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Morphodynamics of fluid–fluid displacement in 3D deformable granular media

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Abstract

We study experimentally the displacement of one fluid by another in a granular pack, to uncover relationships between fluid invasion and medium deformation. We develop an experimental setup that allows us to reconstruct the coupled invasion–deformation dynamics in 3D, and simultaneously characterize with unprecedented fidelity the fluid invasion pattern and to document a transition from fluid–fluid displacement in pores to the formation of conduits by grain motion. We rationalize the findings in terms of a simple poromechanics model that indeed captures this transition as a result of a balance between viscous and frictional forces. These results contribute to elucidating the role of three-dimensionality in the timing, mode, and morphology of fluid–fluid displacement and injection-induced deformation in porous media.

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I. INTRODUCTION

Multiphase flow in porous media plays a fundamental role in many natural and engineered processes, which include enhanced oil recovery [1, 2], geothermal energy production [3], geologic CO₂ sequestration [4, 5], water infiltration in soil [6, 7], and water dropout in fuel cells [8], just to name a few. Of particular complexity and importance are processes that involve the simultaneous displacement of fluids and deformation of the host medium, with applications as varied as shale-oil and shale-gas production via hydraulic fracturing [9, 10], methane venting from organic-rich sediments [11], hydrate formation and dissociation [12–14], volcanic eruptions [15], desiccation cracks in soil [16], air sparging for subsurface contaminant remediation [17, 18], and patterning of carbon nanotube forests [19].

Studies investigating the morphology of deformation of granular packs from fluid injection have almost exclusively been conducted in quasi-2D experimental setups. For single-phase fluid flows, early experiments demonstrated a transition from viscous fingering to fracturing in clay slurries [20, 21], and recent work has shown the emergence of inelastic deformation from the collective rearrangement of a 2D monolayer of elastic frictionless hydrogel particles [22]. Fracture patterns have been observed also in loose and dense systems such as particle rafts as a result of surfactant spreading [23, 24], and colloidal suspensions as a result of drying [25, 26]. Two-phase flow experiments of air injection into liquid-filled granular packs have elucidated a transition from Saffman–Taylor-type fingering to fracturing as the packing ratio increases [27]. This type of fluid–fluid–particle displacement of a granular suspension can lead to a variety of fascinating invasion patterns, including labyrinth structures [28], coral morphologies, and stick–slip bubbles [29].

Of particular interest to us are systems at high packing density, which display a transition from pore-scale fingering to hydrocapillary fracturing, as a result of the interplay between viscous and capillary forces and frictional forces [30–32]. This balance of forces at the pore scale permits rationalizing observations in a wide range of systems, including gas injection into water-saturated granular packs [33–36] and clays [37], liquid imbibition into sand [38], desiccation cracks [39], CO₂ migration in porous rocks [40], and methane venting from soft sediments [11, 41, 42].

Three-dimensional visualization of multiphase flow in porous media under dynamic conditions has relied on direct optical visualization [43], planar laser-fluorescence imaging similar to the one we employ here [44–47], confocal microscopy [48], magnetic resonance imaging [49] and high-energy X-ray computed tomography [50, 51]. While these 3D observations have been instrumental

in characterizing fluid–fluid displacements—including temporally-resolved imaging of pore-scale invasion events [50, 51]—they have been limited, however, to small sampling volumes and to rigid porous media that left medium deformation unexplored.

Here, we perform 3D imaging of two-phase flow in a deformable porous medium, with an emphasis in understanding the morphology and dynamics of fluid invasion and medium deformation. We construct a porous cell made of borosilicate glass beads initially filled with glycerol to achieve refractive index matching, and inject a less-viscous silicon oil that is also index-matched. We employ a planar laser fluorescence imaging technique, in which a laser sheet mounted on a moving stage shines on the medium and excites fluorescent dyes pre-mixed with the defending and invading fluids. To control the deformability of the medium, we apply a confining weight to the top lid of the cell. To elucidate the morphology of fluid invasion under different confining weights, bead sizes and injection rates, we perform two different types of measurements and analyses:

1. Macroscopic (cell-scale), in which we track the displacement of the top lid. We develop a coupled poromechanics model for the onset of frictional failure and the evolution of post-failure overall cell deformation, as a balance between fluid–fluid displacement and cavity expansion. Our model allows us to identify the relevant poromechanical parameters for our model system across a wide range of experimental conditions.
2. Microscopic (pore-scale), in which time-lapse 3D imaging allows us to reconstruct in detail the morphology of the invading fluid and bead displacement. This allows us to delineate the experimental conditions under which fluid invasion into the granular pack is controlled by either pore-scale fingering or conduit opening.

II. LABORATORY EXPERIMENTS

A. Experimental setup

We built a porous-medium cell consisting of an acrylic box (horizontal dimensions $L \times L$). The box, initially open at the top, is filled with borosilicate glass beads (nominal diameter d) and a lid is placed at the top of the bead pack (initial height H_0). This lid is drilled with holes (hole diameter 1 mm) so as to allow uniform seepage of fluid through it while retaining the beads. A weight W is placed on top of the lid to control the confining stress that the bead pack is subject to. The cell has a hole drilled at the center of the bottom face, to allow injection of fluid. A needle

(internal diameter 1.7 mm) is glued to the inside of this hole, and fluids are injected using a syringe pump (PHD 2000, Harvard Apparatus).

Initially, we pour the beads inside the container, and place the lid on top, and then the confining weight. The dry bead pack is then filled with the initial defending fluid, glycerol, at a rate of 1 mL min⁻¹. Glycerol is wetting to the glass beads with respect to air. Thus, injection of glycerol is a viscously-stable, gravity-stable, slow imbibition that results in a complete displacement of air from the porous medium. The absence of air bubbles is checked optically. We stop injection once the glycerol has reached the top of the lid. We then inject the invading liquid at a rate q .

We employed cells of different dimensions ($L = \{2, 6\}$ cm, $H_0 = \{2.5, 4.5\}$ cm), beads of different diameters ($d = \{1, 3\}$ mm), and a range of values for the other control parameters ($0.045 < W < 23.9$ N, $0.03 < q < 100$ mL min⁻¹) to study fluid–fluid displacement under diverse experimental conditions.

B. Imaging

We performed two types of experiments, according to the kind of imaging that was conducted: (1) direct imaging, and (2) planar laser induced fluorescence (PLIF) imaging. In each type of experiment, we employed slightly different fluids, with therefore slightly different properties, which we describe next.

1. Direct imaging

In this set of experiments, we imaged the porous cell from one side using a digital CMOS camera (Hamamatsu ORCA-Flash4.0). The defending fluid, glycerol, has the same refraction index as borosilicate glass ($n = 1.430$), thereby rendering the porous medium transparent. The invading fluid is silicone oil (Sigma Aldrich), dyed with a blue dye (Orcosolve Oil Blue AP), and direct imaging records a 2D projection of the 3D fluid invasion pattern (Figure 1). The properties of the fluids are given in Table I.

