto -\mathrm{i}\,\mathrm{sgn}(\omega)/\left(J^2 t^2 |\omega_n|\right)^{-1/5}$ for the frequencies $(t/J)^3 \ll \omega_n/J \ll (t/J)^{4/3}$, that are achievable in the weak tunneling limit $t \ll J$. Note that the frequency window strictly depends on the coupling t. For $p \gg 1/2$ the Green's function of the c–fermions in the low-frequency limit is $G_c(\mathrm{i}\omega_n) \propto -\mathrm{i}\omega_n$ [47], that leads to the density of states $-\pi^{-1}\mathrm{Im}G_c(\mathrm{i}\omega_n \to \omega + \mathrm{i}0^+) \simeq 0$ in the SYK sector vanished. Therefore, at large p, the normal phase is given by the free fermions in the ψ–dot, whose Green's function is $G_+(\mathrm{i}\omega_n) = -\mathrm{i}/\omega_n$. To follow the frequency scaling of the Green's function $G_c(\mathrm{i}\omega_n)$ while changing p, we introduce the logarithmic derivative $\nu = \partial \ln G_c/\partial \ln \omega_n$ plotted in Figure 1 at low temperatures. Summarizing, the normal phase in the infrared limit is described by the inverse Green's function of the SYK model at small p, whereas it crosses over to free fermions for large p values.

The gap equation (9) at $\Delta = 0$ makes a boundary in between the normal phase and the superconducting one by setting the critical temperature T_c as a function of the coupling rate t. Let's notice that the SYK model (4) does not have a spin degree of freedom after disorder averaging [44]. Thus, it may be thought of as spin polarised. It suppresses superconductivity similar to magnetic impurities: Increase of the coupling to the SYK subsystem decreases the critical temperature [49]. There exists a critical coupling t_c:

$$\frac{1}{U} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \left(\omega + \frac{\mathrm{i}t_c^2}{\sqrt{p}} G_c(\omega)\right)^{-2}, \tag{13}$$

such to abolish superconductivity at zero temperature. The constraint (13) follows from gap equation (9) when $\Delta, T = 0$.



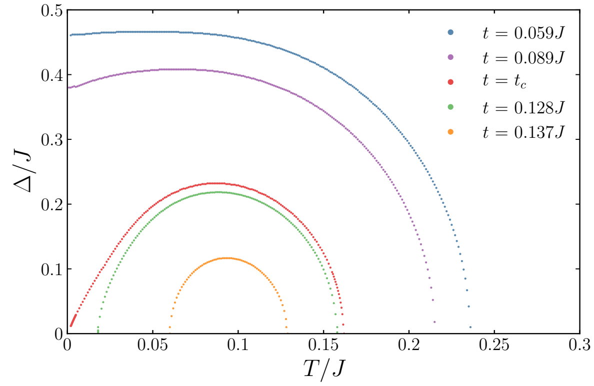


FIG. 2. **Left panel: Critical temperature** as a function of the coupling strength to the SYK dot. The curves for $p < 0.5$ are bent at low temperatures. This illustrates the presence of **two critical temperatures**. At $p = 0.5$ the bend disappears whereas for the values of $p > 0.5$ a single critical temperature decays to zero asymptotically. **Right panel: The pairing gap** as a function of temperature at $p = 0.02$. The critical coupling value is $t_c \approx 0.127J$. U is set equal to J in both panels.