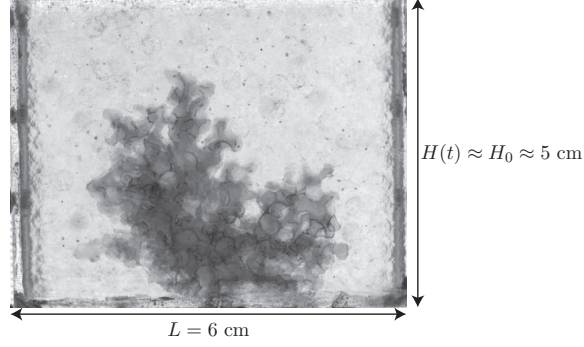


FIG. 1. Image taken by the camera during a direct-imaging experiment in a cell with horizontal dimension $L = 6$ cm and beads of diameter $d = 3$ mm. The dark pattern is the injected silicone oil.

2. Planar laser-induced fluorescence (PLIF) imaging

In a second set of experiments, we used the planar laser induced fluorescence (PLIF) technique, which allows to see through a transparent porous medium [44, 46, 47]. In this case, the invading fluid is also index-matched to the borosilicate glass beads. We used a silicone oil that is a mix of two different products: 60.9% of Dow Corning 556 and 39.1% of Dow Corning 550. The proportions were determined experimentally so that the refractive index of the mix is the closest possible to the one of the beads.

A laser sheet is used to illuminate a vertical plane of the bead pack (Z-Laser ZM18, emitting at 532 nm). Both the defending and invading liquids are dyed using different fluorescent dyes that are excited by the wavelength of the laser. The defending fluid (glycerol) is seeded with Rhodamine 590 Chloride (Exciton) at a concentration 0.12 mg L^{-1} . The invading fluid (silicone oil) is seeded with Pyrromethene 567A (Exciton) at a concentration 2.7 mg L^{-1} . The fluids properties are given in Table I. The two dyes emit at a similar wavelength, around 560 nm. We place a long-pass filter, keeping only light with wavelength longer than 550 nm, ensuring that the light emitted directly by the laser does not reach the camera. We use different dye concentrations so that, in the illuminated plane, we can distinguish the beads and the two fluids: the beads appear black, the defending fluid gray, and the invading fluid white. The laser and camera are mounted on a motorized stage to scan the entire porous cell and obtain a 3D reconstruction of the dynamics of invasion (Figure 2). Images are taken at 1024×1024 pixels, giving a resolution of 80 to 100 $\mu\text{m}/\text{pixel}$. The maximum width of the laser sheet is 0.3 mm, which is well below one bead diameter. Thus, individual pore volumes are lit during a given scan, and we believe that the measurements would not be different

TABLE I. Fluids properties for both types of experiments: density ρ , dynamic viscosity η , viscosity contrast M , and interfacial tension γ .

	Direct Imaging		PLIF Imaging	
Defending fluid	Glycerol	$\rho_d = 1260 \text{ kg m}^{-3}$ $\eta_d = 1410 \text{ mPa.s}$	Glycerol with Rhodamine 590 Chloride	$\rho_d = 1260 \text{ kg m}^{-3}$ $\eta_d = 1410 \text{ mPa.s}$
Invading fluid	Silicon oil with Orcosolve Blue	$\rho_i = 960 \text{ kg m}^{-3}$ $\eta_i = 48 \text{ mPa.s}$	Dow Corning mix with Pyrromethene 567A	$\rho_i = 1013 \text{ kg m}^{-3}$ $\eta_i = 53 \text{ mPa.s}$
Fluid-pair interaction		$M = 29$ $\gamma = 26 \pm 1 \text{ mN m}^{-1}$		$M = 27$ $\gamma = 19 \pm 1 \text{ mN m}^{-1}$

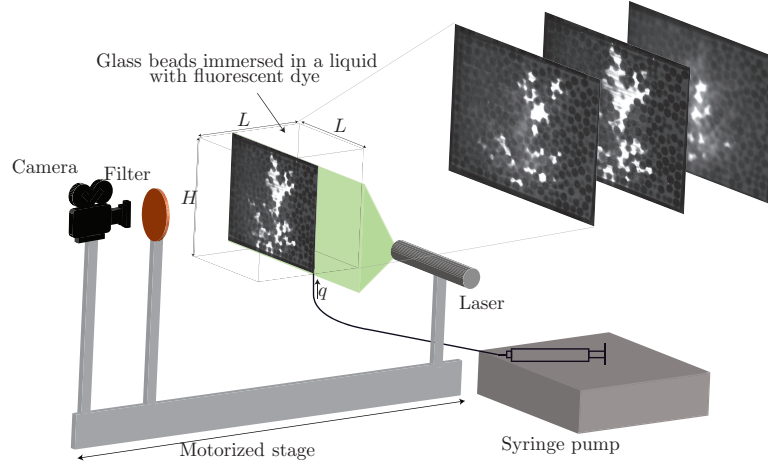


FIG. 2. Schematic of the experimental setup for PLIF imaging. The cubic cell is filled with borosilicate glass beads. We first inject glycerol, which is the defending fluid. Then we inject a mix of Dow Corning silicone oil at a constant injection rate q with a syringe pump. We shine a laser sheet through the cell that excites the defending and invading fluids, and image using a camera filtering out the light emitted directly by the laser. The beads appear black, the defending fluid gray, and the invading fluid white. The laser and the camera are mounted on a motorized stage, which allows scanning the entire cell.

if the laser sheet width was smaller. The frequency of images is such that the separation between planes is 0.5 mm for the large cell ($L = 6 \text{ cm}$) and 0.2 mm for the small cell ($L = 2 \text{ cm}$). The time lapsed between 3D-imaging cycles of the cell is 4.0 s for the large cell, and 4.5 s for the small cell.

C. Wetting properties

125

We employ borosilicate glass beads (Ref Corning 7268 from Sigma Aldrich for the 3-mm beads; Ref CG-1101 from Chemglass for the 1-mm beads), which we reuse from one experiment



FIG. 3. Image from the goniometer during measurement of contact angle. We deposit a droplet of glycerol (dark gray) on a glass bead immersed in silicone oil.

to the next. We ensure reproducibility of wetting properties of the beads by following a 6-step cleaning process before each experiment: (1) wash with acetone to remove most of the silicone oil; (2) wash with isopropanol to dilute the glycerol, let soak overnight; (3) wash with a solvent
 130 remover, soap and water; (4) let dry on a hot plate at 75°C for 12 hours; (5) wash with a piranha solution (mix of 75% hydrogen peroxide and 25% sulfuric acid); (6) let dry in the oven at 75°C for 24 hours. Following this cleaning procedure, the beads are exposed to a hand-held plasma for 10 min, or to an ozone cleaner in order to alter their wetting properties.

We measured the contact angle between the two fluids directly on the glass beads using a
 135 goniometer (ramé hart, Model 590). The contact angle was the same, within the measurement uncertainty, for the two pair of fluids (glycerol/silicone oil and glycerol/Dow Corning mix). To do this, we immersed a glass bead into silicone oil, deposited a droplet of glycerol on top of it, and then measured the static contact angle. The contact angle through the dense glycerol phase was $\theta = 35 \pm 8^\circ$ (Figure 3).

140 Therefore, our fluid–fluid displacement experiments are in the drainage, unfavorable-mobility regime: a less wetting, less viscous fluid (oil) displacing a more wetting, more viscous fluid (glycerol).

D. Bead-pack properties

We determine the flow properties of the glass-bead pack: porosity and permeability. We de-
 145 termine porosity by weighting the cell before and after filling it with glycerol, a fluid of known density. The porosity determined in this way is $\phi_0 = 0.48 \pm 0.02$ for both bead diameters used ($d = 1$ and 3 mm).

We measure the intrinsic permeability k_0 of the bead pack using the constant-head method

[52]. We use a rectangular cell with a sieve at the bottom, so that the beads stay in the cell while
 150 the liquid can escape. We use a syringe pump to keep the level of liquid at the top of the cell
 constant. The cell is put inside a container with a fixed level of liquid, which can overflow in a
 larger container, and we measure the rate of liquid overflowing.

For the 3-mm beads, the liquid used is glycerol. We measure $k_0/\eta = 9.5 \pm 0.2 \times 10^{-9} \text{ m s kg}^{-1}$.
 Taking the dynamic viscosity $\eta = 1.41 \text{ Pa s}$, we obtain $k_0 = 1.3 \times 10^{-8} \text{ m}^2$. For the 1-mm beads,
 155 the liquid used is a mixture of water and glycerol of viscosity $\eta = 0.143 \text{ Pa s}$, and we measure
 $k_0 = 1.3 \times 10^{-9} \text{ m}^2$.

We can compare this value with the one given by the Kozeny–Carman equation [52]:

$$k_{kc} = \frac{d^2}{180} \frac{\phi_0^3}{(1 - \phi_0)^2}, \quad (1)$$

which gives $k_{kc} = 2.0 \times 10^{-8} \text{ m}^2$ for $d = 3 \text{ mm}$, and $k_{kc} = 2.3 \times 10^{-9} \text{ m}^2$ for $d = 1 \text{ mm}$ —a
 160 discrepancy of about 40% with respect to the experimental values that can easily be attributed to
 lack of monodispersity of the granular pack.

III. POROMECHANICS MODEL

Fluid injection into the fluid-filled granular cell pressurizes the fluid in the cell, which has two
 main effects: (1) fluid invasion and displacement of the immiscible defending fluid; and (2) defor-
 165 mation of the granular pack. These two effects are interdependent, and this coupling is generally
 known as *poromechanics* [53–56].

Here, we develop a simplified poromechanics model that will allow us to rationalize the be-
 havior observed in the experiments, reported later in Section IV. In simple terms, the behavior is
 as follows: the granular pack experiences small deformations initially, behaving elastically, up to
 170 a point at which a cavity near the injection port forms, displacing the grains and opening a con-
 duct that is observable macroscopically. Fundamentally, we are interested in characterizing and
 predicting the onset of fluid-driven granular failure, and the macroscopic evolution of deformation
 after failure.

In what follows, we develop this model, staging the development into pre-failure, onset of
 175 failure, and post-failure.

A. Poroelastic model before frictional failure

Before the granular pack experiences large irreversible deformations, the evolution of pore pressure and skeleton deformation can be described by the field theory of poroelasticity in small deformations [53–56]. The equation for the evolution of pore pressure p takes the form

$$\left(\frac{b^2}{K_d} + \frac{1}{M} \right) \frac{\partial p}{\partial t} + \frac{b}{K_d} \frac{\partial \sigma_V}{\partial t} + \nabla \cdot \mathbf{v} = 0, \quad (2)$$

where K_d is the dry bulk modulus, b is the Biot coefficient, M is the Biot modulus, σ_V is the volumetric stress, and \mathbf{v} is the seepage velocity of the fluid with respect to the solid. For incompressible fluids and grains (that is, when all the deformation can be attributed to grain rearrangement), $M \rightarrow \infty$, $b \rightarrow 1$, and we can define the piezometric head $h = p/(\rho_f g)$ where ρ_f is the fluid density and g is the gravitational acceleration.

If we assume constant load conditions, $\sigma_V \approx \text{const}$, and using Darcy’s law for the seepage velocity,

$$\mathbf{v} = -K \nabla h, \quad (3)$$

where $K = k \rho_f g / \eta_f$ is the hydraulic conductivity. Defining the hydraulic diffusivity, $D = (k/\eta) K_d = K/(\rho_f g) K_d$, the poroelastic pressure equation reduces to:

$$\frac{\partial h}{\partial t} - D \nabla^2 h = 0, \quad (4)$$

which under these assumptions is decoupled from the linear momentum balance governing medium deformation.

This canonical diffusion equation has been solved under many different geometries and boundary conditions [e.g., 55, 57, 58]. Here, we put forward a simplified configuration that reasonably represents the flow in our porous cell.

Given the point injection, the cubic-like dimensions of the cell, and the uniform seepage through the top boundary, we assimilate the physical problem to a pressure-diffusion problem on a hollow half-sphere (outer radius R and inner radius a) under radial symmetry, with prescribed flowrate q at the inner boundary, and constant piezometric head equal to the initial piezometric head (taken as reference, $h = 0$) at the outer boundary. We have confirmed, via 3D finite-element simulations of Equation (4) with appropriate boundary conditions, that the behavior of the solutions in the actual box geometry and the half-sphere geometry are very close to each other

(within 5% for radial distances $r < 10$ mm), thereby justifying adopting the half-sphere geometry
 205 as a valid approximation for our purposes.

Mathematically, the initial and boundary value problem takes the form:

$$\text{PDE: } \frac{\partial h}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(Dr^2 \frac{\partial h}{\partial r} \right), \quad a < r < R, \quad t > 0, \quad (5)$$

$$\text{IC: } h(r, 0) = 0, \quad a < r < R, \quad (6)$$

$$\text{BC: } h(R, t) = 0, \quad t > 0, \quad (7)$$

$$4\pi a^2 K \left. \frac{\partial h}{\partial r} \right|_{r=a} = -2q, \quad t > 0. \quad (8)$$

The solution to this IBVP can be obtained via the change of variable $u := h r$ and using the method of separation of variables [59]:

$$h(r, t) = \frac{2q}{4\pi K} \frac{1}{r} \cdot \left[\left(1 - \frac{r}{R} \right) - 2 \sum_{n=1}^{\infty} \frac{1}{1 + \frac{R-a}{a}(1 + a^2 \lambda_n^2)} \frac{\sin(\lambda_n(R-r))}{\sin(\lambda_n(R-a))} \exp(-D \lambda_n^2 t) \right], \quad (9)$$

210 where the eigenvalues λ_n satisfy the nonlinear equation

$$\tan(\lambda_n(R-a)) + a\lambda_n = 0, \quad n = 1, 2, \dots \quad (10)$$

For illustration, we plot the solution as a function of radial distance r at different times for parameters corresponding to the large cell ($R = 4.5$ cm) filled with the large beads ($d = 3$ mm) and an effective dry bulk modulus $K_d = 10$ kPa. The solution $h(r, t)$ exhibits a fast transient
 215 reaching steady-state within ~ 10 s (Figure 4, top). By plotting the time derivative $\dot{h}(r, t)$ we observe a pressure pulse that propagates from the injection point ($r = a$) outwards (Figure 4, bottom).