There are three competing phases contributing to the denominator of the self-consistency relation (9): SYK non-Fermi liquid, free fermions, and superconducting condensate Δ. If there are enough of the SYK fermions ($N > 2M$), Δ interplays with the non-Fermi liquid at zero temperature. The latter one falls off with the temperature rise making a room for the superconducting phase beyond the critical coupling, which results in the growth of the critical temperature. Indeed, Figure 2 (left) shows the bend of the critical temperature in the vicinity of the critical coupling [50]. This phenomenon resembles the reentrant superconductivity [31, 32] in superconductors with Kondo impurities [33–35]. The pairing gap goes down at low temperatures with the coupling increase as in Figure 2 (right). Achieving the critical coupling when Δ vanishes at zero temperature leads to the appearance of the lower critical temperature. In contrast, the reentrant superconducting regime is absent for $N < 2M$, since the normal phase behaves as the conventional Fermi liquid at low temperatures and large p, as was noticed earlier. In Figure 2 (left), we show [50] that $p = 1/2$ ($N = 2M$) separates the regions with one or two critical temperatures. Similarly, consideration of the random free fermions model $\sum_{ij} J_{ij} c_i^\dagger c_j$ instead of the SYK model does not give the reentrance effect. In this case, the self-energy equation (6) changes to $\Sigma_c(\mathrm{i}\omega_n) = J^2 G_c(\mathrm{i}\omega_n) + 2\sqrt{p}\, t^2 G_+(\mathrm{i}\omega_n)$. The results for the critical temperature are presented in Figure 3. It is still possible to suppress the superconductivity at zero temperature providing sufficient impurities, but there is only a single critical temperature as the normal phase is always set by the free fermions [51].

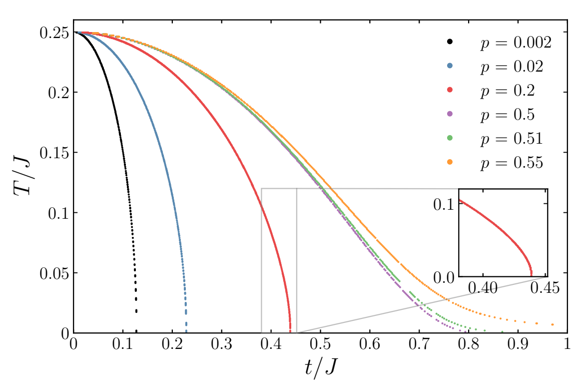


FIG. 3. **Critical temperature** as a function of the coupling strength to the **random free fermions** model.

From Figure 2 (right), one notices the pairing gap at the critical coupling is $\propto T$ at low temperatures. We numerically examine [50] Δ in the reentrant phase $p < 1/2$ for several values of p and U (see Figure 4). The gap saturates $2\pi T$ almost irrespective of parameters of the problem. Units recovery brings us to the relation (1) announced above so that the gap is set by the inverse Planckian time $1/t_{\mathrm{P}}$ multiplied by $\hbar$.

This observation seems as reminiscent of quite a peculiar feature of an Andreev billiard [53]: In a clean chaotic cavity proximate to a superconductor, the induced gap equals to $\hbar/t_{\mathrm{E}} = \hbar/\left(t_{\mathrm{L}} \ln \frac{p_{\mathrm{F}} l}{\hbar}\right)$ [38–40], where t_{E} is the Ehrenfest time (the typical time scale of quantum dynamics), t_{L} is the Lyapunov time, p_{F} is the Fermi momentum, and l is the characteristic cavity length. The effect is predicted in the regime of the Ehrenfest time far exceeds τ – the typical lifetime of an electron/hole excitation in the cavity. Oppositely, if $t_{\mathrm{E}} \ll \tau$ the gap behaves as $\hbar/\tau$, where τ does not depend on the Planck constant [36, 37]. In the SYK model the Lyapunov time coincides with the Planckian relaxation time