B. Onset of frictional failure

We estimate, based on simple considerations of frictional failure, the critical value of piezomet-
 220 ric head change, h_f , that will lead to particle rearrangement within the granular pack. We assume that the initial stress state of the granular pack is controlled by the confining weight on top of the lid, the weight of the beads and the interstitial fluid, and the lateral confinement provided by the vertical walls of the cubic pack. Thus, the total vertical stress is geostatic, $\sigma_v = (W/L^2) + \rho_b g z$, the

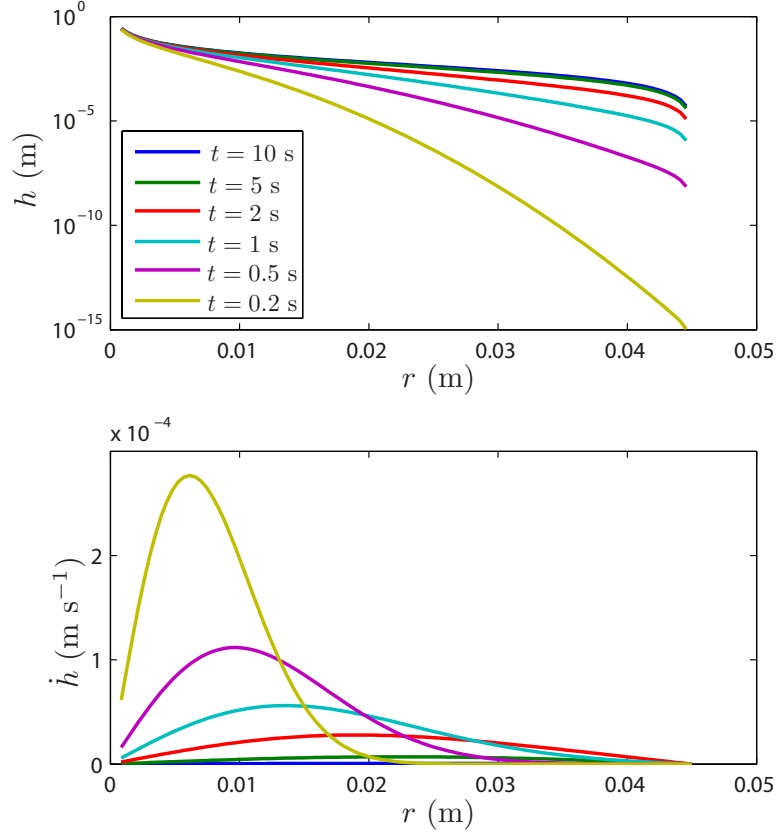


FIG. 4. Behavior of the solution [Eq. (9)] to the poroelastic diffusion IBVP [Eqs. (5)–(8)]. Top: piezometric head $h(r, t)$. Bottom: time derivative $\dot{h}(r, t)$. See text for details of the parameters used.

pore pressure is hydrostatic, $p = \rho_f g z$, and therefore the vertical and horizontal effective stresses at the bottom of the cell are:

$$\sigma'_v = \frac{W}{L^2} + \rho' g H_0, \quad (11)$$

$$\sigma'_h = K_0 \sigma'_v. \quad (12)$$

where $\rho' = \rho_b - \rho_f$ is the buoyant density, and K_0 is the coefficient of lateral stress at rest.

We employ the Mohr–Coulomb failure criterion for cohesionless granular materials:

$$\tau = \mu_f \sigma'_n, \quad (13)$$

where τ is the shear stress, σ'_n is the normal effective stress, and μ_f is the coefficient of friction.

Using the common Mohr circle interpretation, failure is attained once the pressure is increased

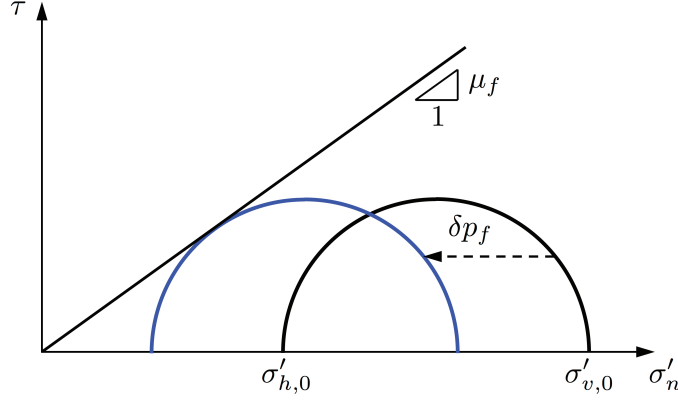


FIG. 5. Mohr circle interpretation of the Mohr–Coulomb failure criterion [Eq. (13)]. Because the diagram is expressed in terms of effective stresses, $\sigma'_n = \sigma_n - p_f$, an increase in fluid pressure ($\delta p_f > 0$) results in a decrease in effective normal stress σ'_n , shifting the Mohr circle to the left, towards the Mohr–Coulomb failure line.

sufficiently to move the Mohr circle against the failure line (Fig. 5). From simple geometry, and employing Eqs. (11)–(12), we obtain the critical pressure increase

$$\delta p_f = \frac{1}{2} \left(\frac{W}{L^2} + \rho' g H_0 \right) \left((1 + K_0) - \frac{1 - K_0}{\sin(\tan^{-1} \mu_f)} \right). \quad (14)$$

Defining the frictional parameter

$$\alpha_f := (1 + K_0) - \frac{1 - K_0}{\sin(\tan^{-1} \mu_f)}, \quad (15)$$

and expressing in terms of piezometric head increment, $h_f = \delta p_f / (\rho_f g)$, we arrive at the expression

$$h_f = \frac{1}{2} \frac{\frac{W}{L^2} + \rho' g H_0}{\rho_f g} \alpha_f. \quad (16)$$

In this simplified analysis, we conclude that failure leading to conduit opening will take place the piezometric head increase at steady-state predicted by the poroelastic model at a distance \bar{r} a few bead diameters away from the injection point,

$$h_{ss}(\bar{r}) \equiv h(\bar{r}, t \rightarrow \infty) = \frac{2q}{4\pi K} \frac{1}{\bar{r}} \left(1 - \frac{\bar{r}}{R} \right), \quad (17)$$

is greater than the threshold failure head, h_f . Given that this condition is satisfied, we estimate the time of failure as the time at which $h(\bar{r}, t_f) = h_f$. Combining Eqs. (9) and (16), we arrive at the

245 nonlinear equation for the *time of failure*, t_f :

$$\sum_{n=1}^{\infty} \frac{1}{1 + \frac{R-a}{a}(1 + a^2 \lambda_n^2)} \frac{\sin(\lambda_n(R - \bar{r}))}{\sin(\lambda_n(R - a))} \exp(-D \lambda_n^2 t_f) = \left(1 - \frac{\bar{r}}{R}\right) - \frac{\bar{r}}{2q/(4\pi K)} h_f. \quad (18)$$

C. Cavity/conduit opening after frictional failure

Once the failure condition has been reached, the granular pack is mobilized. This leads to the opening of a conduit near the inlet, and macroscopic deformation of the skeleton—an effect that
250 results in displacement of the top lid to accommodate this deformation.

Given that the constituents of the porous pack (beads and fluids) are assumed to be incompressible, fluid mass conservation integrated over the entire cell dictates that

$$q(t) = q_{\text{out}}(t) + L^2 \dot{H}, \quad (19)$$

where q_{out} is the flow rate out the cell by seepage through the porous top lid, and \dot{H} is the top-lid
255 displacement velocity. This poses the simple question of what is the balance between the two terms on the right hand side of Eq. (19): (1) if $q \approx q_{\text{out}}$, fluid invasion takes place by fluid displacement without cavity expansion; (2) if $q \approx L^2 \dot{H}$, fluid invasion is accommodated exclusively by cavity expansion. Therefore, Eq. (19) implicitly reflects a balance between viscous resistance and frictional resistance.

260 Given the fast transient of the poroelastic solution $h(r, t)$ towards steady state (Fig. 4) and that \bar{r} is close to the injection port, we assume that the piezometric head is given by the steady-state solution, $h_{\text{ss}}(\bar{r})$ but limited to h_f , i.e.,

$$h_f = h_{\text{ss}}(\bar{r}) = h(\bar{r}, t \rightarrow \infty) = \frac{2q_{\text{out}}}{4\pi K} \frac{1}{\bar{r}} \left(1 - \frac{\bar{r}}{R}\right). \quad (20)$$

From Eqs. (19) and (20), we obtain a nonlinear equation involving the *displacement rate* \dot{H} :

$$265 \quad \frac{1}{L^2} \left(q - \frac{2\pi K \bar{r}}{1 - \bar{r}/R} h_f \right) = \dot{H}, \quad (21)$$

where h_f is dependent on W and the frictional parameter α_f through Eq. (16).

Qualitatively, the balance between viscous resistance and frictional resistance can be expressed as a *frictional failure number*, N_f , defined as the ratio of the piezometric head increase from the

poroelastic solution h_{ss} (Equation (17)) and the piezometric head increase for frictional failure h_f (Equation (16)):

$$N_f := \frac{h_{ss}}{h_f}. \quad (22)$$

If $N_f \ll 1$, friction is dominant and there is no deformation. This situation is favored by low injection rate ($q \downarrow$), high medium permeability ($k \uparrow$), low fluid viscosity ($\eta \downarrow$), and high confining weight ($W \uparrow$). If $N_f \gg 1$, the opposite is true: viscous pressure drop is dominant, causing cavity expansion.

IV. RESULTS: MACROSCOPIC DEFORMATION

A. Evolution of lid position

The macroscopic deformation of the granular pack is reflected in the displacement of the top lid, $H(t) - H_0$. The evolution of this quantity for a typical experiment shows that the displacement is negligible for a short period of time, and it then increases at an approximately constant rate (Figure 6). Such behavior is congruent with our poroelastic–Coulomb-failure model, in which there is an induction time in which pressure increases due to injection in a small-deformation configuration, and a frictional-sliding stage in which (constant-rate) injection leads to (constant-rate) displacement of the top lid.

B. Model fit to failure onset and cavity expansion

We now probe whether the conceptual and mathematical model of poroelastic evolution followed by frictional failure and conduit opening is a valid representation of fluid–fluid displacement in a deformable granular pack observed in the experiments. To perform this interrogation of the data from the lens of the model, we identify the parameters that are common to all experiments and known ($\rho_f, \eta_f, \rho', g, \gamma$), the parameters that are experiment-specific and also known (H_0, d, k, W, q), and the parameters that are experiment-specific and *unknown*: the frictional parameter α_f , the effective radius of granular failure \bar{r} , and the effective dry bulk modulus K_d .

To constrain these three parameters, we utilize the experimental data of lid position as a function of time binarized into two quantities for each experiment: the failure time t_f and the lid displacement rate \dot{H} . We perform a least-squares minimization of the parameters of the model in

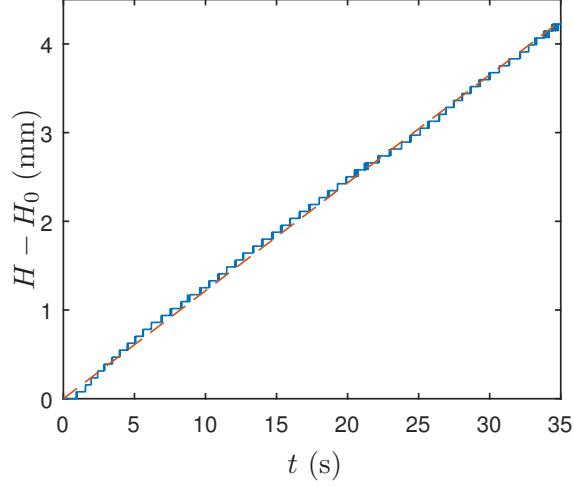


FIG. 6. Displacement of the top lid of the cell as a function of time, $H(t) - H_0$. The solid line represents the experimental data, the dashed line is a linear fit. The data corresponds to an experiment with $L = 6$ cm, $d = 1$ mm, $W = 18.83$ N, $H_0 = 51$ mm and $q = 45$ mL/min.

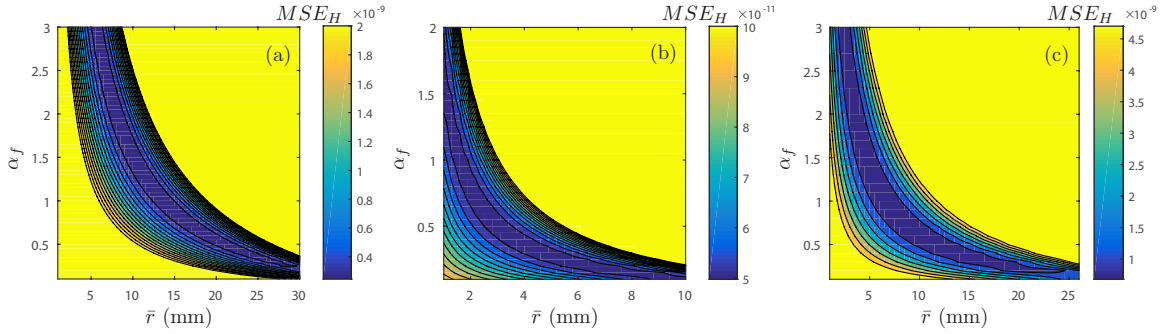


FIG. 7. Mean square error for \dot{H} in (\bar{r}, α_f) -space, illustrating that there is a Pareto curve that minimizes the MSE. Shown are the three sets of experimental data: (a) $L = 6$ cm, $d = 1$ mm; (a) $L = 2$ cm, $d = 1$ mm; (a) $L = 6$ cm, $d = 3$ mm.

terms of these two quantities using Equations (18) and (21).

Because of the scatter in the experimental measurements, it proves useful to perform this least-squares minimization in two steps. The first is the identification of ranges of joint pair values (\bar{r}, α_f) that best match the measurements of \dot{H} . This global minimization leads to a Pareto curve in (\bar{r}, α_f) -space for all three experimental sets (Fig. 7).

Identification of K_d from minimization of the model and the t_f data [Eq. (18)], however, turned out to be challenging—likely because of the inherent uncertainty in the identification of the failure time, which results in a large range of possible values of the effective dry bulk modulus. This addi-

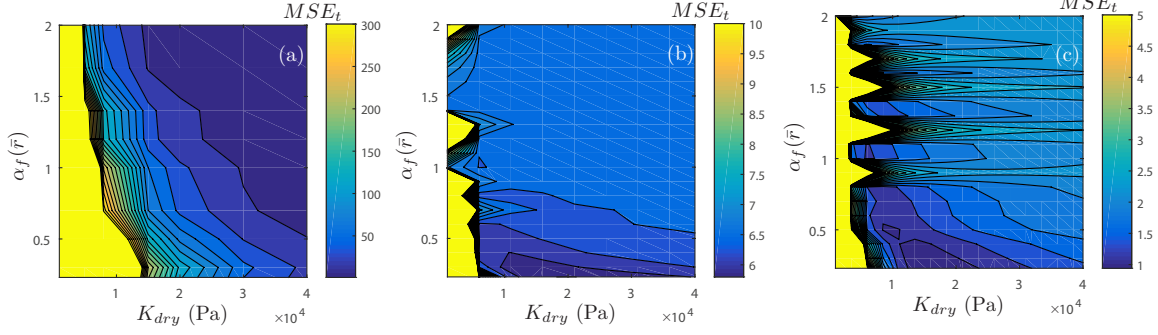


FIG. 8. Mean square error for t_f in (K_d, α_f) -space, where the relationship $\alpha_f = \alpha_f^{\text{opt}}(\bar{r})$ from the Pareto curve in Fig. 7 has already been incorporated. The plots illustrate that K_d is not well constrained by the data. Shown are the three sets of experimental data: (a) $L = 6$ cm, $d = 1$ mm; (a) $L = 2$ cm, $d = 1$ mm; (a) $L = 6$ cm, $d = 3$ mm.

tional data set does allow, however, to provide additional constraints on the frictional parameter α_f (Fig. 8).