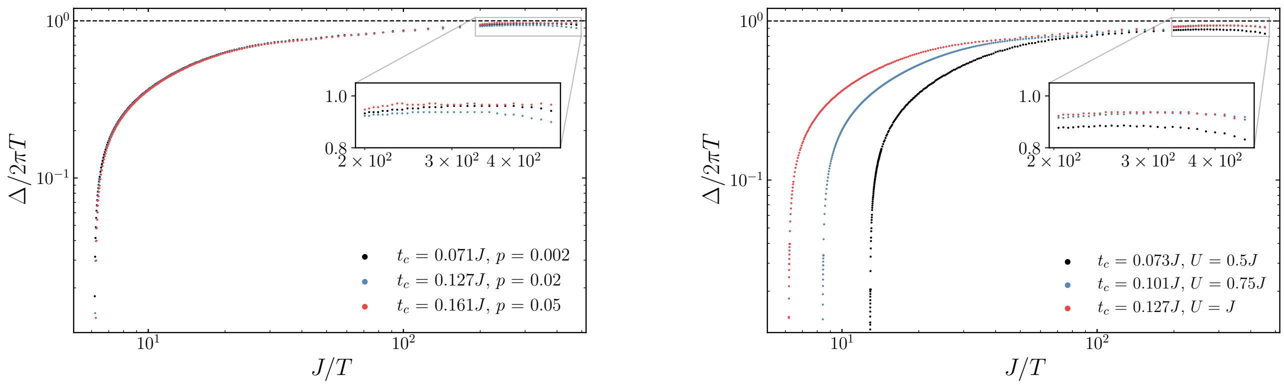


FIG. 4. **The gap to temperature ratio** as a function of inverse temperature at the critical coupling depends on neither the mode ratio p (fixed $U = J$, **left panel**) or the Richardson interaction strength U (fixed $p = 0.02$, **right panel**). In both cases, Δ saturates $2\pi T$ at low temperatures [52]. In the right panel, we notice that decrease of the interaction in the superconducting dot reduces the critical temperature as in the bare Richardson model (3).

$t_{\mathrm{L}} = \hbar/\left(2\pi k_{\mathrm{B}} T\right) = t_{\mathrm{P}}$ [13, 14], although those are different physical quantities [54]. However, the Ehrenfest time is $t_{\mathrm{L}} \ln N \gg \hbar/(2\pi k_{\mathrm{B}} T)$, which differs from t_{P} predicted in the pairing gap (1) by $\ln N$.

To estimate the gap behaviour at the critical coupling we consider the equations (6-8) at finite Δ:

$$\left(\mathrm{i}\omega_n - \Sigma(\mathrm{i}\omega_n)\right) G_c(\mathrm{i}\omega_n) = \frac{\left(\mathrm{i}\omega_n - \frac{t^2}{\sqrt{p}} G_c(\mathrm{i}\omega_n)\right)\left(\mathrm{i}\omega_n - \frac{t^2(1-2p)}{\sqrt{p}} G_c(\mathrm{i}\omega_n)\right) - |\Delta|^2}{\left(\mathrm{i}\omega_n - \frac{t^2}{\sqrt{p}} G_c(\mathrm{i}\omega_n)\right)^2 - |\Delta|^2}, \tag{14}$$

whereas the self-energy equation (10) stays unchanged. The right-hand side of the equation (14) tends to unity for $p \ll 1/2$. Thus it is sufficient to substitute the SYK Green's function in the gap equation (9) in this regime.

As we look for a low-temperature correction to zero Δ at the critical coupling, we expand the gap equation (9) in powers of Δ up to the second order:

$$\frac{1}{U} \simeq 2T \sum_{n=0}^{+\infty} \frac{1}{\left(\omega_n + \frac{\mathrm{i}t_c^2}{\sqrt{p}} G_c(\omega_n)\right)^2} \left(1 - \frac{|\Delta|^2}{\left(\omega_n + \frac{\mathrm{i}t_c^2}{\sqrt{p}} G_c(\omega_n)\right)^2}\right). \tag{15}$$

The SYK Green's function diverges at low frequencies as $1/\sqrt{\omega_n}$ and decays as $1/\omega_n$ in the ultraviolet. Hence the principal contribution to the sum (15) from the high frequencies is given by the bare ω_n in the denominator. On the other hand, divergent Green's function is crucial at low frequencies. Assuming G_c decays fast enough in comparison to ω_n, we replace G_c with the infrared SYK Green's function $G_{SYK}(\mathrm{i}\omega_n) = -\mathrm{i}\pi^{1/4} \mathrm{sgn}\left(\omega_n\right)/\sqrt{|J\omega_n|}$ in the expression (15).