We interpret the results of parameter fitting as follows:

1. The frictional parameter α_f is a measure of the frictional resistance of the pack. It is in principle a material parameter dependent on the friction coefficient μ_f and the coefficient of lateral stress at rest K_0 , and typical values of these two coefficients would render $\alpha_f \in [0, 1]$. For experiment series 2 and 3 ($L/d \approx 20$), the experimental data further suggest that a value of $\alpha_f \approx 0.5$ provides the best fit—a value that comes naturally from taking $\mu_f \approx 0.6$ and $K_0 \approx 0.5$. For experiment series 1 ($L/d \approx 60$), the best fit occurs for a higher value, $\alpha_f \approx 1$.
2. The effective failure distance \bar{r} is in the order of 5–10 mm, or a few bead diameters, as it would be expected from breaking arches of force chains around the injection point.
3. The effective dry bulk modulus K_d remains relatively unconstrained by the data, but is in the order of 5–20 kPa, with values in the low end of this range for experiment series 2 and 3—with lower L/d and likely looser packing due to wall effects—and in the high end of the range for experiment series 1.

The results of the model fit against experimental values of \dot{H} and t_f are shown in Figs. 9 and 10, respectively. It is apparent that while the frictional sliding model explains post-failure deformation

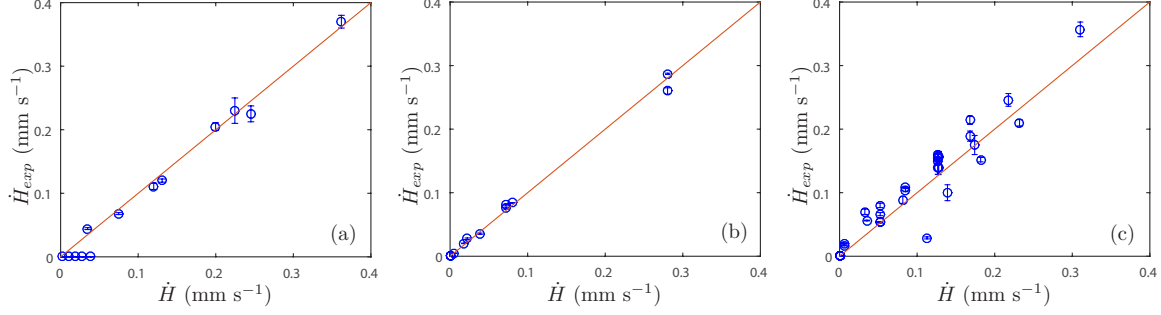


FIG. 9. Model fit for lid displacement rate \dot{H} [Eq. (21)]. The red solid line indicates the 1 : 1 line denoting perfect match between theory and experiment. Shown are the three sets of experimental data: (a) $L = 6$ cm, $d = 1$ mm; (b) $L = 2$ cm, $d = 1$ mm; (c) $L = 6$ cm, $d = 3$ mm.

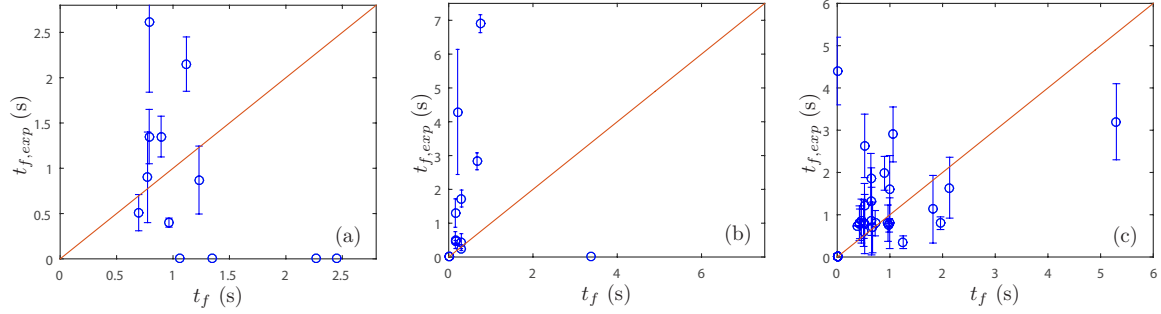


FIG. 10. Model fit for time of failure t_f [Eq. (18)]. The red solid line indicates the 1 : 1 line denoting perfect match between theory and experiment. Shown are the three sets of experimental data: (a) $L = 6$ cm, $d = 1$ mm; (b) $L = 2$ cm, $d = 1$ mm; (c) $L = 6$ cm, $d = 3$ mm.

(\dot{H}) nicely, the onset of failure (t_f) in these granular systems is plagued with scatter, making its prediction challenging. Overall, however, the model offers a good parsimonious explanation—over a wide range of experimental conditions—of the pressurization, frictional failure and conduit opening as a result of fluid–fluid displacement in our granular system (Fig. 11).

V. RESULTS: PATTERNS OF FLUID INVASION AND MEDIUM DEFORMATION

To gain insight into the details of fluid–fluid displacement and the morphodynamics of deformation, we resort to 3D dynamic imaging of the granular pack at the sub-pore scale, using the PLIF imaging technique described in Section II B.

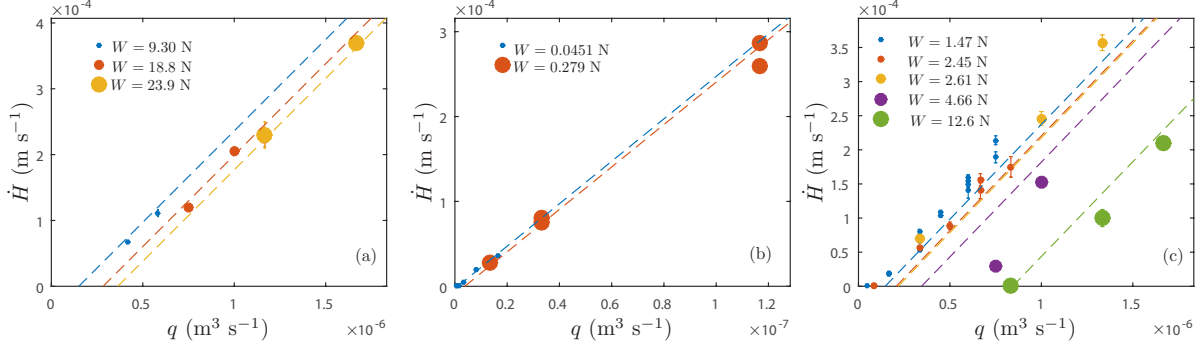


FIG. 11. Model prediction and experimental data for lid displacement rate \dot{H} as a function of injection rate q [Eq. (21)]. Different symbols represent different confining weights W . Shown are the three sets of experimental data: (a) $L = 6$ cm, $d = 1$ mm; (b) $L = 2$ cm, $d = 1$ mm; (c) $L = 6$ cm, $d = 3$ mm.