The low-temperature version of the relation (15) can be written by means of the Euler-Maclaurin formula [55]:

$$\frac{1}{U} \simeq \int_0^{+\infty} \frac{d\omega}{\pi} \frac{1}{\left(\omega + \frac{\mathrm{i}t_c^2}{\sqrt{p}} G_{SYK}(\omega)\right)^2} \left(1 - \frac{|\Delta|^2}{\left(\omega + \frac{\mathrm{i}t_c^2}{\sqrt{p}} G_{SYK}(\omega)\right)^2}\right) - \frac{pT}{t_c^4 G_{SYK}(\pi T)^2} \left(1 + \frac{2\pi T}{3} \frac{\partial G_{SYK}(\pi T)/\partial\omega}{G_{SYK}(\pi T)}\right), \tag{16}$$

where we expand up to T^2 keeping in mind that $\Delta \propto T$ [56] at the critical coupling. Finally, one notices two terms in the top row of the equation (16) match the critical coupling condition (13). Therefore, we obtain [57]:

$$\Delta(T) \simeq \sqrt{6}\pi T. \tag{17}$$

Although, this estimate gives $\eta \approx 1.22$ that exceeds the found numerical value $\eta \approx 0.96$ for the pairing gap $\Delta \approx \eta\, \hbar/t_{\mathrm{P}}$, derived low-temperature gap behavior (17) is independent of the problem parameters as in Figure 4.

Conclusion.—In this manuscript, we considered the superconducting proximity effect for the Sachdev-Ye-Kitaev model. We have shown, that the superconducting dot coupled to the complex SYK model possesses reentrant superconductivity. At the critical coupling, that gives rise to the occurrence of the lower critical temperature, the pairing gap disappears at $T = 0$ and grows linearly with the temperature increase. The linear–T growth of the gap is given by $\hbar/t_{\rm P}$, where $t_{\rm P} = \hbar/(2\pi k_{\rm B}T)$ is the Planckian relaxation time. The same time scale serves as an ultimate bound on many-body quantum chaos [8], saturated in strongly coupled systems without quasiparticle excitations. Thereby a natural question arises whether the pairing gap is an appropriate physical observable for the Lyapunov spectrum [58] of the SYK model. Accurate studies of the OTOC in the proposed system (2) might shed light on that. On its own, $\Delta \approx \eta\, \hbar/t_{\rm P}$ may be used to characterize the cSYK quantum dots [59, 60]. However, this requires consideration of a more realistic setup such as a superconducting lead attached to the particular realization of the complex SYK model.

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[1] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[2] A. Altland and B. D. Simons, *Condensed Matter Field Theory*, 2nd ed. (Cambridge University Press, 2010).
[3] T. Senthil, Phys. Rev. B **78**, 035103 (2008).
[4] B. Keimer, S. A. Kivelson, M. R. Norman, S. Uchida, and J. Zaanen, Nature **518**, 179186 (2015).
[5] C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A. E. Ruckenstein, Phys. Rev. Lett. **63**, 1996 (1989).
[6] S. Sachdev, *Quantum Phase Transitions*, 2nd ed. (Cambridge University Press, 2011).
[7] J. Zaanen, Nature **430**, 512513 (2004).
[8] J. Maldacena, S. H. Shenker, and D. Stanford, Journal of High Energy Physics **2016**, 106 (2016).
[9] S. H. Shenker and D. Stanford, Journal of High Energy Physics **2015**, 132 (2015).
[10] D. A. Roberts and B. Swingle, Phys. Rev. Lett. **117**, 091602 (2016).
[11] A. I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP **28**, 1200 (1969).
[12] S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
[13] A. Kitaev, KITP Program: Entanglement in Strongly-Correlated Quantum Matter (2015).
[14] J. Maldacena and D. Stanford, Phys. Rev. D **94**, 106002 (2016).
[15] S. Sachdev, Phys. Rev. X **5**, 041025 (2015).
[16] K. Bulycheva, Journal of High Energy Physics **2017**, 69 (2017).
[17] R. A. Davison, W. Fu, A. Georges, Y. Gu, K. Jensen, and S. Sachdev, Phys. Rev. B **95**, 155131 (2017).
[18] X.-Y. Song, C.-M. Jian, and L. Balents, Phys. Rev. Lett. **119**, 216601 (2017).
[19] H. Takagi, B. Batlogg, H. L. Kao, J. Kwo, R. J. Cava, J. J. Krajewski, and W. F. Peck, Phys. Rev. Lett. **69**, 2975 (1992).
[20] L. Taillefer, Annual Review of Condensed Matter Physics **1**, 51 (2010).
[21] A. A. Patel and S. Sachdev, Phys. Rev. Lett. **123**, 066601 (2019).
[22] J. A. N. Bruin, H. Sakai, R. S. Perry, and A. P. Mackenzie, Science **339**, 804 (2013).
[23] A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, T. L., and C. Proust, Nature Physics **15**, 142147 (2019).
[24] Y. Nakajima, T. Metz, C. Eckberg, K. Kirshenbaum, A. Hughes, R. Wang, L. Wang, S. R. Saha, I.-L. Liu, N. P. Butch, D. Campbell, Y. S. Eo, D. Graf, Z. Liu, S. V. Borisenko, P. Y. Zavalij, and J. Paglione, (2019), arXiv:1902.01034 [cond-mat.str-el].
[25] Y. Cao, D. Chowdhury, D. Rodan-Legrain, O. Rubies-Bigordà, K. Watanabe, T. Taniguchi, T. Senthil, and P. Jarillo-Herrero, (2019), arXiv:1901.03710 [cond-mat.str-el].
[26] A. A. Patel, M. J. Lawler, and Eu.-A. Kim, Phys. Rev. Lett. **121**, 187001 (2018).
[27] I. Esterlis and J. Schmalian, Phys. Rev. B **100**, 115132 (2019).
[28] Y. Wang, (2019), arXiv:1904.07240 [cond-mat.str-el].
[29] D. Chowdhury and E. Berg, (2019), arXiv:1908.02757 [cond-mat.str-el].
[30] L. Kouwenhoven and C. Marcus, Physics World **11**, 35 (1998).
[31] M. Maple, W. Fertig, A. Mota, L. DeLong, D. Wohlleben, and R. Fitzgerald, Solid State Communications **11**, 829 (1972).
[32] Y. B. Simons, O. Entin-Wohlman, Y. Oreg, and Y. Imry, Phys. Rev. B **86**, 064509 (2012).
[33] E. Müller-Hartmann and J. Zittartz, Phys. Rev. Lett. **26**, 428 (1971).
[34] G. Riblet and K. Winzer, Solid State Communications **9**, 1663 (1971).
[35] E. Mller-Hartmann, B. Schuh, and J. Zittartz, Solid State Communications **19**, 439 (1976).
[36] J. A. Melsen, P. W. Brouwer, K. M. Frahm, and C. W. J. Beenakker, Physica Scripta **T69**, 223 (1997).
[37] H. Schomerus and C. W. J. Beenakker, Phys. Rev. Lett. **82**, 2951 (1999).
[38] A. Lodder and Yu. V. Nazarov, Phys. Rev. B **58**, 5783 (1998).
[39] İ. Adagideli and C. W. J. Beenakker, Phys. Rev. Lett. **89**, 237002 (2002).
[40] M. G. Vavilov and A. I. Larkin, Phys. Rev. B **67**, 115335 (2003).
[41] R. Richardson, Physics Letters **3**, 277 (1963).
[42] J. von Delft, A. D. Zaikin, D. S. Golubev, and W. Tichy, Phys. Rev. Lett. **77**, 3189 (1996).