A. Fluid invasion pattern

We first confirm that we can reconstruct the 3D dynamic fluid invasion pattern with fidelity. The process of 3D reconstruction involves thresholding the images, with a depth-dependent threshold that accounts for the inevitable finite transparency of the beads and fluids—images on a plane farther from the camera are less bright than those on a plane closer to the camera. Because of the sharp contrast between the invading fluid and the defending fluid and beads, the thresholding employed for binarizing the images was straightforward to select and did not have any significant impact on the results. The other inevitable artifact in the 3D dynamic imaging is the finite time that it takes to perform a scan (in our case, on the order of 1.2 s), and we assume that the invasion pattern does not change much during that period of time. Using the thresholded images pertaining to one full scan we reconstruct the 3D pattern for each experiment [see Fig 12(b) for a typical example].

To check the validity of our image analysis, we compute the volume of fluid injected V_{inj} by counting the voxels inside the injected pattern, and compare this volume with the known injected volume from the pump, q [Fig 12(a)]. The good agreement between the two volumes gives us confidence that the reconstructed fluid invasion is well captured by the 3D dynamic imaging. In particular, for experiments with injection rate $q < 40$ mL/min, the front advances less than one bead diameter during a scan, and the assumption that the pattern does not change during one scan is valid for practical purposes.

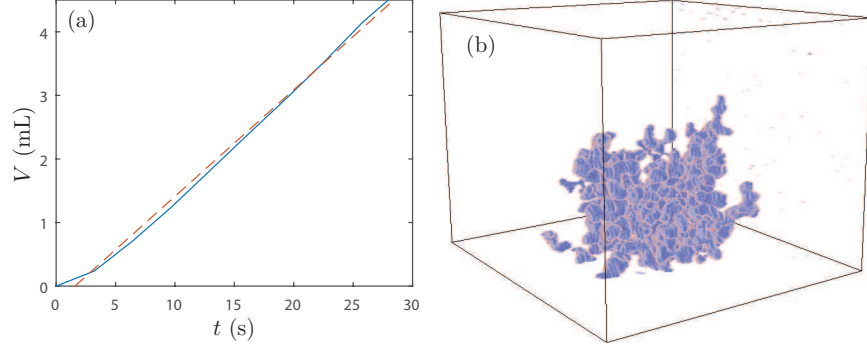


FIG. 12. Fluid invasion pattern. (a) Injected volume as a function of time. The blue solid line represents the volume reconstructed from 3D dynamic imaging; the red dashed line is a fit with an imposed slope equal to the injection rate q . (b) Reconstructed 3D injection pattern, at $t = 16.2$ s. The base of the cube is of size $L = 6$ cm. Experiment with $q = 10 \text{ mL min}^{-1}$, $d = 3 \text{ mm}$, and $W = 1.47 \text{ N}$.

B. Fractal dimension

The fractal dimension of a pattern has been employed widely to categorize dynamic processes into “universality classes” [60–64]. Most analyses of fractal dimension in fluid–fluid displacements in porous media have been restricted to 2D problems [e.g., 65–80]. Determination of the fractal dimension in 3D displacements has been hindered by either direct imaging, which only permits a 2D projection of the invading pattern [43], or by limited spatial and temporal resolution of the 3D scans [45]. Recent studies using either PLIF imaging [46, 47], confocal microscopy [48], or high-energy X-ray imaging [50, 51], resolve the displacements at the pore level, but the imaged volume is insufficient to ascertain the macroscopic pattern and the fractal dimension.

We determine the fractal dimension from the sequence of 3D images in our experiments. For a given experiment and at a given time t , we compute the volume V_s of injected fluid inside a semi-sphere centered at the injection port and of radius r_s . By plotting V_s (re-scaled by the characteristic volume V_p of a single pore) as a function of r_s (re-scaled by the nominal bead diameter d), we obtain curves that display a power-law behavior that extends with time, up to the point when the morphology of the invading pattern starts to be affected by the cell boundaries [Fig. 13(a)]. We determine the fractal dimension D_f of the invading pattern as the slope of that power law, and obtain the results shown in Fig. 13(b). The experimental data exhibits sufficient scatter that it is difficult to observe clear trends of D_f as a function of the two control variables (injection rate q and confining weight W) for the range probed experimentally. We interpret this behavior as being the

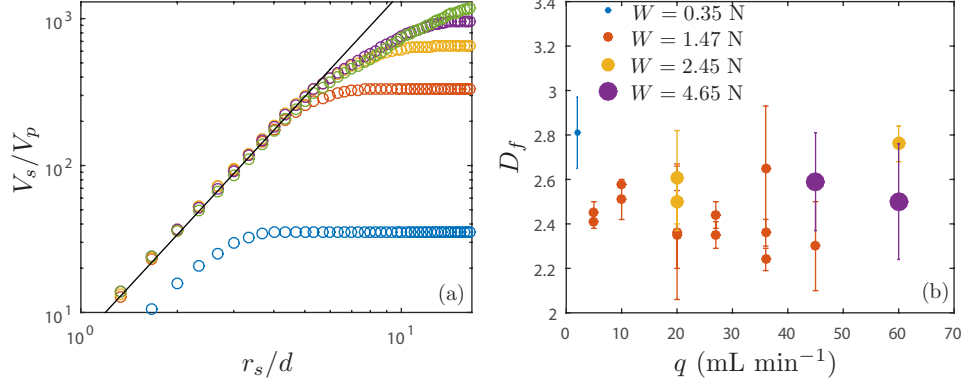


FIG. 13. Fractal dimension. (a) V_s/V_p as a function of r_s/d , where V_s is the volume of injected liquid inside a half sphere of radius r_s centered at the injection point. V_p is the characteristic volume of a single pore, and d is the nominal bead diameter. The different colors correspond to scans at different times t . The solid line represents a linear fit for $r_s < 18$ mm, which we consider is unaffected by boundary effects and has, in this case, a slope of 2.50. The plot corresponds to an experiment with $d = 3$ mm, $L = 6$ cm, $W = 1.47$ N and $q = 36$ mL/min. (b) Fractal dimension D_f estimated for different experiments with different injection rate q and confining weight W , with $L = 6$ cm and $d = 3$ mm.

result of: (1) large variability among experiments due to the relatively small volume available for sampling, and (2) the fluid–fluid displacement is affected by fingering, regardless of the presence or absence of deformation of the granular pack.

For intermediate times within each experiment, we can evaluate the capillary number directly as $\text{Ca} = \eta_{\text{def}} v_{\text{front}} / \gamma$. For our experiments, this results in values in the range $\text{Ca} \in [0.03, 1]$, for which we expect viscous-dominated displacements. In this sense, the values of fractal dimension that we measure ($D_f \sim 2.5$) are consistent with values expected from viscous fingering [43].