[43] K. A. Matveev and A. I. Larkin, Phys. Rev. Lett. **78**, 3749 (1997).
[44] See supplemental material.
[45] S. Banerjee and E. Altman, Phys. Rev. B **95**, 134302 (2017).
[46] X. Chen, R. Fan, Y. Chen, H. Zhai, and P. Zhang, Phys. Rev. Lett. **119**, 207603 (2017).
[47] S.-K. Jian and H. Yao, Phys. Rev. Lett. **119**, 206602 (2017).
[48] L. P. Gor'kov, Sov. Phys. JETP **7**, 505 (1958).
[49] P. De Gennes, *Superconductivity of Metals and Alloys* (Benjamin, 1966).
[50] The full self-consistent scheme (6-9) is solved numerically with the adaptive golden ratio algorithm [61,44].
[51] Earlier it was shown that the superconducting instability in the unparticle system leads to the reentrance effect as well [62], whereas restoration of the quasiparticles makes the critical temperature a single-valued function of the pairing strenth.
[52] The gap decrease in Figure 4 at very low temperatures (see the enlarged segments) has a numerical origin. As Matsubara frequencies are $\propto T$, achieving temperatures close to zero requires a sufficient increase of the numerical grid. This leads to the accuracy reduce due to the computer memory overflow [44].
[53] C. W. J. Beenakker, "Andreev billiards," in *Quantum Dots: a Doorway to Nanoscale Physics*, edited by W. Dieter Heiss (Springer Berlin Heidelberg, Berlin, Heidelberg, 2005) pp. 131–174.
[54] The equilibration time and the Lyapunov time are *a priori* different physical quantities. Nevertheless, the fact that both quantities are subjected to the same bound raises the question of whether those two seemingly independent quantities might be related. This hypothesis has been intensively investigated in the context of the AdS/CFT correspondence, in large–N vector models and spin systems (see [63] and the references therein). In systems with a small parameter (large–N quantum field theories or weakly coupled field theories) where a regime of exponential growth is present in the OTOC, however, they are set by the same physics even though they are quantitatively different [64,65].
[55] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (Dover, 1964).
[56] Similarly, in large–N models, the primary contribution of low Matsubara frequencies $\omega_n = \pm\pi T$ to the gap equation leads to $\Delta \propto T$ [66].
[57] We use $\int_0^{+\infty}\frac{d\omega}{\pi}\left(\omega+\frac{\mathrm{i}t_c^2}{\sqrt{p}}G_{SYK}(\omega)\right)^{-4} = \frac{pJ}{9\pi^{3/2}t_c^4}$.
[58] A. Romero-Bermúdez, K. Schalm, and V. Scopelliti, Journal of High Energy Physics **2019**, 107 (2019).
[59] A. Chen, R. Ilan, F. de Juan, D. I. Pikulin, and M. Franz, Phys. Rev. Lett. **121**, 036403 (2018).
[60] I. Danshita, M. Hanada, and M. Tezuka, Progress of Theoretical and Experimental Physics **2017**, 8, 083I01 (2017).
[61] Yu. Malitsky, Math. Program. (2019), doi:10.1007/s10107-019-01416-w.
[62] A. Karch, K. Limtragool, and P. W. Phillips, Journal of High Energy Physics **2016**, 175 (2016).
[63] J. Couch, S. Eccles, P. Nguyen, B. Swingle, and S. Xu, (2019), arXiv:1908.06993 [cond-mat.stat-mech].
[64] S. Grozdanov, K. Schalm, and V. Scopelliti, Phys. Rev. Lett. **120**, 231601 (2018).
[65] S. Grozdanov, K. Schalm, and V. Scopelliti, Phys. Rev. E **99**, 012206 (2019).
[66] Y.-M. Wu, A. Abanov, Y. Wang, and A. V. Chubukov, Phys. Rev. B **99**, 144512 (2019).