C. Pore opening and conduit pattern

To confirm this hypothesis, we image simultaneously and in high resolution the invasion of the injected fluid and the deformation of the granular pack. First, we determine the position of the individual beads before the beginning of injection of the invading fluid (Fig. 14). The bead pack structure can be resolved throughout the experiment. At any given time t_n , the pattern of the injected fluid is all within the pore volume. At the time of the next scan, t_{n+1} , the injected pattern will comprise a larger volume, and a fraction of that volume may consist of regions where beads

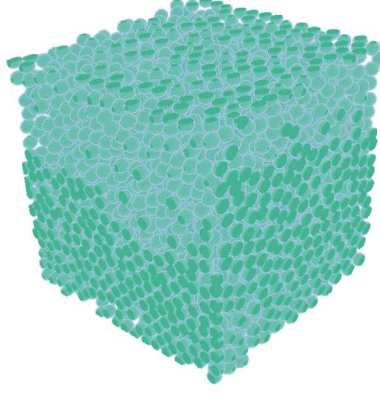


FIG. 14. Position of the glass beads at the beginning of fluid injection. Experiment with $L = 6$ cm and $d = 3$ mm.

were displaced: we denote that volume of displaced solid between scans the “conduit volume”.

In Fig. 15, we show 3D renderings of the injection pattern (in white) and the conduit volume (in red) for experiments corresponding to $L = 6$ cm, $d = 3$ mm, $W = 1.47$ N, and different injection rates q . It is apparent that, as the injection rate increases, the amount of bead displacement also increases, from virtually no bead displacement at the lowest q [Fig. 15(a)] to a conduit volume fraction of $\approx 45\%$ for $q = 40$ mL/min [Fig. 15(f)]. The numeric values of this trend between conduit volume fraction and injection rate are shown in Fig. 16.

This direct pore-scale observation is consistent with the macroscopic measurements of lid displacement and it is compatible with our model of frictional failure.

VI. CONCLUSIONS

In this work, we elucidate the role of grain displacement in the evolution of injection patterns in granular media by means of direct 3D observations of the dynamic process at the pore scale. We reconcile observations of fluid invasion morphology with the underlying pore-scale physics. We find that the onset and evolution of macroscopic displacement can be captured with a simple model of frictional failure.

Our results open the door to better understanding, quantification and eventual prediction of processes in which poromechanics of multiphase mixtures plays a central role, such as soil desiccation [16, 39], magma degassing [15], methane venting from sediments [11, 14, 42], and hydraulic fracture of hydrocarbon formations [81]. Our 3D PLIF experimental setup is also allowing us to study

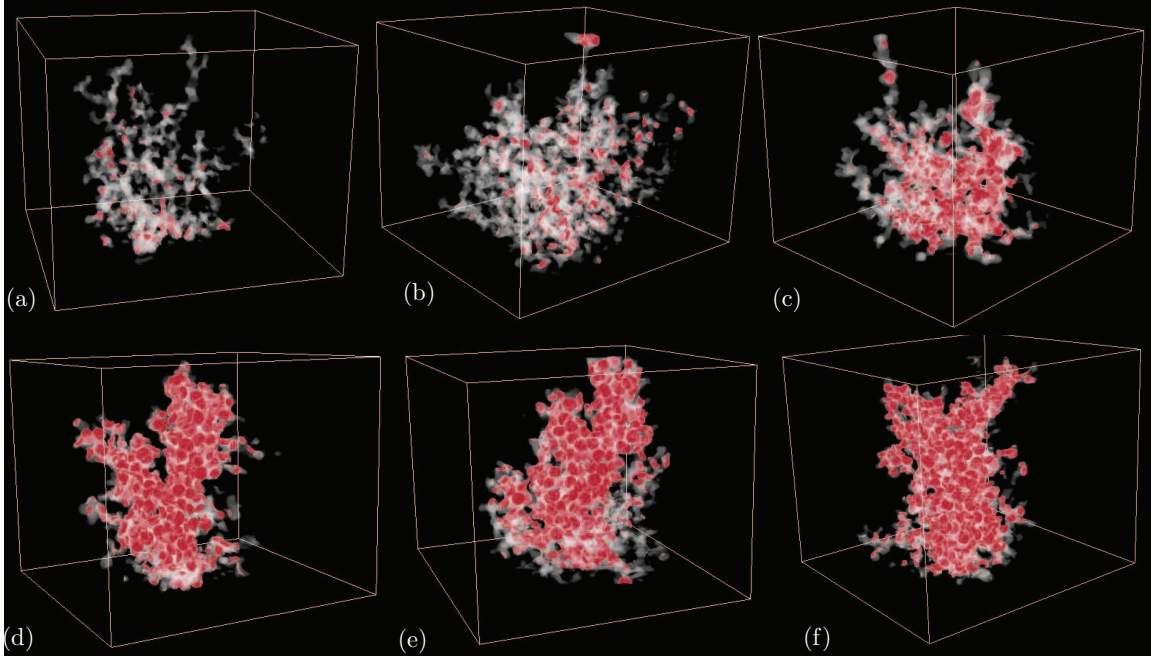


FIG. 15. Injection pattern (white) superimposed to the conduit volume (in red) near the time of breakthrough, for experiments with $L = 6$ cm, $d = 3$ mm, $W = 1.47$ N. (a) $q = 2$ mL/min, at $t = 108$ s and $V = 2.1$ mL; (b) $q = 5$ mL/min at $t = 61$ s and $V = 4.7$ mL; (c) $q = 10$ mL/min at $t = 29$ s and $V = 4.7$ mL; (d) $q = 20$ mL/min, $t = 12.8$ s and $V = 3.8$ mL; (e) $q = 27$ mL/min, $t = 9.6$ s and $V = 3.6$ mL; (f) $q = 36$ mL/min, $t = 12.7$ s and $V = 6.8$ mL.

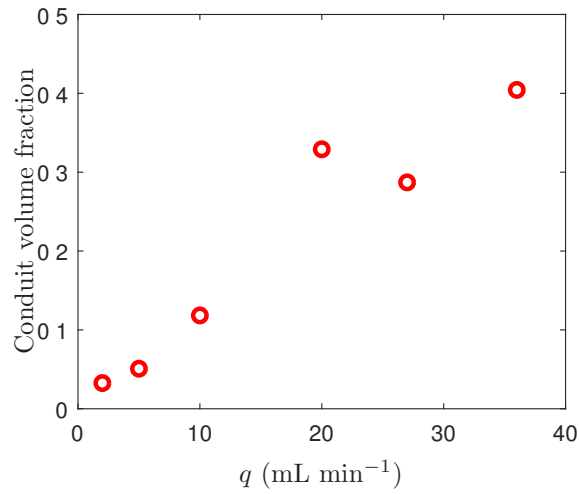


FIG. 16. Fraction of the fluid invasion volume that corresponds to volume involving bead displacement, for different injection rates q .

other multiphase porous-media processes such as buoyancy-driven fingering, an instability that controls rainwater infiltration in soils [e.g., 6, 7, 82]) and hydrocarbon migration in sedimentary basins [e.g., 49, 75, 83].

We plan to extend our investigation in a number of directions. For example, we are currently exploring ways to alter the wettability of the system [84–88] to investigate the impact of wettability on the emergence and morphology of conduits—the technical challenge being to maintain index matching among the two fluids and the beads. We are also conducting experiments where we measure the injection pressure, to discern if the pressure signature associated with pore-invasion events [e.g., 73, 89–92]) is different from that of conduit opening.

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